

ECON 103: Midterm Solutions

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November 9, 2025

Outline

Midterm Solutions: Free Response Questions

Midterm Solutions: Multiple Choice Questions

Midterm Grading and Common Errors

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Midterm Grading and Common Errors

Question 1

$$\mathbf{Y} = \beta_1 + \beta_2 \mathbf{X} + \mathbf{e}$$

Suppose that, from a sample of 62 observations, the least squares estimates are given by:

$b_1 = 2.0$ and $b_2 = 6.0$, and $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$, and $\text{cov}(b_2, b_1) = -2.0$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

- a) $H_0 : \beta_2 = 0$
- b) $H_0 : \beta_1 + 2\beta_2 = 5$

Given:

- ▶ $t_{(0.975,60)} = 2.00$
- ▶ $t_{(0.950,60)} = 1.69$

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2 \mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Q: Using $\alpha = .05$, test:

- a) $H_0 : \beta_2 = 0$
- b) $H_0 : \beta_1 + 2\beta_2 = 5$

Prerequisites

1. One side or two sided?
2. Variance of combination of parameters:

$$\text{Var}(ab_1 + bb_2) = a^2 \text{Var}(b_1) + b^2 \text{Var}(b_2) + 2ab \text{Cov}(b_1, b_2)$$

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2\mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Solution for (a): This is a two sided test: $H_0 : \beta_2 = 0$ vs $H_a : \beta_2 \neq 0$, so we use $t_{0.975,60} = 2$.

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2\mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Solution for (a): This is a two sided test: $H_0 : \beta_2 = 0$ vs $H_a : \beta_2 \neq 0$, so we use $t_{0.975,60} = 2$.

The T statistic is:

$$t = \frac{b_2 - 0}{\sqrt{\text{var}(b_2)}} = \frac{6 - 0}{\sqrt{4}} = 3$$

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2 \mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Solution for (a): This is a two sided test: $H_0 : \beta_2 = 0$ vs $H_a : \beta_2 \neq 0$, so we use $t_{0.975,60} = 2$.

The T statistic is:

$$t = \frac{b_2 - 0}{\sqrt{\text{var}(b_2)}} = \frac{6 - 0}{\sqrt{4}} = 3$$

Is $|t| > t_{1-\alpha/2,60}$?

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2 \mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Solution for (a): This is a two sided test: $H_0 : \beta_2 = 0$ vs $H_a : \beta_2 \neq 0$, so we use $t_{0.975,60} = 2$.

The T statistic is:

$$t = \frac{b_2 - 0}{\sqrt{\text{var}(b_2)}} = \frac{6 - 0}{\sqrt{4}} = 3$$

Is $|t| > t_{1-\alpha/2,60}$? $|3| > 2$, so we reject H_0 .

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2\mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Solution for (b):

This is a two sided test: $H_0 : \beta_1 + 2\beta_2 = 5$ vs $H_a : \beta_1 + 2\beta_2 \neq 5$, so we still use $t_{0.975,60} = 2$.

Question 1

Model: $\mathbf{Y} = \beta_1 + \beta_2 \mathbf{X} + \mathbf{e}$

Given: $b_1 = 2.0$, $b_2 = 6.0$, $\text{var}(b_1) = 2.0$, $\text{var}(b_2) = 4.0$, $\text{cov}(b_1, b_2) = -2.0$,
 $\text{cov}(b_2, b_1) = -2.0$, $t_{(0.975,60)} = 2.00$ and $t_{(0.950,60)} = 1.69$

Solution for (b):

This is a two sided test: $H_0 : \beta_1 + 2\beta_2 = 5$ vs $H_a : \beta_1 + 2\beta_2 \neq 5$, so we still use $t_{0.975,60} = 2$.
Compute the T statistic:

$$t = \frac{b_1 + 2b_2 - 0}{\sqrt{\text{Var}(b_1 + 2b_2)}} = \frac{2 + 2 \cdot 6 - 5}{\sqrt{10}} = \frac{9}{\sqrt{10}}$$

$$\sqrt{\text{Var}(b_1 + 2b_2)} = \sqrt{\text{Var}(b_1) + 4 \text{Var}(b_2) + 4 \text{Cov}(b_1, b_2)} = \sqrt{10}$$

We have

$$\left| \frac{9}{\sqrt{10}} \right| = 2.846 > 2$$

so we reject H_0 .

Question 2

A study looked at the relationship between household food expenditures (FEXP) and weekly income (INC). The study suggested that a simple linear regression model given by:

$$FEXP = 83.416 + 10.209 * INC$$

Given:

1. $se(b_1) = 43.41$, $se(b_2) = 2.09$, $N = 40$
2. $t_{0.975,38} = 2.024$, $t_{0.95,38} = 1.686$
3. food expenditures (in \$), weekly income (in \$100)

Question

- A) Construct a 95% confidence interval for b_2 .
- B) Test the null hypothesis that β_2 is zero with an alternative hypothesis that it is greater than zero.
- C) Make the assumption that SR6 holds and that $cov(b_1, b_2) = -85.903$ and then compute $se(b_1 - 10b_2)$.

Prerequisites

Question 2

Model: $FEXP = 83.416 + 10.209 * INC$

Given: $se(b_1) = 43.41$, $se(b_2) = 2.09$, $N = 40$, $t_{0.975,38} = 2.024$, $t_{0.95,38} = 1.686$

Q: 95% CI for β_2 , $H_0 : \beta_2 = 0$, $H_a : \beta_2 > 0$, find $se(b_1 - 10b_2)$.

Prerequisites

1. One tailed or two tailed?
2. CI formula: $\hat{\beta}_2 \pm t_{\alpha/2, n-2} \cdot se(\beta_2)$
3. Variance formula for parameters:
$$\text{Var}(ab_1 + bb_2) = a^2 \text{Var}(b_1) + b^2 \text{Var}(b_2) + 2ab \text{Cov}(b_1, b_2)$$

Question 2

Model: $FEXP = 83.416 + 10.209 * INC$

Given: $se(b_1) = 43.41$, $se(b_2) = 2.09$, $N = 40$, $t_{0.975,38} = 2.024$, $t_{0.95,38} = 1.686$

Q: 95% CI for β_2 , $H_0 : \beta_2 = 0$, $H_a : \beta_2 > 0$, find $se(b_1 - 10b_2)$.

Solution for (a)

To construct a 95% two-sided CI for β_2 , we use $t_{1-.05/2,38} = t_{0.975,38} = 2.024$

$$b_2 \pm t_{0.975}(38) \text{ s.e. } (b_2) = 10.209 \pm 4.23 \approx [5.98, 14.44]$$

Question 2

Model: $FEXP = 83.416 + 10.209 * INC$

Given: $se(b_1) = 43.41$, $se(b_2) = 2.09$, $N = 40$, $t_{0.975,38} = 2.024$, $t_{0.95,38} = 1.686$

Q: 95% CI for β_2 , $H_0 : \beta_2 = 0$, $H_a : \beta_2 > 0$, find $se(b_1 - 10b_2)$.

Solution for (b)

$$H_0 : \beta_2 = 0 \text{ vs } H_a : \beta_2 > 0$$

This is a **one sided test**, so we use $t_{1-\alpha,38} = t_{0.95,38} = 1.686$ Compute the t statistic:

$$t = \frac{b_2 - 0}{\text{s.e. } (b_2)} = \frac{10.209}{2.09} \approx 4.89$$

$$|t| = |4.89| > 1.686 = t_{1-\alpha,38} \implies \text{reject } H_0$$

Question 2

Model: $FEXP = 83.416 + 10.209 * INC$

Given: $se(b_1) = 43.41$, $se(b_2) = 2.09$, $N = 40$, $t_{0.975,38} = 2.024$, $t_{0.95,38} = 1.686$

Q: 95% CI for β_2 , $H_0 : \beta_2 = 0$, $H_a : \beta_2 > 0$, find $se(b_1 - 10b_2)$.

Solution for (c)

$$\text{Var}(b_1 - 10b_2) = \text{Var}(b_1) + 100 \text{Var}(b_2) - 20 \text{Cov}(b_1, b_2)$$

To compute the standard error, we use the formula:

$$se(b_1 - 10b_2) = \sqrt{\text{Var}(b_1 - 10b_2)} = \sqrt{43.41^2 + 100 \cdot 2.09^2 - 20 \cdot (-85.903)} \approx 63.56$$

Question 3

For wage denoting daily wage and edu denoting years of education, consider the model:

$$WAGE = \beta_1 + \beta_2 * (EDU)^3 + e$$

We estimate the model by linear regression and obtain:

- ▶ $b_1 = 1.00$, $se(b_1) = 1.00$, $b_2 = 0.05$, $se(b_2) = 0.01$
- ▶ $WAGE$ is in units of \$1,000

Q:

- A) Expected wage for a person with $EDU = 10$
- B) Marginal effect of one additional year of education at $EDU = 10$
- C) Standard error of the marginal effect in (B)

Prerequisites

1. Formula for marginal effect: $\frac{\partial WAGE}{\partial EDU} = 3\beta_2(EDU)^2$
2. Variance of marginal effect: $\text{Var}\left(\frac{\partial WAGE}{\partial EDU}\right) = \text{Var}(3\beta_2(EDU)^2) = 9\beta_2^2(EDU)^4$

Question 3

Model: $WAGE = \beta_1 + \beta_2 * (EDU)^3 + e$

Given: $b_1 = 1.00$, $se(b_1) = 1.00$, $b_2 = 0.05$, $se(b_2) = 0.01$, units are \$1,000.

Solution for (A)

Expected wage at $EDU = 10$

$$\mathbb{E}[WAGE \mid EDU = 10] = b_1 + b_2 \cdot 10^3 = 1 + 0.05 \cdot 1000 = 51$$

The expected wage is \$51,000.

Question 3

Model: $WAGE = \beta_1 + \beta_2 * (EDU)^3 + e$

Given: $b_1 = 1.00$, $se(b_1) = 1.00$, $b_2 = 0.05$, $se(b_2) = 0.01$, units are \$1,000.

Solution for (B)

Marginal effect at $EDU = 10$

$$M.E.(EDU) = \frac{\partial WAGE}{\partial EDU} = 3\beta_2(EDU)^2$$

Question 3

Model: $WAGE = \beta_1 + \beta_2 * (EDU)^3 + e$

Given: $b_1 = 1.00$, $se(b_1) = 1.00$, $b_2 = 0.05$, $se(b_2) = 0.01$, units are \$1,000.

Solution for (B)

Marginal effect at $EDU = 10$

$$M.E.(EDU) = \frac{\partial WAGE}{\partial EDU} = 3\beta_2(EDU)^2$$

Plug-in estimate at $EDU = 10$:

$$\widehat{M.E.} = 3 \cdot b_2 \cdot 10^2 = 3 \cdot 0.05 \cdot 100 = 15$$

\Rightarrow The marginal effect is \$15,000 per additional year (at $EDU = 10$).

Question 3

Model: $WAGE = \beta_1 + \beta_2 * (EDU)^3 + e$

Given: $b_1 = 1.00$, $se(b_1) = 1.00$, $b_2 = 0.05$, $se(b_2) = 0.01$, units are \$1,000.

Solution for (C)

Since $\widehat{M.E.} = 3(EDU)^2 b_2$, we find the standard error as:

$$se(\widehat{M.E.}) = 3(EDU)^2 \cdot se(b_2) = 3(10)^2(0.01) = 3$$

Question 3

Model: $WAGE = \beta_1 + \beta_2 * (EDU)^3 + e$

Given: $b_1 = 1.00$, $se(b_1) = 1.00$, $b_2 = 0.05$, $se(b_2) = 0.01$, units are \$1,000.

Solution for (C)

Since $\widehat{M.E.} = 3(EDU)^2 b_2$, we find the standard error as:

$$se(\widehat{M.E.}) = 3(EDU)^2 \cdot se(b_2) = 3(10)^2(0.01) = 3$$

\Rightarrow The standard error of the marginal effect is \$3,000 per year.

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Question 4

What does R^2 , the goodness-of-fit, measure?

- A) The probability of the true value falling within \pm its standard error.
- B) The p -value on the coefficient we are using to test our hypothesis of interest
- C) The confidence interval of the error terms as determined by the coefficients
- D) The proportion of the variation in y explained by x within the regression model
- E) The sum of squares total (SST), as defined in class.

Solution: D

R^2 , the goodness-of-fit, is defined as:

$$R^2 = 1 - \frac{SSR}{SST}$$

Equivalently, R^2 measures the proportion of variation in y explained by the regression model:

$$R^2 = \frac{\text{Explained Variation}}{\text{Total Variation}} = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

Question 5

How do you reduce the probability of committing a Type I error?

- A) Reduce α
- B) Increase α
- C) Use a two-tailed test
- D) Increase the rejection region
- E) None of the above will reduce a Type I error.

Solution: A

The significance level, denoted by α , is the probability of rejecting the null hypothesis when it is actually true:

$$\alpha = P(\text{Type I Error}) = P(\text{Reject } H_0 \mid H_0 \text{ is true})$$

Question 6

In the regression of y on x , the error term (residuals) exhibit heteroskedasticity if:

- A) the residuals have constant variance
- B) $\text{Var}(y \mid x)$ is constant.
- C) $\text{Var}(y \mid x)$ varies with the estimate of either β_1 or β_2
- D) $\text{Var}(y \mid x)$ varies with x
- E) y is a function of x

Solution: D

Heteroskedasticity occurs when the variance of the error term is not constant across observations:

$$\text{Var}(\varepsilon \mid X) = \sigma^2 h(x), \quad h(x) \neq 1$$

Question 7

In which case would testing the null hypothesis involve a two-tailed statistical test?

- A) H_1 : Incentive pay for teachers does not increase student achievement.
- B) H_1 : Higher sales tax rates does not reduce state tax revenues.
- C) H_1 : Extending the duration of unemployment benefits does not increase the length of joblessness.
- D) H_1 : Smoking does not reduce life expectancy.
- E) None of the above involve a two-tailed statistical test.

Solution: E

- A) "does not increase" \implies one-tailed test
- B) "does not reduce" \implies one-tailed test
- C) "does not increase" \implies one-tailed test
- D) "does not reduce" \implies one-tailed test

Question 8

Let Y denote annual income in thousands of dollars, and $FEMALE = 1$ if an individual is female and $FEMALE = 0$ otherwise. We estimate the model:

$$Y = \beta_1 + \beta_2 FEMALE + e$$

and determine that $b_1 = 44.75$ and $b_2 = 11.25$. Note that b_1 and b_2 are measured in \$1,000 s. What is your estimate of expected income among females?

- A) \$44,750
- B) \$56,000
- C) \$11,250
- D) Not enough information to answer this question.

Solution: B

First note that in this model $E[Y | FEMALE] = \beta_1 + \beta_2$. Our estimate for $\beta_1 + \beta_2$ is $b_1 + b_2 = 44.75 + 11.25 = 56$. Because income is measured in thousands, the correct answer is **\$56,000**.

Question 9

You estimate a simple linear regression model using a sample of 78 observations and obtain the following results:

$$Y = 97.25 + 33.74 * X$$

where: $b_1 = 97.25$, $b_2 = 33.74$, $se(b_1) = 3.86$, $se(b_2) = 9.42$

You want to test the following hypothesis:

$$H_0 : \beta_2 = 0, \quad H_1 : \beta_2 \neq 0$$

Assume $\alpha = 5\%$. What is the probability of a Type II Error?

- A) between 0.05 and 0.10
- B) between 0.01 and 0.025
- C) between 0.02 and 0.05
- D) It is impossible to determine without knowing the true value of β_2
- E) None of the above are correct.

Solution: D

$$\text{Type II error: } P(\text{Fail to reject } H_0 \mid H_A \text{ true}) = \beta$$

Question 10

There were 64 countries in the 2024 Olympics that won at least one medal. Let *MEDALS* be the total number of medals won by a country and let *GDPB* be the GDP (in billions of 2024 dollars). A linear regression model explaining the number of medals won is given by:

$$\text{Medals} = \beta_1 + \beta_2 * \ln(\text{GDPB}) + e$$

We want to test the hypothesis that a one-billion dollar increase in GDP leads to an increase in the expected numbers of medals won by 0.015 , against an alternative hypothesis that is does not.

What are the null and alternative hypotheses in terms of the model parameters?

- A) $H_0 : \beta_2 = 0.015$ and $H_1 : \beta_2 < 0.015$
- B) $H_0 : \beta_2 = 0.015$ and $H_1 : \beta_2 > 0.015$
- C) $H_0 : \beta_2 = 0.015$ and $H_1 : \beta_2 \neq 0.015$
- D) $H_0 : \beta_2 \neq 0.015$ and $H_1 : \beta_2 = 0.015$
- E) $H_0 : \beta_2 \neq 0.015$ and $H_1 : \beta_2 < 0.015$

Solution: C

Question 11

Suppose we are interested in the relationship between students' test scores and class size, and we have conducted an experiment in which we randomly assign students either to a large class or a small class, and we consider the model:

$$(\text{test score}) = \beta_1 + \beta_2 * (\text{LARGE CLASS}) + e$$

where $\text{LARGE CLASS} = 1$ if the student is in a large class and $\text{LARGE CLASS} = 0$ if the student is in a small class.

Which of the following statements is false?

- A) Because the classroom assignment is random, we can say that the assumption that $E[e \mid \text{LARGE CLASS}] = 0$ is credible.
- B) β_1 measures the expected test score for students in a small class.
- C) β_2 measures the expected test score in a large class.
- D) β_2 measures the difference between expected scores in large and small classes.
- E) All of the above are True.

Solution: C

Question 12

All other things equal, using a t -approximation gives a smaller confidence interval than using a standard normal (Z) approximation.

A) True

B) False

Solution: B

The t -distribution is more spread out than the standard normal distribution.

Question 13

In testing $H_0: \beta_2 = c$ (where $c = a$ constant) using a 0.05 probability of a Type I error, you determine that the p-value = 0.38. What should you conclude?

- A) H_0 is true, $\beta_2 = c$.
- B) H_0 should be rejected and is unlikely to be true since the p-value < 0.50 .
- C) It is impossible to know for certain, but we can say that there is a 0.38 probability that $\beta_2 = c$
- D) There is sufficient evidence to reject H_0 , so we accept the null hypothesis by default.
- E) None of the above are true.

Solution: E

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Some Common Issues

Common Mistakes

- ▶ A few students left some multiple choice questions blank—always guess at the end!
- ▶ We don't "fail to reject H_A ". We only "Reject H_0 " or "Fail to reject H_0 ".
- ▶ Plugging in 11 instead of 10 in the marginal effect.
- ▶ Math errors / going too quickly. Ex: $(10)^2 \cdot 2 = 20$
- ▶ Mixing up the one sided vs. two sided test—draw a picture to help!

Grading

- ▶ We gave partial credit on the free response questions, even though it takes much longer to grade, which raised the scores significantly!

Thank you!

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