MGS3100: Exercises – Decision Analysis

1. In the following payoff table (in $ million), 1) identify decision alternatives, states of nature, and payoffs; 2) what does the number 29 mean? -12?

	High Demand (H)	Low Demand (L)
Produce 30,000	29	-12
Produce 20,000	18	8
Produce 12,000	3	11

2. Your company is testing a site for drilling for oil. You may hit a dry well, a small oil well, or a large oil well. Consider the following payoff table (in $ thousand) for your situation:

	Dry Well	Small Oil Well	Large Oil Well	Maximax	Maximin	Laplace
Drill	-5000	1000	6000			
Do not Drill	0	0	0			

1) What are the decision alternatives? What are the states of nature?
2) What would be the best decisions under each of the following criteria (Show your work by filling in the blank cells in the table above)?
   a) Maximax
   b) Maximin
   c) Laplace

3. Consider now that you have the following expectations based on your analysis of the site. You believe that there is a 50% chance of hitting a small well and a 20% chance of hitting a large well.

	Dry Well	Small Oil Well	Large Oil Well
Drill	-5000	1000	6000
Do not Drill	0	0	0
Probability		0.5	0.2

Calculate the following:
1) Expected Returns for both decisions (which one is better?)
2) Expected Return Under Perfect Information (EVUPI or ERwPI)
3) Expected Value of Perfect Information

4. You own a pizza shop in a downtown mini-mall. It is Saturday morning, and you are trying to decide how many pizzas to make to meet today's lunch hour demand. Based upon your experience with Saturdays, you think that the probability of being able to sell 20 pizzas is 0.2, of being able to sell 40 pizzas is 0.3, and of being able to sell 50 pizzas is 0.5.

   Suppose a pizza sells for $10 and has an incremental cost of $4.25. If you have leftover pizzas, you can sell them to the homeless shelter for $1.25 each. If demand exceeds the number of pizzas you have prepared, every disappointed customer costs you $0.25 worth of lost customer goodwill.

   1) Define the decisions, alternatives, states of nature, outcomes, revenues, costs, and payoffs for this problem. Then construct the payoff table.
   2) What is the decision making situation based on the knowledge about the state of nature?
   3) Assume you did not have the probability knowledge. Identify the decision you would make using each of the following criteria: Maximax, Maximin, and LaPlace.
   4) Find the ER for each alternative. Based upon the ER criterion, what is the best decision? Also find the ER with PI and the EVPI.
   5) Draw the decision tree that this problem calls forth and solve it.

5. General Motors (GM) is planning their production strategy for their next model. Three alternatives are being considered for their model Malibu: 30,000, 20,000, and 12,000. GM decides to categorize the demand for Malibu for the next year as either High (H) or Low (L). The payoffs measured in millions of dollars and probabilities of states of nature are presented in the table below.

	States of nature	
Decision Alternatives	High (H)	Low (L)
Produce 30K	29	-12
Produce 20K	18	8
Produce 12K	3	11
Probabilities	0.62	0.38

For this problem, if we want to do a decision tree,
1) How many decision nodes are required?
2) How many branches come out of <u>each</u> decision node?
3) How many chance nodes are required?
4) How many branches come out of <u>each</u> chance node?
5) Draw the decision tree. Label each branch completely including probabilities and payoffs.
6) Solve the decision tree and find the best production strategy.

6. For the above GM problem and the decision tree, it can hire a marketing research firm to help estimate the demand more accurately. Consider the reliabilities of the marketing research firm given below,
   1) compute the posterior probabilities,
   2) draw the revised decision tree (a blank tree is provided below).
   3) compute EVPI and EVSI.

RELIABILITIES

	High	Low
Encouraging	0.80	
Discouraging		0.70

PRIOR PROBABILITIES

High	Low
0.62	0.38

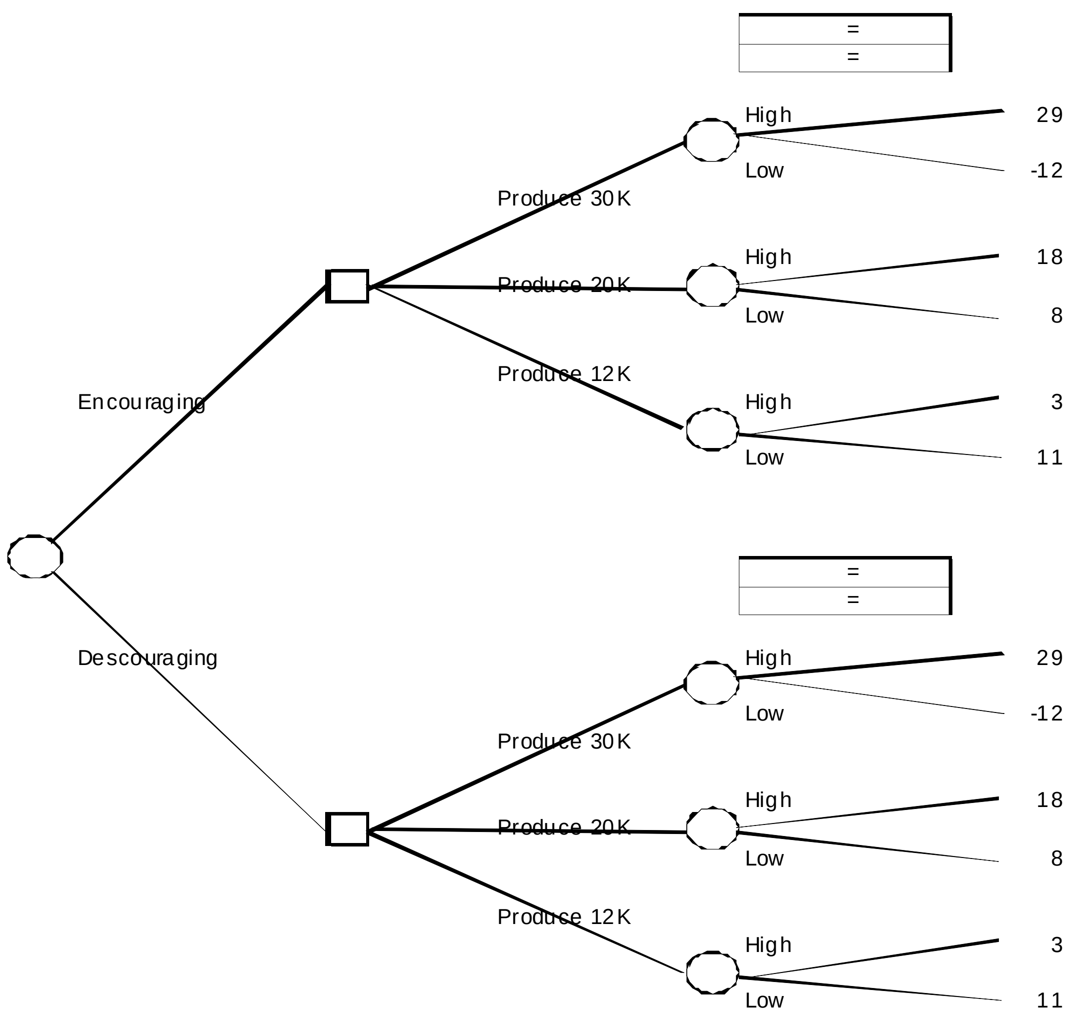
JOINT & MARGINAL PROBABILITIES

	High	Low
Encouraging		
Discouraging		

POSTERIOR PROBABILITIES

	High	Low
Encouraging		
Discouraging		

MGS 3100 Bayes' Theorem



7. Guido's EZ Loan Company pigeonholes credit applicants as Good and Bad risks. On the average, 10% of applicants are Bad risks. Guido uses a sophisticated computerized credit scoring model to attempt to discriminate between these 2 groups. A study of their experience in using the model yielded interesting results. Guido's gives credit to good risks 90% of the time. Bad risks get credit only 20% of the time.

   Assume that a person selected at random from potential applicants to Guido's applies for credit and is granted a loan. What is the probability that he/she is bad risk, P(Bad Risk|Credit Granted)?

8. Your friend Tenfour Goodbuddy is an independent trucker whose truck is empty after his most recent haul. He has found a deal to take a load to Birmingham, with a return load included, for a total of $3000. Another deal would give him a load to Charlotte for $2500. Tenfour thinks there's a 50-50 chance of finding a $2500 return load from Charlotte. Otherwise he must return empty, with no revenue. Assume that the cost of a round trip to either city is the same, and that he sees no other loads presently available.

   Sometimes in the past he has called the Thunder Road Truck Stop (TRTS) in Charlotte to get information. His friend there is very chatty, so it always costs him at least $10 for the call. 90% of the time when he had ended up with a return load, TRTS had told him "It's busy". 80% of the time when he had ended up with no return load (a "load of postholes"), TRTS had said "It's slow".

   To summarize, the available probabilities are:

P(RL) = .5	P(B|RL) = .9	P(S|RL) = .1
P(NL) = .5	P(B|NL) = .2	P(S|NL) = .8

   B = busy S= slow RL = return load NL = no return load

   1) The above scenario implies a decision tree. Neatly draw that tree. Include all decisions and chance occurrences, and all payoffs. You may ignore the $10 cost of the call (and perhaps deal with it later).
   2) Compute P(B), P(S), P(RL|B), P(RL|S), P(NL|B), and P(NL|S).
   3) Solve the decision tree, and state the resulting decision rule. How much, if anything, is the $10 phone call to Charlotte worth?

Solutions

1. 1)

	States of Nature	
Decision Alternatives	High Demand (H)	Low Demand (L)
Produce 30,000	29	-12
Produce 20,000	18	8
Produce 12,000	3	11

Payoffs

2) 29 means that if you choose to produce 30,000 units and the demand is high, then you can make $29 million. -12 means that if that if you choose to produce 30,000 units and the demand is low, then you will lose $12 million.

2.

	Dry Well	Small Oil Well	Large Oil Well	Maximax	Maximin	Laplace
Drill	-5000	1000	6000	6000	-5000	666.67
Do not Drill	0	0	0	0	0	0

1) Decisions: Drill and Do not Drill; States of nature: Dry, Small, and Large Oil Well.

2) Maximax – Drill
Maximin – Do Not Drill
Laplace – Drill

3.

	Dry Well	Small Oil Well	Large Oil Well	**ER**
Drill	-5000	1000	6000	**200**
Do not Drill	0	0	0	**0**
Probability	0.3	0.5	0.2	

1) Expected Returns for both decisions are in the last column of the above table.
Drilling is better with ER = 200.

2) Expected Return Under Perfect Information
EVUPI = 0*0.3 + 1000*0.5 + 6000*0.2 = 1700

3) Expected Value of Perfect Information
EVPI = EVUPI – EV w/o PI = 1700 – 200 = 1500

4. 1). Decisions: since we assume that the decision of opening the pizza shop for this particular Saturday has been made, the only decision here is to determine how many pizzas to make *today*.

Alternatives: there are three alternatives the pizza shop should consider (based on the possible demand levels): make 20 pizzas, make 40 pizzas, and make 50 pizzas.

States of nature: the uncertainty involved with this decision is the demand level. Hence the three states of nature are: demand of 20, 40, and 50.

Outcomes: the outcomes can be understood as the states of nature or as the consequence of the combination of different alternatives and states of nature.

Revenues: the total income of the pizza shop for any combination of a particular alternative (make quantity) and a particular state of nature (demand). It is calculated as:

Revenue = 10 min(demand, make quantity) + 1.25 max(0, make quantity - demand).

Costs: the total cost for any combination of a particular alternative (make quantity) and a particular state of nature (demand). It is defined as:

Cost = 4.25(make quantity) + 0.25 max(0, demand - make quantity).

Payoffs: the net income (profit) which is calculated as: Payoff = Revenue - Cost.

Payoff Table

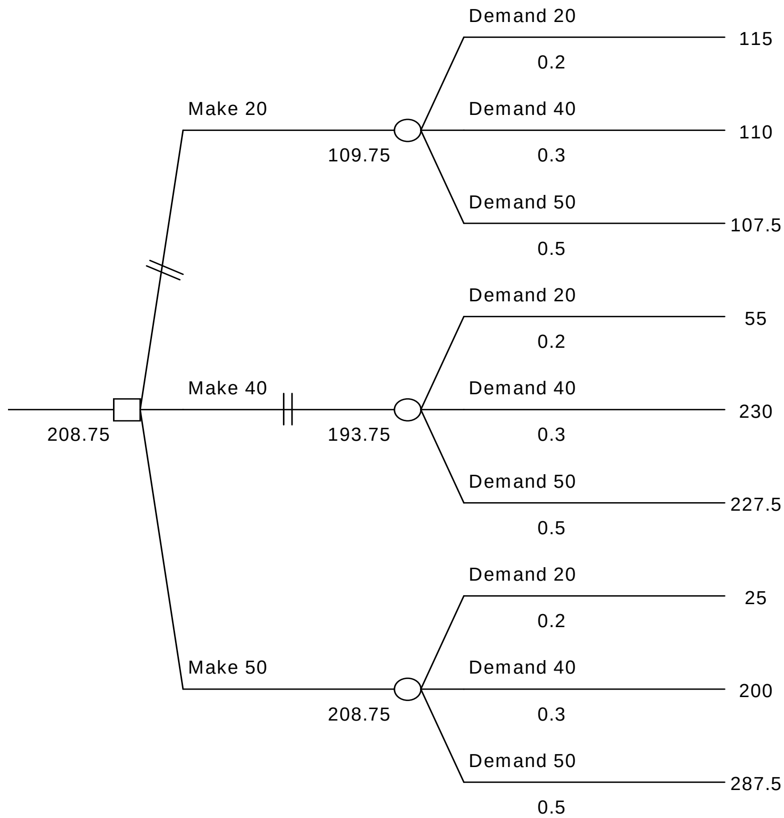
Make Quantity	**Demand**		
	20	**40**	**50**
20	$115.00	$110.00	$107.50
40	$55.00	$230.00	$227.50
50	$25.00	$200.00	$287.50

2). Decision making under risk.

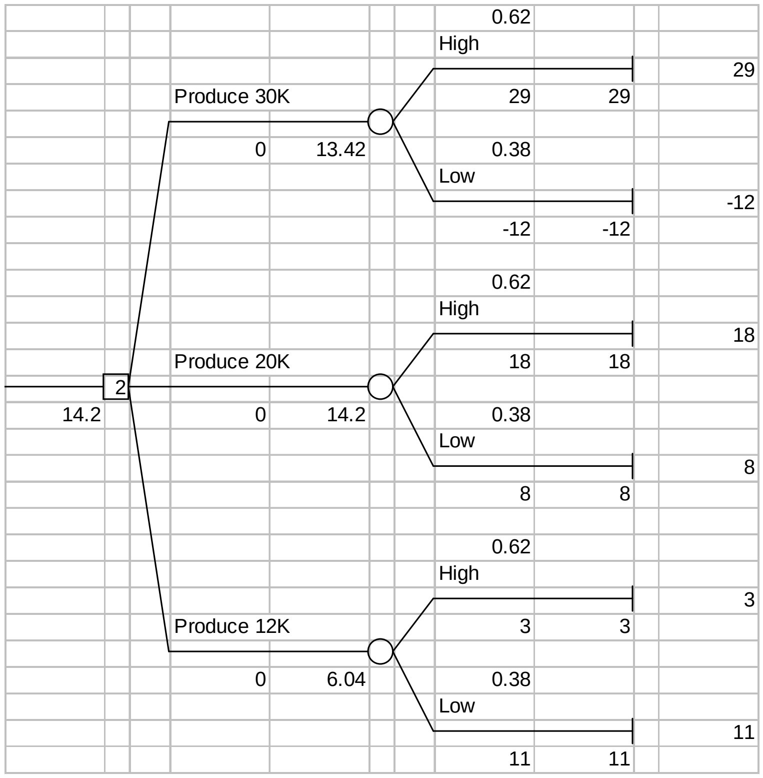
3). Maxmax: make 50
Maxmin: make 20
Laplace: make either 40 or 50

4). Expected returns for the three alternatives are $109.75, $193.75, and $208.75 respectively. The best alternative is to make 50 pizzas. With perfect information, the ER is $235.75. EVPI = $27.

5). The decision tree with the solution is given below:



5. 1) How many decision nodes are required? **1**
   2) How many branches come out of each decision node? **3**
   3) How many event nodes are required? **3**
   4) How many branches come out of each event node? **2**
   5) Draw the decision tree. Label each branch completely including probabilities and payoffs.



6) Solve the decision tree and find the best production strategy.
<u>Produce 20K (EV=14.2)</u>

6. 1)

Encouraging = E
Discouraging = D
High = H
Low = L

RELIABILITIES

	High		Low	
Encouraging	P(E|H)=	0.80	P(E|L)=	0.30
Discouraging	P(D|H)=	0.20	P(D|L)=	0.70

PRIOR PROBABILITIES

High	Low
0.62	0.38

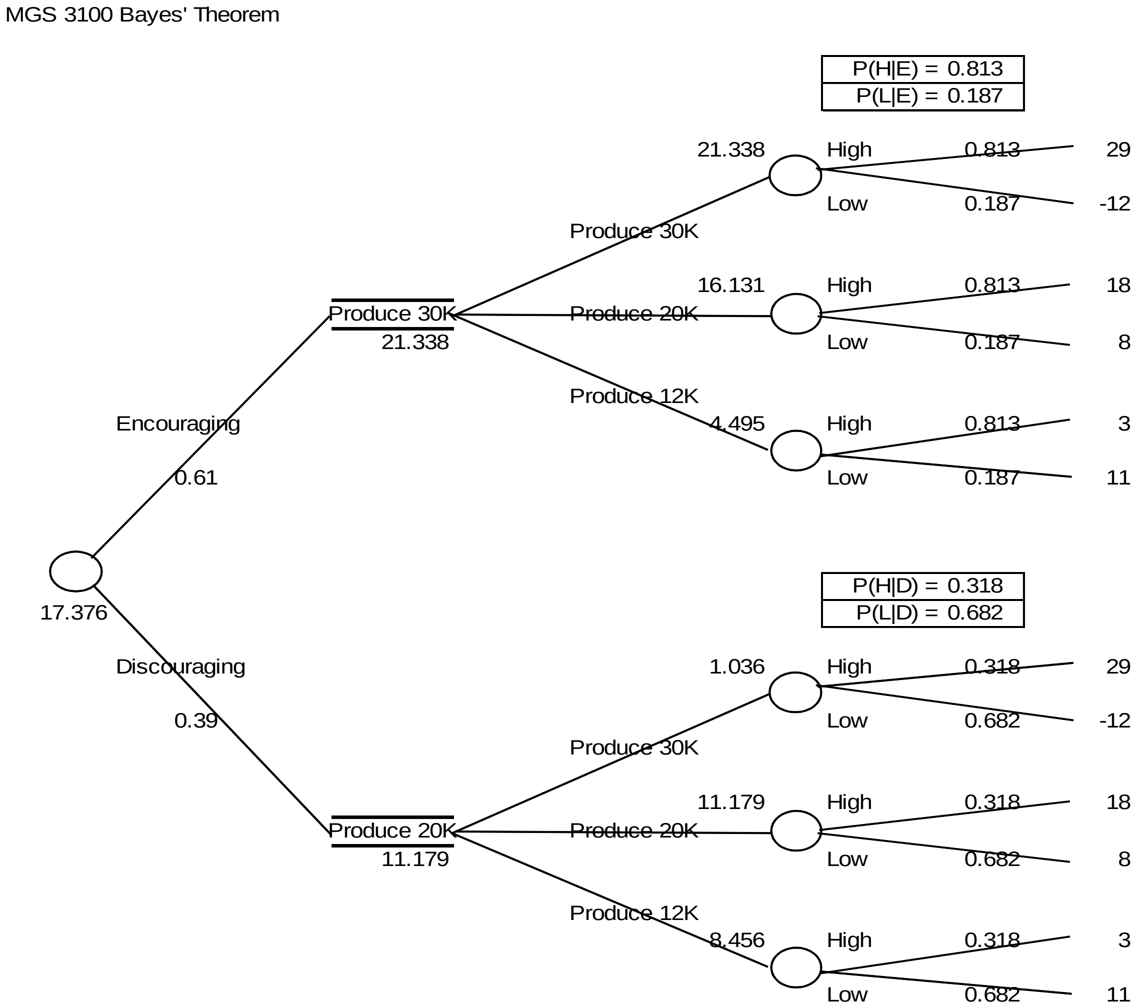
JOINT & MARGINAL PROBABILITIES

	High	Low	
Encouraging	P(E&H)= 0.496	P(E&L)= 0.114	P(E)= 0.610
Discouraging	P(D&H)= 0.124	P(D&L)= 0.266	P(D)= 0.390
	P(H)= 0.620	P(L)= 0.380	

POSTERIOR PROBABILITIES

	High		Low	
Encouraging	P(H|E)=	0.813	P(L|E)=	0.187
Discouraging	P(H|D)=	0.318	P(L|D)=	0.682

2)



3) EVSI = Max ER with sample info – Max ER w/o sample info = 17.376 – 14.2 = 3.176
EVPI = EV w/ PI – EV w/o PI = 29(0.62) + 11(0.38) – 14.2 = 7.96

7. Let BR = Bad Risk, GR = Good Risk, CG = Credit Granted
From the problem, we know the following: P(BR) = 0.10, P(CG|GR) = 0.90, and P(CG|BR) = 0.20. So P(GR) = 1 - P(BR) = 1 - 0.10 = 0.90.

Using the Bayes' Theorem:

$$
\begin{aligned}
P(BR \mid CG) &= \frac{P(CG \mid BR)P(BR)}{P(CG)} \\
&= \frac{P(CG \mid BR)P(BR)}{P(CG \mid BR)P(BR) + P(CG \mid GR)P(GR)} \\
&= \frac{0.20 \times 0.10}{0.20 \times 0.10 + 0.90 \times 0.90} \approx 0.0241
\end{aligned}
$$

That is, the probability that the person is a bad risk given that he/she has been granted a loan is about 0.0241 or 2.41%.

Note: if you are not comfortable with the above formula based calculation, you may use the following tabular form as:

Prior probabilities

BR	GR
0.1	0.9

Likelihoods

	BR	GR
CG	0.2	0.9
CNG	0.8	0.1

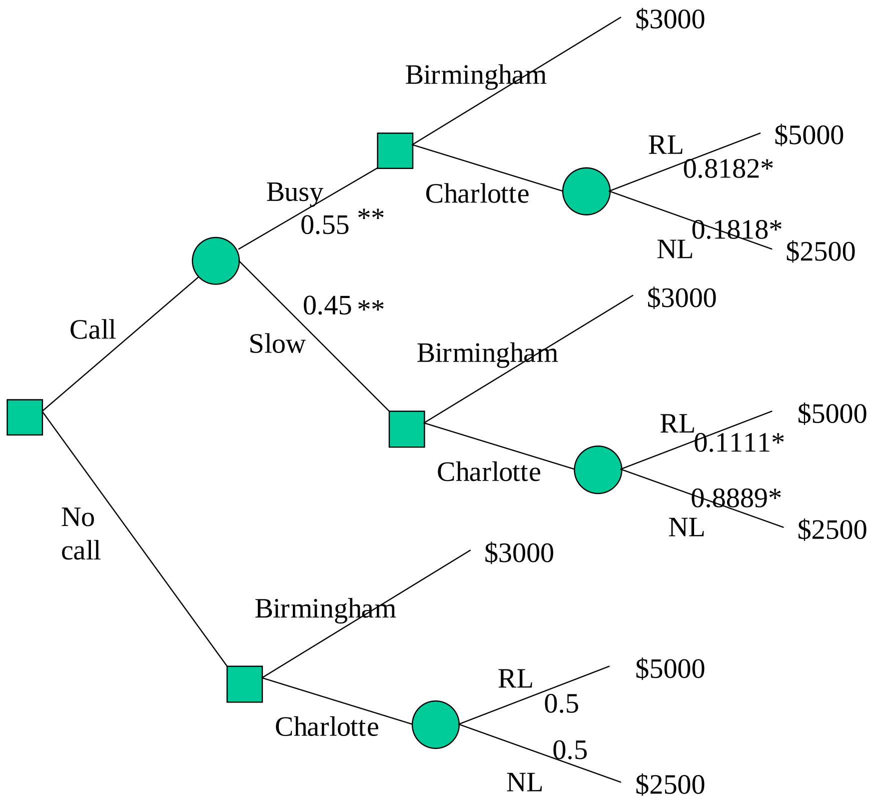
Joint probabilities (multiply priors by likelihoods)

	BR	GR	Marginal probabilities
CG	0.02	0.81	0.83
CNG	0.08	0.09	0.17

Posteriors (joint divided by marginal probabilities)

	BR	GR
CG	**0.024096**	0.975904
CNG	0.470588	0.529412

6. 1). The decision tree is given below. Notice that the marginal probabilities (P(B), P(S)) and the posterior probabilities (P(RL|B) etc.) have been included in the tree. These probabilities are obtained based on the calculation of Part (b).



* -- posterior probability; ** -- marginal probability

2). The calculation of all these probabilities is listed below. Make sure you understand the logic of the calculation and the meaning of each number.

Prior probabilities

RL	NL
0.5	0.5

Likelihoods

	RL	NL
B	0.9	0.2
S	0.1	0.8

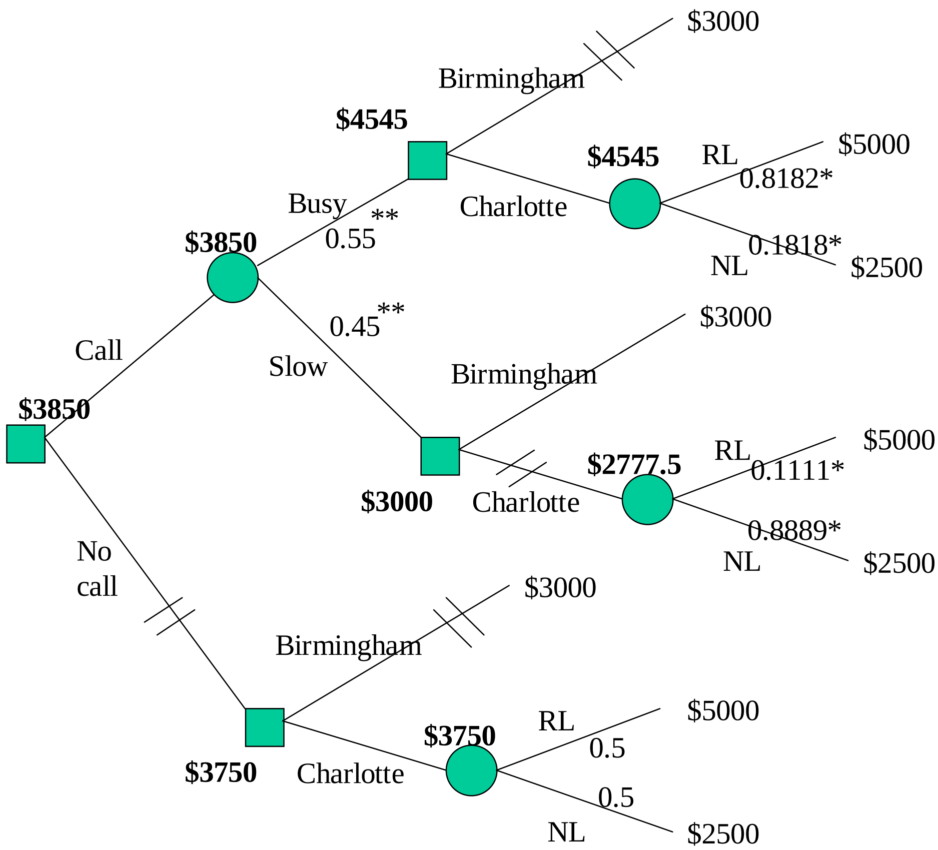
Joint probabilities (multiply priors by likelihoods)

	RL	NL	Marginal probabilities
B	0.45	0.1	0.55
S	0.05	0.4	0.45

Posteriors (joint divided by marginal probabilities)

	RL	NL
B	0.8182	0.1818
S	0.1111	0.8889

3). The solved tree is given below.



The decision rule is: Call Thunder Road. If things are slow in Charlotte, take the Birmingham road; If things are busy in Charlotte, take the Charlotte load.

Without the call, you would just go to Charlotte, with an expected return of $3750. With the call, your ER becomes $3850. So, EVSI = 3850-3750=$100. That is, the call can, *on the average*, improve profit by $100. Since the call costs $10, the net worth of the call is $90.