Stochastic Modeling of Asset Behavior in Market Crises: A Comparative Analysis of Diverse Portfolio Resilience

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Market crashes, an inevitable part of financial investing, often erase gains achieved during stable periods. Portfolios, even those showing robust growth, are vulnerable to substantial losses in such downturns. This uncertainty underscores the critical need to understand portfolio behaviors across different market conditions. This report addresses the critical need to understand portfolio resilience in these turbulent times by analyzing four investment portfolios—Growth, Aggressive, Balanced, and Conservative—each containing a unique allocation of stocks, bonds, real estate, commodities, and cash. Stochastic models, specifically Geometric Brownian Motion, the Ornstein-Uhlenbeck process, and the Vasicek model, are employed to simulate the behavior of these assets across normal market conditions and during market crashes. The primary findings indicate that the Growth portfolio experiences the highest growth during stable periods and the steepest decline during crashes, while the Conservative portfolio exhibits the most stability but longer recovery times compared to the other portfolios. Notably, the Aggressive, Balanced, and Growth portfolios demonstrate similar recovery patterns. A key observation is the impact of crash duration on portfolio returns; longer crashes result in lower final returns for all portfolios, with the Conservative portfolio maintaining the highest returns during significantly prolonged downturns. Additionally, the difference in the four portfolios studied in this paper show that volatility in portfolio value is proportional to recovery time. The study highlights the balance between growth potential and stability across different market scenarios. Moreover, the simulation tool created for this project provides a flexible and robust approach for portfolio analysis. It enables the customization of portfolios and crash parameters, thereby acting as a valuable resource for planning investment strategies in diverse economic environments.

I. INTRODUCTION

In financial mathematics, modeling stock market behaviors using stochastic processes is crucial for understanding asset dynamics, especially during market turbulence. These models are vital for both academic research and practical applications in risk management and strategic financial planning.

Portfolio management, a key aspect of finance, involves balancing various financial assets to achieve specific objectives while minimizing risks. Historical market crashes like the 1929 Great Depression, 1987 Black Monday, the early 2000s dot-com bubble, and the 2008 financial crisis, along with the COVID-19 pandemic, highlight the need for resilient investment strategies in times of crisis [1].

This report analyzes the resilience of diverse investment portfolios under different market crash scenarios. It examines four portfolios with varying risk exposures and asset allocation strategies: Conservative, Aggressive, Balanced, and Growth.

A. The Finance Market

Financial markets, including stocks, bonds, commodities, cash, and real estate, are complex and reflect economic strength. The stock market, influenced by numerous factors, serves as an economic barometer. Understanding market behavior during crises is crucial for portfolio resilience and effective investment strategies.

Market crashes typically lead to rapid asset value declines and heightened volatility, which essentially means fluctuations in an asset's price. The 2008 financial crisis and the COVID-19 pandemic have shown the diverse responses of different assets during such times.

In financial mathematics, two models are particularly noteworthy: Geometric Brownian Motion (GBM) and the Vasicek model. These models are pivotal in financial mathematics, particularly in understanding market dynamics. GBM is adept at modeling stock price volatility, capturing the stochastic nature of financial markets. The Vasicek model, meanwhile, is valuable for its insights into interest rate behaviors, a crucial aspect of financial planning and analysis. Both models are instrumental for strategizing and risk management in finance, offering a mathematical foundation to predict and analyze market trends and asset performance under various economic conditions.

Stock market simulations using these models are crucial for investors and analysts to test strategies and understand market behaviors. They are also essential in financial education and for institutions to assess extreme market event impacts on portfolios.

B. Related Research and Novel Inputs

Research on stock market simulations and portfolio performance provides insights into financial modeling and strategy, emphasizing the need to complement simulations with other financial tools for a comprehensive market understanding.

Significant studies in this domain include the adaptation of the GBM model for more realistic stock price simulations [2] and the novel methodology for modeling stock market indexes in different economic states using GBM [3]. These contributions highlight the role of GBM and stochastic models in financial research.

Our study stands out by simulating various market crash scenarios and analyzing recovery times for different portfolios, examining both short-term and long-term crashes and their impact on a range of assets. This approach offers a nuanced understanding of market crashes, their effects on diverse investments, and portfolio recovery dynamics.

C. Assets

In this study, the different portfolios comprises a mix of assets including bonds, stocks, commodities, and real estate. These assets are chosen for their distinct roles and impacts on investment strategies.

Bonds: In our portfolio, government bonds, particularly zero-coupon bonds, stand out for their reliability during economic downturns [4]. These bonds are unique: investors buy them for less than their final value and get the full amount back when they mature, without any interest payments in between (hence the name zero-coupon) [5]. Their value is dependent to changes in interest rates, making an accurate interest rate model essential. The Vasicek model is utilized for this purpose. It effectively captures how interest rates tend to fluctuate and return to an average over time [6]. By applying this model, along with the bond pricing formulas from [7], we can accurately simulate the value of these bonds in varying economic scenarios.

Stocks and Commodities: Stocks, representing company ownership, are integrated into the portfolio for their potential for high returns through capital gains and dividends. Their volatility is a reflection of factors such as company performance and economic conditions. Commodities, ranging from precious metals to agricultural products, are included for their tangible nature and their distinct market dynamics, which often diverge from stock market trends [8]. The inclusion of both asset types enriches the portfolio with a spectrum of risk and return profiles. The Geometric Brownian Motion model, applied in this context, effectively simulates the random nature of these asset prices, encapsulating the essence of market volatility.

Real Estate: Real estate assets, particularly private residential villas, are chosen for their relative market stability and consistent growth, typically ranging from 2-6% annually [9]. This sector adds a layer of diversification to the portfolio, balancing the volatility of stocks and commodities with its steady growth and lower risk profile. The decision to focus on residential real estate over more complex sectors like commercial or industrial properties ensures a more streamlined and accurate analysis within the study's framework.

D. Risk Versus Expected Return

In financial markets, "risk" reflects the fluctuation in an asset's value, with a direct correlation to expected return. Harry Markowitz's theory provides a foundation for this relationship, proposing that optimal portfolios can be created by balancing risk against expected return. These portfolios lie on the "efficient frontier", a concept implying that for each level of risk, there exists a portfolio that maximizes expected return. Portfolios below this frontier are suboptimal, as they do not yield the highest possible return for their risk level. The theory emphasizes the importance of asset allocation in portfolio optimization to achieve a desirable balance of risk and return [10].

II. METHODOLOGY

A. Stocks and Commodities

To model the behavior of stocks and commodities, the stochastic differential equation (SDE) of the Geometric Brownian Motion (GBM) is utilized. GBM is defined by the following equation:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{1}$$

where μ denotes the drift, σ denotes volatility, and $dW(t)$ is the incremental Wiener process [11].

$$dW(t) = \sqrt{dt}w(t) \tag{2}$$

A Wiener process, expressed as in Equation 2, is a stochastic process whose increment $w(t)$ is a Gaussian random number with mean zero and variance one.

$$S(t) = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right) \tag{3}$$

The GBM model in Equation 3 is often used to simulate the random movements of stock and commodity prices [12]. The term $\mu - \frac{\sigma^2}{2}$ is referred to the drift term in this case. The drift represents the average rate of return over a given time period.

B. Real estate

To model the real estate market, the Euler–Maruyama Method (EMM) was utilized for discretizing the Ornstein–Uhlenbeck process (OUP). The EMM is a numerical scheme for approximating solutions to stochastic differential equations, making it suitable for simulating processes like the OUP [13]. The OUP, fundamentally based on Brownian motion, is distinguished by its mean-reverting characteristic, which implies that values tend to return to a long-term mean over time.

$$\mu_{rev} = \theta(\mu S(0) - S(t)) \tag{4}$$

$$dS(t) = \mu_{rev} dt + \sigma dW(t) \tag{5}$$

The OUP is defined by Equations 4 and 5, where θ represents the rate of mean reversion, and μ_{rev} denotes the mean reversion term, as outlined in [14]. Equation 4 defines the mean reversion dynamic, and Equation 5 describes the overall Ornstein-Uhlenbeck process.

Equations 4 and 5 are integrated into the standard GBM framework (Equation 1). The integrated equation is then discretized using the EMM to adapt it for simulation.

$$S(t + \Delta t) = S(t) + \mu_{rev}\Delta t + \sigma S(t)\sqrt{\Delta t}w(t) \tag{6}$$

The discretized form of the equation is presented by Equation 6 and represents the final model for the real estate market, which forms the basis of our simulation.

C. Interest Rate and Bond Pricing Model

The Vasicek model is employed to simulate the stochastic behavior of interest rates over time. The model is characterized by the SDE [6]:

$$dr(t) = a(b - r(t))dt + \sigma dW(t), \tag{7}$$

where $r(t)$ represents the interest rate at time t, a is the mean reversion rate indicating the velocity at which interest rates adjust towards the mean, b is the equilibrium interest rate level that rates revert to over time, σ represents the volatility of interest rate, and $dW(t)$ signifies the incremental Wiener process, which introduces randomness reflecting the unpredictable nature of interest rate movements.

To make the equation suitable for simulation, the SDE is discretized via the Euler-Maruyama method into the following form:

$$r(t + \Delta t) = r(t) + a(b - r(t))\Delta t + \sigma\sqrt{\Delta t}w(t), \tag{8}$$

where Δt is the time step, and $w(t)$ is a Gaussian random number with mean zero and variance one.

The bond pricing is determined by the analytical solution to the Vasicek model, expressed in terms of the risk-neutral measure. The price formula for a zero-coupon bond is given by [7]:

$$B(t, T) = \exp\left(D(t, T) - A(t, T)r(t)\right) \tag{9}$$

where functions $A(t, T)$ and $D(t, T)$ are specified as:

$$A(t, T) = \frac{1 - e^{-a(T-t)}}{a} \tag{10}$$

$$D(t, T) = \left(b - \frac{\sigma^2}{2a^2}\right)[A(t, T) - (T - t)] - \frac{\sigma^2 A(t, T)^2}{4a} \tag{11}$$

For a detailed derivation of the bond price formula, see [7].

D. Parameters Estimation

The estimation of volatility (σ) and drift (μ) parameters is a fundamental aspect across various financial modeling methods, including the Geometric Brownian Motion, Ornstein-Uhlenbeck process, and interest rate models. This section discusses the methodologies for estimating these parameters.

Volatility is calculated using a standard formula, applicable in various market conditions, including financial crashes. The formula, as per [15], is expressed as:

$$\hat{\sigma}(t) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r(t) - \bar{r})^2} \tag{12}$$

where $r(t)$ is the return of an asset at time t and $\bar{r}$ denotes the average return of the asset during the entire time period T.

The drift parameter (μ), representing the expected rate of change in market values, is estimated through fundamental analysis. This involves determining the annual expected return and converting it to a daily rate based on the number of trading days in a year. This approach is consistent across different modeling techniques.

E. Portfolios

The four distinct portfolios analyzed in this report are selected based on four specific risk levels. As stated earlier, these varying risk levels require different asset allocations. Portfolios categorized under higher risk levels necessitate a larger allocation towards more volatile investments. The strategy for allocating these assets to attain both high and low variation is determined through an analysis of the parameters used in modeling each asset. This analysis leads to the conclusion that stocks typically exhibit higher variation, whereas bonds tend to show lower variation. Utilizing this insight, the four portfolios under examination in this study, each tailored to a unique risk profile, are presented in Figure 1. For the sake of simplicity, the allocation towards commodities, cash and real-estate are kept at a similar level for the first three portfolios: Conservative, Balanced and Aggressive. In these three portfolios only the ratio between the bonds and the stocks are varied in order to vary the volatility. The final portfolio, the Growth portfolio, is an extreme case were even the three assets: commodities, cash and real-estate are minimized in order to allocate a vast majority towards stocks.

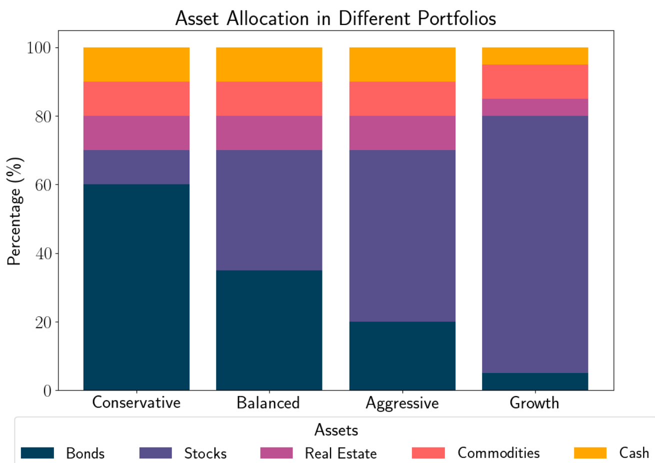

FIG. 1. Asset allocation percentages in Conservative, Balanced, Aggressive, and Growth portfolios, showing the distribution across five asset classes: Bonds, Stocks, Real Estate, Commodities, and Cash.

The portfolios are designated as follows, each representing a distinct risk level:

- **Conservative Portfolio**: Primarily focused on stability, this portfolio allocates a significant portion to bonds (60%). Stocks are minimally included (10%), with the rest distributed evenly among cash, commodities, and real estate.

- **Balanced Portfolio**: As a middle ground, this portfolio equally balances the allocation between stocks and bonds (35% each), providing a blend of growth potential and stability. The remainder is evenly split among real estate, commodities, and cash.

- **Aggressive Portfolio**: Oriented towards growth, this portfolio has a higher tilt towards stocks (50%), with a reduced allocation in bonds (20%). The allocation of the other assets remains consistent with the Conservative portfolio, ensuring some diversification.

- **Growth Portfolio**: Highly focused on stock market exposure (75%), this portfolio significantly reduces the percentages in other assets. Bonds, cash, and real estate are minimally included, each constituting smaller fractions (5% each), with a slightly higher allocation in commodities (10%).

F. Overview of Market Crash Models

This study examines two distinct types of market crashes: a short-term crash and a long-term crash. Each scenario is characterized by a 40% decrease in the stock market and an interest rate increase from 2% to 7%. Additionally, a key parameter in both crash types is the increasing volatility, which varies across different assets. This approach acknowledges that volatility does not rise uniformly across all assets during a market crash, reflecting the complex and nuanced nature of real-world market dynamics. The values chosen for the market decrease and interest rate hike are based on historical data, aiming to imitate real crash scenarios that have historically occurred, thus ensuring the study's scenarios are grounded in realistic market behavior.

Both the short-term and long-term market crashes are distinguished primarily by their duration. The short-term crash spans 40 trading days, corresponding to approximately two months, while the long-term crash extends over 252 trading days, roughly one year.

G. Market Crashes Simulation Details

Real Estate Market: In the real estate sector, the simulation incorporates adjustments to key parameters to mimic market dynamics, as derived from data sources such as Statistics Sweden [16], and Swedish Real Estate Statistics [17]. The volatility parameter (σ) is calibrated to increase from 0.01 under normal conditions to 0.02 during market crashes. The mean level (μ), indicative of general market trends, shifts from a positive growth rate of 7% (1.07) in stable periods to a decline of 10% (0.90) in downturns. Additionally, the rate of mean reversion (θ) is consistently set at 0.10, reflecting the market's tendency to revert to its long-term mean.

Stocks and Commodities: For the stock market and commodities, volatility parameters (σ) have been calculated for each asset, utilizing data derived from historical market behaviors, especially during the Subprime crisis (2007-2009), following Equation 12. The S&P 500, which represents stocks in this study, shows a calculated increase in volatility from 0.0136 under normal conditions to 0.0283 during market crashes. In the realm of commodities, calculated volatility adjustments are evident in assets like Gold, Crude Oil, Silver, Copper, Natural Gas, Corn, and Oat. These adjustments reflect changes in historical data, with Gold's volatility calculated to shift from 0.0143 to 0.0188, and Crude Oil's from 0.0223 to 0.0471 during crash scenarios.

Similarly, for drift parameters (μ), critical for assessing average market returns, the calculated values for both stocks and commodities are based on a simulated

40% market decrease during crashes. The drift parameter is calculated to increase from 0.397×10^{-3} in normal conditions to 1.190×10^{-3} during crash scenarios for both asset classes. This calculated approach ensures a realistic and dynamic simulation of the market, effectively capturing the distinct behaviors of different assets under varying market conditions.

Bonds and Interest Rates: In the bond market simulation, the equilibrium interest rate level (b), derived from historical data [18], is adjusted to increase from 2% to 7%. This change reflects the typical shifts in interest rates observed during periods of market crash. Concurrently, the volatility parameter (σ) for bonds is modified from 0.02 in stable market conditions to 0.07 in times of market turbulence. This adjustment in volatility mirrors the increased market risk and investor uncertainty, effectively capturing the essence of bond market dynamics across various economic scenarios. Additionally, the mean reversion rate (a) is calibrated to rise from 0.1 to 0.4, indicating a faster reversion to equilibrium in volatile markets.

H. Simulation Scenarios

The study explores four distinct market crash scenarios:

- **Two Short Crashes:** This scenario investigates the market's response to consecutive rapid and intense downturns. It is crucial for understanding the resilience of the market and its ability to recover from a series of short-term crises.
- **Two Long Crashes:** Concentrating on sustained economic challenges, this scenario explores how markets manage and recuperate from extended periods of decline. It mirrors situations similar to long-term recessions or deep economic depressions.
- **Short Crash Followed by Long Crash:** This combined scenario provides an insight into the market's capacity for adaptation. It examines the market's response to an initial, abrupt crash followed by an extended phase of economic downturn.
- **Long Crash Followed by Short Crash:** Contrasting the previous scenario, this setup evaluates the impact of a long-term downturn followed by a sudden and sharp crisis. It is crucial for understanding the cumulative effects of prolonged stress on market stability, followed by a rapid shock.

These scenarios are designed to analyze the market's reaction and resilience under different sequences of economic downturns.

I. Market Crash Rationale

The parameters chosen for the market crash simulations are rooted in historical data to ensure a realistic representation of market behaviors during economic downturns. Sources like Statistics Sweden and Swedish Real Estate Statistics provide foundational data for parameter calibration, [16][17]. The rationale behind these specific values and scenarios is to offer a nuanced understanding of market dynamics and to facilitate the development of effective risk management strategies tailored to varying economic conditions.

III. RESULTS AND DISCUSSION

In the conducted simulation, which spanned a decade, the four distinct scenarios were analyzed to observe the behavior of investment portfolios during market crashes. Over this 10-year period, each portfolio, starting with the same initial value, was subjected to these varying market conditions, and their performance was tracked over 2520 trading days. These results are visualized in figure 2 and 3.

Key Findings:

- The Growth portfolio exhibited the most volatility, showing the highest returns in stable markets and the steepest declines during crashes, see Figures 2 and 3.
- The Conservative portfolio, while being resilient during downturns, demonstrated the slowest recovery pace compared to the other portfolios, which showed similar behaviors during recovery periods, see Figure 4 (left).
- Balanced and Aggressive portfolios showed intermediate behavior, neither as volatile as the Growth portfolio nor as stable as the Conservative one.
- A noteworthy finding is the presence of a threshold in crash duration. Beyond this threshold, characterized by an exceptionally prolonged crash (1650 trading days), the Growth portfolio yielded lower returns than the Conservative portfolio, see Figure 4 (right). This observation highlights a critical point where conservative investment strategies start outperforming riskier, growth-oriented ones in extended market downturns.

Figures 2 and 3 showcasing the change in portfolio value for all four scenarios show similar behavior and patterns. The most volatile portfolio, the Growth portfolio, and the least volatile portfolio, the Conservative portfolio, show the most interesting results while the remaining two graphs are a middle ground between these two risk-levels, which is represented in all four graphs. The Growth portfolio always has either the highest value or the lowest value depending on the

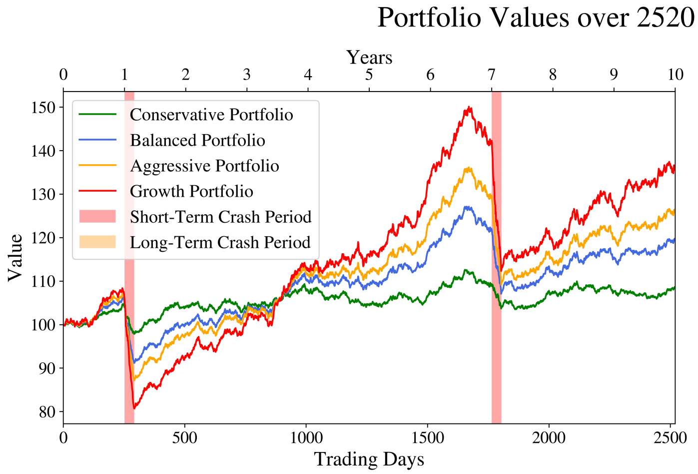

FIG. 2. Portfolio value over 2520 trading days (10-year period). Left: two consecutive short-term crashes, each enduring 40 trading days; the first initiating at year 1, and the second at year 7. Right: two consecutive long-term crashes, each spanning 252 trading days; the initial crash commences at year 1, followed by the second at year 7.

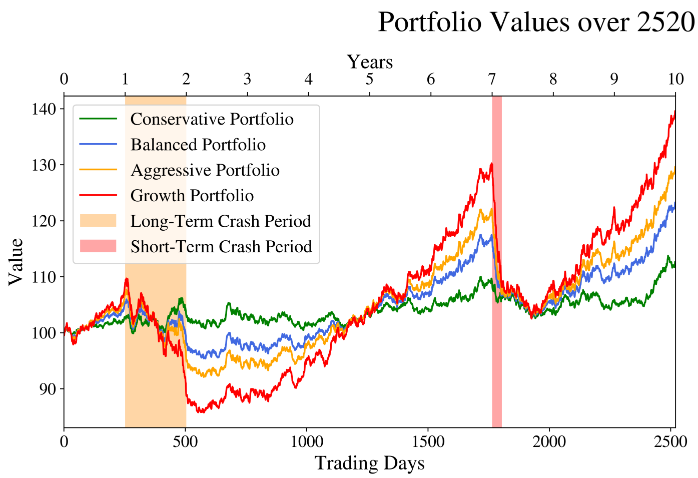

FIG. 3. Portfolio value over 2520 trading days (10-year period). Left: a long-term crash lasting 252 trading days starting at year 1, succeeded by a short-term crash of 40 trading days initiating at year 7. Right: a short-term crash lasting 40 trading days beginning at year 1, followed by a long-term crash extending over 252 trading days starting at year 7.

period. In the early stages, before the first crash, the Growth Portfolio tends to either increase or decrease in value more than other portfolios. Whether it increases or decreases is random as this process is stochastic, accurately mimicking the behavior of real assets. During the crash the value of this portfolio suffers the most out of these four portfolios. During the periods without a crash the Growth portfolio also grows the most. The conservative portfolio shows the opposite patterns, during a crash it is the most resilient and during the recovery period it is the least growing. The patterns are consistent in all four scenarios. It is important to note that while at times the value of all four portfolios may decrease below their initial value, at the end of the decade-long period they all have a value above the initial value.

A. Parameter Analysis

Analyzing the impact of market crashes on investment portfolios is critical to understanding risk management. This section examines the correlation between the duration of a market crash and portfolio returns. The return R on a portfolio is calculated using the formula [19]:

$$R = \left(\frac{V_f}{V_i} - 1\right) \times 100 \tag{13}$$

where V_f is the final value of the portfolio, and V_i is the initial value of the portfolio.

Figure 4 (right) reveals a downward trend in the final returns of all investment portfolios as market crash

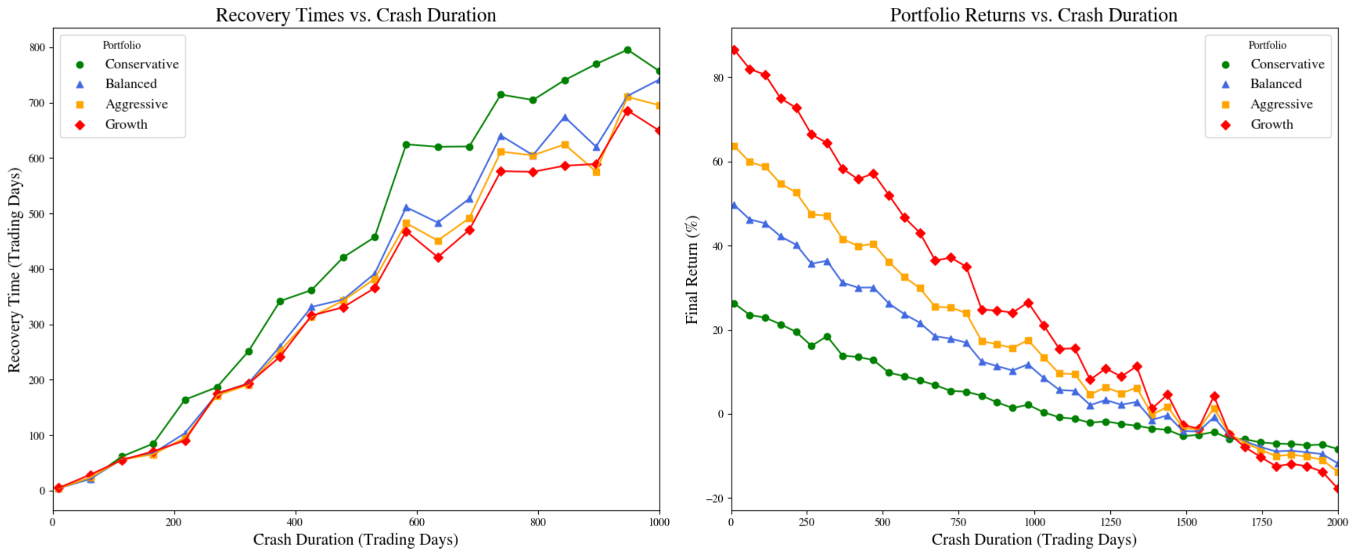

FIG. 4. Left: Recovery time in trading days post-crash for a portfolio to regain its pre-crash value, plotted against crash duration. Right: Final return (%) of Conservative, Balanced, Aggressive, and Growth investment portfolios over increasing crash duration, illustrating the risk-return trade-off.

duration extends. This overall decline reflects the sensitivity of portfolio performance to prolonged economic downturns.

Notably, the Conservative portfolio exhibits the least decline, suggesting resilience during economic stress. On the contrary, the Growth portfolio, though initially offering high returns, declines sharply, indicating heightened sensitivity to market downturns.

As crash duration extends, the Balanced and Aggressive portfolios follow intermediate paths. Their decline rates are distinct, yet both reflect a moderate decrease when compared to the Growth portfolio. This suggests that while they may bear risk, they do not do so to the extent of the Growth portfolio.

Figure 4 (left) is generated by simulating a period with a crash a number of times with different crash duration. The recovery time is then calculated for each simulation by calculating the amount of days between the end of the crash duration and the day that the portfolio recovers the value it had the day before the crash. This graph clearly shows that recovery time increase when crash duration increases. The figure also shows that the recovery time is the highest for low risk portfolios, suggesting that high risk portfolios has a lower recovery time.

B. Implications of Crash Duration on Portfolio Performance

While the Growth portfolio suffers the most during a crash, interestingly it shows a tendency to recover the fastest suggesting that volatility is proportional to the recovery rate, this relationship can be seen in Figure 4 (left). Here we see the conservative portfolio, at all times, has the highest recovery time of all portfolios. Inversely, the graphs for portfolio value also suggest that resilience of a portfolio is inversely proportional to the volatility of a portfolio. This conclusion is based on the fluctuations during the crash. This observation is of course as expected but it further proves the model is realistic.

A critical observation from Figure 4 (right) is the crossover point, around trading day 1650. During shorter crashes (before the crossover point), the Growth portfolio maintains its superiority in terms of returns. However, when the crash lasts longer than the crossover point, the Conservative portfolio outperforms all others, delivering the highest returns. This inflection point is crucial for investors, marking the juncture at which conservative strategies may begin to yield superior outcomes in the context of prolonged market downturns.

IV. CONCLUSION

This study presents a comprehensive analysis of four distinct investment portfolios—Conservative, Balanced, Aggressive, and Growth—under various market crash scenarios. We explore how portfolio composition and risk tolerance influence performance during downturns. Our key findings are as follows:

- **Conservative Portfolio:** Characterized by its low volatility and substantial allocation to bonds, this portfolio demonstrated remarkable resilience during market crashes. However, it exhibited the longest recovery time post-crash, reflecting its conservative nature.

- **Balanced and Aggressive Portfolios:** These portfolios, with a more equitable distribution of assets, displayed intermediate behavior in terms of both volatility and recovery time. They struck a balance between stability and growth, underlining the importance of diversified asset allocation in risk management.
- **Growth Portfolio:** Dominated by stocks, this portfolio experienced the highest growth during stable periods but suffered the most significant losses during market downturns. Notably, it also showed the fastest recovery, highlighting the high-risk, high-reward nature of stock-heavy portfolios.

The study distinctly reveals that high volatility is directly proportional to shorter recovery periods, while low volatility correlates with longer recovery times. Notably, there's a key threshold at around 1650 trading days, after which conservative strategies begin to outperform growth-oriented ones in terms of return.

Highlighting the intricacies of portfolio management, particularly in volatile markets, this study provides valuable insights into the risk-return trade-offs inherent in different investment strategies. Although the study is based on theoretical stochastic models, it's vital to acknowledge their limitations in accurately predicting actual financial market behaviors, which are affected by numerous unpredictable elements.

Our findings serve as a crucial guide for investors in understanding the dynamics of portfolio performance during market fluctuations and in making informed decisions about asset allocation and risk management.

V. CONTRIBUTIONS

Fadi: Modeled real estate using the Ornstein-Uhlenbeck process, wrote associated code and text, developed and analyzed crash scenarios.

Abubakar: Took part in constructing portfolio models and the research for this. Wrote associated code and text, results, discussions and conclusion.

Francisco: Modeled bonds using the Vasicek model for interest rate, priced bonds, wrote associated code and text, developed and analyzed crash scenarios.

Mustafa: Conducted the primary research for portfolio constructions, authored the introduction, and performed the literature review.

Sky: Modeled commodities, cash, and stocks using Geometric Brownian Motion, wrote associated code and text, developed and analyzed crash scenarios.

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