3-way tables: Alcohol Cigarette, and Marijuana Use

Survey asked 2276 students in their final year of high school in a nonurban area near Dayton, Ohio whether they ever used alcohol, cigarettes, or marijuana.

Alcohol Use	Cigarette Use	Marijuana Use	
		Yes	No
Yes	Yes	911	538
	No	44	456
No	Yes	3	43
	No	2	279

This is example of a $2 \times 2 \times 2$ contingency table. Shorthand: A=alcohol, C=cigarette, M=marijuana.

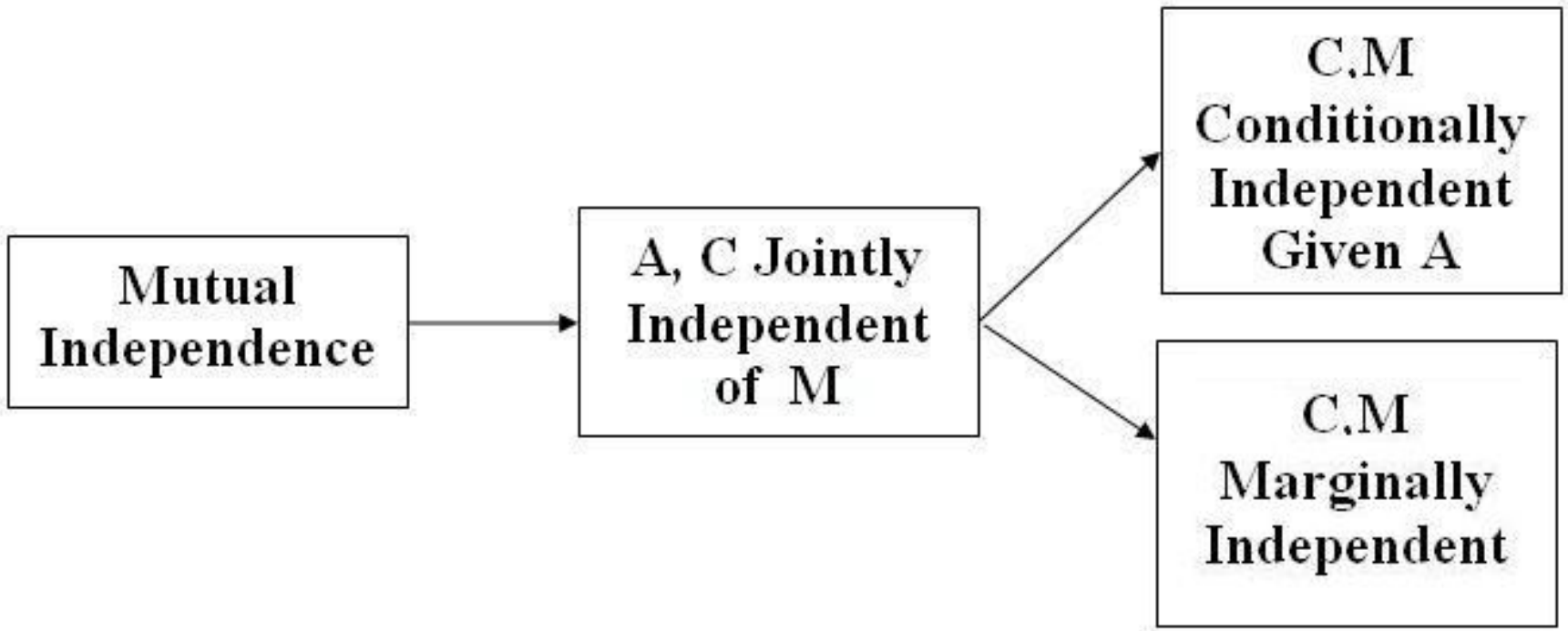
Contingency Tables: Death Penalty Example

The 674 subjects where defendants in murder cases in Florida between 1976 and 1987.

Victim's Race	Defendant's Race	Death Penalty Yes	Death Penalty No	Percent Yes
White	White	53	414	11.3
	Black	11	37	22.9
Black	White	0	16	0.0
	Black	4	139	2.8
Total	White	53	430	11.0
	Black	15	176	7.9

3-way tables: Types of Interaction

Symbol	Interpretation
(A,C,M)	Mutual Independence
(AC,M)	AC jointly independent of M
(AC,AM)	M, C conditionally independent given A
(AC,AM,CM)	Homogeneous association of each pair.



Marginal independence: fit 2×2 table.

3-way tables: Types of Interaction

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$$Y_{ijk} \sim Poisson(\lambda_{ijk})$$

Conditioned on total (N) $Y_{ijk} \sim Multinom(N, \pi_{ijk})$.
π_{i++} be probability of row $A = i$,
π_{ij+} be probability of $A = i, C = j$, etc.

1. A,C, and M **mutually independent**

$$\lambda_{ijk} = \lambda\lambda_i^A\lambda_j^C\lambda_k^M$$

$$\pi_{ijk} = \pi_{i++}\pi_{+j+}\pi_{++k}$$

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1. M is **jointly independent** of A, C

$$\lambda_{ijk} = \lambda\lambda_i^A\lambda_j^C\lambda_k^M\lambda_{ij}^{AC}$$

$$\pi_{ijk} = \pi_{ij+}\pi_{++k}$$

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1. C and M **conditionally independent** given A

$$\lambda_{ijk} = \lambda\lambda_i^A\lambda_j^C\lambda_k^M\lambda_{ij}^{AC}\lambda_{ik}^{AM}$$

$$\pi_{jk|i} = \pi_{j+|i}\pi_{+k|i}.$$

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1. Each pair of A,C, and M has **homogeneous association**.

$$\lambda_{ijk} = \lambda\lambda_i^A\lambda_j^C\lambda_k^M\lambda_{ij}^{AC}\lambda_{ik}^{AM}\lambda_{ik}^{CM}.$$

e.g. the dependence relationship of A, C does not depend on M.

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1. **Saturated Model.**

$$\lambda_{ijk} = \lambda\lambda_i^A\lambda_j^C\lambda_k^M\lambda_{ij}^{AC}\lambda_{ik}^{AM}\lambda_{ik}^{CM}\lambda_{ijk}^{ACM}.$$

Analysis of 3-way tables

1. Fit log-linear model (Poisson GLM) for each of the models.
   1. Criterion: maximum likelihood.
   2. Fitting method: Newton Raphson.
2. Use a model selection criterion to choose the best one.
   1. AIC, BIC.
   2. Use Deviance χ^2 test to choose between nested models.

`glm(..., family=poisson)`, `loglm(MASS)` in `R`.

Contingency Tables: Death Penalty Example

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Victim's Race	Defendant's Race	Death Penalty Yes	Death Penalty No	Percent Yes
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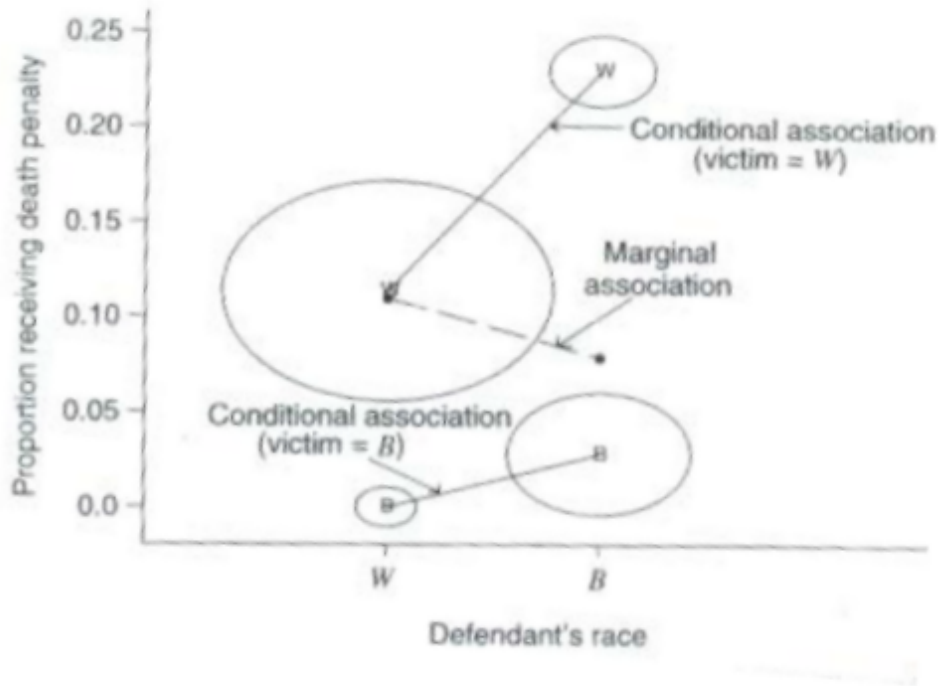
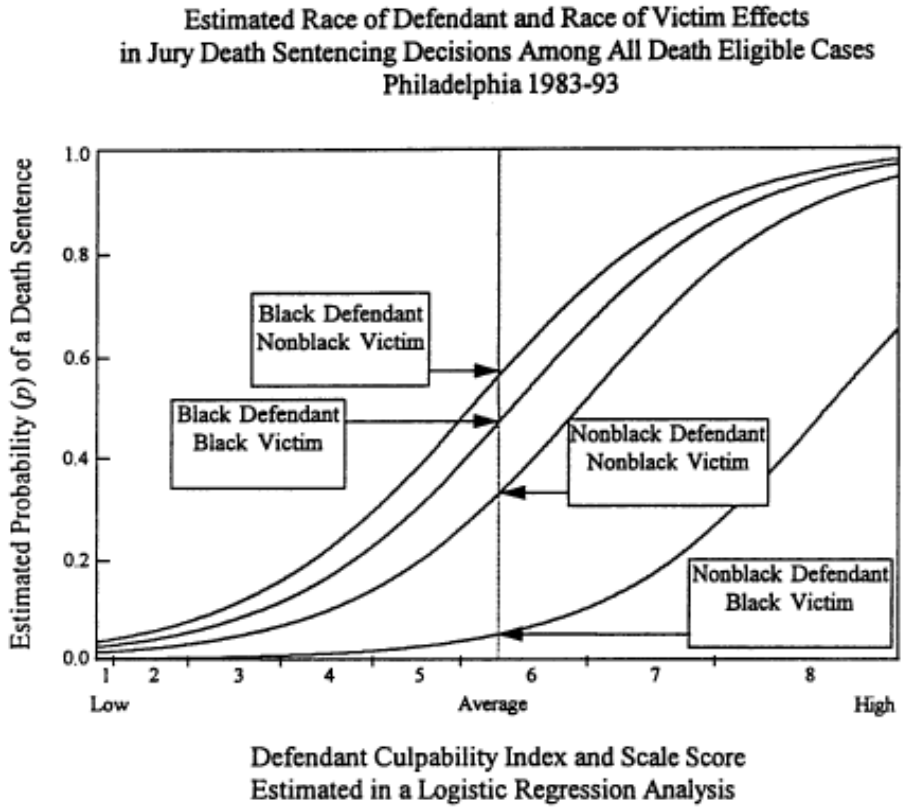
Simpson's Paradox



Repercussions

This analysis was influential in drawing attention to racial bias in the US court system. Similar studies were done in other cities.



See:

`http://66.39.33.150/death-penalty-black-and-white-who-lives-who-dies-who-decides`

3-way tables: Drug Use Data

Alcohol Use	Cigarette Use	Marijuana Use	
		Yes	No
Yes	Yes	911	538
	No	44	456
No	Yes	3	43
	No	2	279

Conditional Odds Ratios

Estimates of odds ratios from the various models:

	Conditional Association			Marginal Association		
Model	AC	AM	CM	AC	AM	CM
(A,C,M)	1.0	1.0	1.0	1.0	1.0	1.0
(AC,M)	17.7	1.0	1.0	17.7	1.0	1.0
(AM,CM)	1.0	61.9	25.1	2.7	61.9	25.1
(AC,AM,CM)	7.8	19.8	17.3	17.7	61.9	25.1