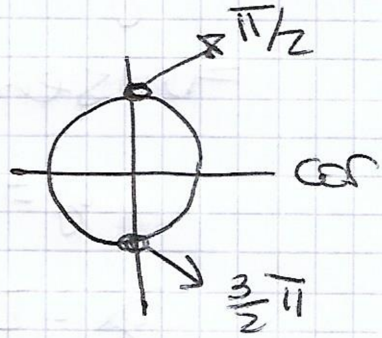
IRM- clase 12

Hallar las raíces de las siguientes funciones en el intervalo I indicado:

a) $f(x) = 2\cos(2x-2\pi)$, $I = \mathbb{R}$

$$2\cdot\cos(2x-2\pi) = 0$$

$$\cos(2x-2\pi) = \frac{0}{2}$$



$$\cos(2x-2\pi) = 0$$

Llamo $y = 2x-2\pi$

$$\Rightarrow \cos(y) = 0$$

$$y = \frac{\pi}{2} + k\pi \quad , k\in\mathbb{Z}.$$

$$2x-2\pi = \frac{\pi}{2} + k\pi$$

$$2x = \frac{\pi}{2} + \underbrace{2\pi + k\pi}_{(k+2)\cdot\pi} \quad , k\in\mathbb{Z}.$$

$$2x = \frac{\pi}{2} + (k+2)\cdot\pi \quad , k\in\mathbb{Z}.$$

llamo $j = k+2$.

<u>obs</u>: $k\in\mathbb{Z} \iff j\in\mathbb{Z}$.

$$2x = \frac{\pi}{2} + j\pi \quad , j\in\mathbb{Z}.$$

$$\boxed{x = \frac{\pi}{4} + j\cdot\frac{\pi}{2} \quad , j\in\mathbb{Z}}$$

b) $f(x) = -2\,\text{sen}\left(3x - \frac{\pi}{2}\right)$ $\quad I = [\pi, \pi]$

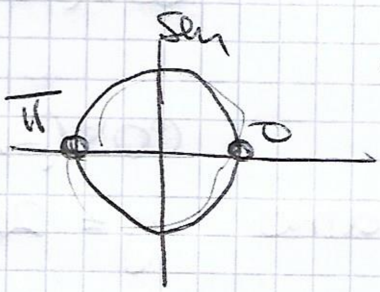
$$-2\,\text{sen}\left(3x - \frac{\pi}{2}\right) = 0$$

$$\text{sen}\left(3x - \frac{\pi}{2}\right) = \frac{0}{-2}$$

$$\text{sen}\left(3x - \frac{\pi}{2}\right) = 0$$

Llamo $y = 3x - \frac{\pi}{2}$

$$\Rightarrow \text{sen}(y) = 0.$$



$$y = \pi + k\pi, k \in \mathbb{Z}$$

$$3x - \frac{\pi}{2} = \pi + k\pi$$

$$3x = \pi + \frac{\pi}{2} + k\pi$$

$$3x = \frac{3}{2}\pi + k\pi$$

$$x = \frac{\frac{3}{2}\pi + k\pi}{3}$$

$$x = \frac{\cancel{3}}{2 \cdot \cancel{3}}\pi + k \cdot \frac{\pi}{3}, \quad k \in \mathbb{Z}.$$

$$\boxed{x = \frac{\pi}{2} + k \cdot \frac{\pi}{3}, \quad k \in \mathbb{Z}}$$

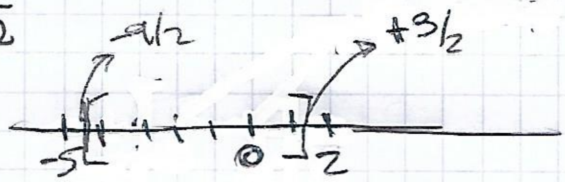
$$-\pi \leqslant \frac{\pi}{2} + k \cdot \frac{\pi}{3} \leqslant \pi$$

$$\frac{-\cancel{\pi}}{\cancel{\pi}} \leqslant \frac{\cancel{\pi} \cdot \left(\frac{1}{2} + \frac{k}{3}\right)}{\cancel{\pi}} \leqslant \frac{\cancel{\pi}}{\cancel{\pi}}$$

$$-1 \leqslant \frac{1}{2} + \frac{k}{3} \leqslant 1$$

$$-1-\frac{1}{2} \le \frac{k}{3} \le 1-\frac{1}{2}$$

$$3\cdot\left(-\frac{3}{2}\right) \le k \le \left(\frac{1}{2}\right)\cdot 3$$



$$-\frac{9}{2} \le k \le \frac{3}{2}$$

O sea, de los valores en $\left[-\frac{9}{2}, \frac{3}{2}\right]$ me tengo que quedar con los enteros:

$k=-4$, $k=-3$, $k=-2$, $k=-1$, $k=0$, $k=1$

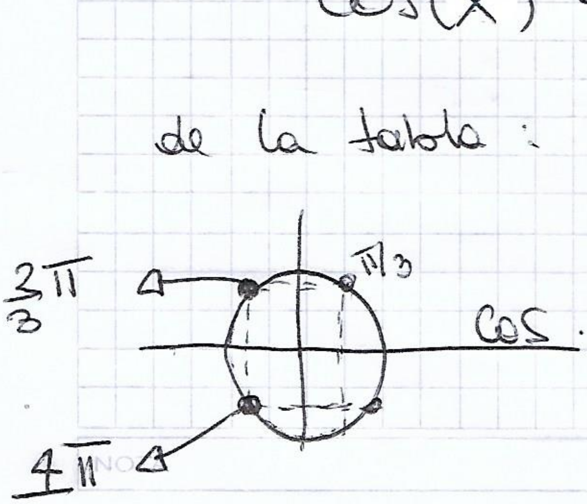
$k=-4 \longrightarrow x = \pi/2 - 4\cdot\pi/3 = (-5/6)\pi$
$k=-3 \longrightarrow x = \pi/2 - 3\cdot\pi/3 = (-1/2)\cdot\pi$
$k=-2 \longrightarrow x = \pi/2 - 2\cdot\pi/3 = (-1/6)\pi$
$k=-1 \longrightarrow x = \pi/2 - \pi/3 = (1/6)\pi$
$k=0 \longrightarrow x = \pi/2 - 0 = \pi/2$
$k=1 \longrightarrow x = \pi/2 + \pi/3 = (5/6)\pi$.

Rta: $x=-\frac{5}{6}\pi$; $x=-\frac{1}{2}\pi$; $x=-\frac{1}{6}\pi$; $x=\frac{1}{6}\pi$;
$x=\frac{\pi}{2}$; $x=\frac{5}{6}\pi$.

c) $f(x) = 6\cos(x)+3$, $I=[-\pi, 0]$

$$6\cdot\cos(x)+3 = 0$$
$$6\cos(x) = -3$$
$$\cos(x) = \frac{-3}{6}$$
$$\cos(x) = -\frac{1}{2}.$$

de la tabla: $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.



$$\cos(x) = -\frac{1}{2}$$

$\boxed{x=\frac{2}{3}\pi+2k\pi}$ $\boxed{x=\frac{4}{3}\pi+2k\pi}$

Me fijo, para cada "rama" de soluciones, cuáles caen en el intervalo deseado

$$-\pi \leq \frac{2}{3}\pi + 2k\pi \leq 0$$

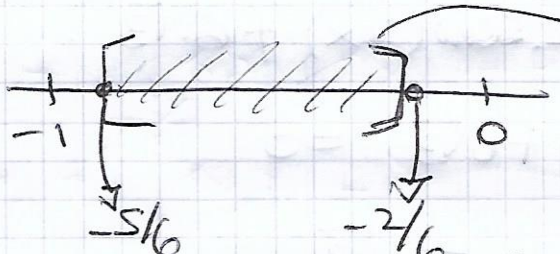
$$-\frac{\cancel{\pi}}{\cancel{\pi}} \leq \frac{\left(\frac{2}{3}+2k\right)\cancel{\pi}}{\cancel{\pi}} \leq 0$$

$$-1 \leq \frac{2}{3} + 2k \leq 0$$

$$-1-\frac{2}{3} \leq 2k \leq -\frac{2}{3}$$

$$-\frac{5}{3} \leq 2k \leq -\frac{2}{3}$$

$$-\frac{5}{6} \leq k \leq -\frac{2}{6}.$$



No hay ningún entero en este intervalo.

$$-\pi \leq \frac{4}{3}\pi + 2k\pi \leq 0$$

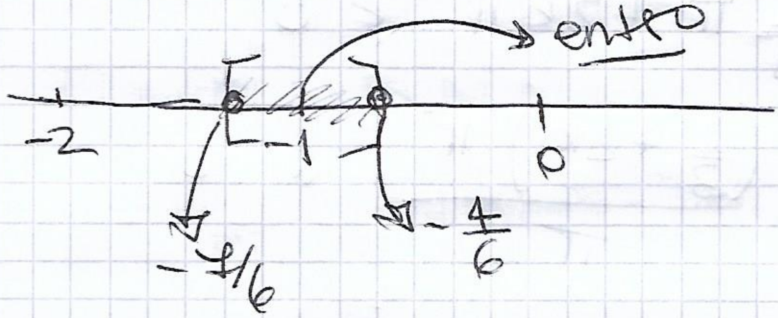
$$-\frac{\cancel{\pi}}{\cancel{\pi}} \leq \frac{\left(\frac{4}{3}+2k\right)\cancel{\pi}}{\cancel{\pi}} \leq 0$$

$$-1 \leq \frac{4}{3} + 2k \leq 0$$

$$-1-\frac{4}{3} \leq 2k \leq -\frac{4}{3}$$

$$-\frac{7}{3} \leq 2k \leq -\frac{4}{3}$$

$$-\frac{7}{6} \leq k \leq -\frac{4}{6}$$



el único entero en $\left[-\frac{7}{6}, -\frac{4}{6}\right]$ es $k=-1$

$k=-1 \xrightarrow{\quad}$ (reemplazo en x) $x = \frac{4}{3}\pi - 2\pi = -\frac{2}{3}\pi$

$$x = -\frac{2}{3}\pi$$

Rta : $x = -\frac{2}{3}\pi$.

d) $f(x) = 12.\text{sen}(2x) + 6.\sqrt{3}$. $I = [0, \pi]$

$$12.\text{sen}(2x) + 6\sqrt{3} = 0$$

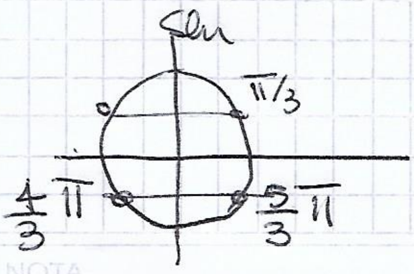
$$12.\text{sen}(2x) = -6\sqrt{3}$$

$$\text{sen}(2x) = -\frac{\cancel{6}\sqrt{3}}{\cancel{12}}$$

$$\text{sen}(2x) = -\frac{\sqrt{3}}{2}$$

$y = 2x$

$$\text{sen}(y) = -\frac{\sqrt{3}}{2}$$



$\boxed{y = \frac{4}{3}\pi + 2k\pi}$ ó $\boxed{y = \frac{5}{3}\pi + 2k\pi}$

$2x = \frac{4}{3}\pi + 2k\pi$ ó $2x = \frac{5}{3}\pi + 2k\pi$

$x = \frac{4}{6}\pi + \frac{\cancel{2}k\pi}{\cancel{2}}$ $\qquad$ $x = \frac{5}{6}\pi + \frac{\cancel{2}k\pi}{\cancel{2}}$

$\boxed{x = \frac{2}{3}\pi + k\pi}$ $\qquad$ $\boxed{x = \frac{5}{6}\pi + k\pi}$

Miro cada "rama" cuáles caen en el intervalo I:

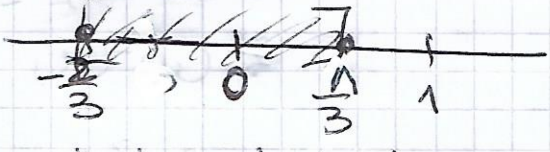
$0 \leq \frac{2\pi}{3} + k\pi \leq \pi$

$0 \leq \frac{\cancel{\pi}\cdot\left(\frac{2}{3} + k\right)}{\cancel{\pi}} \leq \frac{\cancel{\pi}}{\cancel{\pi}}$

$0 \leq \frac{2}{3} + k \leq 1$

$-\frac{2}{3} \leq k \leq 1 - \frac{2}{3}$

$-\frac{2}{3} \leq k \leq \frac{1}{3}$.



el único entero en este intervalo: $k=0$.

$k=0 \xrightarrow[\text{(reemplazo en x)}]{} x = \frac{2\pi}{3} + 0 = \frac{2\pi}{3}$.

La otra "rama" en I:

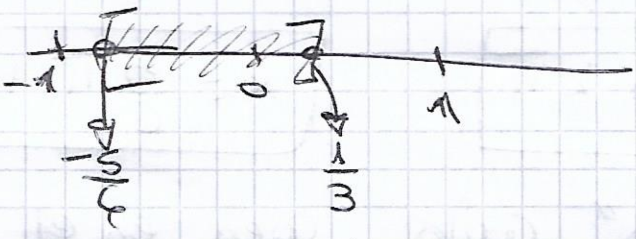
$0 \leq \frac{5}{6}\pi + k\pi \leq \pi$

$0 \leq \frac{\left(\frac{5}{6} + k\right)\cdot\cancel{\pi}}{\cancel{\pi}} \leq \frac{\cancel{\pi}}{\cancel{\pi}}$

$$0 \leq \frac{5}{6} + k \leq 1$$

$$-\frac{5}{6} \leq k \leq 1 - \frac{5}{6}$$

$$-\frac{5}{6} \leq k \leq \frac{1}{3}$$



en este intervalo $\left[-\frac{5}{6}, \frac{1}{3}\right]$ el único entero es $k=0$.

$$k=0 \xrightarrow[\text{(reemplazo en x)}]{} x = \frac{5}{6}\pi + 0 = \frac{5}{6}\pi$$

Rta: $x = \frac{2\pi}{3}$ y $x = \frac{5}{6}\pi$.

e) $f(x) = 4\cos^2(x) - 1$, $I = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$4\cos^2(x) - 1 = 0$$

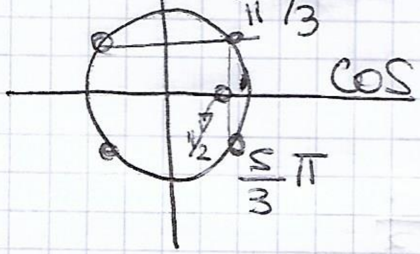
$$4 \cdot \cos^2(x) = 1$$

$$\cos^2(x) = \frac{1}{4}$$

$$\cos(x) = \frac{1}{2} \qquad \cos(x) = -\frac{1}{2}$$

Resuelvo cada una de estas DOS igualdades y luego tengo que UNIR las soluciones.

$\cos(x) = \frac{1}{2}$. tabla: $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$



(1) $\boxed{x = \frac{\pi}{3} + 2k\pi \quad , k \in \mathbb{Z}}$

(2) $\boxed{x = \frac{5}{3} + 2k\pi \quad , k \in \mathbb{Z}}$.

Me fijo de (1) y (2) cuáles "caen" en I:

(1):

$$-\frac{\pi}{2} \leq \frac{\pi}{3} + 2k\pi \leq \frac{\pi}{2}$$

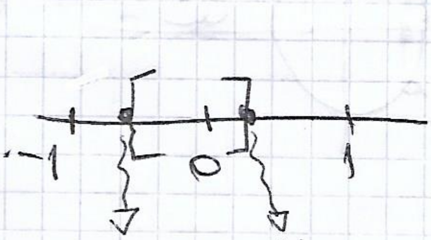
$$-\frac{\cancel{\pi}}{2\cancel{\pi}} \leq \frac{\cancel{\pi}\cdot\left(\frac{1}{3}+2k\right)}{\cancel{\pi}} \leq \frac{\cancel{\pi}}{2\cancel{\pi}}$$

$$-\frac{1}{2} \leq \frac{1}{3} + 2k \leq \frac{1}{2}$$

$$-\frac{1}{2} - \frac{1}{3} \leq 2k \leq \frac{1}{2} - \frac{1}{3}$$

$$-\frac{5}{6} \leq 2k \leq \frac{1}{6}$$

$$-\frac{5}{12} \leq k \leq \frac{1}{12}$$



el único k entero en $\left[-\frac{5}{12}, \frac{1}{12}\right]$ es $k=0$.

$k=0 \xrightarrow[\text{(en x)}]{} x = \frac{\pi}{3} + 0 = \frac{\pi}{3}$

Me fijo de (2) cuáles "caen" en I:

$$-\frac{\pi}{2} \leq \frac{5}{3}\pi + 2k\pi \leq \frac{\pi}{2}$$

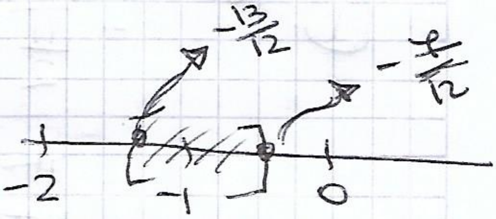
$$-\frac{\cancel{\pi}}{2\cancel{\pi}} \leq \frac{\cancel{\pi}\cdot\left(\frac{5}{3}+2k\right)}{\cancel{\pi}} \leq \frac{\cancel{\pi}}{2\cancel{\pi}}$$

$$-\frac{1}{2} \leq \frac{5}{3} + 2k \leq \frac{1}{2}$$

$$-\frac{1}{2} - \frac{5}{3} \leq 2k \leq \frac{1}{2} - \frac{5}{3}$$

$$-\frac{13}{6} \leq 2k \leq -\frac{7}{6}\pi$$

$$-\frac{13}{12} \leq k \leq -\frac{7}{12}\pi$$

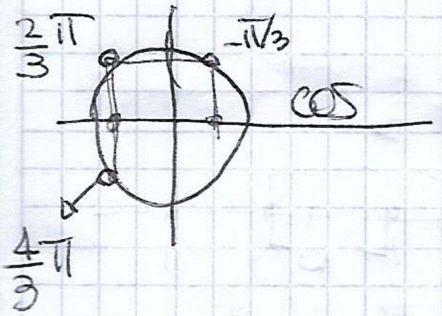


el único k en este intervalo es $k=-1$.

$$k=-1 \xrightarrow[\text{(en } x)]{} x = \frac{5}{3}\pi - 2\pi = -\frac{\pi}{3}$$

Tengo que resolver la otra igualdad:

$$\cos(x) = -\frac{1}{2}.$$



① $\boxed{x = \frac{2}{3}\pi + 2k\pi}$

② $\boxed{x = \frac{4}{3}\pi + 2k\pi}$

Me fijo de ① y ② cuales "caen" en I:

① : $-\frac{\pi}{2} \leq \frac{2}{3}\pi + 2k\pi \leq \frac{\pi}{2}$

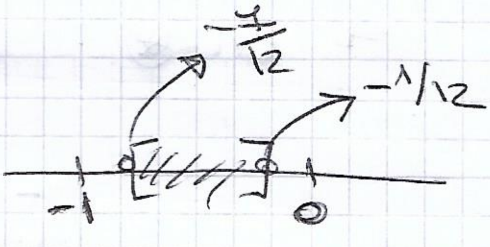
$$-\frac{\cancel{\pi}}{2\cancel{\pi}} \leq \frac{\cancel{\pi}\cdot\left(\frac{2}{3}+2k\right)}{\cancel{\pi}} \leq \frac{\cancel{\pi}}{2\cancel{\pi}}$$

$$-\frac{1}{2} \leq \frac{2}{3} + 2k \leq \frac{1}{2}$$

$$-\frac{1}{2} - \frac{2}{3} \leq 2k \leq \frac{1}{2} - \frac{2}{3}$$

$$-\frac{7}{6} \leq 2k \leq -\frac{1}{6}$$

$$-\frac{7}{12} \leq k \leq -\frac{1}{12}.$$



<u>No</u> hay ningún entero en $\left[-\frac{7}{12}, -\frac{1}{12}\right]$

② :
$$-\frac{\pi}{2} \leq \frac{4}{3}\pi + 2k\pi \leq \frac{\pi}{2}$$

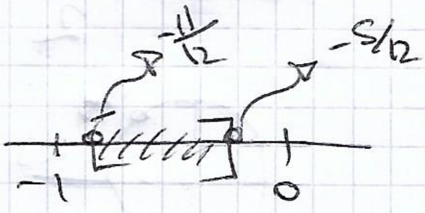
$$-\frac{\cancel{\pi}}{2\cancel{\pi}} \leq \frac{\cancel{\pi}\cdot\left(\frac{4}{3}+2k\right)}{\cancel{\pi}} \leq \frac{\pi}{2\pi}$$

$$-\frac{1}{2} \leq \frac{4}{3} + 2k \leq \frac{1}{2}$$

$$-\frac{1}{2} - \frac{4}{3} \leq 2k \leq \frac{1}{2} - \frac{4}{3}$$

$$-\frac{11}{6} \leq 2k \leq -\frac{5}{6}$$

$$-\frac{11}{12} \leq k \leq -\frac{5}{12}$$



No hay ningún entero en $\left[-\frac{11}{12}, -\frac{5}{12}\right]$

<u>Rta</u> : $x = \frac{\pi}{3}$, $x = -\frac{\pi}{3}$.

f) $f(x) = 2\,\text{sen}^2(x) + 3\cos(x) - 3$, $I = [0, 2\pi]$.

$(*)$ $2\,\text{sen}^2(x) + 3\cos(x) - 3 = 0.$

Uso que: $\left.\begin{array}{l} \cos^2(x) + \text{sen}^2(x) = 1 \\ \Rightarrow \quad \text{sen}^2(x) = 1 - \cos^2(x) \end{array}\right\}$ pongo todo en función de "cos".

$\Rightarrow$ en $(*)$: $2\left(1 - \cos^2(x)\right) + 3\cos(x) - 3 = 0$

$$2 - 2\cos^2(x) + 3\cos(x) - 3 = 0$$

$$-2\cos^2(x) + 3\cos(x) - 1 = 0.$$

Si llamo $\boxed{y = \cos(x)}$.

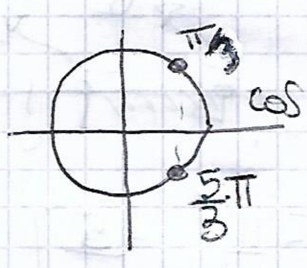
$$-2 \cdot y^2 + 3y - 1 = 0.$$

$$y = \frac{-3 \pm \sqrt{9 - 4 \cdot (-2)(-1)}}{2 \cdot (-2)}$$

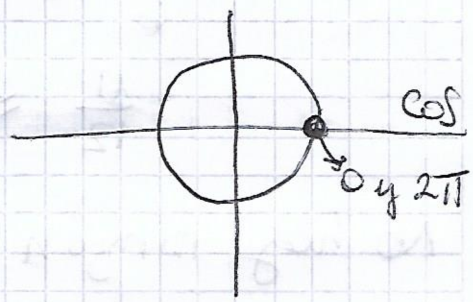
$$y = \frac{-3 \pm 1}{-4} \quad \begin{cases} y = \dfrac{-3+1}{-4} = \dfrac{-2}{-4} = \dfrac{1}{2} \\[2ex] y = \dfrac{-3-1}{-4} = \dfrac{-4}{-4} = 1 \end{cases}$$

Tengo que resolver:

$\cos(x) = \dfrac{1}{2}$ y $\cos(x) = 1$



en $[0, 2\pi]$ las sol. son $x = \dfrac{\pi}{3}$ y $x = \dfrac{5}{3}\pi$



en $[0, 2\pi]$ $x = 0$ y $x = 2\pi$

<u>Rta</u>: $x = \dfrac{\pi}{3}$, $x = \dfrac{5}{6}\pi$, $x = 0$, $x = 2\pi$.

NOTA