Exponential suppression of the topological gap in self-consistent intrinsic Majorana nanowires

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Abstract

Predictions of topological p-wave superconductivity and Majorana zero modes (MZMs) in hybrid superconductor-semiconductor nanowires have been difficult to realize experimentally. Consequently, researchers are actively exploring alternative platforms for MZMs. Here, we theoretically study depleted nanowires with intrinsic superconductivity (as opposed to proximity-induced). Using a **self-consistent Hartree-Fock-Bogoliubov mean field theory**, we compute the topological phase diagram versus Zeeman field and filling for intrinsic wires with attractive interactions. We find that, although intrinsic wires could be less vulnerable than hybrids to topology-adverse effects, such as disorder and metallization, they are hindered by a fundamental limitation of their own. Although a **topological p-wave gap** is indeed possible, it is far less robust than in hybrid Majorana nanowires. Instead of remaining stable beyond the topological transition, it is found to **decay exponentially with Zeeman field,** greatly reducing the parameter region with an appreciable topological gap.

Statement of the problem

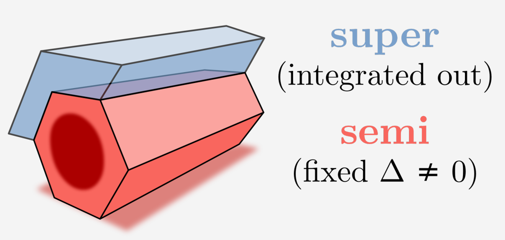
- The Oreg-Lutchyn recipe [1,2] for Majorana zero modes (MZMs) is based on **hybrid superconductor-semiconductor** (SM/SC) nanowires
- Key idea: **topological p-wave SC** and MZMs emerges for Zeeman field exceeding a **critical value**
- Hybrid systems are complex and prone to multiple issues, such as **disorder**, **metallization** and **interface effects**. MZMs not convincingly demonstrated yet
- Could **nanowires with intrinsic SC** develop p-wave without these shortcomings?

Majorana nanowire models

We consider three Majorana nanowire tight-binding models of increasing complexity. These are implemented on a quasi-1D nanowire, either infinite or of finite length L

Oreg-Lutchyn (OL) model

Hybrid SM/SC nanowire with SC integrated out (self-energy from the SC approximated at $\omega = 0$)

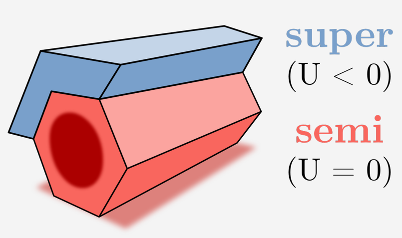


- Effective mass m^*
- Spin-orbit coupling α
- Zeeman field V_Z
- Local s-wave pairing Δ

$$H^{\mathrm{OL}} = H_0 + H_{\mathrm{SOC}} + H_Z + H_{\Delta^{ii}}$$

Self-consistent hybrid nanowire model

Hybrid SM/SC nanowire with the SC included self-consistently via an attractive **on-site interaction U**

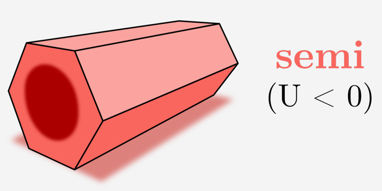


- SM like OL, but without Δ
- SC with $\alpha = 0$ and local U<0
- Coupling τ between the two

$$H^{\mathrm{hyb}} = H_0^{\mathrm{SM}} + H_Z^{\mathrm{SM}} + H_{\mathrm{SOC}}^{\mathrm{SM}} + H_0^{\mathrm{SC}} + H_Z^{\mathrm{SC}} + \Sigma_{\mathrm{HFB}}^{\mathrm{SC}} + H^{\text{SC-SM}}$$

Self-consistent intrinsic nanowires

Low-density nanowire with SOC and **intrinsic SC** via an attractive on-site interaction U



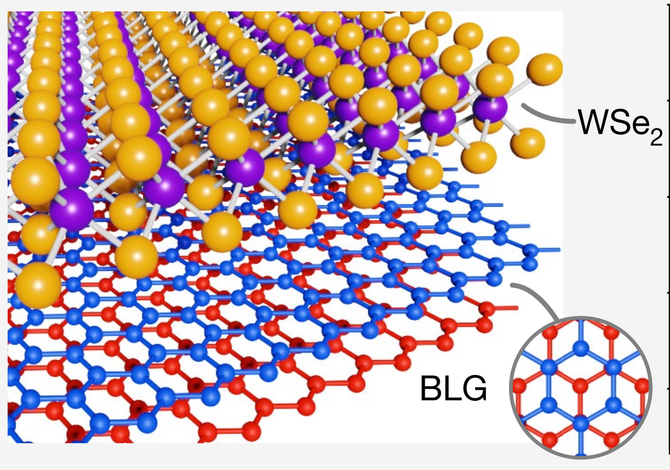
- SM like OL, but without Δ
- Local interaction U<0, or longer ranged $v(\boldsymbol{r}_i - \boldsymbol{r}_j)$

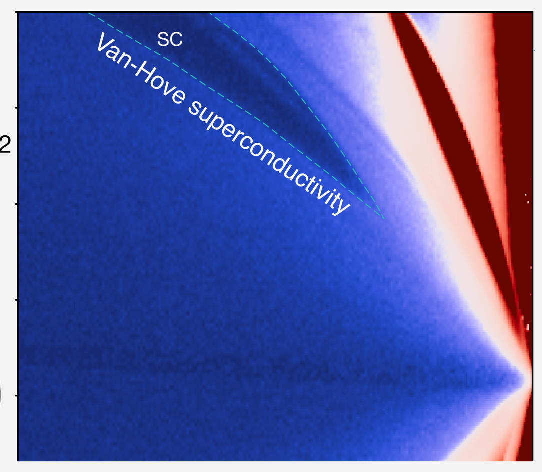
$$H^{\mathrm{int}} = H_0 + H_{\mathrm{SOC}} + H_Z + \Sigma_{\mathrm{HFB}}$$

Note on low-density SCs

The Oreg-Lutchyn recipe requires **low density systems**, since the critical Zeeman grows as the Fermi energy. **Can these develop intrinsic SC?**

While traditional low-density semiconductors have strong SOC, they do not develop superconductivity at any reasonable temperature. However, some new materials (notably **2D heterostructures**) do [3].

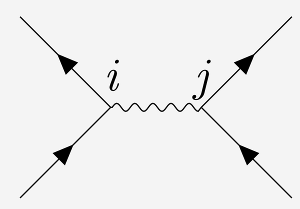




(left) Bilayer graphene/WSe$_2$ heterostructure. (right) Phase diagram vs carrier density and displacement field

Interactions

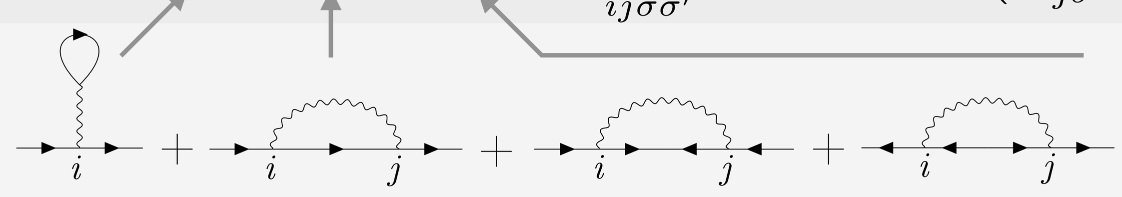
Electron-electron interactions are modelled by v^{ij}



$$H^{\mathrm{int}} = \sum_{ij} n_i v^{ij} n_j = \sum_{ij\sigma\sigma'} c^\dagger_{i\sigma} c_{i\sigma} v^{ij} c^\dagger_{j\sigma'} c_{j\sigma'}$$

We approximate this by a self-consistent mean field $H^{\mathrm{int}} \rightarrow \Sigma_{\mathrm{HBF}}$ (including anomalous terms)

$$\Sigma_{\mathrm{HFB}} = \Sigma_H + \Sigma_F + \Sigma_\Delta = \frac{1}{2}\sum_{ij\sigma\sigma'} (c^\dagger_{i\sigma}, c_{i\sigma}) \check{\Sigma}^{ij}_{\sigma\sigma'} \begin{pmatrix} c_{j\sigma'} \\ c^\dagger_{j\sigma'} \end{pmatrix}$$



Hartree　　Fock　　Bogoliubov (anomalous)

The Nambu self energy matrix depends on the Nambu density matrix as

$$\check{\Sigma}^{ij} = \frac{1}{2}\delta_{ij}\tau_z \sum_l v^{il}\mathrm{Tr}(\tau_z \tilde{\rho}^{ll}) - v^{ij}\tau_z\tilde{\rho}^{ij}\tau_z \qquad \rho^{ij}_{ee} = \langle c^\dagger_{\boldsymbol{j}} \otimes c_{\boldsymbol{i}} \rangle$$

$$\tilde{\rho}^{ij} = \begin{pmatrix} \rho^{ij}_{ee} & \rho^{ij}_{eh} \\ \rho^{ij}_{he} & -(\rho^{ji}_{ee})^T \end{pmatrix} \qquad \begin{aligned} \rho^{ij}_{eh} &= \langle c_{\boldsymbol{j}} \otimes c_{\boldsymbol{i}} \rangle \\ \rho^{ij}_{he} &= \langle c^\dagger_{\boldsymbol{j}} \otimes c^\dagger_{\boldsymbol{i}} \rangle \end{aligned}$$

Self-consistency equations

The mean field depends on the density matrix, and the density matrix depends on the mean field. Self-consistency is obtained by Anderson-accelerated fixed-point iteration.

The self-consistency equations for a SC without SOC ($\alpha = 0$) can be written out explicitly. This generalizes the BCS gap equation to:

$$\Delta = -\frac{U}{4N}\sum_k \frac{\Delta}{\tilde{E}_k}\left[\tanh\left(\frac{\tilde{E}_k + \tilde{V}_Z}{2k_BT}\right) + \tanh\left(\frac{\tilde{E}_k - \tilde{V}_Z}{2k_BT}\right)\right],$$

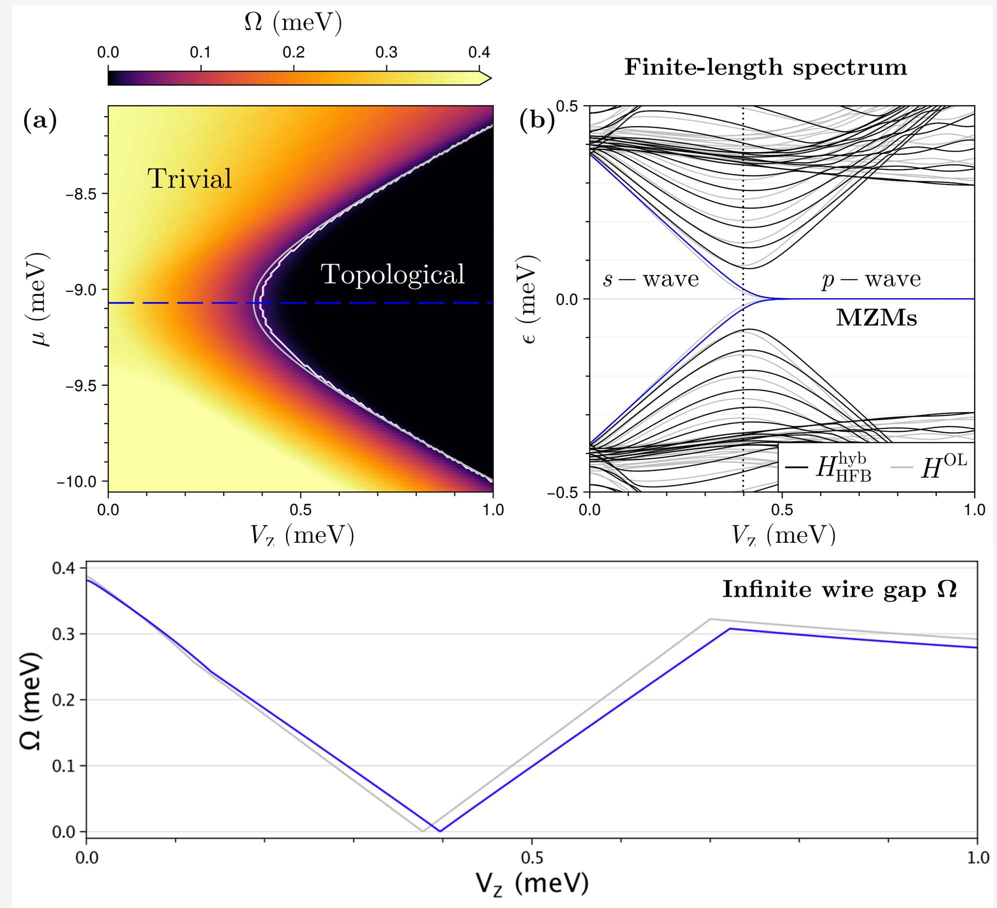
$$\delta\mu = -\frac{U}{2} + \frac{U}{4N}\sum_k \frac{\tilde{\epsilon}_k}{\tilde{E}_k}\left[\tanh\left(\frac{\tilde{E}_k + \tilde{V}_Z}{2k_BT}\right) + \tanh\left(\frac{\tilde{E}_k - \tilde{V}_Z}{2k_BT}\right)\right]$$

$$\delta V_Z = \frac{U}{2N}\sum_k \frac{\sinh(\tilde{V}_Z/k_BT)}{\cosh(\tilde{V}_Z/k_BT) + \cosh(\tilde{E}_k/k_BT)}.$$

$$\tilde{E}_k = \sqrt{\Delta^2 + \tilde{\epsilon}_k^2} \quad \tilde{\epsilon}_k = \epsilon_k - \mu - \delta\mu \quad \tilde{V}_Z = V_Z + \delta V_Z$$

OL vs Hybrid

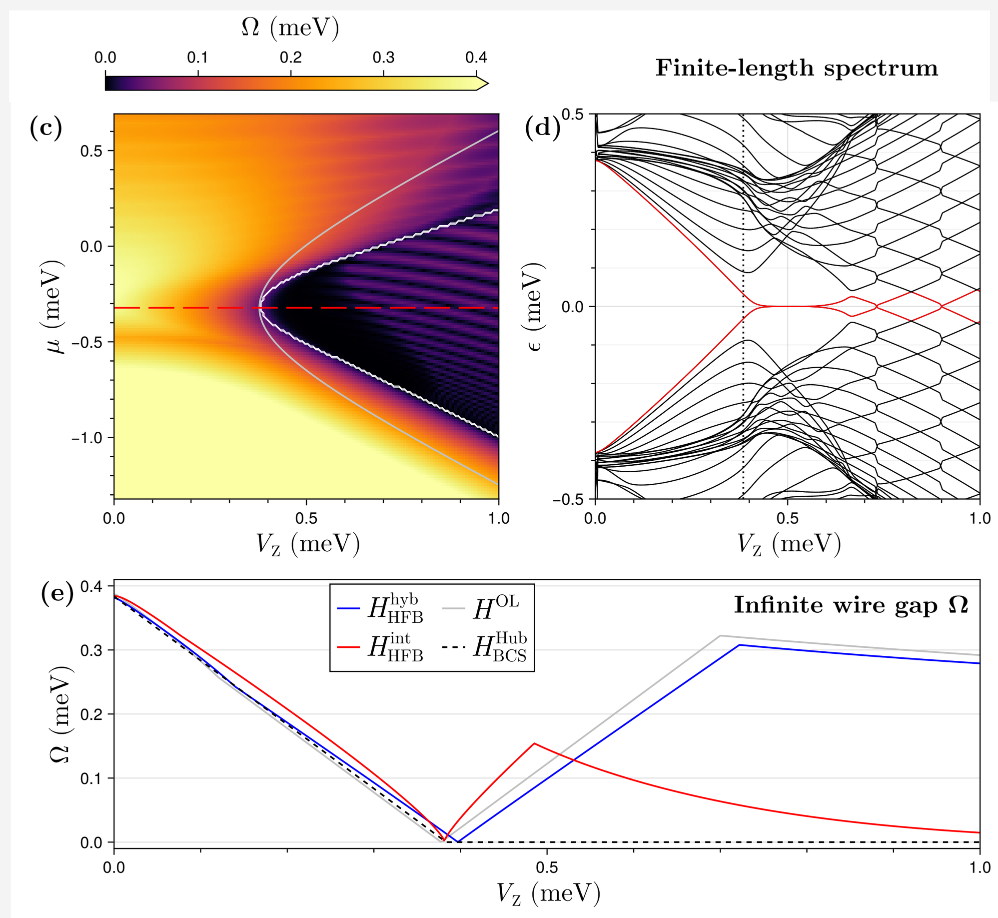
The two models are essentially identical



Intrinsic U<0 nanowire

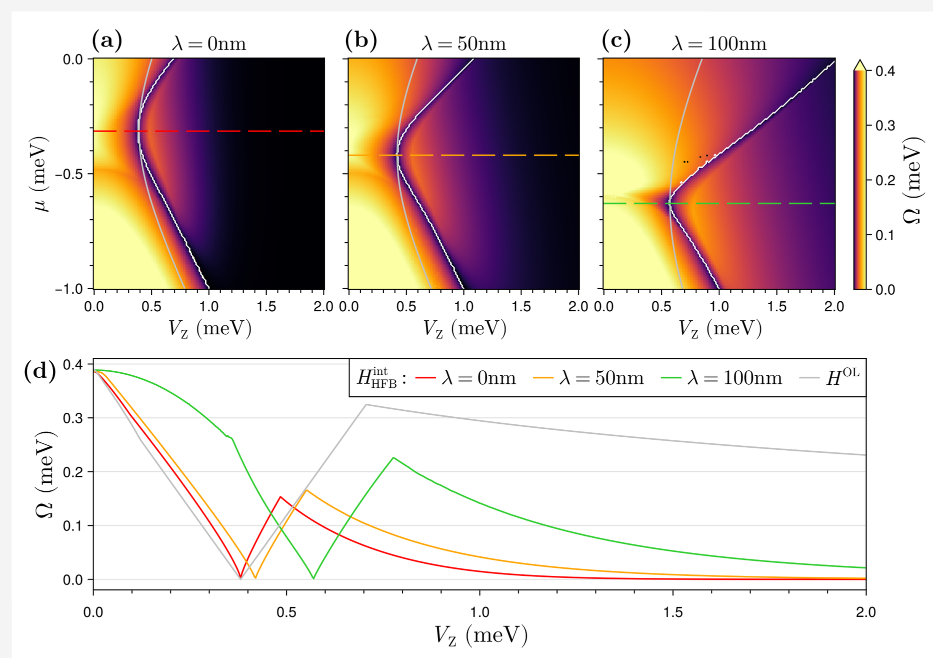
Intrinsic nanowires are qualitatively different

- Topological p-wave region is reduced
- The gap Ω decays exponentially with V_Z
- MZMs are destabilized
- Conclusion: hybrid is better than intrinsic!



Long-range interactions

Increasing the interaction range λ helps, but does not solve the exponential gap decay



[1] Oreg, Refael, von Oppen. *PRL* ***105***, *177002 (2010)*

[2] Lutchyn, Sau, Das Sarma. *PRL* ***105***, *(7)077001 (2010)*

[3] Zhang, Polski et al. *Nature* ***613***, *268--273 (2023)*