Dark Solitons and Breathers as solutions of the NonLinear Schrödinger Equation

BSc in Physics Research Project

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NonLinear Schrödinger Equation

$$i\frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + \sigma|\psi(x,t)|^2\psi(x,t) = 0$$

Abstract:

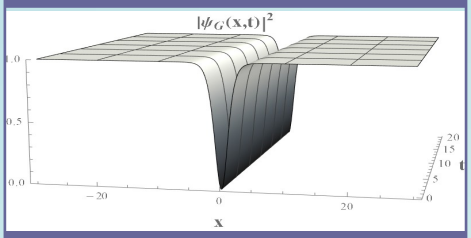
This project deals with the nonlinear dynamics of solitons - localized waves that maintain their shape due to balance between its natural diffraction and self-focusing (or self-defocusing) nonlinearity - and the nonlinear dynamics of breathers - strongly localized solitonic structures that oscillate periodically in either space, time or both.

Firstly, we modulate the instability of a plane wave background in the presence of small perturbations and show the appearance of two different medium, the self-focusing medium (σ>0) and the self-defocusing medium (σ<0) .

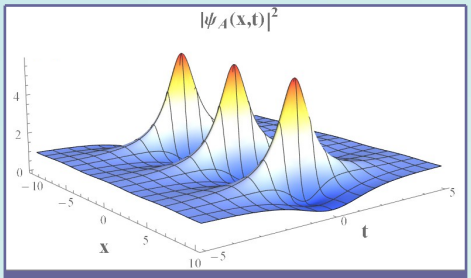
Secondly, we derive analytically from the NonLinear Schrödinger Equation with cubic nonlinearity in a self-defocusing media the exact spatial solutions of the dark and gray soliton.

For last, we derive analytically, this time in a self-focusing medium, another types of exact solutions: the Akhmediev breather, the Kuznetsov-Ma breather, and the Peregrine soliton.

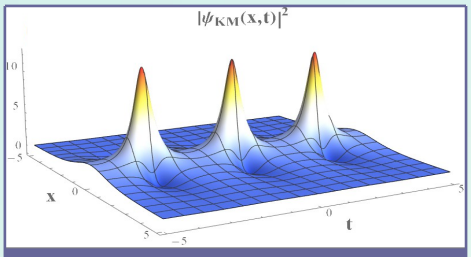
$$\psi_D(x,t) = \pm\sqrt{\Omega}\tanh\left[\sqrt{\frac{\Omega}{2}}(x-x_0)\right]e^{-i\Omega t}$$



The dark soliton is a stationary solution with a characteristic tanh profile and an abrupt π shift at the dip's center.

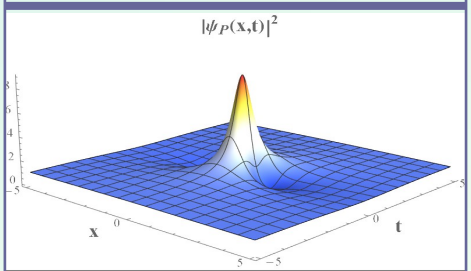


The Akhmediev breather solution is localized in time but oscillates in space



The Kuznetsov-Ma breather solution is localized in space but oscillates in time

$$\psi_P(x,t) = \left[1 - 4\frac{1+2it}{1+2x^2+4t^2}\right]e^{it}$$



The Peregrine soliton is localized both in time and space.