Liability Design with Information Acquisition

Francisco Poggi and Bruno Strulovici
Northwestern University*

November 17, 2020

1 Introduction

In 2019, a California court sentenced paint maker Sherwin-Williams to pay hundreds of millions of dollars to address the dangers caused by lead paint. The sentence was remarkable because even though lead paint became banned in 1978, the suit concerned damage caused during the decades before the ban and centered on the accusation that paint makers were aware of the dangers caused by lead paint long before the ban.

In essence, the court's argument was that Sherwin-Williams and other paint makers knew or should have known the dangers caused by lead paint.

While it is difficult for a regulator to guess a firm's private information, it is perhaps easier to assess due diligence: did paint makers research the risk of lead paint sufficiently well before marketing it?

Formally, the problem is not just one of private information, but also one of information acquisition: how can a regulator make sure that agents learn sufficiently well before taking actions?

One may model this question as a delegated Wald problem (Wald (1945)): the principal is a regulator, who relies on the agent (the firm) to acquire information before deciding whether to launch a product or abandon it if it is too risky.

*Emails: franciscopoggi2015@u.northwestern.edu and b-strulovici@northwestern.edu. Strulovici gratefully acknowledges financial support from the National Science Foundation (grant 1151410)

If the regulator could unrestrictedly penalize an agent for the damages caused, he could force the agent internalize all damage and implement the socially optimal level of information acquisition and the optimal decision.

For various reasons, liability may be capped, however, which precludes the full transfer of damages to the agent. In this case, the regulator can punish the agent if and only if damage occurs, and charge him a penalty that depends on the information available to the regulator after the damage has occurred.

In this project, we study this problem in a Brownian version of the Wald Problem: the agent observes an arithmetic Brownian whose drift depends on the state of the world, i.e., on the riskiness of the product. Information acquisition is costly. The first-best policy is to search until the riskiness of the product becomes sufficiently clear, and launch the product if this riskiness is low and abandon it if the riskiness is high.

We characterize the optimal liability rule when the agent has private information, liability is capped, and the regulator can penalize the agent only when damage occurs. As part of this characterization, we provide a simple condition under which the identifiability condition of our companion paper (Poggi and Strulovici (2020)) applies, which explains why we can focus without loss of generality on policies that do not extract the agent's initial private information: the Taxation Principle with Non-Contractible Events of that paper applies.

2 Baseline Model

A firm must decide whether to launch a product or abandon its development. If launched, the product may cause some damage with positive probability. The firm has some private information about the product's riskiness and can acquire additional information before deciding between launching the product and abandoning it.

A regulator wishes to encourage the launch of low-risk products and deter the launch of high-risk ones and to encourage the firm to acquire sufficient information before making its decision.

The regulator faces two constraints. First, the firm has limited liability: the social cost caused by product damage is $L > 0$ and the firm's liability is capped at some lower level $l < L$. Second, the regulator can penalize the firm only if some damage occurs: it cannot penalize firms that acquired too little information and took an overly risky decision unless such risks result in some damage.

The timing of the game is as follows:

1. The firm is endowed with a prior $\theta \in \Theta \subset [0,1]$ about the product's riskiness $y \in \{0,1\}$, with $\theta = \Pr(y=1)$.

2. The firm can acquire additional information about y.

3. The firm adopts or abandons the product.

4. If the firm adopts the product, it causes some damage if the product was risky ($y=1$) and doesn't if the product was safe ($y=0$).

5. In case of damage, the firm pays a penalty $\psi \leq l$ set by the regulator.

The assumption that a risky product causes damage with probability 1 is without loss of generality: if this probability were less than 1, the same analysis would apply using expected damage and expected penalties.

Information structure: During the information-acquisition stage, the firms observes a process X given by:

$$X_t = (-1+2y)t + \sigma B_t$$

where B is the standard Brownian motion. The drift of X depend symmetrically on the product's riskiness y: the drift is $+1$ if the product causes damage and -1 if it doesn't. Therefore, observing X gradually reveals y. This revelation is progressive due to the stochastic component of X.

The firm stops acquiring information at some time τ that is adapted to filtration of X.

The regulator has a prior $\lambda \in \Delta(\Theta)$ about the firm's private information. She observes nothing about X except if some damage occurs, in which case she observes the last value X_τ taken by the process at the time of the firm's decision. X_τ is a measure of the firm's due diligence to assess the product's riskiness before launching it.

In this Brownian model, it is well-known (though not immediate) that for each $t > 0$, the variable X_t is a sufficient statistic for the information about y contained by the path $\{X_s\}_{s \leq t}$ of the process X until time t. Mathematically, the likelihood ratio of y associated with a path of X from time 0 to t is only a function of X's value at time t.

Because the stopping time τ is chosen endogenously by the firm, which has private information about y, X_τ is not a sufficient statistic for y once the firm's strategic timing is taken into account. Our assumption that the regulator observes X_τ instead of the entire path $\{X_t\}_{t\leq\tau}$ captures the idea that the regulator does not perfectly observe all the decisions made by the firm during the information acquisition stage. Intuitively, the regulator observes the most informative signal about y contained by the path of X that is independent of the firm's private information.

Payoffs: The firm incurs a running cost c from acquiring information, and a profit π if it launches the product. Let $d = 1$ if the firm launches the product and $d = 0$ if it abandons it, and τ denote the time spent acquiring information. The firm's realized payoff is

$$u = d(\pi - y\psi) - c\tau$$

where π is the firm's profit from the launch in the absence of damage. The regulator's objective internalizes the entire damage caused by the product:

$$v = d(\beta - yL) - c\tau$$

where β is the social benefit from the launch in the absence of a damage.

To gain some intuition about the role of limited liability, it is sometimes useful to set $\pi = \beta$, which means that the firm extracts the entire social surplus from the product in the absence of damage.

3 Preliminary Analysis

First Best: If θ were publicly known, the regulator's optimal information acquisition strategy would have a simple structure, consisting in abandoning the product as soon as the process X exceeds some upper threshold $\bar{x}^*(\theta)$ and launching it as soon as X drops below some lower threshold $x^*(\theta)$.

Moreover, if the liability cap l satisfied $l \geq L$ and $\pi = \beta$, the regulator could achieve the first best by setting $\psi \equiv L$ and align the firm's interest perfectly with the social objective.

Tariffs: Given a tariff $\psi : \mathbb{R} \to \mathbb{R}$, a firm with prior θ chooses a stopping time τ and a launch/abandonment decision $d \in \{0, 1\}$ that maximizes its expected utility

$$E\left[d(\pi - y\psi(X_\tau)) - c\tau \mid \theta\right]. \tag{1}$$

It is straightforward to check that the solution to this problem consists of cutoffs $\underline{x}_\theta^\psi < \bar{x}_\theta^\psi$ such that the firm acquires information until X reaches either cutoffs, abandons the product if X reaches $\bar{x}_\theta^\psi$ and launches it of X reaches $\underline{x}_\theta^\psi$.

Limited liability affects incentives in two ways. First, since the firm does not fully internalize damages, it is willing to take riskier decisions than is socially optimal for a given belief about the product's safety. Second, the value of information is different. For example, if the tariff is $\psi \equiv 0$, the firm has no incentive to acquire any information. The firm always launches the product immediately.

The regulator could set a uniform tariff at the ceiling: $\psi(x) = l$ for all $x \in \mathbb{R}$. This would incentivize the firm to acquire some information but this acquisition would be less than is socially optimal, as the next result shows. Let $x^l(\theta)$ denote the adoption threshold used by type θ when $\psi \equiv l$.

Proposition 1 (Under-exploration) *$x^*(\theta) < x^l(\theta)$ for all $\theta \in \Theta$.*

Proof. We fix some prior $\theta \in \Theta$ throughout the proof and let x^* and x^l the socially optimal and firm optimal thresholds, respectively, when $\psi \equiv l$ and given this prior.

Given a current evidence level x, the firm's expected payoff if it adopts at x is:

$$u(x) = \pi - p(x)l$$

where $p(x) = \Pr(y = 1|x, \theta)$. The regulator's expected payoff if the firm stops at x is:

$$v(x) = \beta - p(x)L.$$

Since $l < L$, we have

$$v(x) - v(x') > u(x) - u(x') \tag{2}$$

for all $x < x'$

Now consider the optimal thresholds x^l and $\bar{x}$ for the firm, starting at some level x, and let $\tau = \inf\{t : X_t \notin (x^l, \bar{x})\}$. For $x \in (x^l, \bar{x})$, acquiring information is optimal for the firm, which means that

$$u(x) \leq f(x)u(x^l) - cE_x[\tau] \tag{3}$$

where $f(x)$ is the probability that $X_\tau = x^l$ (as opposed to $\bar{x}$) and $E_x[\tau]$ is the expected value of τ when the process X starts at x. The regulator also wishes to acquire information at x if

$$v(x) \leq f(x)v(x^l) - cE_x[\tau].$$

This inequality holds strictly satisfied from (2) and x is close enough to x^l: we have

$$u(x) - v(x) > u(x^l) - v(x^l).$$

By continuity, this implies that

$$u(x) - v(x) > f(x)(u(x^l) - v(x^l)).$$

if $f(x)$ is close enough to 1 or, equivalently, if x is close enough to x_l. Rearranging the last inequality, we have

$$v(x) < f(x)v(x^l) + u(x) - f(x)u(x^l).$$

Combining this with (3) then yields

$$v(x) < f(x)v(x^l) - cE_x[\tau].$$

This shows that adopting the product is strictly suboptimal for x in some interval $(x^l, x^l + \eta)$ where $\eta > 0$, since the regulator could do strictly better even if it chooses the abandonment threshold $\bar{x}$, which is suboptimal. Equivalently, this shows that $x^* \leq x^l$.

To show that the inequality is strict, notice that if the regulator were forced to use the same abandonment threshold as the firm, from (2), the smooth pasting property could not be satisfied at x^l for the regulator's problem if it is satisfied for the firm. Therefore, the optimal threshold for the regulator would be strictly lower even if it were forced to use the firm's abandonment threshold. When this constraint is relaxed, the regulator's value function can only increase, which means that stopping at x^l is also strictly suboptimal. ■

Proposition 1 shows that the firm always performs too little due diligence before launching the product, compared to the social optimum.

When $\psi \equiv l$, the firm's only reason to acquire information is to make sure that the risk of damage is sufficiently low when it launches its product. Since the penalty ψ is flat, the firm does not care per se about the level of X_τ when it adopts the product; it only cares about the posterior probability $p(x, \theta)$ that a damage occurs when it launches the product given a prior θ and a signal x. This implies the following result.

Lemma 1 *$x^l(\theta)$ is strictly decreasing in θ.*

Proof. As noted, the firm's optimal strategy when ψ is flat is only a function of its posterior belief when it stops. Let $\underline{p} < \bar{p}$ denote the optimal adoption and abandonment thresholds for the firm expressed in terms of posterior beliefs. Expressed in terms of the process X, the adoption threshold $x^l(\theta)$ is then such that $p(x^l(\theta), \theta) = \underline{p}$. Since the function $p(\cdot, \cdot)$ is strictly increasing in both arguments, this implies that $x^l(\theta)$ is strictly decreasing in θ. ■

Although the uniform tariff $\psi(\cdot) \equiv l$ brings the firm closest to fully internalizing the damage that its product might cause, this tariff may be suboptimal: the regulator could increase social welfare by reducing ψ to reward the firm if it acquired more information. The optimal policy is studied next.

4 Optimal Policy Design

Suppose that the regulator can contract with the firm after the firm observes the initial private information and before it takes any action, and the regulator has full commitment.

Definition 1 *A* direct liability mechanism *is a menu $M = (\{\tau_\theta, d_\theta, \psi_\theta\}_{\theta \in \Theta})$ such that for all $\theta \in \Theta$:*

(i) The stopping time τ_θ is measurable with respect to the filtration $\{\mathcal{F}_t^X\}_{t \geq 0}$ generated by X;

(ii) The decision d_θ is measurable with respect to the information at time τ, i.e., to the σ-algebra $\mathcal{F}_{\tau_\theta}^X$;

(iii) The tariff $\psi_\theta(\cdot)$ is uniformly bounded above by l.

Since the regulator has full commitment, the Revelation Principle guarantees that it is without loss of generality to focus on direct liability mechanisms.

Given a liability mechanism, the firm chooses an item $f_{\hat{\theta}} = (\tau_{\hat{\theta}}, d_{\hat{\theta}}, \psi_{\hat{\theta}})$ from the menu. Faced with the tariff $\psi = \psi_{\hat{\theta}}$, the firm chooses a stopping time and a decision to maximizes its expected utility as given by (1).

Definition 2 *A liability mechanism M is* incentive compatible *if for each $\theta \in \Theta$ it is optimal to chooses the item f_θ from M and the strategy (τ_θ, d_θ).*

Because the regulator is often unable to contract with the agent ex ante, we wish to determine when liability mechanism can be implemented by tariffs that are type independent.

Definition 3 *A direct liability mechanism is a* tariff mechanism *if the tariffs* $\{\psi_\theta\}_{\theta\in\Theta}$ *are independent of* θ*.*

Theorem 1 *Any IC liability mechanism can be implemented by a tariff mechanism. Moreover, the firm's expected payoff conditional on its type is unchanged across both mechanisms.*

Proof. Consider any direct liability mechanism M and let $\underline{x}_\theta = \underline{x}_\theta^{\psi_\theta}$ and $\psi_\theta = \psi_\theta(\underline{x}_\theta)$ denote the firm's adoption threshold and penalty in case of damage that are implemented under mechanism M when the firm has type θ.

We introduce a ceiling mechanism $\tilde{M}$ as follows: for each θ, $\tilde{\psi}_\theta$ gives the maximal penalty l for all x except at $\underline{x}_\theta$, where it gives ψ_θ. The ceiling mechanism $\tilde{M}$ is IC and implements the same thresholds $\underline{x}_\theta$, because under M the firm faces the penalty only when it adopts the product and higher penalties at other levels can only reduce the incentive to deviate.

If M prescribes the same threshold $\underline{x}$ to types $\theta \neq \theta'$, the penalties ψ_θ and ψ_θ' must be identical. Otherwise, one type would want to misreport its type and M would not be incentive compatible.

We define the tariff ψ as follows:

$$\psi(\underline{x}_\theta) = \psi_\theta$$

for all $\theta \in \Theta$ and

$$\psi(x) = l$$

otherwise.

This tariff is type independent. Moreover, it implements the same adoption thresholds as M, as is easily checked. ■

Theorem 1 shows that any liability mechanism can be implemented by a tariff. From now on, we invoke Theorem 1 and focus without loss of generality on mechanisms that are implemented by tariffs.

From Proposition 1, we know that the regulator would like to implement lower thresholds than the firm when the firm faces with a uniform penalty, regardless of the firm's private information. The next proposition shows that under these circumstances, it is without loss of generality to focus on tariffs that are increasing in X_τ, i.e., that are lower, the more due diligence is done by the firm.

Proposition 2 *Consider an IC mechanism implemented by some tariff ψ such that each θ stops at some threshold $x^{\psi}(\theta) \leq x^{l}(\theta)$. Then, there is a tariff $\hat{\psi}$ that is nondecreasing in x and implements the same actions as ψ.*

Proof. Let $X(\psi) = \{x^{\psi}(\theta) : \theta \in \Theta\}$ denote the set of adoption thresholds that are use by at least one type of the firm given the tariff ψ. Notice that we can assume without loss of generality that ψ is equal to its ceiling l for all $x \notin X(\psi)$: this does not affect the firm's equilibrium strategy as this change only makes deviations costlier. We will use the following observation:

$$\text{For all } x, x' \in X(\psi) \text{ such that } x < x', \text{ we have } \psi(x) < \psi(x'). \tag{4}$$

Suppose this were not the case: there exists a type θ who adopts at x and faces a penalty ψ when he could adopt at $x' > x$ and face a penalty $\psi' \leq \psi$. If $x' \leq x^{l}(\theta)$, this is strictly suboptimal for θ because the only reason that type θ adopts below $x^{l}(\theta$ is to receive a lower penalty in case of damage, which is achieved more efficiently by stopping at x'. If $x' > x^{l}(\theta)$, it means that there is a more optimistic type θ' that uses x'

After doing this reset, we define $\hat{\psi}$ as follows:

$$\hat{\psi}(x) = \inf\{\psi(x') : x' \in X(\psi), x' \geq x\},$$

where this infimum is by convention set equal to l is the set is empty. It is straightforward to check that $\hat{\psi}$ is nondecreasing in x. Precisely, $\hat{\psi}$ is a step function whose discontinuity points form a subset of $X(\psi)$.

By construction, $\hat{\psi}(x) \leq \psi(x)$ for $x \in X(\psi)$

The next result characterizes the set of implementable adoption thresholds. It is worth noting that the monotonicity condition that characterizes the set of implementable allocations is of different nature than the classical monotonicity condition that is necessary for incentive compatibility in other quasilinear settings. In this setting, the monotonicity would break if the designer could charge a transfer from those who adopt but don't generate damage.

Proposition 3 *A mechanism is incentive compatible only if the adoption thresholds are non-increasing. For any non-increasing adoption thresholds, there are transfers and abandonment thresholds such that the mechanism is incentive compatible.*

The proof of the only if part is based on the following lemma.

LEMMA 2 *For any ψ and at any $x < 0$, the set of types who wish to continue is an upper set of* Θ.

Proof. We want to show that for any tariff function ψ and level x, if θ prefers to continue, then so does any $\theta' > \theta$.

Suppose that $X_0 = x$ and that the firm considers the strategy of adopting at $\underline{x} < x$ or stopping at $\bar{x} > x$. Let ψ denote the penalty at x and $\underline{\psi}$ denote the penalty at $\underline{x}$. Let $p = \Pr(y = 1|\theta)$.

If θ stops immediately he gets:

$$1 - p\psi. \tag{5}$$

Let T^g, f^g denote the expected hitting time and the probability of hitting $\underline{x}$ if $y = 0$ (the product is good), and T^b and f^b be defined similarly if $y = 1$ (the product is damaged). If θ continues until hitting $\underline{x}$ or $\bar{x}$, his expected payoff is

$$p(f^b(1 - \underline{\psi}) - cT^b) + (1 - p)(f^g \times 1 - cT^g). \tag{6}$$

Comparing (5) and (6), continuing is optimal if

$$p(f^b(1 - \underline{\psi}) + \psi - cT^b) + (1 - p)(f^g - cT^g) \geq 1. \tag{7}$$

The left-hand side is a convex combination of two terms: $a = f^b(1 - \underline{\psi}) + \psi - cT^b$ and $b = f^g - cT^g$. The second term, b is less than 1, because f^g is a probability. Therefore, (7) can hold only if the first term, a, is greater than 1.

Rewriting (7), a type with probability p wishes to continue if

$$p(a - b) \geq 1 - b.$$

Since $a > b$, the coefficient of p is strictly positive. This implies that any type who assigns a probability $p' > p$ to $y = 1$ also wishes to continue.

To conclude the proof of Proposition 3, suppose that some θ wishes to continue given a tariff function ψ and x, and let $\underline{x}, \bar{x}$ denote this type's optimal strategy. All types $\theta' > \theta$, wish to continue with this strategy, and hence also for their optimal strategy.

■

The if part of of Proposition 3 can be shown by construction.

Consider adoption thresholds $x_1 > x_2 > ... > x_n$. We want to find $\psi_1, ..., \psi_n$ such that each type chooses her corresponding adoption threshold.

Start with a the most optimistic type θ_1. Let ψ_1 be such that type θ_1 is indifferent between adoption at x_1 and the optimal adoption with maximal liablity.

$$\theta_1 \pi^b(x_1, \bar{x}_1, \psi) + (1-\theta_1)\pi^g(x_1, \bar{x}_1) = \theta_1 \pi^b(\underline{x}_1, \bar{x}_1, \psi) + (1-\theta_1)\pi^g(\underline{x}_1, \bar{x}_1)$$

Where $\pi^b(\underline{z}, \bar{z}, u)$ is the expected payoff when the state is b and $\underline{z}, \bar{z}$ the adoption and abandonment thresholds, with liability u. And $\pi^g(\underline{z}, \bar{z})$ the expected payoff when state is g for adoption thresholds $\underline{z}, \bar{z}$.

If $x_1 < \underline{x}_2$, then θ_2 prefers x_1 by the same argument used in the proof of the Lemma 2. If $x_1 > \underline{x}_2$, it depends. Let (z_2, l_2) be the prefered adoption threshold and associated liability level for type θ_2 among (x_1, ψ_1) and (x_1, l).

Let ψ_2 be such that the type θ_2 is indifferent between using that threshold and a different one.

$$\max_z \left\{ \theta_2 \pi^b(x_2, z, \psi_2) + (1-\theta_2)\pi^g(x_2, z) \right\} = \max_z \left\{ \theta_2 \pi^b(z_2, z, l_2) + (1-\theta_2)\pi^g(z_2, z) \right\}$$

(A caveat: the solution to the two problems is the same adoption threshold, since the agent is indifferent.)

If $l_2 = x_1$ and since type θ_2 is indifferent between (x_2, ψ_2) and (z_2, l_2), it must be that type θ_1 strictly prefers (x_1, ψ_1). If $l_2 = \underline{x}_2$, then type θ_1 strictly prefers $(\underline{x}_1, l)$ by definition of $\underline{x}_1$ and (x_1, ψ_1) by transitivity.

We continue with (z_3, l_3) being the prefer adoption for type θ_3 among (x_2, ψ_2) and $(\underline{x}_3, l)$ (We know that type θ_3 prefers (x_2, ψ_2) to (x_1, ψ_1) since type θ_2 also does it). And we set ψ_3 so that type θ_3 is indifferent between (x_3, ψ_3) and (z_3, l_3), and so on.

By constuction, the result of this process is IC and implements the adoption thresholds.

PROPOSITION 4 *In the optimal mechanism the lowest type θ_1 is indifferent between his adoption threshold x_1^* and $x^l(\theta_1)$. Each other type θ_k is indifferent between her adoption threshold x_k^* and either the adoption threshold of the type immediately below x_{k-1}^*, or $x^l(\theta)$.*

By Proposition 3, any IC mechanism will have non-increasing adoption thresholds. Let $x_1^* \geq x_2^* \geq ... \geq x_n^*$ be the adoption thresholds of the optimal mechanims. Take type θ_k. For any $\psi_k \leq l < L$, the abandonment threshold is inefficiently high. In other words, let

$$z(\psi) := \arg\max_{z\geq 0} \left\{\theta_k \pi^b(x_k^*, z, \psi) + (1-\theta_k)\pi^g(x_k^*, z)\right\}$$

$z(\psi) \geq z(L)$. The planner wants to choose high transfers not because he cares about the transfers per se (he does not) but because in this way the incentives to abandone are more aligned with his payoff. Fortunatelly, there is no trade-off across types: higher transfers also relax the incentive compatibility constraints. Thus, the transfers constructed in the end of the proof of Proposition 3 are the optimal transfers for any given implementable adoption thresholds.

5 Taxation Principle with Identifiable Information Acquisition

When an IC mechanism implements distinct thresholds for distinct types, the conclusion of Theorem 1 is a corollary of the Taxation Principle with Non-Contractible Events of our companion paper (Poggi and Strulovici (2020)).

According to that paper, a mechanism is *identifiable* if satisfies two conditions that we translate into the present setting. Let A denote the set of all possible strategies by the firm. Each element of A consists of a pair (τ, d), where τ is a stopping time adapted to the filtration of X and d is measurable with respect to $\mathcal{F}_\tau^X$. For any subset A' of A, let $X(A')$ denote the set of observable outcomes by the regulator if the firm chooses an action $a \in A'$ and causes some damage.

Definition 4 *An IC mechanism M is identifiable if there exists a partition $\mathcal{A} = \{A_k\}_{k=1}^K$ of A such that*

(i) $X(A_k) \cap X(A_{k'}) = \emptyset$ for all $k \neq k'$.

(ii) All types θ who choose an action in A_k under the mechanism choose the same action of A_k.

Proposition 5 *If M implements distinct adoption thresholds for all types, then it is identifiable.*

Proof. For each θ, let A_θ denote the set of firm strategies that use adoption threshold $\underline{x}_\theta$, and let $A_0 = A \setminus (\cup_{\theta \in \Theta} A_\theta)$. By assumption on M, $\underline{x}_\theta \neq \underline{x}_{\theta'}$ for all $\theta \neq \theta'$. Therefore, A_θ and A'_θ are disjoint for all $\theta \neq \theta'$ and $\mathcal{A} = \{A_0, A_\theta : \theta \in \Theta\}$ forms a partition of A. Condition (ii) is trivially satisfied since for each cell of $\mathcal{A}$ there is at most one type taking action in that cell. Moreover Condition (i) is also satisfied by construction of the partition: $X(A_\theta) = \{\underline{x}_\theta\}$ for all $\theta \in \Theta$ and, hence, $X(A_\theta) \cap X(A_{\theta'}) = \emptyset$ for all $\theta \neq \theta'$. ■

Corollary 1 *If an IC mechanism M implements distinct adoption threshold for all types, it can be implemented by a tariff mechanism.*

Proof. Proposition 5 implies that M is identifiable. The result then immediately follows from Theorem 1 in Poggi and Strulovici (2020) ■