A Taxation Principle with Moral Hazard

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Origin of the Taxation Principle

- Guesnerie (1981, 1995), Hammond (1979)
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Linnemer (2019), *Annals of Economics and Statistics*

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Goal: to implement

- social choice function (scf) $f : \Theta \to A$
- transfer schedule $t : \Theta \to \mathbb{R}$

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- Focus of this paper.

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- When considering action a, $\tilde{t}(a)$ is the only transfer that matters.
- Proposing a single schedule $\tilde{t}$ instead of $\{t_\theta\}_{\theta\in\Theta}$ doesn't affect incentives and yields same transfers.

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- Not possible with a single tax schedule $\Rightarrow$ Substantive TP Fails.
  - What are the *right* conditions for the principle to hold?

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- **Agents payoff**: $v(\theta, a, z) - q(\theta, a) \cdot d(t, z)$, where q is positive-valued.
- **Contractible outcomes** $C \subseteq Z$ such that for all $z \notin C$, we assume w.l.o.g. that t_0 is the only available tax.

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- $z = 0$ is non-contractible, $t = 0$.

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- Without it, easy to build examples where TP Fails.

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- If sufficiently harsh penalties are available, the planner's optimum can be implemented.
- With a tax mechanism, transfers are independent of agent's type and the planner's optimum cannot be implemented.

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- $\mathcal{A}$ is **invariant** if for any cell A_i, the map μ_a is constant over A_i.

Main Result

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- When $\mathbf{A}$ is invariant, the principal can identify, for each contractible outcome realization, the distribution of contractible outcomes that is associated with the action a.
- Asking the agent to report his private information becomes redundant.

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- We defined a tax mechanism $\tilde{t}$.
- **Next**: check that $\tilde{t}$ yields same incentives as $\{t_\theta\}_{\theta\in\Theta}$.

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- By construction, $\tilde{t}$ was preferred by some type chosing an action in A_i, so $\tilde{t}$ is preferred by all such types, and delivers same payoff as the direct mechanism.
- No type gains by deviating from f under $\tilde{t}$ because payoffs from other actions were already available under the direct mechanism.

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Weakest condition

If **A** is not invariant, there is a set of types Θ, a set of feasible penalties $\Gamma : Z \to \mathbb{R}$, a utility function u, and a social choice function f such that f is implementable but not tax implementable.

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- Private type θ includes the probability of $\omega = 1$ and preference parameters (cost of experiments and care).

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Taxation Principle applies! Penalty as a function of e and s is wlog.

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Theorem 2

Suppose that independence holds and f is implementable. If $\mathbf{A}$ is f-invariant then f is implementable by a tax mechanism.

Extension: Dynamic Version

- Two periods: $\tau = 1, 2$.
- State θ_τ at time τ.
- Action $a_\tau \in A_\tau$ at time τ.
- Outcome $z_\tau \in Z_\tau$ at time τ.
- Set of penalties $\Gamma : Z_1 \times Z_2 \to T$.

We would like to implement $f = (f_1, f_2)$ where

- $f_1 : \Theta_1 \to A_1$
- $f_2 : \Theta_1 \times Z_1 \times \Theta_2 \to A_2$

Other Applications and Extensions

- Applications:
  - Liability design.
  - Plea bargaining.
  - Pre-existing conditions and health insurance.
  - Scoring mechanisms.
  - etc.

- Extensions.
  - Multiple agents with independent types.
  - Dynamic contracting.
  - etc.