Advanced Microeconomics III

Francisco Poggi

Information about the course

- Lectures: Mondays (10:15) and Tuesdays (15:30).
- Office hours: by email.
- Exercise session on Thursdays — please work on the solutions on Wednesday (or Tuesday evening)
- You can collaborate in groups of at most 3, but you should submit your answer sheet individually (indicating your group members).
- Final exam: June 13.

Course material

- Slides will be hosted on my website:

  franciscopoggi.com/courses/microIII

- **Textbook**: "Microeconomic Theory" by Mas-Colell, Whinston, and Green, Oxford University Press, 1995 (**MWG**).
- The course covers Ch. 13, Ch. 14, and Ch. 23 D-F.

Course plan

- Week 1 (Apr 4): Akerlof
- Easter Break
- Week 2 (Apr 25): Spence
- Week 3 (May 2): Rothchild-Stiglitz
- Week 4 (May 9): Moral Hazard.
- Week 5 (May 16): Bayesian Implementation/Envelope Theorem
- Week 6 (May 23): Revenue Maximizing Auctions.
- Week 7 (May 30): Efficient Mechanisms.

Information economics

- What is “information”?
  - Informally: the ability to exclude some states of the world.
- What is “asymmetric information”?
- **Asymmetric information is present in many economic relationships**
  - Trade of used goods or novel goods
  - Labour markets
  - Financial Markets
  - Provision of public goods
  - Expert advise
- What is “economics of information”?
  - economics of markets with asymmetric information, i.e., welfare and distributional aspects of equilibria.

Modeling information

- Ω: state space.
- A (deterministic) signal $\sigma : \Omega \to S$ where S is the set of signal realizations.
- A partition of Ω is a collection $\mathcal{E}$ of nonempty disjoint subsets whose union is Ω.
- The partition generated by signal σ is the collection

$$\mathcal{E} = \{\sigma^{-1}(\{s\}) : \ s \in S^*\}$$

where S^* is the range of σ.

Modeling information

- Suppose $\mathcal{E}_1$ and $\mathcal{E}_2$ are partitions of Ω. $\mathcal{E}_1$ is finer (coarser) than $\mathcal{E}_2$ if every element of $\mathcal{E}_1$ is a subset of some element of $\mathcal{E}_2$. I.e,

$$\text{for every } E \in \mathcal{E}_1, \text{ there exists } E' \in \mathcal{E}_2 \text{ such that } E \subseteq E'.$$

- A: set of actions.
- Let $\mathcal{A}$ be the set of signal-contingent action plans $\alpha : S \to A$.
- Bayesian agent with utility $u : A \times \Omega \to R$ and prior P solves

$$\max_{\alpha \in \mathcal{A}} \sum_{\omega \in \Omega} u(\alpha(\sigma(\omega)), \omega) P(\omega)$$

Theorem

Theorem

Consider $\sigma_1 : \Omega \to S_1$ and $\sigma_2 : \Omega \to S_2$. The following are equivalent:

1. *The partition $\mathcal{E}_1$ generated by σ_1 is finer than the partition $\mathcal{E}_2$ generated by σ_2.*
2. *There exists a function $\gamma : S_1 \to S_2$ such that $\sigma_2 = \gamma \circ \sigma_1$.*
3. *Every Bayesian expected utility maximizer prefers σ_1 to σ_2 for every decision problem.*

- Blackwell Theorem extends this result to stochastic signals.

Akerlof's market for lemons

- QJE (1970).
- Around 40k citations.
- Nobel Prize (2001) with Spence and Stiglitz.

- Before QJE, paper was previously rejected by 3 top journals.
  - AER: trivial.
  - JPE: wrong.
  - REStud: trivial.

Akerlof's market for lemons

- Continuum of sellers (measure N).
- Continuum of buyers (measure larger than N).
- Each seller has a car of quality θ.
- Buyers and sellers have quasiliner preferences:
  - Payoff of a buyer that buys car of type θ at price p:

$$\theta - p$$

  - Payoff of a seller that sells a car of type θ at price p is:

$$p - r(\theta)$$

  - ($r(\theta)$ can be thought of as an opportunity cost.)
- Seller knows the quality of his car.
- $\theta \sim F$ with support in $[\underline{\theta}, \bar{\theta}]$

Efficient allocation

Let $\Theta \subset [\underline{\theta}, \bar{\theta}]$ be the set of qualities that are traded.

- Gains from trade:

$$\int_{\underline{\theta}}^{\bar{\theta}} 1_{\{\theta \in \Theta\}} \cdot [\theta - r(\theta)] \cdot N \; dF(\theta)$$

- Efficient allocation Θ^* solves:

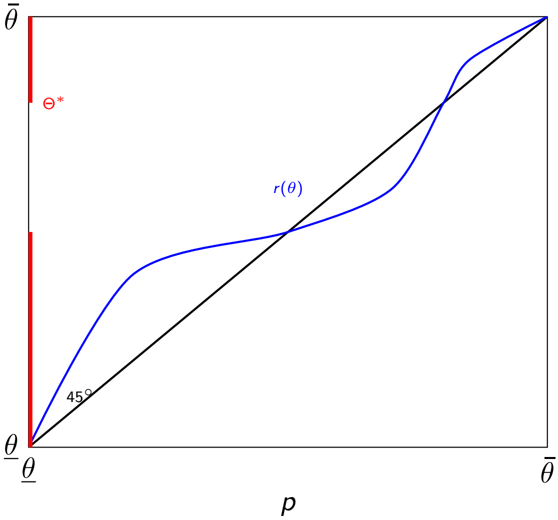
$$\max_{\Theta \in \mathcal{P}([\underline{\theta}, \bar{\theta}])} \int_{\underline{\theta}}^{\bar{\theta}} 1_{\{\theta \in \Theta\}} \cdot [\theta - r(\theta)] \cdot N \; dF(\theta)$$

- Solution:

$$\theta \in \Theta^* \qquad \Leftrightarrow \qquad \theta \geq r(\theta)$$

$$\Theta^* = \{ \; \theta \in [\underline{\theta}, \bar{\theta}] \; : \; \theta \geq r(\theta) \; \}$$

Efficient allocation



Benchmark: symmetric information

- Suppose car quality is observable.
- **Competitive equilibrium**: price $\hat{p}(\theta)$ such that demand quantity and supply quantity is equal for all car qualities.

$$\text{Demand for car of quality } \theta = \begin{cases} 0 & \text{if } \quad p > \theta \\ [0, N'] & \text{if } \quad p = \theta \\ N' & \text{if } \quad p < \theta \end{cases}$$

$$\text{Supply for car of quality } \theta = \begin{cases} N & \text{if } \quad p > r(\theta) \\ [0, N] & \text{if } \quad p = r(\theta) \\ 0 & \text{if } \quad p < r(\theta) \end{cases}$$

Benchmark: symmetric information

- For qualities efficient to trade:

$$\theta > r(\theta) \qquad \Rightarrow \qquad \hat{p}(\theta) = \theta \text{ and } \hat{Q}(\theta) = N$$

- For qualities efficient not to trade:

$$\theta < r(\theta) \qquad \Rightarrow \qquad \hat{p}(\theta) \in (\theta, r(\theta)) \text{ and } \hat{Q}(\theta) = 0$$

Observation

With symmetric information the competitive equilibrium is efficient.
Welfare theorems.

Asymmetric Information: competitive equilibrium

- Since car quality is not observable by the buyers, all car qualities should have the same price.
- A *competitive equilibrium* is a price $\hat{p}$ and a set $\hat{\Theta} \in \mathcal{P}([\underline{\theta}, \bar{\theta}])$ such that
  - Demand:
$$\hat{p} = E[\theta | \theta \in \hat{\Theta}]$$
  - Supply:
$$\hat{\Theta} = \{\theta : r(\theta) \leq \hat{p}\}$$

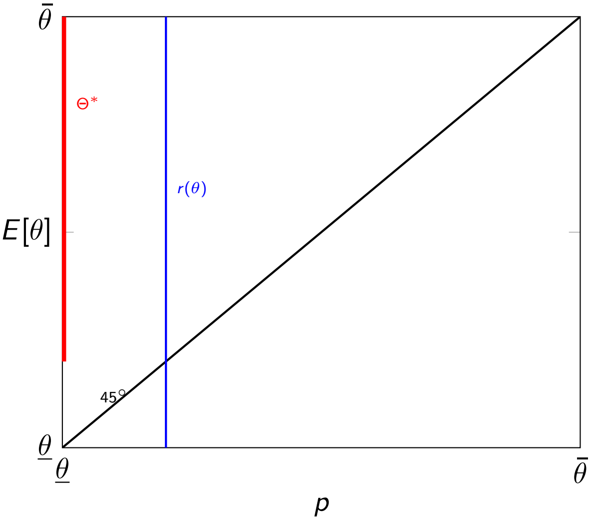
Example

$r(\theta) = \bar{r}$ and $F(r) \in (0, 1)$.

- Case $p > \bar{r}$:
  - then $\Theta = [\underline{\theta}, \bar{\theta}]$.
  - Equilibrium price $\hat{p} = E[\theta]$ if $E[\theta] > \bar{r}$.
  - Inefficient.

- $p < \bar{r}$
  - then $\Theta = \emptyset$.
  - Equilibrium price $\hat{p} < \bar{r}$ if $E[\theta] < \bar{r}$.
  - Also inefficient.

Example

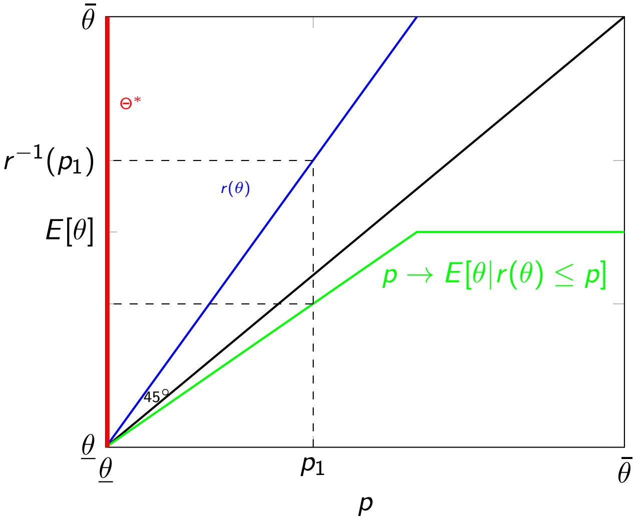

$\bar{\theta}$
Θ^*
$r(\theta)$
$E[\theta]$
45°
$\underline{\theta}$
$\underline{\theta}$
$\bar{\theta}$
p

Adverse selection

- In the previous example:
  - Willingness to sell of sellers is independent of the quality.
  - But efficient allocation depends on the quality.

- *Adverse selection* occurs when $r(\theta)$ is increasing in θ.
  - For any price, only the relatively worse cars ($\theta \leq r^{-1}(p)$) are going to be sold.
- Market may fail completely even when it is efficient that all cars are sold.

Possibility of market breakdown



Existence of CE with no market breakdown

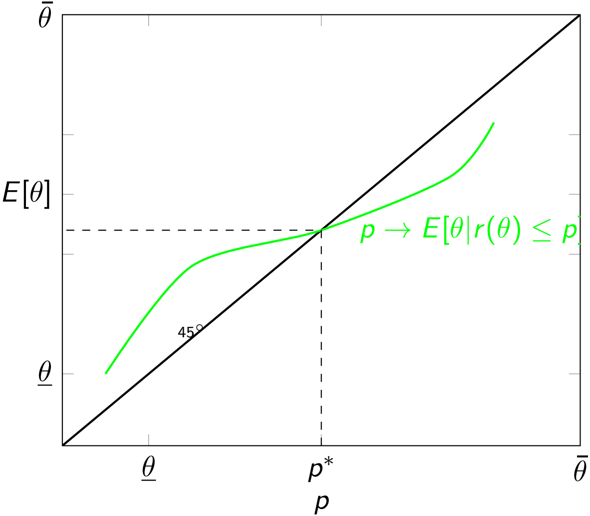
Assumptions:

1. Negative Selection: r is strictly increasing.
2. No atoms: F is continuous.
3. No market breakdown: There exists a price such that $E[\theta|r(\theta) \leq p] > p$.

Proposition

Assume 1-3. Then a competitive equilibrium with some trade exists.

Existence of CE with no market breakdown



Existence of CE with market breakdown

Assumptions:

3'. Market breakdown: $E[\theta | r(\theta) < p]$ for all p.

Proposition

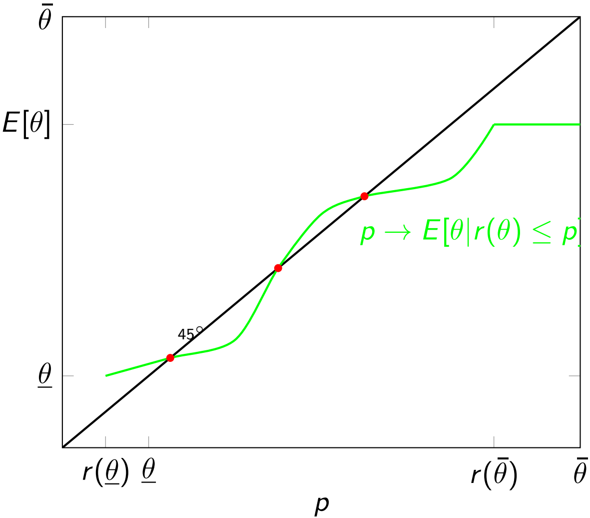
Assume 1, 2 and 3'. Then a competitive equilibrium with no trade exists. Moreover, no equilibrium with a positive mass of trade exists.

Parametric Examples

- Example 1: constant cost.
  - F uniform on $[0, 1]$.
  - $r(\theta) = \bar{r}$.
- For which $\bar{r}$ is the CE efficient?

- Example 2: linear cost.
  - F uniform on $[0, 1]$.
  - $r(\theta) = \alpha \cdot \theta$.
- For which α is the CE efficient?

Equilibrium Multiplicity



Equilibrium Multiplicity

- When there are multiple equilibria, these can be Pareto ranked:
  - Buyers make zero expected profits in all equilibria.
  - in 'higher' equilibria more sellers sell, and those who sell make higher profits.

- Are some of these equilibria more *likely* than others?

Game-theoretic approach

- Same underlying structure with F and $r(\cdot)$ common knowledge.
  - Three players: Buyer 1, Buyer 2, Seller.

- Timing:
  - Buyers offer prices p_1, p_2 simultaneously.
  - Nature chooses car quality θ according to F.
  - Seller decides who to sell, if anybody.

Pure-Strategy Subgame-perfect Nash Equilibria

- We assume: negative selection, no atoms, and no market breakdown.
- Let p^* the highest competitive equilibrium price.
- Extra assumption: *"genericity"*

$$\exists \epsilon > 0 : \qquad \text{for all } p > p^* \qquad E[\theta | r(\theta) \leq p] > p$$

Proposition

Asusme Negative selection, no atoms, no market breakdown and genericity. Then in any SPNE, both buyers offer the price p^*.

Pure-Strategy Subgame-Perfect Nash Equilibria

- At stage 2: in any SPNE the seller
  - sells at price $\max\{p_1, p_2\}$ if $> r(\theta)$
  - keeps the good if $\max\{p_1, p_2\} < r(\theta)$
- Each buyer's SPNE expected payoff is zero.
  - Proof by contradiction.
- Total Payoff:

$$F(r^{-1}(p))[E[\theta|r(\theta) < p] - p]$$

- Then p must be a CE price or below $r(\underline{\theta})$.
- If $p < p^*$ there is a profitable deviation.

Market with one buyer

- Variant: only one buyer (and one seller as before).
    - In general the equilibrium differs from two-buyer case.

- However: under assumptions 'no atoms' and 'market breakdown' we have as before
    - equilibrium with no trade.
    - no equilibrium with trade.

Experimental evidence

- Ball, Bazerman, Carroll (1991): Laboratory Experiment of Akerlof's market with one buyer.
    - One firm (acquirer) is considering making an offer to buy another firm (target).
    - Complication is that acquirer is uncertain about the ultimate value of the firm.
    - Target's management has an accurate estimate of the value.
    - What final price offer should the acquirer make for the target?

Experimental evidence

- Experiment:
  - Subjects play "acquirer".
  - Computer plays "target".
  - Acquirer knows that, under old management, market value of target takes any value between 0 and $100M$ with equal probabilities.
  - Value under new management is 50% higher than under old management.
  - Target knows its value.
  - Acqurer makes a price offer, then target accepts or rejects.
  - feedback: realized profit.
  - play 20 times.
  - profit proportional.

Experimental evidence

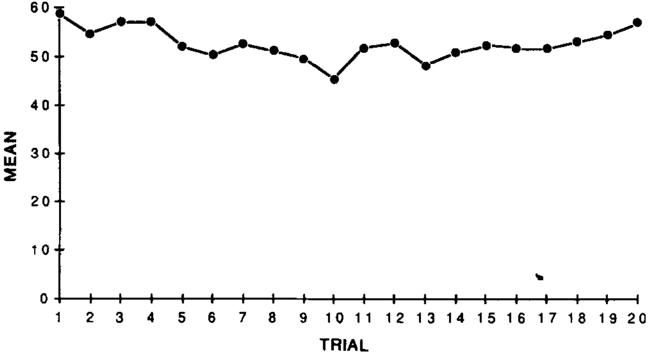


FIG. 1. Mean bids across trials for subjects in Experiment 1.

Experimental evidence

- Possible explanation: feedback too 'weak' to allow market unraveling.
  - Probability of positive profit at $p > 0$?

Relationship between information and trade

- Buyer and Seller can potentially trade a good of uncertain quality.
- Good's quality equally likely to be of three types: $\omega \in \{L, M, H\}$.
- Buyer's valuation:

$$b(\omega) = \begin{cases} 14 & \text{if} \quad \omega = L \\ 28 & \text{if} \quad \omega = M \\ 42 & \text{if} \quad \omega = H \end{cases}$$

- Seller's valuation:

$$b(\omega) = \begin{cases} 0 & \text{if} \quad \omega = L \\ 20 & \text{if} \quad \omega = M \\ 40 & \text{if} \quad \omega = H \end{cases}$$

- Trade is always efficient.

Relationship between information and trade

- **Case 1**: Buyer and Seller are equally uninformed.

$$E[b(\omega)] = 28 > 20 = E[s(\omega)]$$

  - Trade can take place for all qualities at any price between 20 and 28.

- **Case 2**: Seller partially uninformed: {{L}, {M,H}}
  - There is no price at which L, M, H are traded.

$$E[b(\omega)] = 28 < 30 = E[s(\omega)|\omega \in \{M, H\}]$$

  - L can be traded at a price in $[0, 14]$.

Relationship between information and trade

- **Case 3**: Seller is perfectly informed.
  - L and M can be traded at a price in $[20, 21]$.

$$E[b|\omega \in \{L, M\}] = 21 > 20 = E[s|\omega = M]$$

- Example shows that market can expand in the face of greater information asymmetry.
- Relationship between information asymmetry and trade might be nonmonotonic.