

Advanced Microeconomics III

Mechanism Design - 2

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Introduction

- Consider an auction but where the revenue-collector is included as a “agent 0”.
- The revenue-maximizing scf is a constrained Pareto efficient allocation!
 - Not possible to increase the utility of agents without decreasing the utility of agent zero.
- The set of constrained Pareto efficient scf is usually difficult to characterize.
- We ask instead: Does there exist a constrained efficient scf that is Pareto-efficient?

Efficient Mechanisms

- Consider a quasilinear environment with private values:
 - Set of agents I .
 - Type profiles Θ .
 - Preferences given by $u_i(x, \theta_i) - t_i$.
- We say that an allocation rule $\alpha : \Theta \rightarrow X$ is *efficient* iff

$$\alpha(\theta) \in \arg \max_{x \in X} \sum_{i \in I} u_i(x, \theta_i) \quad \forall \theta \in \Theta.$$

Efficient Mechanisms

- We study a set of mechanisms that implement an efficient allocation in the quasilinear setting with private values.
- As before, by virtue of the Revelation Principle, we can restrict attention to DRM.
- Since α is fixed, the mechanisms we consider differ only in the transfer rule.
- Other properties that are interesting beyond efficiency:
 - Incentive Compatibility (BIC and DSIC)
 - Voluntary participation (“individual rationality”)
 - That no money is required to run the mechanism (“Budget-balanced”).
- Usually, tension between the last two.

Example

- One thing one could do is a DRM in which
 - $\alpha^*(\theta)$ is efficient given the reports.
 - Each agent receives a transfer equivalent to the sum of others' payoffs.
 - Given others' reports θ_{-i} , agent chooses:

$$\max_{\hat{\theta}_i \in \Theta_i} u_i(\alpha^*(\hat{\theta}_i, \theta_{-i}), \theta_i) + \sum_{j \neq i} u_j(\alpha^*(\hat{\theta}_i, \theta_{-i}), \theta_j)$$

- Since α is efficient,

$$\sum_{k \in I} u_k(\alpha^*(\theta), \theta_k) \geq \sum_{k \in I} u_k(\alpha^*(\hat{\theta}_i, \theta_{-i}), \theta_k)$$

- Thus, it is dominant to report truthfully.
- Problem: this requires large positive transfers to participants. 'run a deficit'.

VCG Mechanisms

- VCG mechanism (due to Vickrey, Clark and Groves) is a family of DRM in which the allocation α^* is efficient allocation and

$$\tau_i(\theta) = \sum_{j \neq i} u_j(\alpha^*(\bar{\theta}_i, \theta_{-i}), \theta_j) - \sum_{j \neq i} u_j(\alpha^*(\theta), \theta_j)$$

where $\bar{\theta}_i$ is a default type for player i .

- **Intuition:** each agent pays *what other agents can achieve without i minus what the other agents get if i is present*. In other words, the externality imposed on others.
- Note: VCG is a DRM. All θ s should be interpreted as the reports of the agents.

Example: VCG in auctions

- Let $\Theta_i = [0, 1]$ for all i
- let $\bar{\theta} = (0, 0, \dots, 0)$.
- **Efficient allocation:** give the object to the agent that values the item the most.
- VCG payments are:

$$\tau_i(\theta) = \begin{cases} \max_{j \neq i} \theta_j & \theta_i > \theta_j \quad \forall j \\ 0 & \text{otherwise} \end{cases}$$

- Note: The VCG mechanism is equivalent to the second price auction.

Example: Binary public good provision

- Let $X = \{0, 1\}$
 - Let $u_i(x, \theta_i) = x \cdot \theta_i$
 - c : cost of providing the public good.
-
- Consider two agents: $I = \{1, 2\}$.
 - The efficient allocation is:

$$\alpha^*(\theta) = \begin{cases} 1 & \text{if } \theta_1 + \theta_2 \geq c \\ 0 & \text{otherwise} \end{cases}$$

Example: Binary public good provision

- VCG transfers with $\bar{\theta}_i = 0$.

$$t^{VCG}(\theta_i) = 0 - \theta_j \cdot 1_{\{\theta_i + \theta_j > 1\}}$$

- VCG transfers with $\bar{\theta}_i = 1$.

$$t^{VCG}(\theta_i) = \theta_j - \theta_j \cdot 1_{\{\theta_i + \theta_j > 1\}} = \theta_j \cdot 1_{\{\theta_i + \theta_j < 1\}}$$

VCG is DSIC

For any profile of default types $\bar{\theta}$, the VCG mechanism is DSIC.

Proof.

$$\begin{aligned}U_i(\theta) &= u_i(\alpha(\theta_i, \theta_{-i}), \theta_i) - t_i(\theta_i, \theta_{-i}) \\&= u_i(\alpha(\theta_i, \theta_{-i}), \theta_i) - C(\bar{\theta}_i, \theta_{-i}) + \sum_{j \neq i} u_j(\alpha(\theta_i, \theta_{-i}), \theta_j) \\&\geq u_i(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_i) - C(\bar{\theta}_i, \theta_{-i}) + \sum_{j \neq i} u_j(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_j)\end{aligned}$$

Where

$$C(\bar{\theta}_i, \theta_{-i}) := \sum_{j \neq i} u_j(\alpha^*(\bar{\theta}_i, \theta_{-i}), \theta_j)$$

Example: Bilateral Trade

- One buyer and one seller.
- Single object.
- Buyer value for the object is θ_b .
- Seller's cost (alternatively value) is θ_s .
- **Efficient allocation rule:**

$$\alpha^*(\theta) = \begin{cases} \text{trade} & \text{if } \theta_b > \theta_s \\ \text{no trade} & \text{if } \theta_b < \theta_s \end{cases}$$

Example: Bilateral Trade

To construct the VCG mechanism, let's pick default types $\bar{\theta}_b = 0$ and $\bar{\theta}_s = 1$.

- When there is no trade:

$$t_b^{VCG}(\theta) = t_s^{VCG}(\theta) = 0$$

- When there is trade ($\theta_b > \theta_s$),

$$t_b^{VCG}(\theta) = \theta_s$$

$$t_s^{VCG}(\theta) = -\theta_b$$

- The mechanism incurs a deficit whenever there is trade.

Participation and Budget-balanced condition

- Voluntary participation (IR):

$$U_i(\theta) \geq 0 \quad \forall i \in I.$$

- This is an ex-post notion of participation. Each agent-type is happy to participate for all types of others.
- Budget-balanced condition (BB):

$$S(\theta) := \sum_{k \in I} t_k(\theta) = 0 \quad \forall \theta \in \Theta$$

- This is an ex-post notion of budget-balanced, in the belief-free spirit of DSIC.
- If $S(\theta) \geq 0$ for all θ we say that the mechanism *never runs a deficit*.
 - **Note:** this equivalent to voluntary participation of agent zero.

Myerson-Satterthwaite

In the bilateral trade environment there is no mechanism that is efficient, DSIC, satisfies voluntary participation, and that is budget-balanced.

Proof.

- Deferred for later.



- Comments:
 - The result generalizes to environments with asymmetric supports, as long as the supports intersect.
 - Inefficiency persist but gets smaller quickly when the number of traders on each side of the market increases.

Beyond Dominant Strategies

- Budget-balanced is a strong condition. Sometimes we are interested in the mechanism not running a deficit in expectation.
- Likewise, IR requires that all agents are happy to participate on the mechanism ex-post. Sometimes we are interested in interim incentives to participate (IIR).
- Finally, in Bayesian environments, we can relax our solution concept to Bayesian implementation.

Expected Externality Mechanism (AGV)

- **Idea:** instead of making each player pay the realized externality imposed on others, make them pay the expected externality.
- **Extra assumption:** Independent types.
- **AGV mechanism** (Arrow and d'Aspremont and Gerard-Varet):
DRM in which the allocation rule α^* is efficient and the payment rule is given by

$$t_i^{AGV}(\theta) = \frac{1}{N-1} \sum_{j \neq i} \tilde{t}_j(\theta_j) - \tilde{t}_i(\theta_i)$$

where

$$\tilde{t}_i(\theta_i) = E_{\theta_{-i}} \left[\sum_{j \neq i} u_j(\alpha^*(\theta), \theta_j) \right]$$

AGV is budget-balanced

Observe that, for all θ ,

$$\begin{aligned}\sum_{i \in I} t_i^{AGV}(\theta) &= \sum_{i \in I} \left[\frac{1}{N-1} \sum_{j \neq i} \tilde{t}_j(\theta_j) - \tilde{t}_i(\theta_i) \right] \\ &= \frac{1}{N-1} \underbrace{\sum_{i \in I} \sum_{j \neq i} \tilde{t}_j(\theta_j)}_{(N-1) \sum_{i \in I} \tilde{t}_i(\theta_i)} - \sum_{i \in I} \tilde{t}_i(\theta_i) \\ &= 0\end{aligned}$$

AGV is BIC

- If i tells the truth, she gets an interim utility

$$\begin{aligned} U_i(\theta_i) &= E_{\theta_{-i}} \left[u_i(\alpha^*(\theta), \theta_i) - t_i^{AGV}(\theta) \right] \\ &= E_{\theta_{-i}} \left[u_i(\alpha^*(\theta), \theta_i) + E_{\theta_{-i}} \left[\sum_{j \neq i} u_j(\alpha^*(\theta), \theta_j) \right] \right] \\ &\quad - \underbrace{\frac{1}{N-1} E_{\theta_{-i}} \left[\sum_{j \neq i} u_j(\alpha^*(\theta), \theta_j) \right]}_{\text{constant}} \end{aligned}$$

- By the efficiency of α^* , $\sum_{k \in I} u_k(\alpha^*(\theta), \theta_k) \geq \sum_{k \in I} u_k(x, \theta_k) \quad \forall x$.
- Hence, truthful reporting is a BNE of the DRM.

AGV may not be IIR

- There is nothing that guarantees that IIR is satisfied.
- There might be a agent i and type θ_i such that AGV involves

$$U_i(\theta_i) < 0$$

Generalized VCG

- We allow for type dependent outside options $\underline{U}_i(\theta_i)$.
- Given the efficient allocation rule α^* , let

$$\bar{\theta}_i \in \arg \min_{\theta_i \in \Theta_i} E_{\theta_{-i}} \left[\sum_{j=1}^N u_j(\alpha^*(\theta_i, \theta_{-i}), \theta_j) - \underline{U}_i(\theta_i) \right]$$

- We refer to $\bar{\theta}_i$ as the *least charitable type* of agent i .

Generalized VCG

- **GVCG**: DRM in which the allocation rule α^* is efficient and the payments are given by:

$$t_i^{GVCG}(\theta) = \sum_{j \neq i} u_j(\alpha^*(\bar{\theta}_i, \theta_{-i}), \theta_j) + u_i(\alpha^*(\bar{\theta}_i, \theta_{-i}), \bar{\theta}_i) \\ - \sum_{j \neq i}^N u_j(\alpha^*(\theta_i, \theta_{-i}), \theta_j) - \underline{U}_i(\bar{\theta}_i)$$

Theorem (Krishna-Perry)

The GVCG mechanism is IIR and BIC, and maximizes the expected surplus among all mechanisms that are IIR, BIC and implement the efficient allocation rule.

Proof - BIC

Exercise.

Proof - IIR

Exercise.

Proof - Revenue Maximizing

- Consider a mechanism Γ that is efficient, BIC, and IIR.
- Let $\hat{u}_i : \Theta_i \rightarrow \mathbb{R}$ be the payoff function for agent i in Γ . By payoff equivalence:

$$\hat{U}_i(\theta_i) - \hat{U}_i(\bar{\theta}_i) = U_i(\theta_i) - U_i(\bar{\theta}_i)$$

- Where u_i is the utility of agent i in the GVCG mechanism.
- Notice that

$$\hat{U}_i(\bar{\theta}) \geq \underline{U}_i(\bar{\theta}_i) = U_i(\bar{\theta}_i)$$

Where the inequality holds by IIR of Γ .

- Then,

$$\hat{U}_i(\theta_i) - \hat{U}_i(\bar{\theta}_i) \geq U_i(\theta_i) - U_i(\bar{\theta}_i)$$

Proof - Revenue Maximizing

- Thus,

$$\hat{U}_i(\theta_i) \geq U_i(\theta_i)$$

- Since the allocation for Γ and GVCG is the same, it must be that

$$E_{\theta_{-i}} \left[t_i^{\Gamma}(\theta) | \theta_i \right] \leq E_{\theta_{-i}} \left[t_i^{\text{GVCG}}(\theta) | \theta_i \right] \quad \forall \theta_i \in \Theta_i$$

- Taking expectation over θ_i and adding up all types we get the desired result.

Proof - Myerson-Satterthwaite

- In the bilateral trade environment, $\bar{\theta}_b = 0$ and $\bar{\theta}_s = 1$ are the least charitable types.
- Thus, the VCG mechanism we considered coincides with the GVCG mechanism.
- But this mechanism runs a deficit, thus:

There does not exist an efficient, BIC, IIR, ex-ante BB mechanism in the bilateral trade environment.