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On Taxation and Incentives: Further Reflections on the limits to Redistribution

ON TAXATION AND INCENTIVES :

FURTHER REFLECTIONS ON THE LIMITS

TO REDISTRIBUTION

by

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Two pieces of literature which are important for the theoretical understanding of public economics have known a rapid growth in the last fifteen years. The first one is the literature on optimal taxation which taking its roots in the older work of Ramsey F. [1927], Boiteux M. [1956] has received a widespread recognition from the publication of Diamond-Mirrlees [1971] article: The second one is the incentives literature which found a new vitality in the innovative work of Hurwicz L. [1972] , Gibbard A. [1973] , Groves [1973], Maskin E. [1977] and whose success is evidenced through the achievements of the literature on the Clarke Groves Vickrey mechanisms (see Green-Laffont [1979]).

Although the existence of connections between taxation and incentives problems has not been ignored in the past (for example, the optimal taxation approach has repeatedly been justified with arguments appealing more or less explicitely to incentive compatibility), the two pieces of literature did not interact very much and developed rather independantly. There has been few attempts to confront taxation and incentives issues in the most explicit way if one excepts some recent work (in which have to be singled out the sections devoted to this problem in the rich article of P. Hammond [1979]). In the line of such work, it is the purpose of this article to bring out in the most systematic fashion a comparison of taxation systems and incentive compatible procedures. The attention will focus on two main questions : to which extent and in which precise sense are taxation systems incentive compatible procedures ? In the case where they are incentive compatible are taxation systems the "best" possible incentive compatible mechanisms or are they at contrary "rough" mechanisms which are dominated on theoretical grounds by more sophisticated mechanisms ?

These questions will be raised in a full general equilibrium framework which will be described. It should however be mentioned at this point that the model treats only of informational problems of the consumption sector. Hence it is not relevant to the study of Planning of production (with imperfect information) but basically relates to the economic issue of redistribution.

From now it should also be clear to the reader that the fundamental question which lies behind the scene is that of the theoretical status of second best taxation. Many theorists have been reluctant vis a vis second best taxation, considering that second best constraints were insufficiently justified and lacked deep theoretical basis. This position has been in particular defended in a talented way by F. Hahn [1973] . His criticism that we will remind throughout the paper, questions the fact that redistribution is limited to what is allowed by optimal commodity taxes. The controversy clearly relates to the reasons of impossibility of lump sum taxation and to the actual limits to redistribution.

Let us come now to the description of the paper.

Section I presents the model. The framework is that of a general abstract economy where the consumption sector consists of a continuum of consumers. A central Planner is in charge of the organization of the economy, facing informational and observational constraints : particularly, he cannot observe any component of the vector of characteristics which determine the preferences and endowments of the consumers and he cannot observe transactions taking place between consumers within the consumption sector. In counterpart, he has full control of the production sector and can detect any transaction between this sector and any consumer. Also, some intra consumers transactions are too much costly to be possible.

The purpose of the analysis is to compare the relative merits of two different Planning conceptions. The first one which is associated with a (higly) sophisticated Planner rests on the design of abstract incentives mechanisms. In the Hurwicz tradition, information is extracted from "game forms" but a particular emphasis is put here on the observational constraints and on the imperfect control of the Planner on the final allocation.

The (partly!) unsophisticated Planner advocates an economic organization based on the implementation of tax systems, taxes being linear for commodities which can be reallocated between consumers and non linear for others commodities.

Section II and III concentrate on the comparison of allocations obtained with game forms and tax systems in the case where the distribution of characteristics is public information and when game forms are restricted to be fully anonymous. The strong equivalence result which is obtained is in line with the previous analysis of P. Hammond 1979 (which itself relies on the techniques of analysis of incentive compatibility developped in Hurwicz L. [1979]). Conclusions are reinforced through the more systematic and more precise approach which is taken (starting from the basic constraints and focusing on institutional comparisons) and extended in several directions (particularly to explain the mixture of linear and non linear taxes).

Section IV comes back to the comparison when the "game form" is no longer restricted to be anonymous (anonymity being itself divided between recipient anonymity and anonymity in influence). It appears that difficulties may occur in the definition of tax systems and that the correlation which may exist between the trajectory of characteristics of the society (the characteristics of everybody but one) and individual characteristics play a crucial role in the analysis. It is exhibited a class of environments -which is in some sense the largest one where this phenomenon can occur- where the unsophisticated Planner relying on non anonymous tax systems can rival the sophisticated one using fully general incentives mechanisms. A priori, the conclusions are dependant -as in every piece of work on incentives- on the solution concept considered in the game form which is here the concept of Nash equilibrium.

I - THE MODEL

I.A. - Preferences, Technological and informational constraints

We consider an economy with n private commodities.[1)]

Consumers are associated with a vector of characteristics θ, which is finite-dimensional ($\theta \in \mathbb{R}^v$). (This vector summarizes all the information on consumers which is relevant for economic purposes).So, θ describes the endowment of the consumer in each of the commodities of the abstract commodity space $\mathbb{R}^n$. It also describes all the taste parameters [2)] and possibly some parameters that the planner might consider as relevant for the evaluation of social welfare (color of the eyes, the name of the agents).

Once θ is known, the preferences of an agent over the set of transactions are known; they are represented by a utility function defined on a transaction set $Z(\theta)$ and depending upon the vector of transactions z; $U = U(z, \theta)$. U is supposed monotonic in z.

Hence according to this formulation two agents which have the same vector θ have the same preferences on transactions and the same transaction set. The underlying assumption is rather weak : all the differences of the agents can be comprehensively described by a finite dimensional vector. We suppose also that there are "many agents" in the economy. As an idealization of this assumption, we assume the existence of a continuum of agents indexed by $a \in [0,1] = A$; a has to be interpreted as a distinctive personal feature of the agent which allows the planner to identify one agent : it may be for example the name of the agent, under the condition that such a name is not an information relevant in the sense indicated above, or preferably a "number" given to the agent.

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(1): A significant part of the analysis would not be affected by the introduction of public goods. The results might however receive a different interpretation.

(2): Let us notice that the distinction between taste and endowment parameters is not as straightforward as it looks. For example, in the income tax problem, differences in ability can be formalized as differences of tastes (agents are more or less reluctant to provide one hour of "effective" labour) or through endowments parameters (agents have the same preferences and the same endowments but in different types of labor or still different endowments in

In this section, we assume that the profile $\underset{\sim}{\theta}$, i.e. the function $\underset{\sim}{\theta} : a \in A \to \underset{\sim}{\theta}(a)$ which associates to every agent his characteristics, is completely unknown to the Planner. In counterpart the Planner knows exactly the distribution of characteristics in the society, as stated in assumption HI).

HI)- The distribution of characteristics is given by a probability measure μ the support of which is Θ, a subset of $\mathbb{R}^v$ which is contained in a compact set.

Although it anticipates on further development, a comment is needed at this point : the abstract mechanisms that one will design in the following are powerful enough to discover the distribution, even if it is unknown a priori. So the knowledge of the distribution that we assume is unnecessary for the part of the analysis concerning mechanisms. However the design of a tax system does require this knowledge, or at least requires informations on the response of aggregate demand which can be inferred from the distribution of θ. In the absence of an information on something like the distribution, the decision maker would be unable even to decide whether a tax system is admissible or not. HI is the simplest assumption which makes meaningful the comparison between admissible tax systems and admissible game forms.

On the production side, one will only be concerned by the aggregate production possibilities of the economy; they are described by a global production set Y . The Planner has <u>both full information and full control on this production set</u>; in particular the implementation of any global production plan $y \in Y$ is subject to the Planner's approval.

I.B. - <u>Observational constraints and transaction costs</u> :

Another crucial assumption concerns the observation of transactions in the economy.

First we suppose that transactions between the production side and the consumption side are observable at no cost : precisely the planner can detect the trade vector between the production side of the economy and any consumer a . Such a trade vector is denoted z (a).

Clearly the assumption is stronger, although consistent, than the assumption of Government control of the public sector. It implies that the Government can discriminate between any two commodities [3] and recognize the the amount of any trade between the production and the consumption sector, and identify the consumer engaged in the transaction.

Second we take a second polar assumption for the transactions within the consumption side : we assume that they are entirely unobservable to the Government.

However, we complete this assumption by the hypothesis that some of the (unobservable) transactions are impossible within the consumption sector. Precisely, we suppose that commodities are partitioned in two groups L_1, L_2, $L = L_1 \cup L_2$. A vector z will be denoted $z = (z_1 , z_2)$ where $z_1 \in \mathbb{R}^{|L_1|}$, $z_2 \in \mathbb{R}^{|L_2|}$. Trades on commodities of L_1 are unobservable and cannot be forbidden by the Planner if the agents want to engage in these trades.

But trades on commodities L_2 are impossible within the consumption sector. This impossibility has to be understood as resulting from very high transaction costs if transaction occurs between consumers. A kind of commodities we think as belonging to L_2 is electricity. Transferring the electricity obtained from the electricity company to your neighbour involves rather complex operations and a rather high transaction cost (disregarding the fact that they also are illegal in many countries).

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(3) : In fact, the analysis is in part compatible with an imperfect discrimination between commodities; but results have to be reinterpreted accordingly. To give very briefly an intuitive illustration of this assertion, take the case of labour. Strictly speaking, the assumption means that different qualities of labour can be observed. However, if following the standard interpretation of the linear income tax models, we interpret the existence of different productivities as different endowments is an homogeneous "effective labour", then the assumption does not imply that qualities and employment time can be observed, but only that the amount of effective labour supplied, which is in fact the product of productivity by employment time, i.e. in this case income, is observed. With Maskin's categories [1980], both "concealment" (or something like concealment) and "destruction" of endowments can be incorporated in this framework.

In fact all the above assumptions can be viewed as polar cases of observation and transaction costs and they can be summarized as follows :

- Costs of observation of individual θ by the Government	: $+\infty$
- Costs of observation of the distribution of θ by the Govern.	: 0
- Transaction costs between the production and the consumption side for any commodity	: 0
- Costs of observation of trades between the production and the consumption sides	: 0
- Costs of observation of trades between consumers	: $+\infty$
- Transaction costs between consumers for commodities $\ell \in L_1$	: 0
- Transaction costs between consumers for commodities $\ell \in L_2$ [4)]	: $+\infty$

II - ALLOCATION VIA GAME FORMS :

Given the informational and observational constraints that we have just described, what is the adequate economic organization, from the Planner's point of view ? To attempt to answer this question, we will follow the general framework proposed by L. Hurwicz [1972] for the comparative study of economic organizations. Following this framework, different types of organization should be viewed as abstract allocation mechanisms operating on a specified class of environnments. These allocation mechanisms are associated with a game or to put it differently have a "game form". This game has an outcome depending upon the strategies available to the agents and also upon the equilibrium concept which is adopted; it determines the allocation which finally prevails.

(4): In fact, as the above electricity example suggests what we have in mind, for commodities L_2 is a mixture of high transaction costs and low enforcement costs (and hence low observation costs). So it is reasonable to expect that the last assumption (for transaction costs of L_2) could be replaced by an assumption of zero observation costs for transaction on commodities L_2 within the consumption sector.

We should emphasize, at this point, one specific feature of the approach taken here :

The equilibrium concept that we adopt here is the one of Nash equilibrium. In fact when $L_1 = \emptyset$ i.e. when exchange is impossible within consumers, the equilibria of game forms that we consider in the whole paper are also dominant strategies equilibria. When $L_1 \neq \emptyset$, this is no longer true. Also, we restrict our attention to direct games (or revelation games) when the messages sent by the agents are their characteristics. The literature on incentive compatibility has emphasized that direct games were as much efficient as more general games when the solution concept was the one of dominant strategy. This is no longer true with Nash equilibrium. We will indicate in the following how the results will be modified with a more general message space. Let us immediately mention that the equivalence result of this section as well as the spirit of the main result of section IV would not be affected.

Although the general framework à la Hurwicz focus on informational questions, it does not take specifically into account the observability problems that have been emphasized here. These observational limitations play a central role and impose the design of a two-steps game. The first step, whose outcomes are variables observable by the Center, are under the Planner's control when the second one which concerns trades non observable to the Center will be outside the Planner's control.

Let us come to a ~~more~~ precise description, although still informal of this two-steps game : the game form is as follows :

- the first game (Step A) is a revelation game where agents announce their characteristics and where outcomes are net trades ($z^A(a)$) with the production sector.
- the second game (Step B) is played by the consumers and consists in exchange of commodities $\ell \in L_1$.

(5) : This is in contrast with the original suggestion of Hurwicz but follows the example of the recent work of the Clarke-Vickrey-Groves literature. (See Green-Laffont [1974])

We suppose that its outcomes will be additional net trades ($z^B(a)$) which are competitive equilibria of the economy whose initial endowments are the commodity bundles obtained by the agents after the first stage.

This assumption could be justified in different manners (6) ; one will mainly stress that it is the simplest and most classic hypothesis on the outcome of an exchange process between many agents, assuming that the Government has no control on this process. It should also be emphasized that we cannot avoid to describe in an explicit way the second step exchange process : for example, in order to avoid it, it would be tempting to retain the assumption that the final outcome of commodities L_1 is only constrained to be Pareto optimal (conditionnal to the allocation of commodities L_2). But still if people lie, opportunities of exchange appear and we are left with the problem of describing them.

A last point to finish with this general description. We will suppose that in the first step game, the agents as well as the Center have perfect foresight on the outcome of the second step. This hypothesis is reasonable as soon as we do not want to make the design of the allocation dependant on forecasting mistakes by the agents. However, we will have to face the usual trouble created by the perfect foresight assumption when the outcome is multivalued (as it may be here in the second step).

Let us switch to a more formal analysis in order to describe the first step game.

In full generality, the rules of this game should relate the net trade of an agents to its announced characteristics $\hat{\theta}$, to the announced characteristics of all the other players - here to the announced profile that we can denote $\hat{\underset{\sim}{\theta}}$ - and possibly to the "number" of the agent a. Hence, a net trade of agent a

(6) A more basic view of the second step would be for example that it consists in a sophisticated game played by consumers or that it is a game theoretical bargaining yielding core allocations. The sophisticated exchange game yields competitive outcomes when it is constrained to first best Pareto optimality and when the set Θ is connected (see Champsaur-Laroque [1980] for an emphasis on this latter point). Also the requirements of individual rationality and Pareto optimality imply competitive outcomes in the conditions made clear by Hurwicz [1979] .

should be a function $F(a, \hat{\theta}, \hat{\underset{\sim}{\theta}})$ [7)]

We will restrict the generality of this description in two directions :

. First, F will not depend upon a : we will refer to this property as recipient-anonymity.

. Second, F does not depend on the whole profile $\hat{\underset{\sim}{\theta}}$ but only on the distribution of $\hat{\underset{\sim}{\theta}}$. We will refer to this property as anonymity in influence [7)].

When both types of anonymity hold, we say that the game is anonymous [8)].

Here, as the economy is a continuum, the distribution of θ is the same wether a is or is not considered ; as we will restrict ourselves to solutions of game when the truth is announced, the argument which should appear instead of $\hat{\underset{\sim}{\theta}}$ would always be the known distribution μ. As it is fixed throughout this section, we will not mention it explicitely. And we will define a first step anonymous game as a single valued measurable mapping $F : \Theta \to \mathbb{R}^n$, which associates to every announced characteristics a vector of net trades with the production sector. [9)]

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(7): Where $\underset{\sim}{\theta}$ has to be understood as designating an "equivalence class" of functions which includes as well the function $\underset{\sim}{\theta} : a \in [0, 1] \to \underset{\sim}{\theta}(a)$ or the same function restricted to $[0, a[\cup]a, 1]$

(8): Note that the terminology differs from that of Hammond P. [1979] who calls such a property symmetry. In counterpart, the terminology is the one adopted by Mas-Colell [1978]

(9): Let us notice that we could have required in the definition that $F(\theta) \in Z(\theta)$. This would not introduce significant changes in the analysis.

Suppose now that agents are actually induced to truthful announcements in the first step game.

We can then compute the competitive equilibria of the second step game. The price equilibria are vectors of $\mathbb{R}^{|L_1|}$ and the set of competitive price equilibria denoted π^F is defined as follows :

♦ *Definition*

Set of second step price equilibria π^F

$\pi \in \pi^F$ *if and only if* $\exists$ *a measurable function*

$z^B : \theta \to z^B(\theta) = (z_1^B(\theta), 0)$ *such that :*

i) $z^B(\theta)$ *is a solution of*

$$\operatorname*{Max}_{z^B} U(z^B + F(\theta), \theta) \;,\; \pi \,.\, z_1^B \leq 0,\; z_2^B = 0,\; z^B + F(\theta) \in Z(\theta),\; \forall\, \theta \in \Theta$$

ii) $\int_\Theta z^B(\theta)\, d\mu = 0$

In the definition, whose all ingredients are classical, $z^B = (z_1^B, z_2^B)$, where z_1^B, z_2^B are respectively vectors of $\mathbb{R}^{|L_1|}$, $\mathbb{R}^{|L_2|}$.

Let us come now to the maximization problem of a consumer of characteristic θ ; its depends upon the first step function F, the characteristics θ, and the second step price expectation π and is denoted $P(F, \pi, \theta)$.

Definition

Program $P(F, \pi, \theta)$

$$\operatorname*{Max}_{\hat{\theta}} U(z,\theta)$$
$$z = z^A + z^B$$
$$z^A = F(\hat{\theta})$$
$$\pi \,.\, z_1^B \leq 0 \;,\; z_2^B = 0$$
$$z \in Z(\theta)$$

The control variable is the vector $\hat{\theta}$ *of announced characteristics.*

We will often adopt in the following the notation

$$L(\pi, 0) = \{z = (z_1, z_2) \mid \pi \,.\, z_1 \leq 0 \;,\; z_2 = 0\}$$

The constraints of the programm can then be written down more simply:

$$z = z^A + L(\pi \mid 0) \;,\; z \in Z(\theta) \;,\; z^A = F(\hat{\theta})$$

We are now in position to give a formal definition of the game form.

♦ *Definition*

Two steps ADAM

The two steps game F is an anonymous direct admissible mechanism (ADAM) if and only if :

i) $\exists\ \pi \in \pi^F$ *such that* $\hat{\theta} = \theta$ *is a solution of the program* $P(F, \pi, \theta)$, $\forall\ \theta \in \Theta$

ii) $\displaystyle\int_{\Theta} F(\theta)\, d\mu \in Fr\, Y$

The first condition says that whatever the characteristic θ of the agents the truth is an optimal strategy. Optimality is conditionnal to the fact that the second step price equilibrium vector is π. This actually occurs when all other agents tell the truth but it might be wrong if a group (of positive measure) of agents lied even if these lies were restricted not to contradict the actual distribution of characteristics. Hence here we only have a truthful Nash equilibrium implementation.

The second condition expresses the technological feasibility of the first-step aggregate net trade : we restrict ourselves to productively efficient allocations by imposing that y belongs to Frontier of Y : Fr Y.

Another comment on the definition :

In this model, a crucial fact is that if a consumer lies in the first step, he will be able to achieve his desired second step trade without affecting the second step equilibrium price vector. If we think of the continuum assumption as an approximation of a large number of agents, the corresponding asymptotic property is that a consumer who lies will trade in the second step without modifying much the equilibrium price vector. So the argument might not have an asymptotic correspondant in the case when the equilibrium price vector under consideration is critical. We might rule out such critical price vectors in the above definition. However, for reasons that the reader will exhibit by looking at the proofs and referring to classical results on critical economies, the spirit of the equivalence theorems of this section would remain unaffected by the indicated change of definition.

Let us notice that :

An outcome of the ADAM F is a vector function $z : \Theta \to \mathbb{R}^n$ *of transactions s.t.* $z(\theta) = F(\theta) + z^B(\theta)$ *where* $z^B(\theta) = (z_1^B(\theta), 0)$ *is the solution of* $\text{Max } U(F(\theta) + z^B, \theta)$, $z^B \in L(\pi, 0)$ *for some* $\pi \in \Pi^F$.

Let us note it follows from the definitions that z is measurable.

III - TAX SYSTEMS VERSUS GAME FORMS

III.A. Tax systems

We are now going to focus attention on institutional arrangements which look a priori less sophisticated than the two-steps ADAM just defined above.

The allocation is determined through decentralized procedures based on price adjustments. Transactions are made on markets where consumers and producers respectively behave as utility-maximizers and (competitive) profit maximizers. Government intervention consists in taxing the transactions occuring between the production sector and the consumption sector. The tax system is designed in such a way that it is compatible with the basic observational constraints of the problem and that it takes into account the other assumptions concerning intra-consumers transactions.

Precisely, it consists of linear taxes for commodities $\ell \in L_1$ and non linear taxes for commodity $\ell \in L_2$ and has the following implications:

- the production sector is faced with linear prices .[10)]
- the budget set of the consumers has the following shape :

$$q \cdot z_1 + \varphi(z_2) \leq 0$$

where q is a vector of $\mathbb{R}^{|L_1|}$ describing the consumption (linear) prices of commodities L_1 and $\varphi : \mathbb{R}^{|L_2|} \to \mathbb{R} \cup \{+\infty\}$ is a function describing the non-linear taxes on L_2 [11)]. If one refers to some numeraire (i.e. one commodity belonging to L_1 - say commodity 1 - of price 1), the above budget constraint means that a net trade vector z_1 of commodities $\ell \in L_1$ has a cost which is linear, the coefficients being the consumption prices. It should be noted that the cost of the transaction vector z_2 ($z_2 \in \mathbb{R}^{|L_2|}$) is not a separable function (contrarily to the cost of z_1 which is furthermore linear). Hence, its enforcement requires the centralization of the information gathered on each market of L_2. In other words, when linear taxation of commodities L_1 can be implemented market by market, the non linear one requires a global knowledge of transactions on markets L_2 and hence a more complicated administrative organization :

(10): As production sets are convex , there is no-need to use non linear ones.

(11): It should be noted that no assumption is made on φ . A measurability assumption will be made directly on the outcome associated with the tax system. This procedure might be criticized but it contributes greatly to the simplicity of the results obtained here.

Let us now define formally the demand for transactions of the consumer of characteristics θ (with obvious notation).

♦ *Definition* *Consumer Program* $P(q, \varphi, \theta)$

$\mathit{Max}\ U(z\ ,\ \theta)$

$q \cdot z_1 + \varphi(z_2) \leq 0$

$z \in Z\ (\theta)$

The set of solutions of this program is denoted $D(q, \varphi, \theta)$

So $D(.)$ is the demand for transactions of an agent of characteristics θ, faced with the budget constraint associated with q, φ.

Identifying a tax system with a couple of vectors $p \in \mathbb{R}^n_+$, $q \in \mathbb{R}^{|L_1|}$ and a function φ we can define an admissible tax system.

♦ *Definition* *Admissible tax systems*

An admissible tax system (ATS) *consists of a triple* (p, q, φ) *such that* :

i) $\exists\ z(\theta) \in D(q, \varphi, \theta)$, *a measurable selection of* D, *with*

$$\int_\Theta z(\theta)\ d\mu \in Y$$

ii)

$$p \cdot \int_\Theta z(\theta)\ d\mu \geq p \cdot y, \quad \forall\ y \in Y.$$

Condition i) indicates that the vector of total excess demand is (technologically) feasible when condition ii) indicates that it is profit maximizing with respect to p.

<u>An outcome</u> of the ATS (p, q, φ) is a function z satisfying i) and ii).

Clearly the class of tax systems which are called admissible is large. In particular we have no restriction on the function φ . However when the set of commodities L_2 is empty the set of ATS defined here coincides with the set of equilibria of a Diamond-Mirrlees type of model considered in optimal taxation theory [12). Also, when $n = 2$ and when L_2 reduces to the unique commodity "effective labour" the ATS reduce to the set of admissible income tax systems of the income tax literature. Note that when $L_1 = \emptyset$, we will say that the tax system is <u>fully non-linear</u> and when $L_2 = \emptyset$ we will say that it is <u>linear</u>. [13)

II.B. Game forms versus tax systems

In this section, we want to compare the class of allocations generated by two-steps anonymous direct admissible tax systems (ADAM) and by admissible tax systems (ATS).

A first difficulty appears due to the fact that the outcomes of both games and tax systems are not necessarily single valued. Because of step B, if the set of outcomes of some game F is not empty it will "in general" be multivalued. Also the demand function $D(q, \varphi, \theta)$ may be multi-valued either for exceptional values of θ or even for a big subset of θ.

We cannot hence expect a one to one relationship between ADAM and ATS [14). We will have to show however that the set of allocations attainable through ADAM and ATS is basically the same. This is the consequence of the next two theorems.

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(12): If the production sector is not constant returns to scale and privately owned, the definition of ATS given here supposes that pure profits are 100% taxed. This conforms both the principles of the description taken here and the dominant optimal taxation tradition. For a discussion of the structure of tax equilibria in such models, see Fuchs-Guesnerie [1979].

(13): It is important to note that in case where $L_2 = \emptyset$, we do not suppress the function φ from the definition; but it can only be a constant. The corresponding model is a Diamond-Mirrlees model with poll tax.

(14): From this point of view, one could argue for another definition of a game form based both on F and the selection of price equilibria which is chosen. This would increase notational complexity in a way which seems excessive, without assuring a one to one correspondence. Such a one to one correspondence is in any case entirely ruled out if instead of the convenient definition of tax systems taken here, we use the more satisfactory definition of Guesnerie (1979).

- *Theorem 1*

Given any outcome of an admissible tax system, there exists a 2 steps anonymous direct admissible mechanism which has the same outcome.

- *Proof*

Take the ATS (p, q, φ)

We know that there exists $z : \Theta \to \mathbb{R}^n$ a measurable selection of $\mathcal{D}$ such that :

1) $\bar{y} = \int_\Theta z(\theta)\, d\mu \in Y$

2) $\bar{y}$ is profit maximizing(with respect to p).

Define $\bar{F} : \Theta \to \mathbb{R}^n$ by $\bar{F}(\theta) = z(\theta) \quad \forall\, \theta \in \Theta$

We will show that $\bar{F}$ defines an ADAM :

i) $\bar{F}$ is measurable and $\int_\Theta \bar{F}(\theta)\, d\mu \in \mathrm{Fr} Y$ (by definition)

ii) $q \in \pi^{\bar{F}}$: In fact, we are going to see that given the first step transactions $\bar{F}$, if the price vector is q there is an equilibrium with no transactions in the second step.

For that, let us consider $B(q,\varphi) = \{z=(z_1,z_2)/ q.z_1 + \varphi(z_2) \leq 0\}$ and $\bar{F}(\theta) + L(q, 0)$, the sum of the vector $\bar{F}(\theta)$ and of the set $L(q, 0)$. (cf. definition p.12). This set is necessarly included in $B(q, \varphi)$;

Hence, as $\bar{F}(\theta)$ *is a solution of* $\text{Max } U(z,\theta)$, $z \in B(q,\varphi) \cap Z(\theta)$ $\bar{F}(\theta)$ *is also a solution of* $\text{Max } U(z,\theta)$, $z \in \{\bar{F}(\theta) + L(q,0)\} \cap Z(\theta)$. *There is actually a no-transactions equilibrium associated with* q.

iii) The truth is a dominant strategy of $P(\bar{F}, \theta, q)$.

Let us denote $C^{\bar{F}} = \underset{\Theta}{\cup} \bar{F}(\theta)$ *and let* $C^{\bar{F}} + L(q,0)$ *be the sum of the sets* $C^{\bar{F}}$ *and* $L(q,0)$.

The fact that θ *is a solution of :*

$$\underset{\hat{\theta}}{\text{Max}}\ U(z,\theta), \quad z = z^A + z^B,\ z^A = \bar{F}(\hat{\theta}) \quad , q \cdot z_1^B < 0,\ z_2^B = 0,\ z \in Z(\theta)$$

is equivalent to the fact that $\bar{F}(\theta)$ *is a solution of :*

$$\text{Max } U(z,\theta) \quad z \in \{C^{\bar{F}} + L(q,0)\} \cap Z(\theta).$$

This latter property results from the fact that $\bar{F}(\theta) \in D(q, \varphi, \theta)$ *and that* $\{C^{\bar{F}} + L(q,0)\} \subset B(q,\varphi)$.

Q.E.D.

We have then proved that any admissible tax system is an admissible Nash mechanism. In fact, as noted above the equilibrium we consider is (trivially) a dominant strategy equilibrium in the case where $L_1 = \emptyset$. In this latter case, this property can be related with what has been called the "revelation principle". According to this principle (already mentioned p. 8) any indirect game which has dominant strategy solutions is equivalent to a direct game. An economy with a tax system can be considered as an abstract game form where the strategies of the agents are the announcements of net trades. As there are "many" agents the competitive net trades are optimal strategies for each agent. Hence, the existence of an ADAM equivalent to an ATS conforms, roughly speaking, the revelation principle. (For a similar argument in the particular case of the comparison of the Clarke-Groves-Vickrey mechanism with taxation, see J.-J. Laffont [1981]).

It should however be mentioned here that ATS are shown to be equivalent to a special subclass of ADAM precisely mechanisms where no transactions take place in the second round.

We will show now the converse : 2-steps ADAM are equivalent, in the sense defined above, to ATS.

♦ *Theorem 2* :

Given any outcome of a 2 steps anonymous direct admissible mechanism F, there exists an admissible tax system which has the same outcome.

♦ *Proof*

Take F a 2 steps ADAM and consider the dominant strategy outcome z $z(\theta) = F(\theta) + z^B(\theta)$, *with* $z^B(\theta)$ *a solution of* Max $U(z^B + F(\theta), \theta)$, $\pi \cdot z_1^B \leqslant 0$, $z_2^B = 0$, $z^B + F(\theta) \in Z(\theta)$ *where* $\pi \in \pi^F$.

Consider $C^Z = \bigcup_{\theta \in \Theta} z(\theta)$

We are going to show that taking $q = \pi$, *we can find* φ *such that*

$B(q, \varphi) = C^Z + L(q, 0)$

Take $z = (\bar{z}_1, \bar{z}_2) \in C^F$; $\bar{z} = F(\bar{\theta})$ *for some* $\bar{\theta}$ *in* Θ.

I argue first that $\bar{z}_1 (= F_1(\bar{\theta}))$ *is necessarily a solution of the following program* $P(\bar{z}_2)$:

$P(\bar{z}_2)$: Max $\pi \cdot z_1$, $(\bar{z}_1, \bar{z}_2) \in C^Z$

Suppose the contrary. Then $\exists\, z_1'$ *s.t.* $(z_1', \bar{z}_2) \in C^Z$ *and* $\pi \cdot z_1' > \pi \cdot z_1$. *But, as* $(z_1', \bar{z}_2) \in C^Z$, *there exists* $\theta' \in \Theta$ *such that* $(z_1', \bar{z}_2) = F(\theta')$. *But considering the program* $P(F, \pi, \bar{\theta})$, *it follows from the monotonicity of preferences that the announcement of* θ' *dominates the announcement of* $\bar{\theta}$ *(for an agent of characteristic* $\bar{\theta}$*)* .

Now, let us call $V(\bar{z}_2)$ *the value of the program* $P(\bar{z}_2)$ $(= \pi \cdot \bar{z}_1)$ *and define* φ *as follows* :

If the set $\{z = (z_1 , \bar{z}_2) \in C^F\}$ *is empty, we put* $\varphi(\bar{z}_2) = +\infty$
If not, we put $\varphi(\bar{z}_2) = - V(\bar{z}_2)$.

Clearly, for any $\bar{z}_2$ *and* $\bar{\theta}$ *satisfying* $\bar{z}_2 = F_2(\bar{\theta})$,
$F(\bar{\theta}) + L(q , 0) = B(q ,\varphi) \cap \{z_2 = \bar{z}_2\}$.

Taking the union of both sides, we have then to show that :
$z(\theta) = F(\theta) + z^B(\theta)$ *is a solution of* $\text{Max } U(z , \theta)$, $z \in B(q , \varphi) \cap Z(\theta)$.
which given that $B(q ,\varphi) = C^Z + L(q , 0)$ *is equivalent to the fact that* $z(\theta)$ *is a dominant strategy outcome.*

It remains to note that in $\bar{y} = \int_\Theta z(\theta)\, d\mu$ *, which belongs to* $Fr\, Y$, *the convex set* Y *has a supporting hyperplane with respect to some vector* p.

(p, q, φ) *so defined are actually associated with an ATS.*

Q.E.D.

The above statements show that the sophisticated planner (the adept of game forms) and the less sophisticated one (the adept of tax systems) face the same set of opportunities in the sense that they can reach the same set of allocations. However, it is not true that there is a one to one correspondence between games and tax systems. The outcomes of a given game, F are in general associated with several tax systems and for a given outcome of a tax system here are in general an infinity of games F which yield a similar outcome.

Several corollaries of the theorems deserve to be stated.

♦ *Corollary 1*

Suppose $L_1 = \phi$

A function z *is an outcome of some ADAM if and only if it is an outcome of some fully non-linear ATS.*

♦ *Corollary 2*

Suppose $L_2 = \phi$

A function z *is an outcome of some ADAM if and only if it is an outcome of some linear ATS.*

These two corollaries are close in spirit to Hammond's propositions [1979]. However besides the differences of view points and approaches, we emphasized here that in corollary 2 only Nash implementation is obtained.

Another interesting corollary obtains

♦ *Corollary 3*

If $L_2 = \emptyset$, *and* Y *is a cone which contains* 0, *an outcome of some ADAM is first best Pareto optimal only if it is a competitive allocation.*

♦ *Proof*

From corollary 2, the considered outcome is the outcome of some linear ATS. From Pareto optimality $p = q$. *As* Y *is a cone , one shows that* $\varphi \equiv 0$. *Conclusion follows. (Note how the conclusion is modified when* Y *is not a cone).*

The conclusion that first best Pareto optimality implies competitivity differs from the previous results of this type. Hammond's result holds with $L_1 = \emptyset$ but requires to be true that Θ is connected (see Champsaur - Laroque [1980] .

Here, connectedness does not play any role but instead the fact that the second step exchange takes place.

In sóme sense, the view taken here is more basic (since it considers the possibilities of side exchanges which are themselves related to some intrinsic characteristics of the economy) ; but in another sense it is less basic since it has been assumed (and not derived) that the second step exchange conforms the competitive principles.

Finally let us note that the consideration of a more general message space would <u>not</u> modify the nature of the conclusions which are obtained here. There are few difficulties in seing that (mainly redesigning adequately definitions) and this is left to the reader.

III - MORE ON GAME FORMS VERSUS TAX SYSTEMS

In this section, we will come back on some assumptions adopted in the model of the previous section. Successively, we will study the effect of introducing alternative assumptions concerning :

- the linearity of taxes for commodities L_1 `
- the observability of characteristics
- the domain of characteristies.

III.A. The linearity of taxes on commodities of L_1

In the definition of tax system in section II, we assumed, without further discussion that the Planner restricts himself to use non linear taxes for commodities $\ell \in L_1$. Is that actually a restriction ? Or equivalently could the Planner do better by imposing non linear taxes on all commodities ? If we maintains the basic hypothesis according to which the reallocation of commodities $\ell \in L_1$ cannot be forbidden and takes place according to the the competitive mechanism, intuition inclines us to believe that the answer to the preceding question is negative i.e. that the Planner can gain nothing by using non linear taxes on commodities L_1. We can actually substantiate this intuition and hence to show that the formulation of tax systems of section II implied no loss in generality.

Before sketching briefly the argument, let us define a fully non linear tax system (FNLTS) as a function $\Psi : \mathbb{R}^n \to \mathbb{R}$. With the FNLTS Ψ, the budget set of every consumer is $\{z \mid \Psi(z) \leq 0\}$.

A fully non-linear admissible tax system (FNL ATS) is then naturally associated with a function Ψ, a price vector $\pi \in \mathbb{R}^{|L_1|}$ a price vector p and two measurable functions $z^A(\Theta \to \mathbb{R}^n)$, $z^B(\Theta \to \mathbb{R}^{|L_1|} \times \{0\})$ such that :

1) The couple $(z^A(\theta), z^B(\theta) = (z_1^B(\theta), 0))$ is a solution of

$$\text{Max } U(z, \theta),\ z = z^A + (z_1^B, 0),\ \Psi(z^A) \leq 0,\ \pi \cdot z_1^B \leq 0,\ z \in Z(\theta)$$

2) $\bar{y} = \int_\Theta z^A(\theta)\, d\mu \in Y$

3) $p \cdot \bar{y} \geq p \cdot y,\ \forall y \in Y$

4) $\int_\Theta z_1^B(\theta)\, d\mu = 0$

We let the comments to the reader, who will check that the definition translates well the idea that a reallocation of commodities L_1 between consumers takes place according to the competitive mechanism after the implementation of the non-linear tax system.

We let also to the reader to define a precise concept of outcome of a FNL ATS (in a way similar to what has been done in section II, for outcomes of ADAM and ATS).

The "equivalence" between FNL ATS and ATS takes the precise following form :

♦ *Theorem 3 :*

Given any outcome of a FNLATS there exists an ATS which has an identical outcome.

♦ *Proof*

Rather than proving the statement directly, we will show that it derives straightforwardly from the previous theorems.

Takes an FNLATS Ψ, π, p, *and the outcome associated with the measurable functions* z^A , z^B . *Takes the function* F *defined by* $F(\theta) = z^A(\theta)$, $\forall\ \theta \in \Theta$. *We first claim that* F *is an ADAM such that* $\Pi^F \ni \pi$ *(come back to the definition and check that* $\pi \in \Pi^F$ *and that* θ *is an optimal strategy); and* $z(\theta) = F(\theta) + z^B(\theta)$ *is an outcome of the ADAM. According to theorem 2,* $z(\theta)$ *is also an outcome of some ATS.*

Q.E.D. ▪

As announced, by imposing linear taxes on commodities L_1, we dot not restrict the power of the Planner.

III.B. - The observability of characteristics : the case of partial observability

We are going to suppose in this subsection that some coordinates of the vector of characteristics are observable. Writing the vector of characteristics $\theta' = (\underline{\theta}\ ,\ \theta)$, we assume that $\underline{\theta}$ is observable when θ is not. The concepts of games and tax systems that we introduce now take into account the possibilities offered by this new information both for the designer of the game and the designer of the tax system.

We successively consider :

a) 2 steps ADAM conditional to the observation

The observation of $\underline{\theta}$ allows the Planner to make the outcome of the first step dependant upon $\underline{\theta}$. We will then associate the game with a function F depending upon $(\underline{\theta}, \theta)$ and measurable in $(\underline{\theta}, \theta)$. Associated with such a function F, the second step set of price equilibria is defined exactly as in section II. The maximization problem of the consumer has to be indexed by $\underline{\theta}$, the vector of characteristics which cannot be hidden.

This point being made, the program is formally the same as previously and is denoted $P(F, \pi, \underline{\theta}, \theta)$.

Then, a 2 steps game F is a direct admissible mechanism based on observable characteristics (DAM BOC) if and only if :

1) $\exists\ \pi \in \Pi^F$ s.t. $\hat{\theta} = \theta$ is a solution of the programm $P(F, \pi, \underline{\theta}, \theta)$, $\forall\ (\underline{\theta}, \theta) \in \Theta$

2) $\int_{\Theta} F(\underline{\theta}, \theta)\, d\mu \in \mathrm{FrY}$ where μ denotes the measure on the set Θ of $\theta' = (\underline{\theta} \times \theta)$.

b) Tax systems conditional to the observation :

Observing $\underline{\theta}$, it is possible to make the function φ, which describes the non-linear part of the tax system as a function of $\underline{\theta}$. We write it down $\varphi(\underline{\theta}, z_2)$.

The only change in the formal writing of the consummer program is the presence of $\underline{\theta}$ both in the utility function and the budget constraint. The program is denoted then $P(q, \varphi, \underline{\theta}, \theta)$ where $\underline{\theta}$ intervenes both in the utility function and in the budget constraint .

Its solution is denoted $D(q, \varphi, \underline{\theta}, \theta)$

Then,

an admissible tax system based on observable characteristics (ATS BOC) is defined as a triple (p, q, φ) such that :

1) $\exists$ $z(\underline{\theta}, \theta)$ a measurable selection of $D(q, \varphi, \underline{\theta}, \theta)$ such that
$$\bar{y} = \int_{\Theta} z(\underline{\theta}, \theta)\, d\mu \in Y \quad ((\underline{\theta}, \theta) \in \Theta)$$

2) $p \cdot \bar{y} \geq p \cdot y \,, \forall y \in Y$

It is a matter of routine inspection, which is left to the reader, to check that the argument of Theorem 1 and Theorem 2 can be adapted in order to obtain

♦ *Theorem 4*

Given any outcome of an ATS BOC, (resp. DAM BOC), there exists a DAM BOC (resp. ATS BOC) which has the same outcome.

Hence, increased observability enriches in the same way the class of allocations attainable by games or through taxation.

To come back to Franck Hahn's discussion [1973], it is certain that the fact that somebody is a knight or a baron is a priori a relevant information in the sense that its availaibility enriches the set of attainable allocations (both for games and tax systems). However there is no guarantee at all that this enlarged set contains first best allocations. There is even no guarantee that this information should be utilized at the optimum. In fact the dependence of the optimal allocation upon $\underline{\theta}$ is desirable only if there is correlation between observed characteristics and Planner's objectives.

A second important remark is that the knowledge of the set of Admissible Tax Systems Based on Observable Characteristics can be inferred from the knowledge of the conditional distributions $\mu(\theta / \underline{\theta})$.

III.B. - The restricted domain assumption

The analysis has untill now explicitely supposed that the distribution of characteristics (or just the distribution conditional to observation) were public information.

Such an assumption makes possible a fair comparison of tax systems and game forms. In fact designers of tax systems do not generally make explicit reference to the distribution of characteristics. The information needed for the knowledge of the set of admissible tax systems is extracted through various procedures (in particular econometric estimates of aggregate demand). Such an information on demand is not équivalent to the knowledge of the distribution of characteristics but could certainly be derived from it.

It should however be clear that if game forms and tax systems are equivalent in the sense precised above for any Planner who knows the distribution of characteristics, the implementation of the game form does not require this knowledge. In other words, the game form is powerful enough to extract the information on the distribution. Still in other words, instead of defining the above games on the restricted domain where the distribution of characteristics is known, we can extend it to a universal domain.

There is no difficulty in this extension and we will only sketch briefly the argument.

Coming back on the definition of mechanisms, we have to make the outcome of the first step game a function $F : \Theta \times \mathcal{M} \to \mathbb{R}^n$, where $\mathcal{M}$ is the set of probability measures on Θ .

Given F, we can define the set of second step price equilibria conditional to truthful answers in the first step. We denote it Π^F_μ where F_μ is the restriction of F to a given μ . Formally

$\pi \in \Pi^{F_\mu}$ if and only if

i) $\exists\ z^B(\theta, \bar{\mu}) = (z_1^B(\theta, \bar{\mu}), 0)$ measurable in (θ), such that $z_1^B(\theta, \bar{\mu})$ is the solution of $\text{Max } U((z_1^B, 0) + F(\theta, \bar{\mu}), \theta)$
$\pi \cdot z_1^B \leq 0, \quad z^B + F(\theta, \bar{\mu}) \in Z(\theta), \quad \forall\ \theta \in \Theta$

ii) $\int_\Theta z_1^B(\theta, \bar{\mu})\, d\bar{\mu}(\theta) = 0$

It is easy to check that the only change to introduce in the consumer program is the dependance of F upon μ which is in no way affected by the answer of the consumer; this program is denoted $P(F, \mu, \pi, \theta)$.

Then an Anonymous Direct Admissible Mechanism with Universal Domain (ADAMUD), associated with a function $F : \Theta \times \mathcal{M} \to \mathbb{R}^n$, measurable in θ satisfies

1) $\forall\ \theta \in \Theta,\ \mu \in \mathcal{M}, \exists\ \pi \in \Pi^{F\mu}$ such that θ is a solution of the program $P(F, \mu, \pi, \theta)$

2) $\int_\Theta F(\theta, \mu)\, d\mu(\theta) \in \text{Fr } Y, \quad \forall\ \mu \in \mathcal{M}$

Also in the definition of ATS given above, we can make explicit the reference to the distribution of characteristics which is assumed to hold. Without a need for a new definition we will speak about μ - Admissible Tax Systems or also μ - ATS.

It is straightforward to note that if F defines an ADAM UD, F_μ the restriction of F is an ADAM for the class of environments for which the distribution of characteristics is μ.

Calling Λ the set of measurable functions $\Theta \to \mathbb{R}^n$, we can formally define :

An <u>outcome</u> of an ADAMUD (F) is a <u>mapping</u> 0
$0 : \mu \in \mathcal{M} \to 0(\mu) \in \Lambda$ s.t. $0(\mu)$ is an outcome of the ADAM F_μ

A. <u>selection</u> in the set of outcomes of μ- ATS is a mapping
$M : \mu \in \mathcal{M} \to M(\mu) \in \Lambda$ s.t. $M(\mu)$ is an outcome of some μ ATS

The "equivalence" between tax systems and games takes the following form :

♦ <u>*Theorem 5*</u>

Any outcome of an ADAMUD is a selection in the set of outcomes of μ-ATS. Reciprocally any selection in the set of outcomes of μ.ATS is the outcome of an ADAMUD.

<u>*Proof*</u>

For the first part, take an outcome 0 of the ADAMUD F . We have noted that given μ, F_μ is an ADAM and $0(\mu)$ which is an outcome of F_μ is according to Theorem II an outcome of some μ- ATS. Looking back at the definition of a selection of outcomes of μ - ATS, we can conclude

For the second part, considering a selection M , we know from Theorem I that $M(\mu)$ is the outcome of some ADAM that we can denote φ_μ Putting $F(\theta,\mu) = \varphi_\mu(\theta)$, we define an ADAMUD of which M is an outcome.

Q.E.D.

The result extend as the reader will check, to the case of partial observability. From the statement, we can obtain a clear understanding of the nature of the superiority of the abstract game forms over tax systems. In particular the tax designer can decide upon the "best" tax system (with respect to some social objectives only when he knows enough to be able to describe the set of μ ATS, (for example, if he knows μ)when the game form "controller" can implement something equivalent to optimal taxes whatever the unknown distribution μ which prevails.

IV - COMING BACK ON THE ANONYMITY AND CONTINUUM ASSUMPTIONS

The preceding analysis made clear the relationship between abstract games and tax systems. However, two assumptions at least deserve further discussion.

1) We restricted the so called "sophisticated" Planner to use anonymous mechanisms; what happens if we suppress this restriction ?

2) We considered economies with a continuum of agents in such a way that no agent's answer was able to affect the distribution of characteristics; what happens in finite but large economies ?

I do not intend to give a full treatment of both problems. For the first one a rather systematic argument will be presented but it will remain in a looser form than it could be. Only a sketch of an argument will be given to answer the second question.

IV.A. - Coming back on the anonymity assumption

If we give up the anonymity assumption, we have to introduce some precise definitions in order to be able to describe the most general game forms.

First, let be more precise on A. $A = [0,1]$ is here a measure space associated with the σ-algebra of borelians and with the Lebesque measure α.

Given a profile of characteristics (that we define as a function $\underset{\sim}{\theta} : A \to \Theta$), we associate :

$\omega = \prod_{a \in A} \underset{\sim}{\theta}(a)$ that we call a trajectory of characteristics.

The set of a priori possible trajectories is denoted Ω. Clearly this set is analogous to the basic space of states of nature of probability theory. Up to this point we associate a σ-algebra S on Ω.

Furthermore, we define an equivalence relation on trajectories as follows : ω_1 and ω_2 are said to be equivalent if they coincide but possibly on a subset of measure zero of A. The set of equivalence classes associated with this relation is denoted $\underset{\sim}{\Omega}$ and an element is noted $\underset{\sim}{\omega}$.

The most general type of game forms (for direct games) is described by a function Ψ $\quad \Psi : A \times \Theta \times \underset{\sim}{\Omega} \to \mathbb{R}^n$.

Such a function associates with every agent whose "name" a is observable by definition a net trade depending upon his announced vector of characteristics and the profile of announced characteristics of others (up to the equivalence relation defined previously).[16)]

Take now a given trajectory $\omega = \prod_{a \in A} \underset{\sim}{\theta}(a)$; note $\underset{\sim}{\omega}$ the equivalence class of ω in $\underset{\sim}{\Omega}$. If the game Ψ induces truthful revelation, the second step game will determine a set of equilibrium prices denoted $\Pi^{\Psi,\omega}$ and defined as follows :

(16): In fact it would be more satisfactory to define Ψ on a set bigger than Ω. However this definition will be satisfactory for our purpose since the class of truthfully implementable mechanisms that we consider will induce profiles of responses in $\underset{\sim}{\Omega}$.

$\pi \in \Pi^{\Psi,\omega}$ if and only if 17)

i) $\exists\, z_1^B : A \times \Theta \to \mathbb{R}^{|L_1|}$ measurable such that $z_1^B(a\,,\,\theta)$ is a solution of :

$$\text{Max } U((z_1^B,\, 0) + \Psi(a,\, \theta, \underset{\sim}{\omega}),\, \theta)\ ,\ \pi\,.\, z_1^B \leq 0,\ (z_1^B,\, 0) + \Psi(a,\, \theta, \underset{\sim}{\omega}) \in Z(\theta).$$

ii) $\int_A z_1^B(a,\, \underset{\sim}{\theta}(a))\, d\alpha$ exists and equals to zero.

In principle $\int_A f\, d\alpha$ denotes the Lebesque integral with respect to the Lebesque measure α . However, in the following we will consider cases in which the function $\underset{\sim}{\theta}$ is not necessarily a measurable function of a; we will refer then to an unusual concept of integral.

Note also that when z_1^B does not depend upon a , $\int_A z_1^B(\underset{\sim}{\theta}(a))d\alpha$ equals $\int_\Theta z_1^B(\theta)\, d\mu$ where $\mu = \alpha \circ \underset{\sim}{\theta}^{-1}$ is the probability measure induced on Θ by the profile $\underset{\sim}{\theta}$.

We let to the reader to define the consumer program which depends now on his name and on $\underset{\sim}{\omega}$; it is denoted $P(a\,,\, \Psi, \pi\,,\, \underset{\sim}{\omega},\, \theta)$.

A <u>General Direct Admissible Mechanism</u> (GDAM) associated with a function Ψ satisfies :

1) $\forall\, \omega \in \Omega, \exists\, \pi \in \Pi^{\Psi,\omega}$ such that θ is a solution of the program $P(a,\, \Psi,\, \pi,\, \underset{\sim}{\omega},\, \theta)$, $\forall\, a \in A$, $\forall\, \theta \in \Theta$.

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(17): Note that this definition holds for a given trajectory ω and that the function z_1^B depends on ω although it is not explicitely indexed.

2) $\forall\, \omega = \prod_{a \in A} \underset{\sim}{\theta}(a) \in \Omega \quad , \quad \int_A \Psi(a, \underset{\sim}{\theta}(a), \underset{\sim}{\omega})\, d\alpha \in \text{FrY}$

As we saw above, the anonymity assumption of the previous section had two different dimensions which we referred to as Recipient Anonymity and Anonymity in Influence.

If we consider a function Ψ which does not depend upon a, the general mechanism (GDAM) reduces to what we call a <u>Direct Admissible Recipient Anonymous Mechanism</u>. (DARAM).

If Ψ depends only upon $\underset{\sim}{\omega}$ through the induced distribution $\mu = \alpha \circ \underset{\sim}{\theta}^{-1}$, then we call the mechanism a Direct Admissible Mechanism Anonymous in Influence (DAMAI). (When a mechanism is both a DARAM and a DAMAI it is then an ADAMUD).

GDAM is the most general form which can be taken by a mechanism. What is the more general form taken by tax systems ? As we consider mechanisms depending upon the name of the agent (which is observable) it seems fair to consider tax systems which do depend upon a : they will be called <u>Non Anonymous Admissible Tax Systems</u> (NAATS).

The more natural idea to define NAATS is to transpose the definition of ATS BOC given above, the observable characteristic being now a, the name of the agent. However we will introduce another ingredient in order to take into account the fact that the tax system designer can only rely on a crude information (at most the distribution of θ).

A NAATS will be associated with a function φ describing non linear taxation depending upon (a, z_2), and the price systems p and q. The consumer program will be denoted $P(a, \varphi, q, \theta)$ and its set of solutions $D(a, \varphi, q, \theta)$.

We say that (p, q, φ) defines a NAATS with respect to the set of set of environments $\bar{\Omega}$ if and only if :

1) $\exists\, z(a, \theta)$ a measurable selection of $D(a, \varphi, q, \theta)$ such that $\bar{y} = \int_A z(a, \underset{\sim}{\theta}(a))\, d\alpha$ is independant of $\omega = \prod_{a \in A} \underset{\sim}{\theta}(a) \in \bar{\Omega} \subset \Omega$ and $\bar{y} \in \mathrm{Fr}\ Y$

2) $p \cdot \bar{y} \geq p \cdot y, \quad \forall\, y \in Y$

In 1) the important change with respect to previous definitions is that we make clear that feasibility must hold in some given class of environnments which is not (necessarily) the class of environments with fixed distribution of characteristics.

We are now in position to come back to our basic comparison of the (ultra) sophisticated Planner relying on GDAM and the (partly) unsophisticated Planner relying on NAATS. We will proceed in two steps : in the first one we will make some basic remarks and statements; in the second step one will exhibit a class of economic environnments and initial information in which NAATS are as good as GDAM.

IV. A. Preliminary analysis

This preliminary analysis will consist of a certain number of remarks.

1) Consider a mechanism which is recipient anonymous. I will argue that there is no loss to make it anonymous in influence for a planner who has a social welfare function which is anonymous. In order to define anonymity of the social welfare function, consider a utility profile $V : \theta \in \Theta \rightarrow V(\theta) \in \mathbb{R}$ where $V(\theta)$ is the (expected) utility of agent of characteristic θ. The social welfare function W is anonymous if it can be written down $W = \Gamma(V)$ where Γ is a functionnal defined on the set of utility profiles.

Now take Ψ a mechanism which is recipient anonymous i.e. precisely which is a DARAM. We can assert the following :

Assertion : [18]

Let Ψ be a DARAM on the set of environnements Ω ; suppose that the Planner has an anonymous social welfare function. Then given any expected social welfare level associated with an outcome of Ψ, there exists an ADAMUD (whose "universal" domain is Ω) which gives (at least) the same social welfare.

We will sketch an heuristic proof from which a rigorous although more lenghty argument could be obtained :

We note successively

1) *If Ψ is a DARAM, then to any $\omega \in \Omega$, the function $F^{\omega} : \theta \in \Theta \to \Psi(\theta, \tilde{\omega})$ defines an ADAM whose domain is the class of environments with fixed distribution* $\mu_{\omega} = \alpha_0 \tilde{\theta}^{-1} \; (\omega = \prod_a \tilde{\theta}(a))$

2) *Let* $\mathcal{L}(\mu) = \{\omega \in \Omega \mid \mu_{\omega} = \mu\}$

Take an outcome of Ψ : $z(\theta, \tilde{\omega})$ and define $V/\tilde{\omega}$ by $V/\tilde{\omega}(\theta) = U(z(\theta, \tilde{\omega}), \theta)$.

We can define $W(\Psi|\tilde{\omega}) = \Gamma(V/\tilde{\omega})$

Taking $\operatorname{Max} W(\Psi|\tilde{\omega})$, $\omega \in \mathcal{L}(\mu)$, *and supposing that it has a solution $\omega(\mu)$, we can consider $F^{\omega(\mu)}$. When μ varies, it is an ADAM with domain Ω and $z(., \tilde{\omega}(\mu))$ is an outcome which gives a higher expected social welfare that $z(., \tilde{\omega})$.*

■

The intuition behind the above argument is clear : the choice of a DARAM which is not an ADAM amounts to relate the outcome with the realization of $\tilde{\omega}$ and hence to randomize it in the subclass of environments

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18) : The difference between "assertion", "theorems" and proposition holds in the fact that the proof given is partly heuristic Note that with a compact $\mathcal{L}(\mu)$ the proof is immediately made rigorous.

with a given distribution of θ . There are no advantages but only inconveniences to this ex ante randomization.[19)]

2) Hence for a recipient anonymous mechanism it is in some sense useless to drop anonymity in influence. What about mechanisms which are not recipient anonymous ? Several types of remarks are in order :

i) First, it may be useful even for a Planner which has an anonymous social welfare function to give up the requirement of recipient anonymity. It is known for example from Atkinson-Stiglitz [1980] that "equal treatment of equals" is often non optimal even with utilitarian social welfare functions.

ii) Non Anonymous admissible tax systems can be associated with DAMAI in the conditions given by the following proposition.

♦ *Proposition 1*

Let $\mu \in \mathcal{M}$ be a probability measure on Θ. Let $\Omega_\mu \subset \Omega$ be the subset of trajectories such that $\alpha \circ \underset{\sim}{\theta}^{-1} = \mu$

Then if $\Lambda = (p, q, \varphi)$ is a NAATS defined on Ω_μ and if Z $Z : (A \times \Theta) \to \mathbb{R}^n$ is an outcome of Λ then there exists a DAMAI which has the same outcome.

The proof is only a variant of the proof of theorem I and is left to the reader.

iii) However the converse is not true : an outcome of a DAMAI, even restricted to a set of trajectories Ω_μ with given distribution of θ, is not necessarily the outcome of some NAATS (as NAATS have been defined here). Without developing this point in detail, there is one obvious reason why DAMAI are not NAATS.

First, a DAMAI has to meet the following condition
$\forall \omega = \prod_A \underset{\sim}{\theta}(a) \in \Omega_\mu$, $\int \Psi(a, \underset{\sim}{\theta}(a))\, d\alpha \in \mathrm{FrY}$ when we have assumed that outcomes z of tax systems satisfy :

$\int z(a, \underset{\sim}{\theta}(a))\, d\alpha = \bar{y}$ $(\in \mathrm{FrY})$ which is more restrictive (the comparison applies when $L_2 = \emptyset$).

(19): This has not to be confused with the fact that it is in general interesting to use random mechanisms giving an outcome depending on some probability distribution.

Second, for a DAMAI Ψ it is not necessarily true that $\Pi^{\Psi,\omega} = \bar{\pi}$ $\forall\, \omega \in \Omega_\mu$, when this latter independance would be necessary to expect the property that we are discussing. In other words, in a subclass of trajectories of type Ω_μ the implicit taxes which are associated with some DAMAI may vary with the trajectory instead of being independant of it.

ii) The fourth remark focuses on the fact that there are class of environments where obviously mechanisms are superior to tax systems from the welfare point of view : take for example the polar case where it is known a priori that the trajectories of the stochastic process are (almost surely) continuous. Then the knowledge of $\underset{\sim}{\omega}$ the equivalence class of the trajectory implies (almost surely) the knowledge of $\theta(a)$. Hence the Planner knowing the truth for everybody but a, also knows the true characteristics of a and he can impose the allocation he likes (not taking into account the second step trades). In this case a first best solution is certainly attainable through mechanisms when it may be outside the reach of tax systems. <u>More generally, if there are only correlations between the θ of others and a's θ, they can be exploited in the design of mechanisms when they cannot be in the design of tax systems</u>.

For that reason, one can expect similar performances of tax systems and mechanisms only in the case where there is some kind of independance between the (random) trajectory $\underset{\sim}{\theta}$ restricted to $A/\{a\}$ and the (random) variable $\theta(a)$. This independance is one of the ingredients of the class of environments considered in the next subsection.

IV.B. <u>On a class of environments where non anonymous Tax Systems are "as good" as General Mechanisms</u>

Besides the independance requirement, just mentioned, it is necessary to define tax systems on a domain large enough in order to make them comparable to mechanisms. In other words, the fact that we cannot incorporate in the design of tax systems any information on the precise trajectory has to be counterbalanced by the fact that we are sure of strong compensations along the trajectory. Both requirements are satisfied when we consider the following set of environments.

i) For each $a \in [0\ ,\ 1]$, $\theta(a)$ is a random vector drawn from Θ according to the law given by the probability measure μ.

ii) Any sequence $(\theta(a_1), \ldots, \theta(a_n))$ is a sequence of independent random variables.

A standard argument based on the Kolmogorov theorem that the reader will find in Bewley [1980] shows that there actually exists such a stochastic process. (Which defines a "continuum" of independent random variables).

However generally the profile $\underset{\sim}{\theta}$ as a function of a is badly behaved. It has even no reason to be measurable and hence $f(\underset{\sim}{\theta}(a))$ may not be measurable even if f is and the Lebesque integral $\int_\Theta f(\underset{\sim}{\theta}(a))\, d\alpha$ does not exist.

It makes sense to posit here a definition of the integral such that if the mapping $f : (A \times \Theta) \to \mathbb{R}^n$ is measurable; then $\forall\omega = \prod_{a^\sim} \theta(a)$,

$$\int_A f(a, \underset{\sim}{\theta}(a))\, d\alpha \overset{\text{def}}{=} \int_A \left(\int_\Theta f(a\ ,\ \theta)\ d\mu\ (\theta)\right)\ d\alpha$$

According to Fubini theorem, the right hand side also equals $\int_\Theta \left(\int_A f(a\ ,\ \theta)\ d\alpha\ \right)\ d\mu(\theta)$.

This is a definition the intuitive justification of which has to be found in the fact that in any interval $I(a)$ around a, as small as it may be, the empirical distribution of θ found by taking a large sample $\theta(a_1), \ldots \theta(a_n)$, $a_1, \ldots, a_n \in I(a)$ tends according to the law of large numbers towards μ.

When the uncertainty is of this form, in order to check whether a tax system or mechanism is admissible, it is enough to know μ.

So, in the above definition of a Non Anonymous Admissible Tax System (NAATS), (1) changes in (1'):

(1') $\exists$ a measurable selection of $D(a, \varphi, q, \theta)$, $z(a, \theta)$ such that

$$\int_A z(a, \underset{\sim}{\theta}(a))\, d\alpha = \int_A \left[\int_\Theta z(a,\theta)\, d\mu(\theta) \right] d\alpha \in \text{FrY}$$

We can now prove :

Theorem 5

Consider a class of environments where the profile of characteristics is the realization of some stochastic process of a "continuum of independent and identically distributed random variables" in the sense defined above.

Then,

1) $Z : A \times \Theta \to \mathbb{R}^n$ is an outcome of some DAMAI Ψ on Ω if and only if it is the outcome of some NAATS with domain Ω.

2) Given any (random) outcome of any Direct Admissible Mechanism, there is a DAMAI an outcome of which gives a higher social welfare level, whatever the social welfare functional considered.

Proof (Sketch)

To prove (1), the "if" part results from the above proposition. For the "only if" part, we have to show :

a) $\forall \omega \in \Omega \ \Pi^{\Psi,\omega} = \bar{\Pi}$. The intuitive reason for that is clear. The formal argument is directly obtained from definition of $\Pi^{\Psi,\omega}$ and the definition of the integral $\int_A z_1^B (a, \underset{\sim}{\theta}(a))\, d\alpha$ given above

b) But then, we are in a position in which a variant of the argument of theorem 2 (as used in theorem 4) applies.

The proof of part 2 of the theorem closely follows the argument of the proof of the assertion p. 33. This is left to the reader who will note the technical details which have to be considered.

Finally let us note that we only have considered in this section direct mechanisms. The introduction of general indirect mechanisms is outside the scope of this paper, although the equivalence result of theorem 5 might be preserved.

IV.C. Economies with a large but finite number of agents

The results on the "welfare equivalence" of tax systems and mechanisms which hold in continuum economy are interesting only if we can find corresponding results in large but finite economies and point out how they relate asymptotically with the continuum economy results.

This is in itself a programm of research which deserves to be treated independently However keeping the basic informational assumptions of the preceding section (which make general mechanisms and non anonymous mechanisms equivalent), I will sketch a framework for the asymptotic comparison of mechanisms and tax systems and I will indicate the results which can be expected.

In the $E^{(r)}$ economy, there are r agents whose numbers a are $[\frac{1}{r}, \frac{2}{r}, \dots, \frac{r-1}{r}, 1]$. The characteristics θ of agent a are drawn at random from Θ with the probability distribution μ.

If we assume that the tax designer only knows the underlying distribution μ, he cannot know exactly the actual distribution in the economy $E^{(r)}$.

For that reason, feasibility should not be required for the definition of admissible tax systems; instead a tax system should be called admissible when the probability that the norm of excess demand exceeds some small number ε, is small enough (smaller than ε'). This would be called an $\varepsilon - \varepsilon'$ tax system.

Similarly, one should define $\varepsilon - \varepsilon'$ mechanisms where the feasibility constraint would be similarly relaxed.

A quick analysis suggest thats the following results can be expected.

. Given an admissible tax system in the limit economy there is a corresponding $\varepsilon - \varepsilon'$ tax system in $E^{(r)}$ for r large enough. The same holds for admissible mechanisms and $\varepsilon - \varepsilon'$ mechanisms.

. An outcome of some $\varepsilon - \varepsilon'$ tax system in $E^{(r)}$ is an outcome of some $\varepsilon - \varepsilon'$ mechanism but the converse does not longer hold.

Among the interesting questions are how far the outcome of an $\varepsilon - \varepsilon'$ mechanisms are from the outcome of some $\varepsilon - \varepsilon'$ tax system in $E^{(r)}$ or how larger is the set of $\varepsilon - \varepsilon'$ mechanisms when compared with $\varepsilon - \varepsilon'$ tax systems. Also the relationship between 0 - 0 mechanisms ("exact" mechanisms), $\varepsilon - \varepsilon'$ tax systems and mechanisms in the limit seems particularly worth of being studied.

7 - CONCLUSION

In conclusion, the literature on the Clarke-Groves-Vickrey mechanisms strongly suggests that with enough cleverness in the design of mechanisms one can extract much information and solve (at least in theory) problems which have been for a long time considered as unsolvable (as the free rider problem). To some extent the results obtained here in a different context go the other way : the usual taxation schemes allow to do as well as the more sophisticated incentives devices, at least when a minimal amount of information (here the distribution) is available to the Center. However this conclusion is relativized by the analysis of section IV where mechanisms become again more powerful when they are allowed to be at least non anonymous and when stochastic properties of the unknown environments do not fall in a particular class.

One can also try to summarize unformally the way taxation systems fit into general incentives mechanisms.

. Tax systems are incentive compatible but their implementation requires a priori information which is not needed, since it could be in theory extracted, for the implementation of abstract direct mechanisms.

In rather similar words, a given tax system can allocate resources in a rather restricted set of environments when the adaptation to a larger set of environments is incorporated in the definition of a mechanism.

. Even when some taxation schemes are implicit to incentives mechanisms these schemes are more flexible than what is allowed by tax systems as defined in the usual sense of the word . This is a point made clear in Section IV where DAMAI may imply variable wedges between consumers and producers prices within a class of environments where tax systems introduce by definition invariable wedges.

. Mechanisms can incorporate information on the trajectories of characteristics and take it into account if it is relevant to the knowledge of individual characteristics when tax systems cannot. In fact it would be possible to introduce a more general notion of tax systems where <u>the individual budget set would depend</u> not only possibly on the name of the agent but also on <u>the profile of transactions of all others individuals in the society</u>. This would allow to equalize the performances of general mechanisms and non anonymous tax systems but this would no longer cover what is usually meant by the word tax systems.

Finally, it may be thought that the assumptions of section IV on the structure of the stochastic process governing trajectories are reasonable and consequently that optimal (non anonymous) tax schemes mark the limits to redistribution in the context of our problem.

However, it may be interesting to note that the type of correlations which has been described, even if it is unlikely in a consumption context may become realistic when the agents are producers. On the other hand, it should be kept in mind and it is a topics of further research, that the introduction of observability in the context of section IV, does not lead to an imediate generalization of the results (as in section III) in particular when there is a correlation between the random variables conditional to the observation. This correlation issue might not be hence so easily avoided.

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