Advanced Microeconomics III

Spence's Signaling Model

Francisco Poggi

Introduction

- **Akerlof's Market for Lemons**: asymmetric information can lead to inefficient market outcomes.
  - Buyers cannot distinguish good from bad sellers.
  - Good sellers might be driven out of the market (adverse selection).
- To overcome adverse selection, good sellers need a way to convey their type.
- **Signaling**: type can be conveyed, but only through indirect observable actions.

Introduction

- **Examples of signaling**:
  - *Warranties*: Firms use them to signal the quality of durable goods.
  - *Education*: Workers use it to signal their ability to employers.
  - *Advertising*: Companies use to signal product quality.

- **Key questions:**
  - How does signaling occur in equilibrium?
  - What are the welfare implications of signaling?

Spence's model

- **Agents**:
  - A single worker and multiple firms (at least 2).
- **Worker Types**:
  - $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L$.
  - Only the worker knows θ.
  - Firms assign probability q to type θ_H.
- **Production and Payoffs**:
  - If employed by a firm, worker produces output θ.
  - Firm's payoff:
    - $\theta - w$ if it employs the worker at wage w.
    - Zero otherwise.

Spence's model

- **Timing**:
  - Worker chooses education level $e \in [0, \infty)$.
    This is publicly observed by all firms.
  - Firms make wage offers to the worker.
  - Worker chooses a firm to work for.

- **Worker payoff** when having education e and employed at wage w:

$$u(w, e|\theta) = w - c(e|\theta)$$

  Where $c(e|\theta)$ is the cost of education.

- Note that education in this model is unproductive, i.e. it doesn't affect worker's output.

Spence's model

- **Assumptions on the cost of education**:
  - The cost of no education is zero.

$$c(0|\theta) = 0 \qquad \text{for all } \theta$$

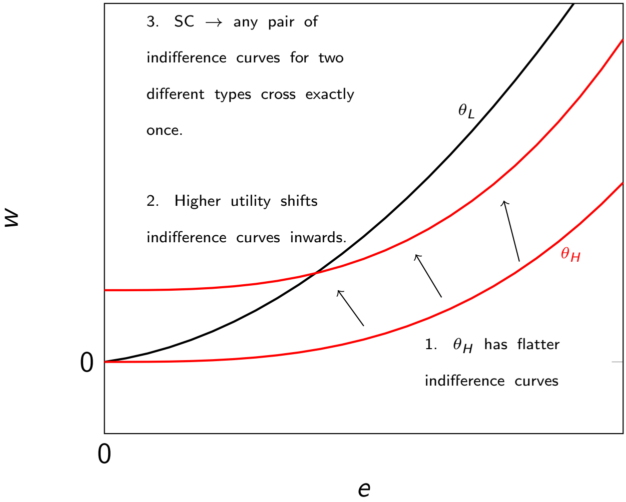
  - The cost of education is str. increasing and str. convex for all θ.

$$c'(e|\theta) > 0 \qquad \text{and} \qquad c''(e|\theta) > 0$$

  - The high-type worker has a smaller marginal cost of education.

$$c'(e|\theta_H) < c'(e|\theta_L) \qquad \forall e > 0 \qquad \text{(Single-crossing)}$$

Indifference curves



Solution concept

- **Solution concept**: Symmetric (Pure-strategy) Perfect Bayesian Equilibrium.
- Consists of:
  - A choice of education level for each worker type: e_L, e_H.
  - Firms' posterior beliefs about the worker being of type H: $\mu(e)$.
  - Wage offers of the firms: $w(e)$.
- Satisfying:
  - Optimal education choice given wage offers.
  - Consistent beliefs whenever possible.
  - Wage offers constitute a Nash equilibrium at each subgame.
  - Firms believe other firms conform to equilibrium wage offer $w(e)$ both on and off the equilibrium path.
- **Symmetry**: All firms hold the same beliefs after observing education. This is **Not** implied by weak PBE.

PBE analysis

- **Wage offers:**
  - Competition among firms leads to the following wage offers (why?):

$$w(e) = E_{\mu(e)}[\theta] = \mu(e) \cdot \theta_H + (1 - \mu(e)) \cdot \theta_L$$

- **Education:**
  - We distinguish two types of pure-strategy equilibria.
    - **Separating equilibria**: $e_H \neq e_L$.
    - **Pooling equilibria**: $e_H = e_L$.

Separating equilibria

- We start characterizing separating equilibria: $e_H \neq e_L$.
  - Bayes' rule where possible:

$$\mu(e_L) = 0 \qquad \mu(e_H) = 1$$

  - By competition:

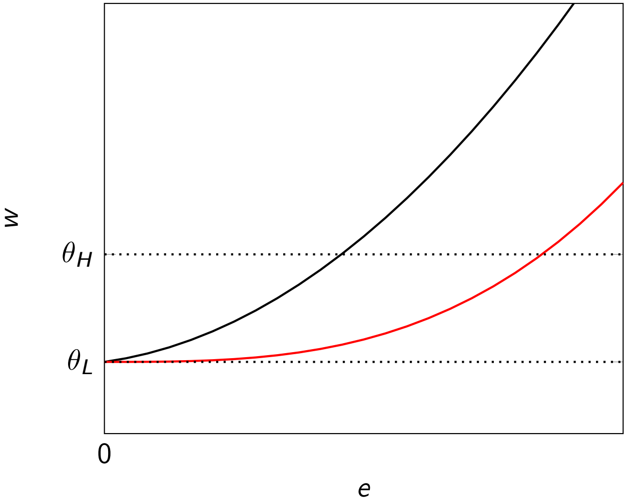
$$w(e_L) = \theta_L \qquad w(e_H) = \theta_H$$

Lemma

In any separating equilibrium, $e_L = 0$.

- PBE implies that $w(e) \in [\theta_L, \theta_H]$.
- So, if $e_L > 0$, the deviation to $e = 0$ is profitable for type θ_L.

Separating equilibria



Separating equilibria: incentive compatibility

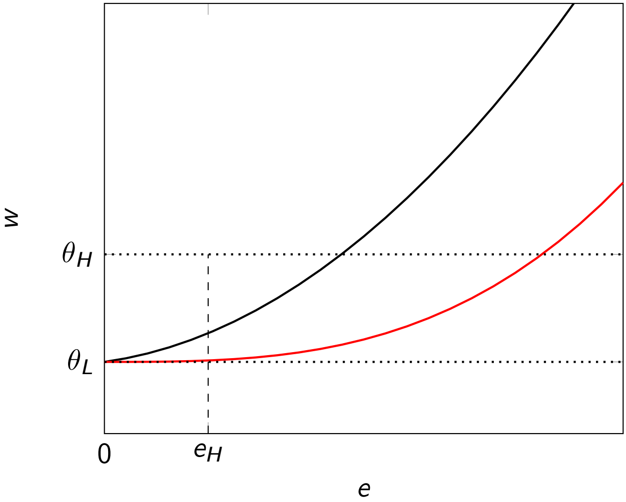
Lemma

In a separating equilibrium, type H chooses $e_H > 0$ such that

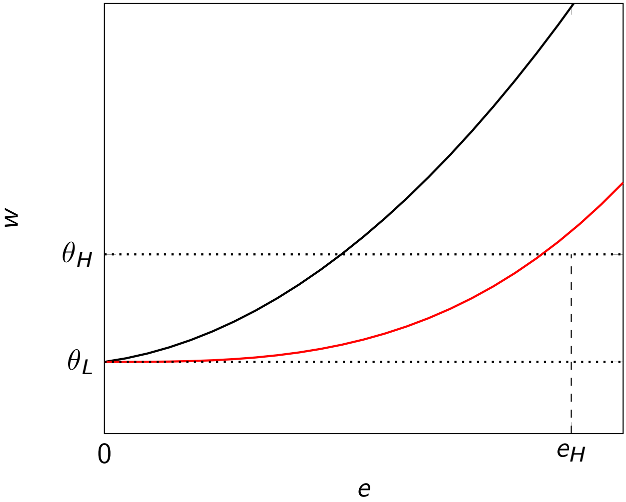
$$\theta_H - c(e_H|\theta_H) \;\geq\; \theta_L \;\geq\; \theta_H - c(e_H|\theta_L) \tag{IC}$$

- First inequality: type H prefers his education e_H rather than zero.
- Second inequality: type L prefers zero rather than e_H.

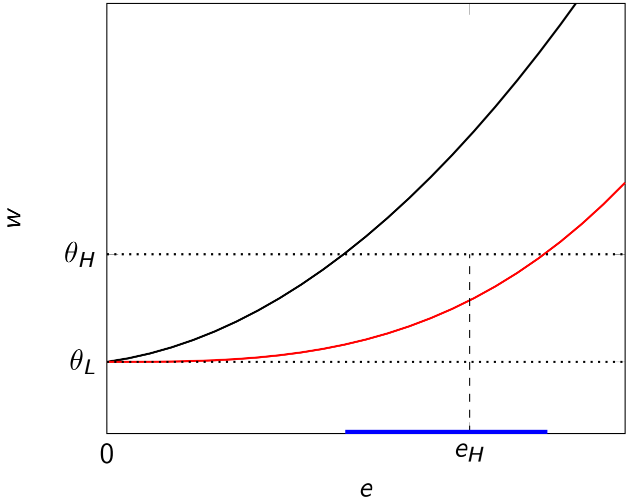
Separating equilibria: IC



Separating equilibria: IC



Separating equilibria: IC



Separating equilibria

- Previous lemmata describe *necessary* conditions for separating equilibrium.

- These are also *sufficient*: remains to specify out-of-equilibrium beliefs.
  - Deviations are considered to be by a low type: $\mu(e) = 0$ for all $e \neq e_H$.
  - Then, consistent wage is θ_L for any $e \neq e_H$.
  - Any deviation is unprofitable.

Equilibrium multiplicity

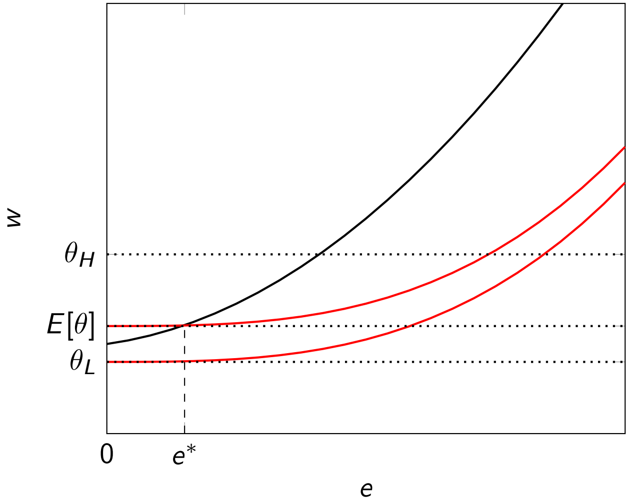
- There are **multiple** separating equilibria.
  - These equilibria can be Pareto ranked.
  - The best separating equilibrium has the lowest education e_H.

$$c(e_H|\theta_L) = \theta_H - \theta_L$$

Pooling equilibria

- Pooling equilibrium: $e_L = e_H = e^*$.
- Bayes' rule where possible: $\mu(e^*) = \Pr(\theta = \theta_H) = q$.
- Competition implies that $w(e^*) = E[\theta]$.
- Out-of-equilibrium beliefs: $\mu(e) = 0$ for $e \neq e^*$.
  - Then $w(e) = \theta_L$ for $e \neq e^*$.

Pooling equilibria



Multiple pooling equilibria

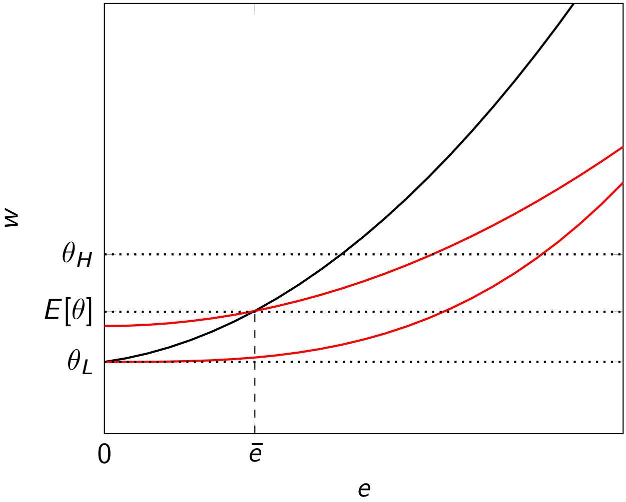
- There are multiple pooling equilibria.
- The best pooling equilibrium is the one with the lowest level of education ($e^* = 0$).

- What about the *worst* one?

$$E[\theta] - c(\bar{e}|\theta_L) = \theta_L$$

$$c(\bar{e}|\theta_L) = E[\theta] - \theta_L$$

Worst pooling equilibrium



Comparing pooling and deparating equilibra

- The best pooling equilibrium may or may not Pareto dominate the best separating equilibrium.
  - High types not always benefit from the availability of a signaling device. Only if their fraction is small enough.

- The best separating equilibrium *never* Pareto dominates the best pooling equilibrium.
  - The low type is always worse-off in a separating equilibrium.

Reasonable beliefs (equilibrium refinements)

- Which equilibrium is more likely to emerge?
  - Pareto dominance is not a game-theoretical argument.

- Forward induction arguments can be used to refine the equilibrium.
  - PBE allows for any beliefs off the equilibrium path.
  - Refinements put conditions on these off equilibrium beliefs.
  - Most refinements in this game uniquely select the least costly separating equilibrium.

Intuitive criterion

- Cho and Kreps (1987) 'Intuitive criterion':
  - Key question: Who might benefit from the deviation?

Definition

A deviation e' is dominated in equilibrium for type θ if, for any sequentially rational response by the receivers $w' = E_{\mu'}[\theta]$ for some beliefs μ', the resulting payoff $u(e', w', \theta)$ is less than the equilibrium payoff $u(e(\theta), w(e(\theta)), \theta)$.

Definition

A PBE *passes the Intuitive Criterion Test (ICT)* if no type θ would be better off deviating to an action $e' \neq e(\theta)$ should the receivers' beliefs following e' assign zero probability to types θ' for whom the deviation is *dominated in equilibrium*.

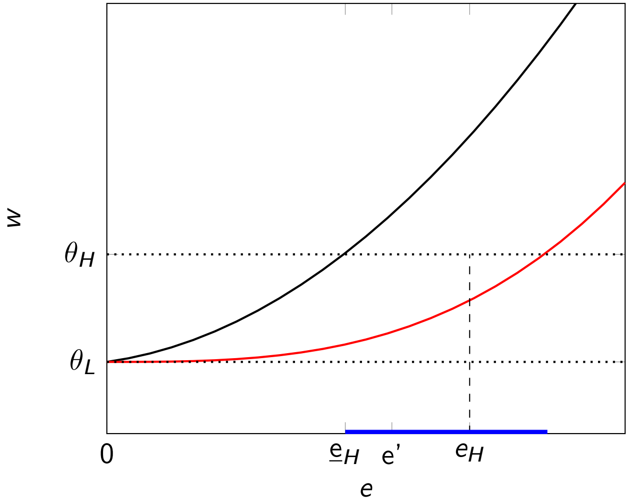
Intuitive criterion: separating equilibrium

- Let $\underline{e}_H$ be the minimal high-type education that can be sustained in a separating equilibrium.
- Starting from a separating equilibrium with $e_H > \underline{e}_H$, we show that ICT is violated.
  - Consider a deviation to $e' \in (\underline{e}_H, e_H)$ (This is off the equilibrium path).
  - A type θ_L can guarantee a payoff of θ_L by following equilibrium strategies. The deviation can bring type θ_L at most:

$$\theta_H - c(e'|\theta_L) < \theta_L$$

  - Thus, a type θ_L would never deviating to e'. Formally e' is dominated in equilibrium for type θ_L.
  - The PBE does not pass the ICT: If $\mu(e') = 1$, type θ_H would benefit from deviating to e'.

Intuitive criterion: separating equilibrium



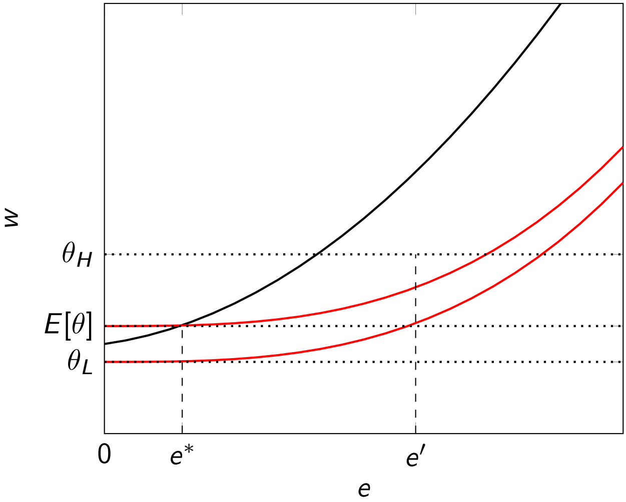
Intuitive criterion: pooling equilibrium

- Let start instead from a pooling equilibrium at e^*.
- **Claim**: there exists e' such that

$$E[\theta] - c(e^*|\theta_H) < \theta_H - c(e'|\theta_L) < E[\theta] - c(e^*|\theta_L)$$

- Deviating to e' is dominated in equilibrium for type θ_L.
- Thus, the pooling PBE does not pass the ICT.
  - If $\mu(e') = 1$, type θ_H would benefit from deviating to e'.

Intuitive criterion: pooling equilibrium



Intuitive criterion

- Only the best separating PBE passes the ICT.
- Notice that sometimes *forced pooling* generates a Pareto improvement.
    - In particular, when the share of high types is sufficiently large.
- Another Pareto improvement can arise with *cross-subsidization*.

Model with continuum of types

- Consider a model with a continuous of types.
  - Support in $[\underline{\theta}, \bar{\theta}]$.
  - Density function f str. positive everywhere in the support.

- **Question**: Is there a separating equilibrium? Is it unique?
  - Parametric assumption: $c(e|\theta) = \alpha \cdot e^2/\theta$.

Empirical evidence

- Bedard (2001) “Human Capital Versus Signaling Models”
  - Study education as a signal of ability, exploiting the effect of constraining access to university in high school graduation levels.
  - **Empirical finding**: Regions with universities have higher high-school drop-out rates.
    - Difficult to explain in a model of human capital.
  - **Signaling explanation**:
    - With no university nearby, more high-ability students stop their education after completing high-school.
    - Low-ability students have incentives to finish high-school to pool with high-ability students.
  - **Policy implications**:
    - Improving access to university might increase drop-out rates and depress wages for some kids.

Other models related to signaling

- Evidence and voluntary disclosure of verifiable information. Grossman (1981) Milgrom (1981) Dye (1985)

- Costless signaling (cheap talk): might work if preferences between sender and receiver are partially aligned. (Crawford Sobel (1982))

Classical evidence models

Seminal model developed by Grossman (1981) and Milgrom (1981)

- Similar to the previous model.
  - One worker, more than 2 firms.
  - Worker has private type θ with cdf F.
  - Firms compete offering wages.

- Instead of choosing a level of education, worker can take a (free) test that perfectly reveals his type.
  - Formally, worker can send a message in $\{\emptyset, \theta\}$.
  - Firms observe the message before making wage offers.

Unraveling

- Let $w(m)$ be the wage that firms offer to an agent that sends message m.
- Let $\Theta_\circ$ be the subset of types that chooses the empty message in equilibrium.
- **Claim**: almost all types take the test: $\Theta_\circ \subseteq \{\underline{\theta}\}$
    - Suppose that $w(\emptyset) > \underline{\theta}$.
    - It must be that $\Theta_\circ = [\underline{\theta}, w(\emptyset))$
    - $w(\emptyset) = E[\theta|\Theta_\circ] < w(\emptyset)$. Abs!
    - So $w(\emptyset) = \underline{\theta}$ and $\Theta_\circ \subseteq \{\underline{\theta}\}$.

Partial unraveling

- Dye (1985) and Jung and Kwon (1988): Worker has evidence with some probability λ, and no evidence otherwise (independent of type).

- **Partial unraveling**:
  - Let w be the wage for a worker in the absence of evidence.
  - Any type with $\theta < w$ will not present evidence.
  - Equilibrium w is the unique solution to:

$$\begin{aligned} w \quad = \quad E[\theta|m=\emptyset] \quad &= \quad E[\theta|\text{ no evidence or } \theta < w]. \\ &= \quad q(w)\cdot E[\theta] + (1-q(w))\cdot E[\theta|\theta < w] \end{aligned}$$

where $q(w) = Pr(\text{ no evidence } | \text{ no evidence or } \theta < w)$.

Partial unraveling

- Example: $\theta \sim U[0, b]$.

$$q(w) = \frac{p}{p + (1-p)F(w)} = \frac{p \cdot b}{p \cdot b + (1-p) \cdot w}$$

- So,

$$E[\theta | m = \emptyset] = \frac{p \cdot b}{p \cdot b + (1-p) \cdot w} \cdot \frac{b}{2} + \frac{(1-p)w}{p \cdot b + (1-p) \cdot w} \cdot \frac{w}{2}$$

- Solving $E[\theta | m = \emptyset] = w$ we get

$$w = \frac{\sqrt{p} \cdot b}{1 + \sqrt{p}}$$