A Theory of Auditability for Allocation Mechanisms

By Aram Grigoryan and Markus Möller

Auditability Measure for Allocation Mechanisms

- **Allocation Environment:**
  - **Individuals:** $\mathcal{I}$
  - **Types:** $\Theta \subseteq \times_{i \in \mathcal{I}} \Theta_i$
  - **Outcomes:** $\Omega \subseteq \times_{i \in \mathcal{I}} \Omega_i$

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Fix ϕ and a group $I \subseteq \mathcal{I}$ with type profile θ_I.
Group I **does not detect** a deviation $\omega \in \Omega$ iff

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Auditability Measure for Mechanism ϕ

Smallest N such that, for every type profile $\theta \in \Theta$ and deviation $\omega \in \Omega$ with $\omega \neq \phi(\theta)$, there is a group of size $n \leq N$ that can detect it.

Illustrative Examples: Single-Object Auctions

- Enviroment:
  - $\Theta_i \subseteq \mathbb{R}_+$.
  - $\Omega_i = \{0, 1\} \times \mathbb{R}$.
  - Single-object: for any $\omega \in \Omega$ the object is allocated exactly to one individual.

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- Three mechanisms: Agents bid θ_i.
  - **FPA**: Agent with the highest bid gets the object and pays the bid.
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Propositions 7 and 8

#FPA and #APA are equal to two. #SPA is equal to the size of $\mathcal{I}$.

Overall Impression

Very nice paper!

- General approach to study auditability.
- Well-motivated. Many important applications:
  - School choice
  - College admisions
  - House allocation
  - Auctions
  - Voting
  - Affirmative action
- Polished and carefully executed.
- **My comments: interpretation and some limitations.**

Comment I: Interpretation

- Interpretation of *type report* $\theta_i \in \Theta_i$.
  - Appears to represent private information.
  - Could alternatively be framed as strategies in a (potentially indirect) mechanism.
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- The auditability measure seems independent of truthtelling, but paper would benefit from explicitely apply a version of the Revelation Principle.

Comment II: Limitations

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Alternative:

Group I **does not detect** a deviation $\omega \in \Omega$ iff $\exists\ \theta_{-I}$ and $\hat{\omega}$ such that

- $(\theta_I, \theta_{-I}) \in \Theta$.
- $\hat{\omega}(i) = \omega(i)$ for all $i \in I$.
- $\hat{\omega}$ in the support of $\phi(\theta_I, \theta_{-I})$.

Comment III: Incentives

- Agnostic approach about principal's deviation incentives.
  - All deviations are treated symmetrically.
  - This is elegant, but in some applications it might make sense to put more weight in some deviations.
    - First vs Second Price Auction.

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- Agnostic approach about agent's incentives to identify deviations.
  - If the deviation is convenient, an agent might not contribute to the detection.
    - First Price Auction example.

Conclusion