Problem Set 3

Spring 2022

Advanced Microeconomics III

Problem 1 Based on MWG 13.D.2.

Consider the following model of the insurance market. There are two types of individuals: high risk and low risk. Each individual starts with an initial wealth W. An accident (e.g., fire) may reduce her wealth by L. The probability of this happening is p_L for low-risk types and p_H for high-risk types, where $p_H > p_L$. Each individual is privately informed about her risk type $p \in \{p_L, p_H\}$.

Individuals are expected-utility maximizers with a Bernoulli utility function $u(w)$ over wealth w. Assume that $u'(w) > 0$ and $u''(w) < 0$ for all w. There are two risk-neutral insurance companies who can offer one or more insurance policies ("contracts"). A contract consists of a premium payment M made by the insured individual to her insurance firm, and a payment R from the insurance company to the insured individual in the event of a loss. Consider a game where the insurance companies first simultaneously offer sets of contracts. Second, nature independently draws a type for each individual, where the probability of low-risk type is $\lambda \in (0, 1)$. Third, each individual accepts at most one contract.

a. What is the expected utility of an individual of type p who accepts a contract $c = (R, M)$? What is the expected payoff for a firm selling the contract c to an individual of type p?

b. In an (R,M) diagram with R on the horizontal axis, sketch a firm's zero-profit line when contracting with a high-risk type, and its zero-profit line when contracting with a low-risk type. Sketch an indifference curve for each type.

Show that for any contract c, the high-risk indifference curve through c is steeper than the low-risk indifference curve.

c. Describ ethe unconstrained Pareto efficient allocations.

d. Describe the unique pure-strategy subgame-perfect Nash equilibrium candidate.

e. Show by example that it is possible that no equilibrium exists.

Problem 2 There are two types of workers $\theta \in \{L, H\}$. An H-type worker passes a standardized test with probability p_H while L-type workers pass the test with probability p_L. Suppose the test can be taken many times. Each time a worker takes the test, she incurs a cost c.

a. What is the expected cost of passing the test for each of the types of worker? (Assume that the probability of passing the test is independent across different takes, and that it remains constant.)

There is a set of firms that offers contracts to the workers. Suppose that firms observe whether the worker passed the test, but not how many times she took it before passing. Thus, in these contracts the firms stipulate two salaries: one if the worker passes the test and one if the worker does not pass the test. The timing is as follows: First, firms simultaneously offer a set of contracts to the pool of workers. Then each worker, knowing his type, chooses a contract from one of the firms. Then the worker takes the test. If the worker doesn't pass, she can take the test again or not, and so on. Once the worker passes or decides not to retake the test, the payoffs are realized.

b. Argue that, for any contract, if a worker takes the test once, then the worker should take the test until passing.

c. What are the PBE in this game? How does the set change with the parameters of the model p_L, p_H and c.

d. Suppose instead that the worker can take the test at most once. What are the PBE in this game?