Advanced Microeconomics III

Mechanism Design - 2

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Introduction

- Consider an auction but where the revenue-collector is included as a "agent 0".
- The revenue-maximizing scf is a constrained Pareto efficient allocation!
  - Not possible to increase the utility of agents without decreasing the utility of agent zero.
- The set of constrained Pareto efficient scf is usually difficult to characterize.
- We ask instead: Does there exist a constrained efficient scf that is Pareto-efficient?

Efficient Mechanisms

- Consider a linear environment with private values:
  - Types Θ.
  - Preferences given by $u_i(x, \theta_i) - t_i$.

- We say that an allocation rule $\alpha : \Theta \to X$ is *efficient* if

$$\alpha(\theta) \in \arg\max_{x \in X} \sum_{i \in I} u_i(x, \theta_i) \qquad \forall theta \in \Theta.$$

Efficient Mechanisms

- We study a set of mechanisms that implement an efficient allocation.
- As before, by virtue of the Revelation Principle, we can restrict attention to DRM.
- Since α is fixed, the mechanisms we consider differ only in the transfer rule.

- Other properties that are interesting beyond efficiency:
  - Incentive Compatibility (BIC and DSIC)
  - Voluntary participation ("individual rationality")
  - That no money is required to run the mechanism ("Budget-balanced").
- Usually, tension in the last two.

Example

- One thing one could do is a DRM in which
  - $\alpha(\theta)$ is efficient given the reports.
  - Each agent receives a transfer equivalent to the sum of others' payoffs.
  - Given others' reports θ_{-i}, agent chooses:

$$\max_{\hat{\theta}_i \in \Theta_i} u_i(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_i) + \sum_{j \neq i} u_j(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_j)$$

  - Since α is efficient,

$$\sum_{k \in I} u_k(\alpha(\theta), \theta_k) \geq \sum_{k \in I} u_k(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_k)$$

  - Thus, it is dominant to report truthfully.
  - Problem: this requires large positive transfers to participants.

VCG Mechanisms

- VCG mechanism is a DRM in which α is an efficient allocation rule and

$$\tau_i(\theta) = \sum_{j \neq i} u_j(\alpha(\bar{\theta}_i, \theta_{-i}), \theta_j) - \sum_{j \neq i} u_j(\alpha(\theta), \theta_j)$$

- Where $\bar{\theta}_i$ is a default type for player i.
- **Intuition**: each agent pays *what other agents can achieve without i* minus *what the other agents get if i is present*. In other words, the externality imposed on others.
- Note: VCG is a DRM. All θs should be interpreted as the reports of the agents.

Example: VCG in auctions

- Let $\Theta_i = [0, 1]$ for all i
- let $\bar{\theta} = (0, 0, ..., 0)$.

- Efficient allocation: to the agent that values the item the most.

- VCG payments are:

$$\tau_i(\theta) = \begin{cases} \max_{j \neq i} \theta_j & \theta_i > \theta_j \quad \forall j \\ 0 & \text{otherwise} \end{cases}$$

- The VCG mechanism is equivalent to the second price auction.

Example: Binary public good provision

- Let $X = \{0, 1\}$
- Let $u_i(x, \theta_i) = x \cdot \theta_i$
- With $I = \{1, 2\}$

$$\alpha^*(\theta) = \begin{cases} 1 & \text{if } \theta_1 + \theta_2 \geq c \\ 0 & \text{otherwise} \end{cases}$$

Example: Binary public good provision

- VCG transfers with $\bar{\theta}_i = 0$.

$$t^{VCG}(\theta_i) = 0 - \theta_j \cdot 1_{\{\theta_i + \theta_j > 1\}}$$

- VCG transfers with $\bar{\theta}_i = 1$.

$$t^{VCG}(\theta_i) = \theta_j - \theta_j \cdot 1_{\{\theta_i + \theta_j > 1\}} = \theta_j \cdot 1_{\{\theta_i + \theta_j < 1\}}$$

VCG is DSIC

For any profile of default types $\bar{\theta}$, the VCG mechanism is DSIC.

Proof.

$$\begin{aligned} U_i(\theta) &= u_i(\alpha(\theta_i, \theta_{-i}), \theta_i) - t_i(\theta_i, \theta_{-i}) \\ &= u_i(\alpha(\theta_i, \theta_{-i}), \theta_i) - C(\bar{\theta}_i, \theta_{-i}) + \sum_{j \neq i} u_j(\alpha(\theta_i, \theta_{-i}), \theta_j) \\ &\geq u_i(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_i) - C(\bar{\theta}_i, \theta_{-i}) + \sum_{j \neq i} u_j(\alpha(\hat{\theta}_i, \theta_{-i}), \theta_j) \end{aligned}$$

□

Example: Bilateral Trade

- One buyer and one seller.
- Single object.
- Buyer values the object is θ_b.
- Seller's cost (alternatively value) is θ_s.

- Efficient allocation rule:

$$\alpha^*(\theta) = \begin{cases} \text{trade} & \text{if } \theta_b > \theta_s \\ \text{no trade} & \text{if } \theta_b < \theta_s \end{cases}$$

Example: Bilateral Trade

We choose default types $\bar{\theta}_b = 0$ and $\bar{\theta}_s = 1$.

- When there is no trade:

$$t_b(\theta) = t_s(\theta) = 0$$

- When there is trade ($\theta_b > \theta_s$),

$$\begin{aligned} t_b(\theta) &= \theta_s \\ t_s(\theta) &= -\theta_b \end{aligned}$$

- The mechanism incurs a deficit whenever there is trade.

Participation and Budget-balanced condition

- Voluntary participation (IR):

$$U_i(\theta_i) \geq 0 \qquad \forall i \in I.$$

- Budget-balanced condition (BB):

$$S(\theta) := \sum_{k \in I} t_k(\theta) = 0 \qquad \forall \theta \in \Theta$$

- This is an ex-post notion of budget-balanced, in the belief-free spirit of DSIC.
- If $S(\theta) \geq 0$ for all θ we say that the mechanism *never runs a deficit*.
  - **Note**: this equivalent to voluntary participation of agent zero.

Myerson-Satterthwaite

In the bilateral trade environment there is no mechanism that is efficient, DSIC, satisfies voluntary participation, and that is budget-balanced.

Proof.

- Deferred for later.

□

- Comments:
  - The result generalizes to environments with asymmetric supports, as long as the supports intersect.
  - Inefficiency persist but gets smaller quickly when the number of traders on each side of the market increases.

Beyond Dominant Strategies

- Budget-balanced is a strong condition. Sometimes we are interested in the mechanism not running a deficit in expectation.
- Likewise, IR requires that all agents are happy to participate on the mechanism ex-post. Sometimes we are interested in interim incentives to participate.
- Finally, in Bayesian environments, we can relax our solution concept to Bayesian implementation.

Expected Externality Mechanism (AGV)

- **Idea**: instead of making each player pay the realized externality imposed on others, make them pay the expected externality.

- **AGV mechanism** (Arrow and d'Aspremont and Gerard-Varet): DRM in which the allocation rule is efficient and the payment rule is given by

$$t_i^{AGV}(\theta) = \frac{1}{N-1} \sum_{j \neq i} \tilde{t}_j(\theta_j) - \tilde{t}_i(\theta_i)$$

$$\tilde{t}_i(\theta_i) = E_{\theta_{-i}} \left[\sum_{j \neq i} u_j(\alpha^*(\theta), \theta_j) \right]$$

AGV is budget-balanced

Observe that, for all θ,

$$\begin{aligned}
\sum_{i \in I} t_i^{AGV}(\theta) &= \sum_{i \in I} \left[\frac{1}{N-1} \sum_{j \neq i} \tilde{t}_j(\theta_j) - \tilde{t}_i(\theta_i) \right] \\
&= \frac{1}{N-1} \underbrace{\sum_{i \in I} \sum_{j \neq i} \tilde{t}_j(\theta_j)}_{(N-1) \sum_{i \in I} \tilde{t}_i(\theta_i)} - \sum_{i \in I} \tilde{t}_i(\theta_i) \\
&= 0
\end{aligned}$$

AGV is BIC

- Expected payment:

$$E_{\theta_{-i}}[t_i^{AGV}(\theta)|\theta_i] = \frac{1}{N-1}\sum_{j\neq i} E_{\theta_{-i}}\left[\sum_{k\neq j} u_k(\alpha^*(\theta),\theta_k)\right]$$

- If i tells the truth, she gets an interim utility

$$\begin{aligned} U_i(\theta_i) &= E_{\theta_{-i}}\left[u_i(\alpha^*(\theta),\theta_i) - t_i^{AGV}(\theta)\right] \\ &= E_{\theta_{-i}}\left[u_i(\alpha^*(\theta),\theta_i) + \sum_{j\neq i} u_j(\alpha^*(\theta),\theta_j)\right] \end{aligned}$$

AGV may not be IIR

- There is nothing that guarantees that IIR is satisfied.
- There might be a agent i and type θ_i such that AGV involves

$$U_i(\theta_i) < 0$$

Generalized VCG

- We allow for type dependent outside options $\underline{U}_i(\theta_i)$.
- Given the efficient allocation rule α^*, let

$$\bar{\theta}_i \in \arg\min_{\theta_i \in \Theta_i} E_{\theta_{-i}} \left[\sum_{j=1}^{N} u_j(\alpha^*(\theta_i, \theta_{-i}), \theta_j) - \underline{U}_i(\theta_i) \right]$$

- We refer to $\bar{\theta}_i$ as the *least charitable type* of agent i.

Generalized VCG

- **GVCG**: DRM in which the allocation rule α^* is efficient and the payments are given by:

$$t_i^{GVCG}(\theta) = \sum_{j \neq i} u_j(\alpha^*(\bar{\theta}_i, \theta_{-i}), \theta_j) + u_i(\alpha^*(\bar{\theta}_i, \theta_{-i}), \bar{\theta}_i)$$

$$- \sum_{j \neq i}^{N} u_j(\alpha^*(\theta_i, \theta_{-i}), \theta_j) - \underline{U}_i(\bar{\theta}_i)$$

Theorem (Krishna Perry)

The GVCG mechanism is IIR and BIC, and maximizes the expected surplus among all mechanisms that are IIR, BIC and implement the efficient allocation rule.

Proof - BIC

TBA

Proof - IIR

TBA

Proof - Revenue Maximizing

TBA

Proof - Myerson-Satterthwaite

- In the bilateral trade environment, $\bar{\theta}_b = 0$ and $\bar{\theta}_s = 1$.
- Thus, the VCG mechanism we considered coincides with the GVCG mechanism.
- But this mechanism runs a deficit, thus:

There does not exist an efficient, BIC, IIR, ex-ante BB mechanism in the bilateral trade environment.