Tariff Mechanisms

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An agent with type $\theta \in \Theta$ must choose an action a from some set A. This action generates an outcome $z \in Z$ that is either contractible ($z \in Z^c$) or not ($z \in Z \setminus Z^c$). Conditional on $z \in Z^c$, the distribution of z is assumed to be independent of θ. This assumption holds, for instance, if θ is a preference parameter of the agent that does not affect outcomes, or if θ affects the probability that the agent is caught (i.e., generates a "contractible" outcome), but not the evidence conditional on being caught. Formally, we assume that

$$F(z|a, z \in Z^c, \theta) = F(z|a, z \in Z^c, \theta')$$

for all $\theta, \theta' \in \Theta$.

A designer wishes to induce specific type-dependent actions. Suppose first that the designer has perfect commitment power and can contract with the agent after the agent has observed his type and before any action is taken. Without loss of generality, the designer can restrict attention to direct revelation mechanisms, in which:

(i) The agent reports his type.

(ii) The mechanism recommends an action to the agent.

(iii) The agent privately chooses an action.

(iv) An outcome is realized.

(v) The agent receives a transfer that depends on his report and on the outcome. If the outcome is not contractible, the transfer is equal to zero.

Formally, a *contractible transfer* is a map $t : \Theta \times Z \to \mathbb{R}$ such that $t(\hat{\theta}, z) = 0$ for any $z \notin Z^c$ and report $\hat{\theta}$.

Given a contractible transfer t, an agent with type θ who chooses action a and reports $\hat{\theta}$ gets expected utility

$$u(\theta, a) + E[t(\hat{\theta}, z)|\theta, a].$$

Since z is independent of θ conditional on (Z^c, a), this expected utility is also equal to:

$$u(\theta, a) + Pr(Z^c|\theta, a)E[t(\hat{\theta}, z)|Z^c, a],$$

Given a transfer t, let

$$\tau(\hat{\theta}, a; t) = E[t(\hat{\theta}, z)|Z^c, a]. \tag{1}$$

If an agent chooses action a, this agent chooses a report $\hat{\theta}(a; t)$ that maximizes (1), regardless of the agent's actual type. This leads to the reduced-form utility:

$$u(\theta, a) + P(Z^c|\theta, a)T(a; t),$$

where $T(a; t) = \max_{\hat{\theta}} E[t(\hat{\theta}, z)|Z^c, a]$.

A social choice function f is a map $f : \Theta \to A$. A social choice function f is:

- *implementable* if there exists a contractible transfer t such that for all $\theta \in \Theta$, $f(\theta)$ maximizes
$$u(\theta, a) + P(Z^c|\theta, a)T(a; t)$$
over $a \in A$;
- *truthfully implementable* if t can be chosen so that reporting $\hat{\theta} = \theta$ is optimal for all $\theta \in \Theta$;
- *tariff implementable* if t can be chosen so as to be independent of θ.

When a contractible transfer is independent of the agent's report, we will call it a *tariff*.

Because the designer can rarely contract with the agent ex ante, we wish to determine when implementable social choice functions are tariff implementable.

For any $A' \subset A$, let $Z(A')$ denote the *contractible consequences* of A': $Z(A')$ is the set contractible outcomes that can be generated by actions $a \in A'$. By assumption, this set is independent of the agent's type.

We start with a straightforward observation:

LEMMA 1 *If f is implementable, it is truthfully implementable.*

Proof. This result, which is a variation on the Revelation Principle, follows by choosing any transfer t that implements f and replacing it by the tariff $\hat{t}(\theta, z) = t(\hat{\theta}(f(\theta); t), z)$ for all $z \in Z(f(\theta))$ and $\hat{t}(\theta, z) = -M$ otherwise, where $-M$ is a lower bound on transfers is such a lower bound is imposed, and arbitrarily negative otherwise. With this new transfer $\hat{t}$, truthtelling is optimal for all types. ■

DEFINITION 1 *f is identifiable if there exists a partition $\mathcal{A} = \{A_k\}_{k=1}^{K}$ of A such that:*

(i) $Z(A_k) \cap Z(A_{k'}) = \emptyset$ for all $k \neq k'$.

(ii) $f(\Theta) \cap A_k$ has at most one element.

In words, f is identifiable if actions can be grouped so that (i) the principal can perfectly detect to which group the action taken by the agent belongs, (ii) each group contains at most one action in the range of f.

The first property imposes a structure on the environment that is independent of f. For example, it rules out situations in which all actions lead to a full support over outcomes. One particular case is when $\mathcal{A}$ is the information partition of the principal, in which case the outcome can be identified with A_k. In general the principal could observe finer information than A_k. The set of partitions that satisfy property (i) is a primitive of the environment.

The second property says that one can choose such a partition, which is specific to f, so that f implements at most one action in each cell of this partition. The second property imposes a hierarchy over actions: within each action cell that is identifiable by the principal, there is one action that is singled out by the social choice function f.

PROPOSITION 1 *If f is implementable and identifiable, then it is tariff implementable.*

Proof. Suppose that t implements f. From Lemma 1, we can assume without loss that t is truthful. We define a tariff $\tilde{t}$ as follows. Let A_f denote the set of actions in the range of f and $Z_f = Z(A_f)$ denote the set of contractible outcomes that may arise when f is implemented. Since f is identifiable, z is generated only by actions in some cell A_k of the partition $\mathcal{A}$. Moreover, For any $z \in Z_f$ the set Θ_k of types who choose an action in A_k given transfer t choose the same action a_k. Finally, all types $\theta \in \Theta_k$ must be indifferent between reporting their own type and any other type in Θ_k, since as noted from Equation (1), they all have the same reporting incentives conditional on taking action a_k. And by truthfulness, they prefer these reports to any other report $\theta' \notin \Theta_k$. Let θ_k denote an arbitrary element of

Θ_k and θ_0 and arbitrary element of Θ. We let

$$\tilde{t}(z) = \begin{cases} t(\theta_k, z) & \text{for all } z \in Z_f \\ t(\theta_0, z) & \text{for all } z \in Z^c \setminus Z_f \end{cases}$$

By construction, $\tilde{t}$ is a contractible transfer, and it is a tariff (i.e., independent of any report). Also by construction, truthtelling is optimal and implements the same function f. ■