Placement with Assignment Guarantees and Semi-Flexible Capacities

By Orhan Aygün and Günnur Ege Bilgin

Summary

Paper studies many-to-one matching problem where there is a conflict between two goals.

- Targeted capacities.
- Assignment guarantees.

Leading application: doctors to hospitals.

Assumption (Responsive Preferences)

Authors propose a modified D-A algorithm (ACA algorithm) and show that it produces "desirable" matchings.

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Matching and Mechanism

Definition

Fixing D and H, a (many-to-one) matching is a set $\mu \subseteq D \times H$ such that

$$(d, h) \in \mu \text{ implies } (d, h') \notin \mu \text{ for all } h' \neq h$$

Let $\mathcal{M}$ denote the set of matchings.

Let (P_D, P_H, q_H, E_H) be the *environment*, set $\mathcal{E}$.

Definition

A *mechanism* is a mapping $\phi : \mathcal{E} \to \mathcal{M}$.

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Desirable Property 1: Fairness

Definition 1.

Given the environment, a matching μ is *stable* if there is no d, d', h such that

$$d \succ_h d' \in \mu(h) \qquad \text{and } h \succ_d \mu(d)$$

A mechanism ϕ is **fair** if $\phi(e)$ is stable for all $e \in \mathcal{E}$.

Desirable Property 2: Capacity Constraint

Definition 2.

Given the environment, a matching μ *satisfies capacity constraint* if

$$|\mu(h)| \leqslant q_h \qquad \forall h \in H$$

A mechanism ϕ satisfies capacity constraint if $\phi(e)$ does it for every $e \in \mathcal{E}$.

Desirable Property 3: Respecting Assignment Guarantees

Definition 3.

Given the environment, a matching μ *respects assignment guarantees* if

$$\mu(d) \succeq_d h \qquad \forall\ h : d \in E_h$$

A mechanism ϕ respects assignment guarantees if $\phi(e)$ does it for every $e \in \mathcal{E}$.

Conflict of goals

Claim

There is no mechanism ϕ that satisfies fairness and respects assignment guarantees without "creating capacities equal to the number of candidates for each program."

Fix ϕ and $e \in \mathcal{E}$, let $|\mu(h)| - q_h$ be the *required new capacities*.

Let $Q^\phi(h)$ be the maximum over all environments.

Claim (revisited).

If ϕ ~~satisfies fairness and~~ respects assignment guarantees, then $Q^\phi(h) = |D|$.

Think of environment such that $E_h = D$, h is favorite hospital for all doctors, and $q_h = 0$.

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Capacity Respecting Fairness

Given this conflict of goals, the paper proposes a relaxed version of fairness and capacity constraint.

Let $U_h(d)$ be the set of doctors that h weakly prefers to d.

Definition

Given an environment $e \in \mathcal{E}$, a matching μ is *q-fair* if there is no (d, h) such that

$$h \succ_d \mu(d) \qquad \text{and} \qquad |U_h(d) \cap \mu(h)| \leqslant q_h$$

Definition

Given an environment $e \in \mathcal{E}$, a matching μ *avoids unnecessary slots* (AUS) if for any d, either

$$d \in E_{\mu(d)} \qquad \text{or} \qquad |\mu(\mu(d)) \cap U_{\mu(d)}(d)| \leqslant q_{\mu(d)}$$

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Avoiding Unnecessary Slots

Let ϕ^* be the mechanism associated with the AGA algorithm.

Proposition 2

ϕ^* satisfies q-fairness, avoids unnecessary slots, and respects assignment guarantees.

Proposition 3

ϕ^* is strategy-proof for doctors.

Theorem 1

ϕ^* is the unique strategy-proof (for doctors) mechanism that respects assignment guarantees, is q-fair, and avoids unnecessary slots.

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Comment 1: Strategy-proofness for hospitals

Paper shows that ϕ^* is strategy-proof for doctors.

- It satisfies the sufficient conditions for a D-A algorithm to be strategy-proof.

For hospitals, ϕ^* might produce "simple" profitable deviations from truth telling.

- E.g. rank doctors with AG very low to hire beyond targeted capacity.
- E.g. rank doctors with AG very high to not hire beyond targeted capacity.

Maybe something can be said about this "simple" deviations.

Caveat: It might require to relax the responsive preferences assumption.

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Comment 2: q-fairness, AUS, and Stability

The paper would benefit from clarifying the relationship between q-fairness, AUS, and "stability".

$D = \{a, b, c\}$ and $H = \{h\}$ with $q_h = 1$ and $E_h = \emptyset$.

Let $a \succ_h b \succ_h c$. Then $\mu(h) = \{a, c\}$ is q-fair.

μ is not stable, but also not AUS.

Stability

A matching is *E-stable* if there is no d, d', h such that

$$h \succ_d \mu(d) \qquad \text{and} \qquad d \succ_h d' \qquad \text{and} \qquad d' \in \mu(h) \cap E_h^c$$

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Claim

q-fairness + AUS $\Rightarrow$ E-stable

Proof.

($\Rightarrow$)

- $h \succ_d \mu(d)$ and $d \succ_h d'$.
- $d' \in \mu(h)$.
- If d' in top q_H ranked doctors in $\mu(h)$, then q-fairness is violated:

$$U_h(d) \subseteq U_h(d') \quad \Rightarrow \quad |U_h(d) \cap \mu(h)| \leqslant |U_h(d) \cap \mu(h)| \leqslant q_h$$

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