Advanced Microeconomics III

Spence's Signaling Model

Francisco Poggi

Introduction

- **Akerlof**: markets with privately informed participants are often inefficient.
- Agents whose information is *favorable* may have an incentive to find means to convey this information.

- **Signaling**: information can be conveyed, but only indirectly.

Introduction

- Examples:
    - A *warranty* may signal good quality of a used car.
    - *education* may signal workers' ability.

- Questions:
    - How can signaling occur in equilibrium?
    - Is signaling always welfare-improving?

Spence's model

- A single worker and many (at least 2) firms.
- Worker can be of two types: $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L$.
- Only the worker knows θ.
- If employed by a firm, worker produces output θ.
- Firm's payoff:
  - $\theta - w$ if employs the worker at wage w.
  - zero otherwise.

Spence's model

- Worker moves first: chooses an observable education level $e \in [0, \infty)$
- Firms observe e (again: not θ).
- Cost of education $c(e|\theta)$.
- Worker payoff when education e and employed at wage w:

$$u(w, e|\theta) = w - c(e|\theta)$$

- Notice that education in this model is unproductive.

Spence's model

- Extra assumptions:

  - Cost of no education is zero.

    $$c(0|\theta) = 0 \qquad \text{for all } \theta$$
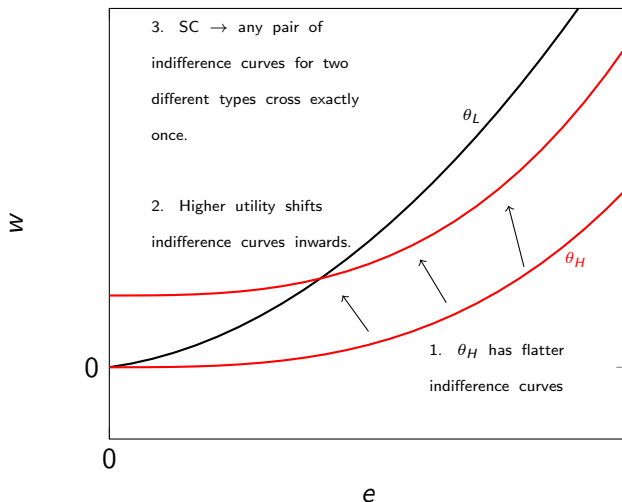
  - Cost of education increasing and convex in education.

    $$c'(e|\theta) > 0 \qquad \text{and} \qquad c''(e|\theta) > 0$$

  - High type worker has a smaller education cost.
    Moreover: High type has a smaller <u>marginal</u> cost of education.

    $$c'(e|\theta_H) < c'(e|\theta_L) \qquad \forall e > 0 \qquad\qquad \text{(Single-crossing)}$$

Indifference Curves

3. SC → any pair of indifference curves for two different types cross exactly once.

2. Higher utility shifts indifference curves inwards.

θ_L

θ_H

1. θ_H has flatter indifference curves

w

e

0

0

PBE Analysis

- **Solution concept**: (Pure-strategy) Perfect Bayesian Equilibrium.

- Described by:
    - A choice of education level for each worker type e_L, e_H.
    - $\mu(e)$ firms' posterior beliefs that worker is of type H.
    - Wage offers of the firms $w(e)$.

- Satisfying:
    - Optimality of education choices given wage offers.
    - Beliefs $\mu(e)$ consistent with Bayes' Rule where possible.
    - Wage offers constitute a Nash equilibrium at each subgame.
    - **Symmetry**: All firms hold the <u>same</u> beliefs after observing e.
        - (Not implied by weak PBE.)
    - Firms believe other firms conform to equilibrium wage offer $w(e)$ both on and off path.

PBE Analysis

- Competition among firms leads to the following wage offers (why?):

$$w(e) = E_{\mu(e)}[\theta] = \mu(e) \cdot \theta_H + (1 - \mu(e)) \cdot \theta_L$$

- Two types of pure-strategy equilibria:

  - **Separating equilibria**: each type chooses a different education level ($e_H \neq e_L$).

  - **Pooling equilibria**: types choose the same education level ($e_H = e_L$).

Separating Equilibria

- $e_H \neq e_L$.
- Bayes' rule where possible: $\mu(e_L) = 0$ and $\mu(e_H) = 1$.
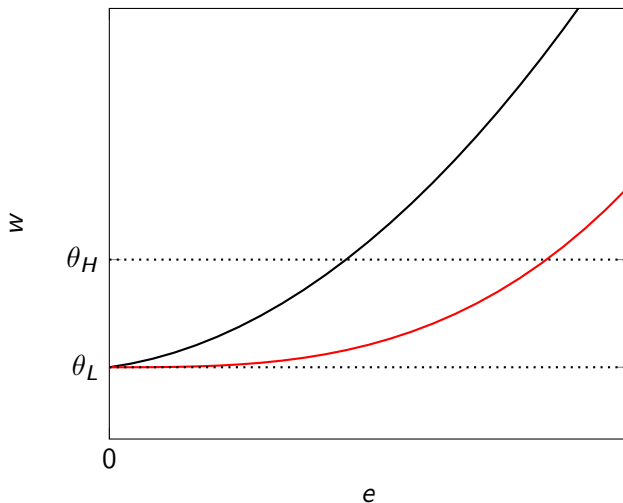- By competition:
$$w(e_L) = \theta_L \qquad w(e_H) = \theta_H$$

Lemma
In any separating equilibrium, $e_L = 0$.

- PBE implies that $w(e) \in [\theta_L, \theta_H]$.
- So, if $e_L > 0$, the deviation to $e = 0$ is profitable for type θ_L.

Separating Equilibria

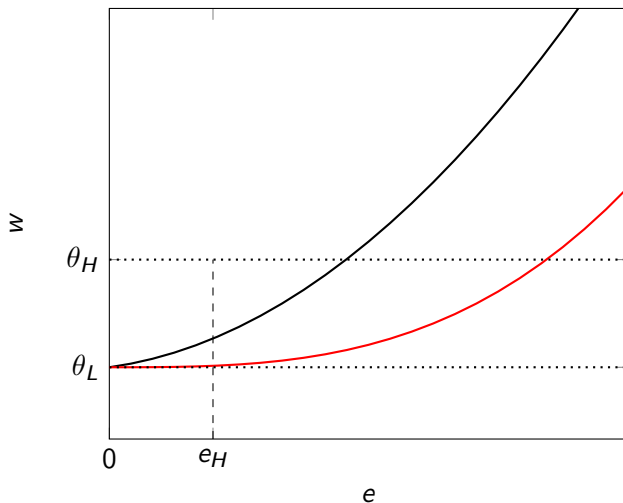# Separating Equilibria: Incentive Compatibility

> **Lemma**
>
> *In a separating equilibrium, type H chooses $e_H > 0$ such that*
>
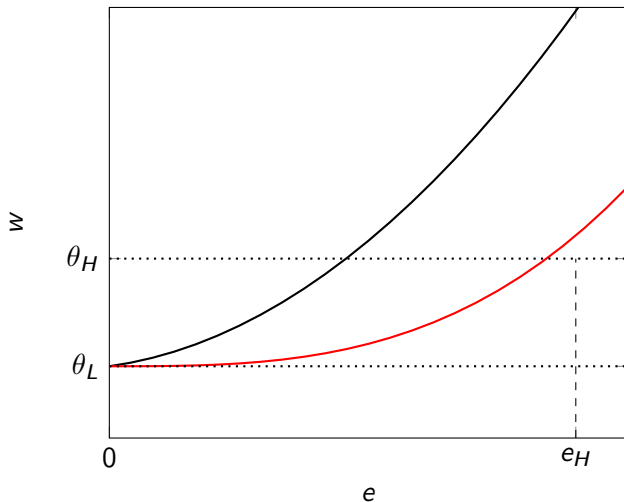> $$\theta_H - c(e_H|\theta_H) \geq \theta_L \geq \theta_H - c(e_H|\theta_L) \tag{IC}$$

- First inequality: type H prefers his education e_H rather than zero.
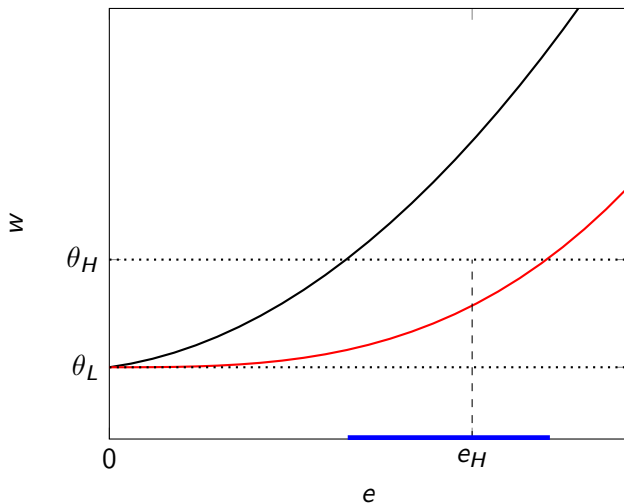- Second inequality: type L prefers zero rather than e_H.

Separating Equilibria

- Previous lemmata describe *necessary* conditions for separating equilibrium.

- These are also *sufficient*: remains to specify out-of-equilibrium beliefs.

  - Suppose any deviation is considered to be by a type L.
  - Then wage would be θ_L for any worker with an education level different than e_H.
  - Any deviation would be unprofitable.
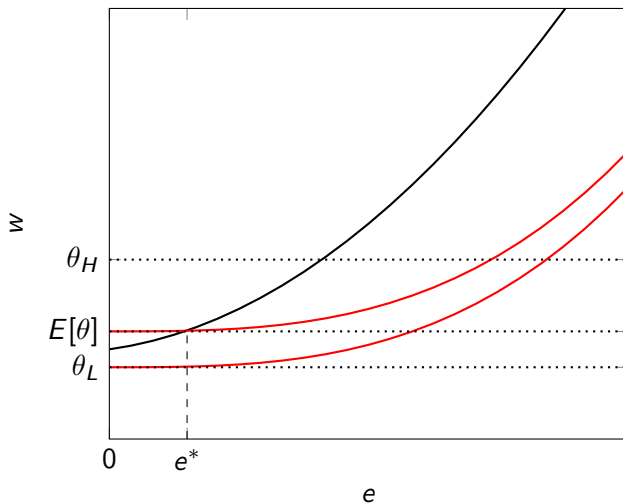
Equilibrium Multiplicity

- We have **multiple** separating equilibria.
    - These equilibria can be ranked in Pareto sense.
    - Best separating equilibrium: the one with lowest education e_H.

$$c(e_H|\theta_L) = \theta_H - \theta_L$$

Pooling Equilibria

- Pooling equilibrium: $e_L = e_H = e^*$.

- Bayes' rule where possible: $\mu(e^*) = \Pr(\theta = \theta_H)$
- Competition implies that $w(e^*) = E[\theta]$.

- Out-of-equilibrium beliefs: $\mu(e) = 0$ for $e \neq e^*$.
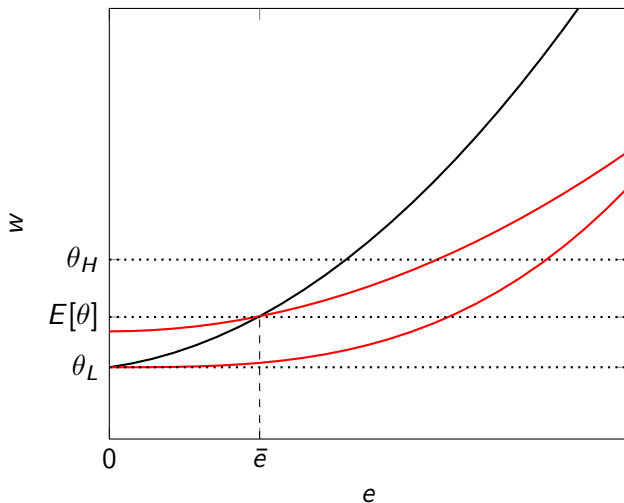 - Then $w(e) = \theta_L$ for $e \neq e^*$.

Multiple Pooling Equilibria

- **Again**: Best pooling equilibrium is the one with the lowest level of education ($e^* = 0$).

- What about the *worst* one?

$$E[\theta] - c(\bar{e}|\theta_L) = \theta_L$$

$$c(\bar{e}|\theta_L) = E[\theta] - \theta_L$$

Worst Pooling Equilibria

Comparing Pooling and Separating Equilibra

- The best pooling equilibrium may or may not Pareto dominate the best separating equilibrium.
  - High types not always benefit from the availability of a signaling device. Only if their fraction is small enough.

- The best separating equilibrium *never* Pareto dominates the best pooling equilibrium.
  - The low type is always worse-off.

Reasonable Beliefs (Equilibrium Refinements)

- There are multiple separating and multiple pooling equilibria.
- Which equilibrium is more likely to emerge?
    - One approach: Pareto dominance. Not a game-theoretical argument.
- Forward induction arguments can be used to refine the equilibrium.
- PBE allows for any beliefs off the equilibrium path.
- Refinements put conditions on these off equilibrium beliefs.
- Most refinements in this game uniquely select the least costly separating equilibrium.

Intuitive Criterion

- Cho and Kreps (1987) 'Intuitive criterion':
    - Key question: Who might benefit from the deviation?

Definition

A deviation e' is dominated in equilibrium for type θ if, for any sequentially rational response by the receivers $w' = E_{\mu'}[\theta]$ for some beliefs μ', the resulting payoff $u(e', w', \theta)$ is less than the equilibrium payoff $u(e(\theta), w(e(\theta)), \theta)$.

Definition

A PBE *passes the Intuitive Criterion Test (ICT)* if no type θ would be better off deviating to an action $e' \neq e(\theta)$ should the receivers' beliefs following e' assign zero probability to types θ' for whom the deviation is *dominated in equilibrium*.
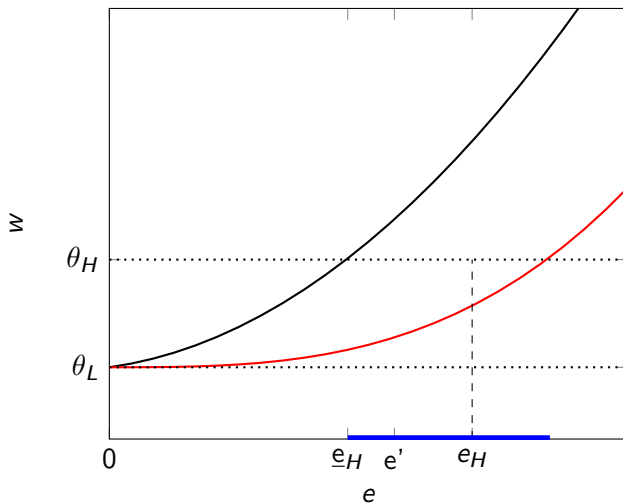
Intuitive Criterion: Separating Equilibrium

- Let $\underline{e}_H$ be the minimal education for the high type that can be sustained in a separating equilibrium.
- Start from a separating equilibrium with $e_H > \underline{e}_H$. We will show that this equilibrium does not satisfy the ICT.
  - Consider a deviation to $e' \in (\underline{e}_H, e_H)$ (This is off the equilibrium path).
  - A type θ_L can guarantee a payoff of θ_L by following equilibrium strategies. The deviation can bring type θ_L at most:

  $$\theta_H - c(e'|\theta_L) < \theta_L$$

  - Thus, a type θ_L would never deviating to e'. Formally e' is dominated in equilibrium for type θ_L.
  - The PBE does not pass the ICT: If $\mu(e') = 1$, type θ_H would benefit from deviating to e'.

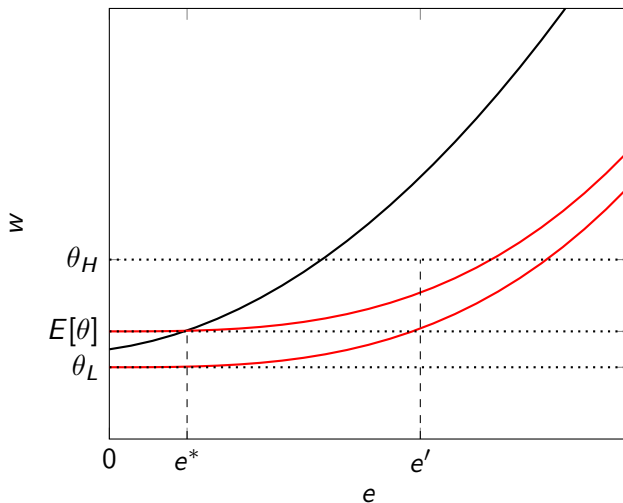# Intuitive Criterion: Separating Equilibrium

Intuitive Criterion: Pooling Equilibrium

- Let start instead from a pooling equilibrium at e^*.
- **Claim**: there exists e' such that

$$E[\theta] - c(e^*|\theta_H) < \theta_H - c(e'|\theta_L) < E[\theta] - c(e^*|\theta_L)$$

- Deviating to e' is dominated in equilibrium for type θ_L.
- So, the pooling PBE does not pass the ICT. If $\mu(e') = 1$, type θ_H would benefit from deviating to e'.

Intuitive Criterion: Pooling Equilibrium

Intuitive Criterion

- Only the best separating PBE passes the ICT.
- Notice that sometimes *forced pooling* generates a Pareto improvement.
  - In particular, when the share of high types is sufficiently large.
- Another Pareto improvement can arise with *cross-subsidization*.

Model with Continuum of Types

- Consider a model with a continuous of types.
  - Support in $[\underline{\theta}, \bar{\theta}]$.
  - Density function f strictly positive everywhere in the support.

- Is there a separating equilibrium? Is it unique?
  - Parametric assumption: $c(e|\theta) = \alpha \cdot e^2/\theta$.

Empirical Evidence

- Education is sometimes used as a signal of ability.
- Bedard, Kelly. Human Capital Versus Signaling Models.
  - Effect of constraining access to university in high school graduation levels.
  - **Finding**: In regions with a university, high school drop-out rates are higher.
  - This would be difficult to explain in model of human capital.
  - Signaling theory provides an explanation:
    - No university is close, more high-ability kids will stop education after high-school.
    - This gives low-ability kids incentives to pool with the high-ability kids.
  - Policy implications:
    - Improving access to university might increase drop-out rates and depress wages for some kids.

Other Models of Signaling

- Costless signaling (cheap talk): might work if preferences between sender and receiver are partially aligned. (Crawford Sobel (1982))

- Evidence and voluntary disclosure of verifiable information. Grossman (1981) Milgrom (1981) Dye (1985)

Classical Evidence Models

Seminal model developed by Grossman (1981) and Milgrom (1981)

- Similar to the previous model.
    - One worker, more than 2 firms.
    - Worker has private type θ.
    - Firms compete offering wages.

- Instead of choosing a level of education, worker can take a (free) test that perfectly reveals his type.
    - Formally, worker can send a message in $\{\emptyset, \theta\}$.
    - Market observes the message before making wage offers.

Unraveling

- Again, our solution concept is PBE.
- let $w(m)$ be the wage that firms offer to an agent that sends message m.
- Let $\Theta_\circ$ be the subset of types that chooses the empty message in equilibrium.
- For simplicity, assume that indifferent types take the test.
- Suppose that $w(\emptyset) > \underline{\theta}$.
- It must be that $\Theta_\circ = [\underline{\theta}, w(\emptyset))$
- $w(\emptyset) = E[\theta|\Theta_\circ] < w(\emptyset)$. Abs!
- So $w(\emptyset) = \underline{\theta}$ and $\Theta_\circ$ is empty.

Partial Unraveling

- Dye (1985) and Jung and Kwon (1988): Worker has evidence with some probability, and no evidence otherwise.

- **Partial unraveling**:
    - Let w be the wage for a worker in the absence of evidence.
    - Any type with $\theta < w$ will not present evidence.
    - Equilibrium w is the unique solution to:

    $$w = \lambda E[\theta|\theta < w] + (1 - \lambda)E[\theta].$$

    - $w > 0$.

Cheap Talk

Classical setup by Crawford and Sobel:

- Two agents, sender and receiver $i \in \{S, R\}$.
- Nature picks θ from prior $F(\cdot)$.
- Sender observes private information $\theta \in [0, 1]$.
- Sender sends a message $m \in M$ to receiver.
- Receiver takes action $a \in R$.

Cheap Talk

- Preferences $U^i(t, a)$.
- $a^i(t)$: preferred action of i when state is i. Solves max $U^i(t, a)$.
- $\bar{a}(t', t'')$ unique solution to

$$\max_a \int_t^{t''} U^R(a, t) dF(t)$$

- Preferences are partially aligned: $a^i(t)$ increases in t.
- Buy not perfectly: $a^S(t) > a^R(t)$.
- Example: quadratic preferences.

$$U^R(a, \theta) = -(a - \theta)^2 \qquad U^S(a, \theta) = (a - \theta - b)^2$$

Cheap Talk

- Key assumption: No commitment.
- Equilibrium:
  - Message for each type: $\mu : [0,1] \to M$. For each θ,

    $$\mu(\theta) \in \arg\max_{m \in M} U^S(\alpha(m), \theta).$$

  - Action for each message: $\alpha : M \to R$. For each $m \in M$,

    $$\alpha(m) \in \max_{a \in R} \int_0^1 U^R(a, \theta)\beta(t|m)dt$$

  - Interpretation of message: $\beta(\theta|\mu)$. Consistent with μ and Bayes' Rule.

Cheap Talk

Proposition

An uninformative equilibrium always exists.

- This is the *babbling* equilibrium:
    - Strategy of sender is state independent.
    - Posterior equals prior.
    - Receiver takes optimal action given prior independently of the message.

- Notice that this is true even for the case in which incentives are perfectly aligned ($b = 0$).

Cheap Talk

- Any informative equilibria?
    - The receiver's best response is:

$$w(\mu(e)) = E[\theta|\mu(e)]$$

- Suppose that on the path all messages of the sender are interpreted to be in one of two categories: l and h.
- Suppose that $w(l) < w(h)$. By single-crossing we have cutoff equilibria.
- There is at most one agent that is indifferent given two actions.

Cheap Talk

Generalizing properties of the equilibrium:

- The equilibrium message strategy is *partitional*: In any PBE there exist k cutoff types $\theta_1, \theta_2, ... \theta_k$ such that for each k,

$$\mu(\theta) = \mu(\theta') \qquad \forall \theta, \theta' \in (\theta_k, \theta_{k+1})$$

- Suppose N is the maximum number of partitions that can be sustained in equilibrium. It is possible to show that:
  - There exist a PBE with n partitions for all $n < N$.
  - The ex-ante expected payoff of both sender and receiver increases in the number of partitions.