Problem Set 5

Spring 2022

Advanced Microeconomics III

Problem 1 For a twice continuously differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ the following three conditions are equivalent. Show that they are also equivalent to supermodularity.

a. $f_x(x,t)$ is nondecreasing in t for all x.

b. $f_t(x,t)$ is nondecreasing in x for all t.

a. $f_{xt}(x,t) \geq 0$ for all (x,t)

Problem 2 Let $f : X \times T \to \mathbb{R}$. Show that if $\arg\max_{x \in S} f(x,t)$ is nondecreasing in t (in the strong set order) for each $S \subseteq X$ then f satisfies single crossing.

(**Note**: this result implies that single crossing is the weakest condition that can ensure MCS on every possible constrained set $S \subseteq X$)

Optional - Problem 3 Consider a parametrized utility function $u(x, y, t)$ where x and y are two levels of consumption and t is a parameter of the consumer's type. Assume that u is continuously differentiable and that u_x and u_y are of constant sign (x and y could be goods or bads).

Definition: u satisfies the Spence-Mirlees single crossing condition if $u_x/|u_y|$ is nondecreasing in t.

This means that the indifference curves on (x, y) are steeper for higher values of the parameter t at all points.

We are going to consider a consumer that chooses x and such that y is determined exogenously as a function of the x chosen. So the utility of the agent of type t that chooses x is $g(x,t) := u(x, h(x), t)$ for some function h.

Recall that $g(x,t)$ satisfies single crossing if for all $t' > t$ and $x' > x$, if parameter t prefers x' to x, then parameter t' also has that preference.

a. Prove that if u satisfies the Spence-Mirlees single crossing condition then $g(x,t) := u(x, h(x), t)$ satisfies single crossing.

b. Prove that if $g(x,t) := u(x, h(x), t)$ satisfies single crossing for all functions $h : \mathbb{R} \to \mathbb{R}$ then u satisfies the Spence-Mirlees single crossing condition.