The Timing of Complementary Innovations

Duke

December, 2021

Introduction

One goal of innovation policy:

Orient resources toward the R&Ds projects that are most (socially) valuable.

Issue: Usefulness of an innovation might be tied to the ***uncertain*** outcome of other developments.

- Diagnosis & treatment for medical condition.
- (Quantum) hardware & software.

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R&D projects carry high levels of uncertainty.

Uncertainty is (partially) resolved as the projects are pursued:

- Some projects are successfully completed.
- Some are found to be more challenging than originally expected.

Question 1: What is the ***efficient timing*** for complementary, uncertain innovations?

A → B?
B → A?

A & B simultaneously?

when to stop?

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In some industries, multiple firms or individuals that compete to develop the innovations.

Two potential inefficiencies:

1. **Underdevelopment**:
   - From imperfect *appropriability* of the rents from subsequent complementary innovations.
2. **Inefficient timing**:
   - Competing firms might focus on *easy* innovations to capture a higher share of total rents.

Question 2: When/how can the efficient timing of complementary innovations be implemented with *decentralized incentives*?

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Contribution

Introduce a dynamic model of R&D.

- **Endogenous timing**: Development requires resources that are not project-specific:
  - time,
  - money,
  - attention,
  - etc.

- **Experimentation**: As the projects are pursued, their prospects change (stochastically).

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Efficient Allocation: (Planner/Single agent problem).

- Multidimensional experimentation problem with interrelated payoffs.
- Not possible to apply a general result (e.g. *Gittins index*).
  - Working on a project reveals information about its viability.
  - Returns from a success depend on the prospects for the remaining project.

- **For complements**: equivalence between the dynamic problem of allocation and a simpler recursive problem.

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Parametric family of problems: **Constant Unknown Rate of Success**.

1. Simple conditions that determine qualitative features of the **efficient allocation**.
   Efficient to complete projects:

   In Sequence **Simultaneously**

2. Conditions under which it is possible to implement the efficient allocation with **decentralized incentives**.

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Prospect dynamics and endogenous timing.

Information acquisition from multiple sources:

Nikandrova and Pancs (2018), Che and Mierendorff (2017), Mayskaya (2019). Olszewski and Wolinsky (2016). Ke and Villas-boas (2019), Liang and Mu (2020)

Complementarities in payoffs.

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Roadmap

General Model

Equivalence Result

CURS Model

- Efficient Allocation
- Competitive Allocation

Extensions

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Model

Two projects $\{A, B\}$.

Time is continuous.

Agent decides: *stopping* and *allocation*.

Each instant before stopping, the agent allocates a unit of *attention* to the projects

$$x_A(t) + x_B(t) \leqslant 1$$

Project i is *completed* when the cumulative attention $X_i(t)$ reaches the *completion amount* τ_i.

Project completion is observable.

$$S_t := \{i : \tau_i \leqslant X_i(t)\}$$

The completion amounts τ_i are random and independent.

$$\tau_i \sim F_i$$

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If the agent stops at time T, he gets $q(S_T)$.

Free disposal: q is increasing.

Developing is costly: flow cost c.

Payoff of an agent that stops at time T and completed projects S_T is:

$$q(S_T) - c \cdot T$$

Agent maximizes expected payoff.

Definition

The projects are *complements* if q is supermodular, i.e.

$$q(A) + q(B) \leqslant q(\{A, B\}) \underbrace{+ \ q(\emptyset)}_{=0}$$

Marginal value increasing in the set of completed projects.

$$q(i, j) - q(j) \geqslant q(i) - q(\emptyset)$$

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Strategies: maps from histories to the set of actions.

WLOO (deterministic) *stationary strategies*: depend only on S_t and $X(t)$.

- Do not depend on:
  - The order in which attention was allocated.
  - The time at which projects in S were completed.

(S, X): *state*. $\mathcal{X}$: Set of stationary strategies with typical element x.

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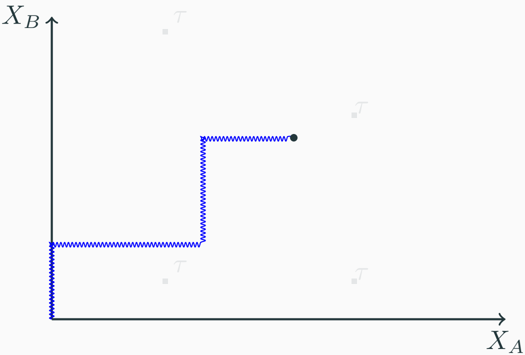
Each stationary strategy delineates a **plan** conditional on no success (*path* + *stopping point*)

After a success, efficient continuation is a simple *threshold*.

For complements, optimal stationary strategy involves higher thresholds than optimal stopping.

Consider a **different plan** with same stopping point.



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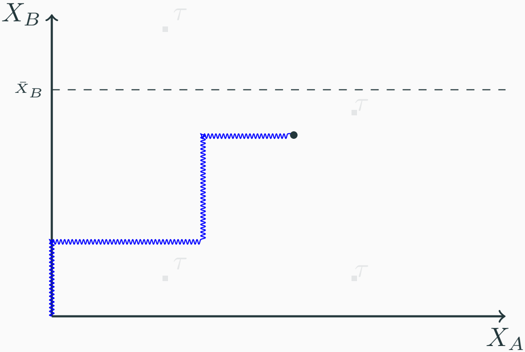

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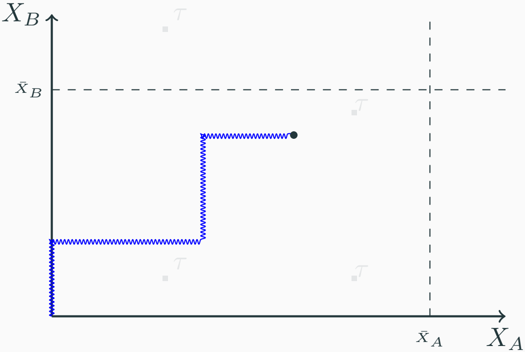

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$q(\{A\}) - c(\tau_A + \bar{X}_B$

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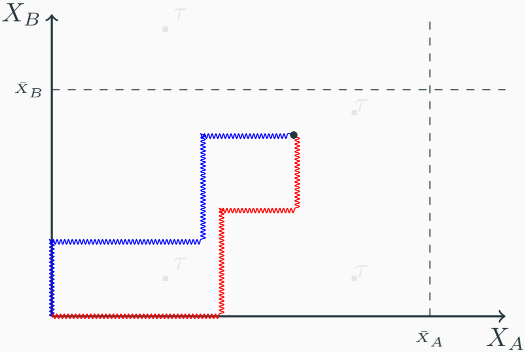

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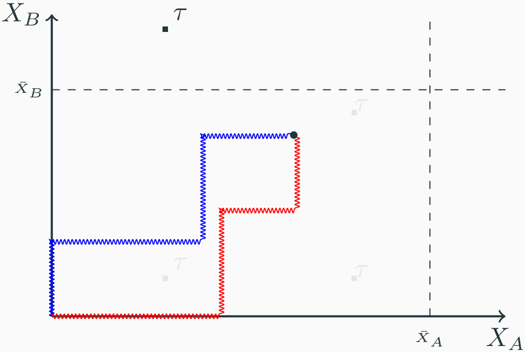

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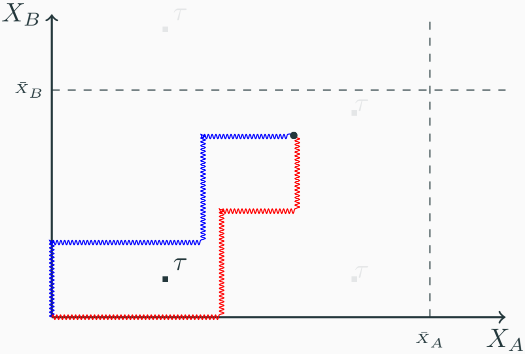

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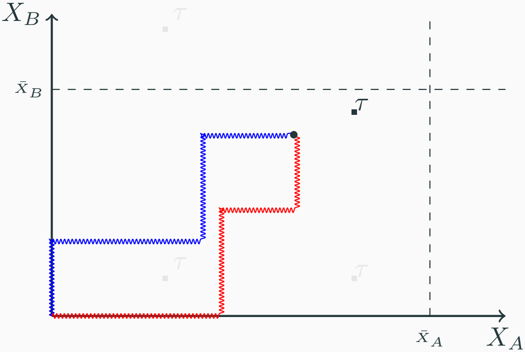

$$-c(X_A^* + X_B^*)$$

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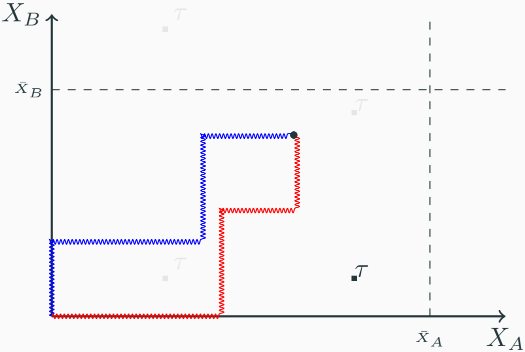

$$q(\{A,B\}) - c(\tau_A + \tau_B)$$

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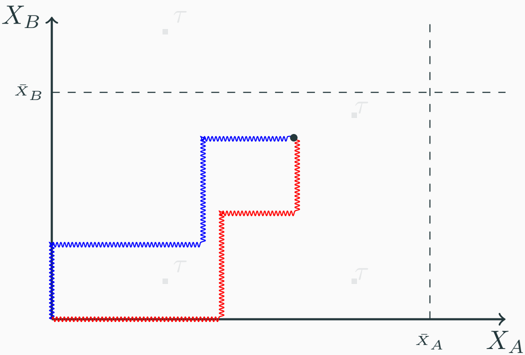
In general, one should treat **Red** and **Blue** *differently*.

For complements, they can be treated like *the same*.



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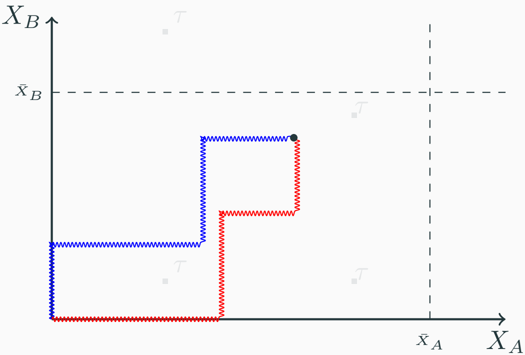

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Equivalence Result

Take a state (S, X).

All uncertainty is in τ. For each stationary strategy x and realization of τ, there is:

- a (deterministic) remaining time $\tilde{T}$
- a (deterministic) set of completed projects at the stopping time $\tilde{S}$.

Allocation Problem:

$$V(S, X) = \max_{x \in \chi} \quad \mathbb{E}_\tau \left[q(\tilde{S}(x, \tau)) - c \cdot \tilde{T}(x, \tau) \,\middle|\, (S, X) \right]$$

Equivalence Result

Take a state (S, X).

All uncertainty is in τ. For each stationary strategy x and realization of τ, there is:

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Order-independent problem:

given a state (S, X), the agent decides how much attention to *pledge* to each remaining project.

- Allocates these pledged resources *independently* (order is irrelevant).
  - If a project is completed before the pledge, he can stop allocating attention to that project.
  - He must complete all pledges to projects that remain uncompleted.

After pledged attention is allocated, if the agent succeeded in (at least) one project he can make a new pledge (otherwise stops).

$$\text{value:} \qquad \hat{V}(S, X)$$

$$\text{optimal pledge:} \qquad X^*(S, X)$$

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Proposition 1.

If the projects are complements then $V \equiv \hat{V}$. *(If the projects are not complements, there exists* $\{F_i\}$ *such that* $V \neq \hat{V}$.*)*

This is useful:

- For complements, it reduces the strategy space to a few parameters.
  - Find X^* with standard optimization tools.
  - Any strategy with such stopping points solves the allocation problem (multiple solutions if optimal pledge is interior).
- No assumptions on F_i, beyond independence.

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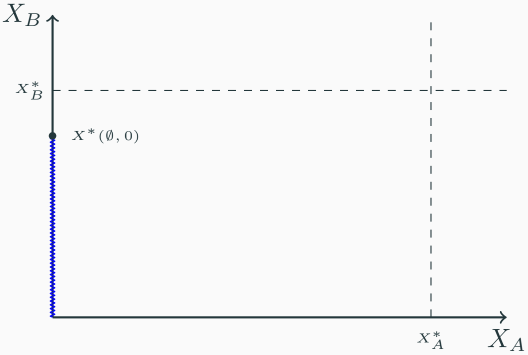
In Sequence

Definition

It is efficient to complete the projects **in sequence** if $X^*(S, X)$ is a corner solution.



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CURS Model

Perfect complements,

- $q(\{A\}) = q(\{B\}) = 0$,
- $q(\{A, B\}) = \gamma$.

Constant unknown hazard rate of success:

- Each project $i \in \{A, B\}$ is either *easy* or *difficult*:

$$\tau_i \sim \text{Exp}(1+\delta) \text{ with probability } p_i, \text{ and}$$
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As attention is allocated to a project and this is not completed, agent assigns higher probability to the project being difficult.

$$p_i(X_i) \text{ decreasing}$$

Subjective completion rate:

$$h_i(X_i) := (1 - p_i(X_i))(1 - \delta) + p_i(X_i)(1 + \delta).$$

After first success: only project i is left, continue as long as

$$h_i(X_i) \cdot \gamma - c \geqslant 0$$

Let $v_i(X_i)$ the continuation value when only project i remains.

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When the rates are **known**, the timing of development does not affect payoff.

Only question: is it worth it to complete the projects or not?

$$\text{Worth completing} \qquad \Leftrightarrow \qquad c \cdot (E[\tau_A] + E[\tau_B]) < \gamma$$

Interesting case:

$$\frac{1-\delta}{2} < \frac{c}{\gamma} < \frac{1+\delta}{2}$$

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Constant Known Rates of Success

If one of the projects is known to be difficult and one easy:

$$\text{Worth completing} \quad \Leftrightarrow \quad \frac{c}{1-\delta} + \frac{c}{1+\delta} < \gamma$$

$$\Leftrightarrow \quad \underbrace{2 \cdot \frac{c}{\gamma}}_{g} + \delta^2 < 1$$

Constant Unknown Rates of Success

Proposition 2.

For uncertain rates,

1. *If $g > 1$,*
   - *it is efficient to develop the projects* ***in sequence****,*
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2. *If $g < 1$, it is efficient to pledge more attention to the* ***most*** *promising project:*

$$X_i^*(\emptyset, X) - X_i > X_j^*(\emptyset, X) - X_j \qquad \Leftrightarrow \qquad p_i(X) > p_j(X)$$

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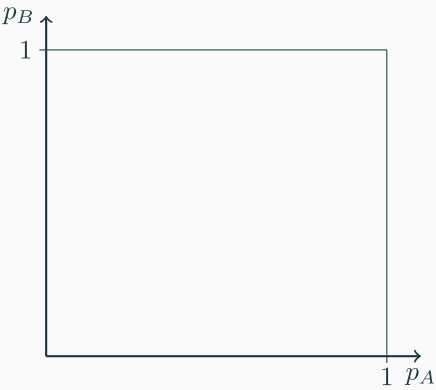
Constant Unknown Rates of Success

Belief space

In **red**, the stopping boundary (before a success).

When $g > 1$, it is efficient to concentrate the attention on the least promising project.



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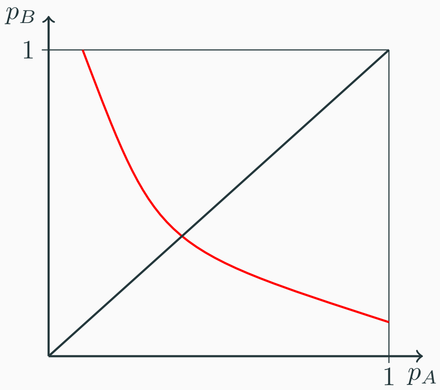

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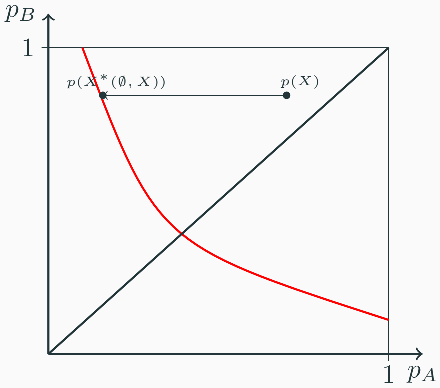

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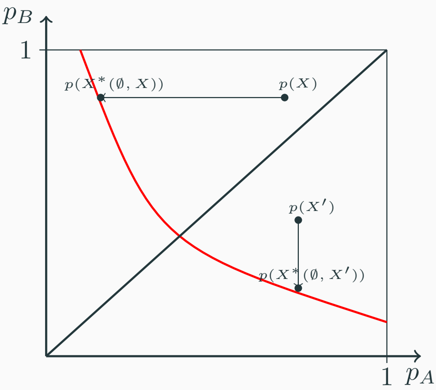

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Constant Unknown Rates of Success

p°: belief at which the agent stops if both projects are below that point.

- If $p \ngeq p^\circ$, it is efficient to work in sequence (starting from the most promising project).
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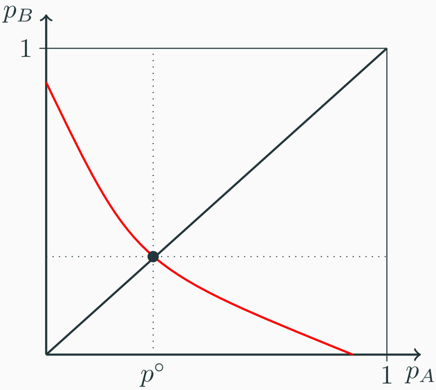

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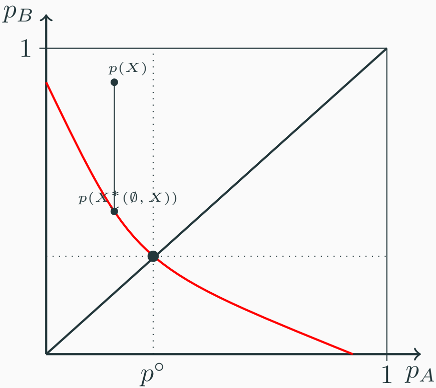

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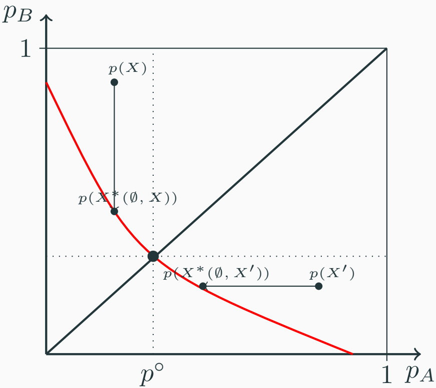

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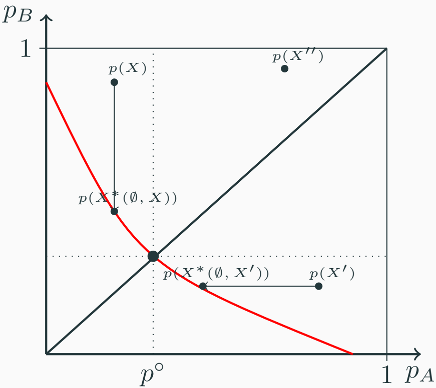

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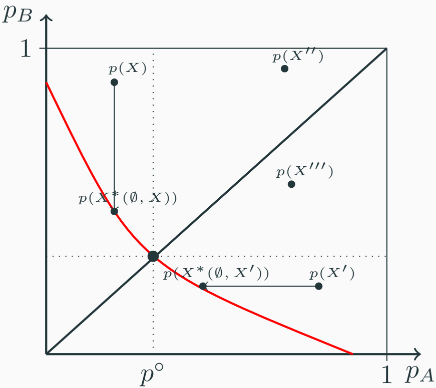

Solution for $g < 1$.

Intuition

g > 1:

Having one difficult project is sufficient bad news to stop.

Relevant info: are both easy or not?

Most efficient way to *learn* about this is to concentrate the resources in one project.

Projects are developed in sequence.

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Proof

As were defined before, we have:

- Value if i is the only project left:

$$v_i(X_i) := V(\{j\}, X_i).$$

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$$h_i(X_i) := p_i(X_i)(1+\delta) + (1 - p_i(X_i))(1-\delta).$$

Both are decreasing. **Key**: how relatively fast.

Proof step 1: $\frac{h_i}{v_i}$ monotonic.
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Proof Sketch - Step 2

$h_i/v_i \nearrow \quad \Rightarrow \quad$ efficient in sequence

Assume X^* is interior.

The red path is efficient.

In the last instant it is better to allocate to A than to B.

$h_A(X_A^* - dt)v_B(X_B^*) - c \geqslant h_B(X_B^*)v_A(X_A^* - dt) - c$

Thus,

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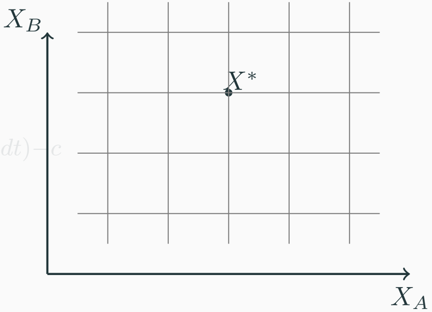

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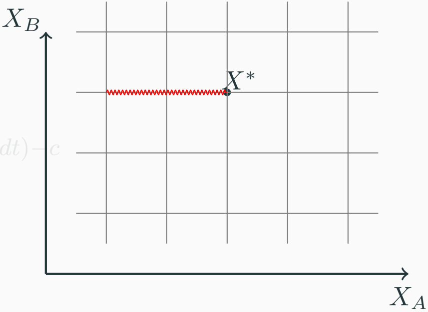

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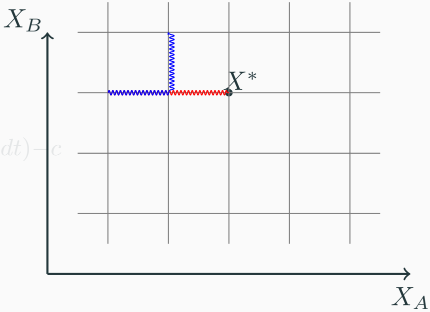

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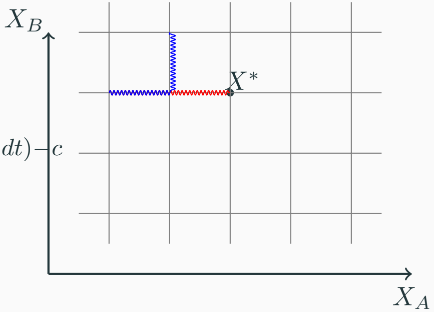

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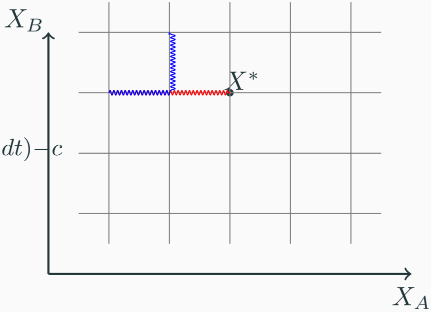

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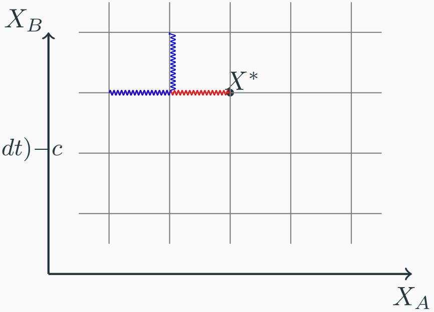

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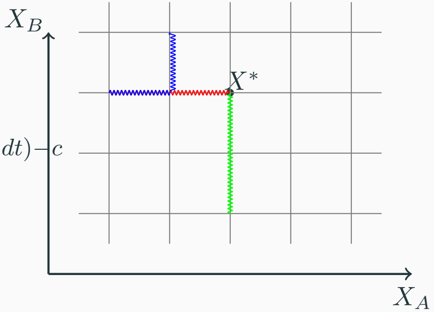

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Competition:

- Pool of n agents.
- At time zero, one agent is picked uniformly at random.
  - For time $[0, d)$ selected agent chooses how to allocate attention and pays the cost of it.
  - All agents observe attention allocation and project completion.
  - At time d a new agent is selected (with replacement), etc.
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Principal wants to implement efficient allocation. Observes:

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Principal knows parameters, but does not know initial beliefs p_A, p_B.

When all agents stop, Principal rewards or taxes the agents.

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- π_1, π_2 rewards associated with 1st and 2nd project completion.

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Definition

A reward scheme is *first-capture* if:

- $E[\pi_1|\tau_1 = t] = v_2(X_2(t))$
- Agents derive zero value from second stage: at τ_1, agents are indifferent between completing the remaining project and stopping.

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Proof Sketch

Consider the problem of a **myopic** agent that maximizes:

$$E\left[\text{Payoff from next } d\right] \tag{1}$$

For d sufficiently small, the myopic agent works on the project with highest flow expected payoff:

$$h_i(X_i) \times E[\pi_i | \tau_i = t] - c$$

For $E[\pi_1 | \tau_1 = t] = v(X_2(t))$, myopic agent pays attention to the project with highest h/v:

$$\begin{aligned} h_A(X_A) \cdot v_B(X_B) - c &\geqslant h_B(X_B) \cdot v_A(X_A) - c \\ \Leftrightarrow \quad \frac{h_A(X_A)}{v_A(X_A)} &\geqslant \frac{h_B(X_B)}{v_B(X_B)} \end{aligned}$$

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If $g > 1$,

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  - The project with highest h/v is the least promising one.

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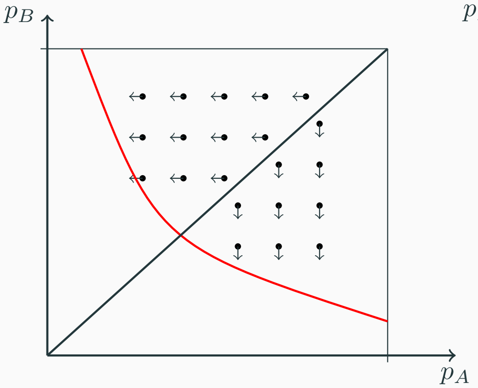
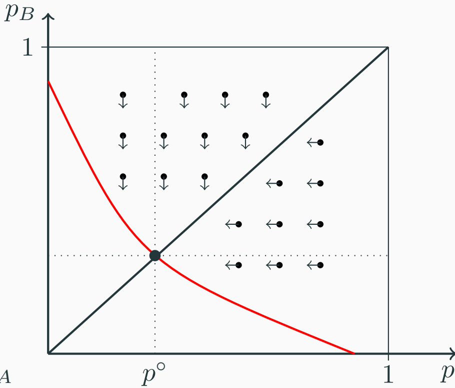
Myopic Allocation



When $g > 1$ agents works always on the least promising project.



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Suppose all future agents will chose the efficient allocation.
Non-myopic agent at t maximizes:

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This is a linear combination of the social welfare and the myopic objective function!

- The solution to this problem must be efficient as well.
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Intuition

Consider

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$$f(x^*) + \alpha \cdot g(x^*) \geqslant f(x) + \alpha \cdot g(x)$$

Implementation

Is there a first-capture reward scheme?

Consider the following:

- Agents that complete projects get

$$\pi_1 = \gamma - c \cdot (\tau_2 - \tau_1)$$
$$\pi_2 = \gamma$$

- Every agent that didn't stop right after τ_1 pays $\frac{1}{n}[\gamma - c \cdot (\tau_2 - \tau_1)]$.

Notice that this reward scheme:

- is first-capture.
- is budget-balanced.
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Different Supports

Project dependent difficulties:

- Rate of completion of project i is $\lambda_i \in \{\lambda_i^L, \lambda_i^H\}$

Proposition

Let $g_i := 2\frac{c/\gamma}{\bar{\lambda}_i} + \left(\frac{\delta_i}{\bar{\lambda}_i}\right)^2$, where $\bar{\lambda}_i := 0.5(\lambda_i^H + \lambda_i^L)$.

- If $g_i > 1$ for both projects, then it is efficient to work on them in sequence.

Different Supports

In this case, first-capture does not guarantee efficiency in the first stage for d small.

Example: **One of the projects has constant success rate** $\lambda_A = 1$.

Proposition

- When $\lambda_A = 1$:
  - It is always efficient to complete the tasks in sequence, starting from B.
  - For $d \to 0$, a first-capture reward scheme does not implement the efficient allocation if $\lambda_B^H < 1$ and p_B is large enough.

General Results and Extensions

Proposition 1 can be used to obtain a solution for supermodular q and general $\{F_i\}$ using standard optimization techniques.

- Moreover, **Proposition 1** can be easily generalized to
  - Discrete time,
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- Monotonicity of the solution implies the equivalence result (Proposition 1).
- However: supermodularity is not sufficient to guarantee monotonicity.
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$h_i/v_i \searrow \quad \Rightarrow$

highest h/v is leads to the efficient pledge

Consider the path that always chooses the project with highest h/v.

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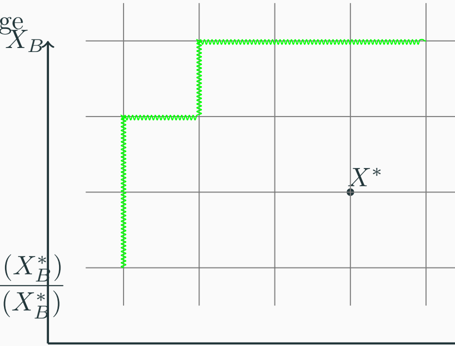

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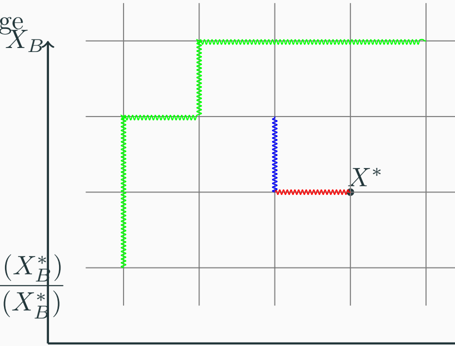

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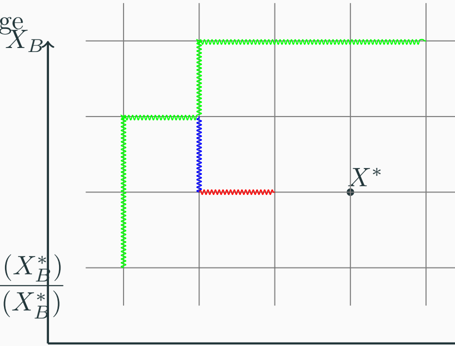

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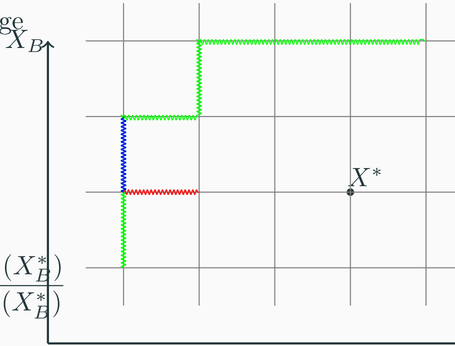
X^* to the right of the path.

$$\frac{h_A(X_A^* - m)}{v_A(X_A^* - m)} > ... > \frac{h_A(X_A^* - 2)}{v_A(X_A^* - 2)} > \frac{h_B(X_B^*)}{v_B(X_B^*)}$$



Conclusion

Model with endogenous timing of complementary innovations.

- Complementarities in the value derived from innovation relates to properties of the solution.
- Connection between properties of the hazard-to-value ratio and the optimal timing of development.

A key insight from sequential innovation is that, to achieve efficiency, early innovators should capture the increase in value of subsequent complementary innovations.

Under some conditions, doing so is efficient even with endogenous timing of developments.

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