Advanced Microeconomics III

Spence's Signaling Model

Francisco Poggi

Introduction

- **Akerlof**: markets with privately informed participants are often inefficient.
- Agents whose information is *favorable* may have an incentive to find means to convey this information.

- **Signaling**: information can be conveyed, but only indirectly.

Introduction

- Examples:
  - A *warranty* may signal good quality of a used car.
  - *education* may signal workers' ability.
- Questions:
  - How can signaling occur in equilibrium?
  - Is signaling always welfare-improving?

Spence's model

- A single worker and many (at least 2) firms.
- Worker can be of two types: $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L$.
- Only the worker knows θ.
- If employed by a firm, worker produces output θ.
- Firm's payoff:
  - $\theta - w$ if employs the worker at wage w.
  - zero otherwise.

Spence's model

- Worker moves first: chooses an observable education level $e \in [0, \infty)$
- Firms observe e (again: not θ).
- Cost of education $c(e|\theta)$.
- Worker payoff when education e and employed at wage w:

$$u(w, e|\theta) = w - c(e|\theta)$$

- Notice that education in this model is unproductive.

Spence's model

- Extra assumptions:
  - Cost of no education is zero.

$$c(0|\theta) = 0 \qquad \text{for all } \theta$$

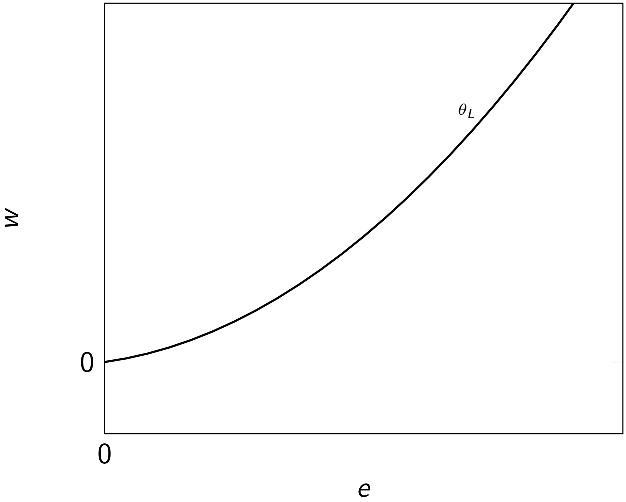
  - Cost of education increasing and convex in education.

$$c'(e|\theta) > 0 \qquad \text{and} \qquad c''(e|\theta) > 0$$

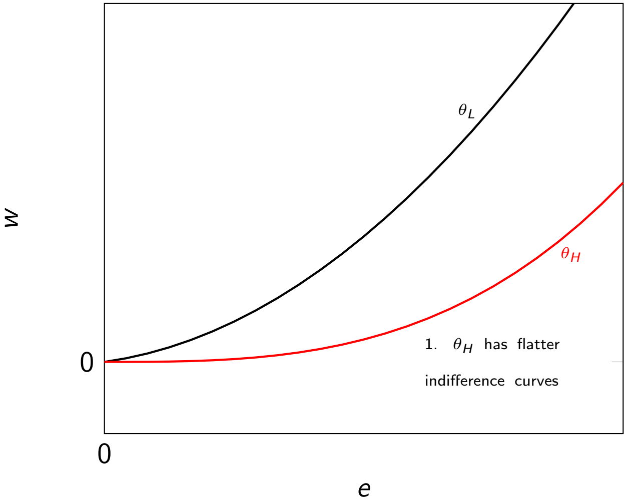
  - High type worker has a smaller education cost.
    Moreover: High type has a smaller marginal cost of education.

$$c'(e|\theta_H) < c'(e|\theta_L) \qquad \forall e > 0 \qquad \text{(Single-crossing)}$$

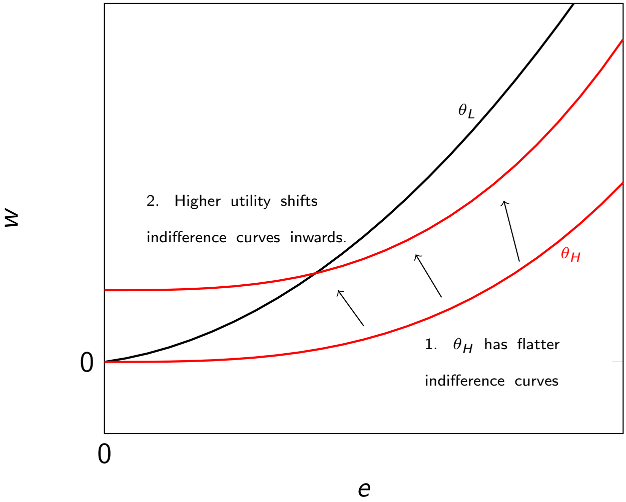
Indifference Curves



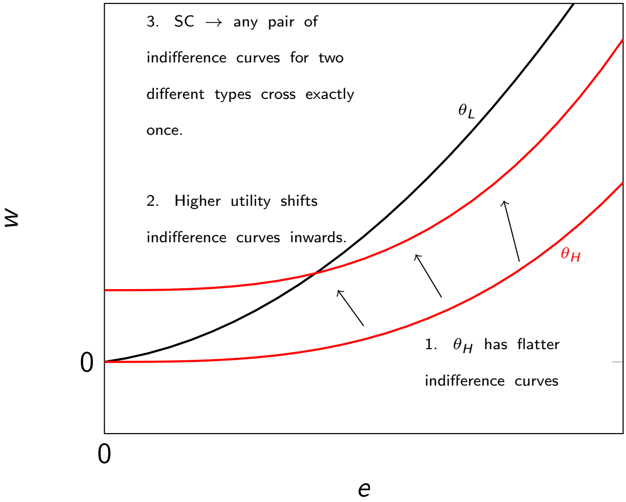
Indifference Curves



Indifference Curves



Indifference Curves



PBE Analysis

- **Solution concept**: (Pure-strategy) Perfect Bayesian Equilibrium.
- Described by:
  - A choice of education level for each worker type e_L, e_H.
  - $\mu(e)$ firms' posterior beliefs that worker is of type H.
  - wage offers of the firms $w(e)$.
- Satisfying:
  - Optimality of education choices given wage offers.
  - Beliefs $\mu(e)$ consistent with Bayes' Rule where possible.
  - Wage offers constitute a Nash equilibrium at each subgame.
  - **Symmetry**: All firms hold the <u>same</u> beliefs after observing e.
    - (Not implied by weak PBE.)
  - Firms believe other firms conform to equilibrium wage offer $w(e)$ both on and off path.

PBE Analysis

- Competition among firms leads to the following wage offers (why?):

$$w(e) = E_{\mu(e)}[\theta] = \mu(e) \cdot \theta_H + (1 - \mu(e)) \cdot \theta_L$$

- Two types of pure-strategy equilibria:
  - **Separating equilibria**: each type chooses a different education level ($e_H \neq e_L$).
  - **Pooling equilibria**: types choose the same education level ($e_H = e_L$).

Separating Equilibria

- $e_H \neq e_L$.
- Bayes' rule where possible: $\mu(e_L) = 0$ and $\mu(e_H) = 1$.
- By competition:

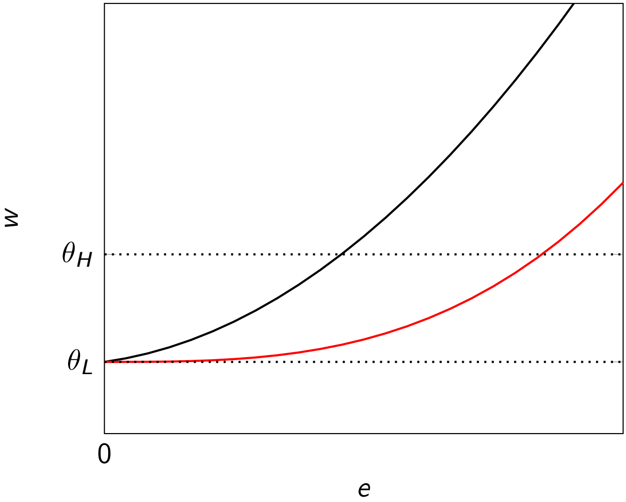
$$w(e_L) = \theta_L \qquad w(e_H) = \theta_H$$

Lemma

In any separating equilibrium, $e_L = 0$.

- PBE implies that $w(e) \in [\theta_L, \theta_H]$.
- So, if $e_L > 0$, the deviation to $e = 0$ is profitable for type θ_L.

Separating Equilibria



Separating Equilibria: Incentive Compatibility

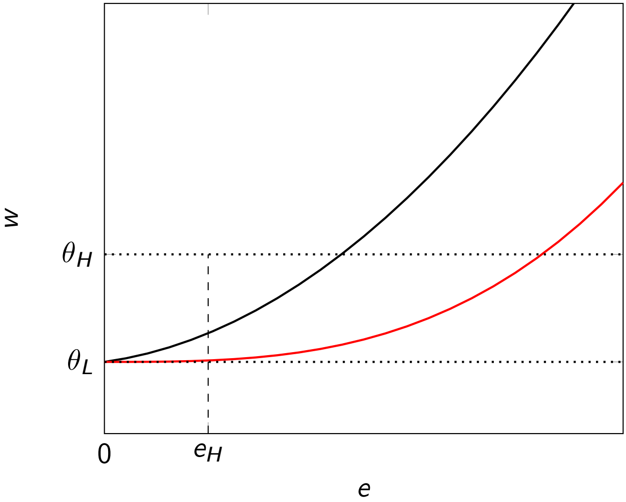
Lemma

In a separating equilibrium, type H chooses $e_H > 0$ such that

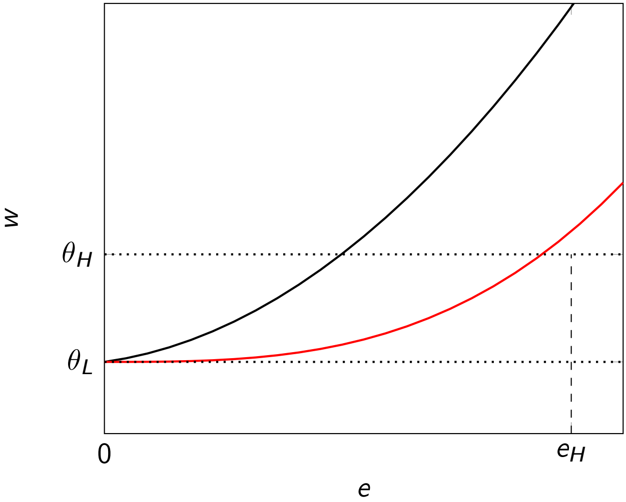
$$\theta_H - c(e_H|H) \;\geq\; L \;\geq\; \theta_H - c(e_H|L)$$

- First inequality: type H prefers his education e_H rather than zero.
- Second inequality: type L prefers zero rather than e_H.

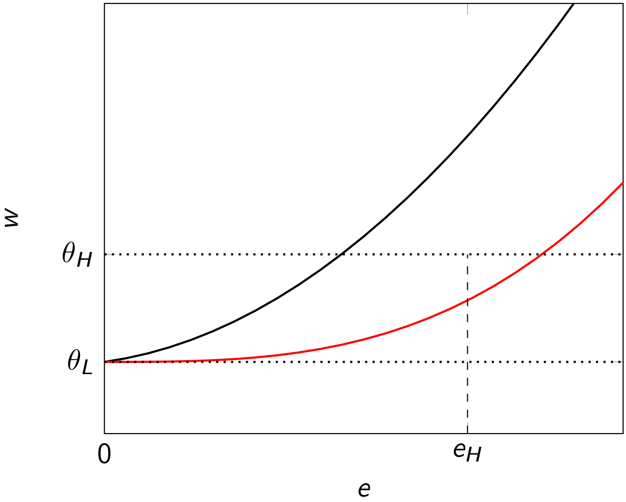
Separating Equilibrium: IC



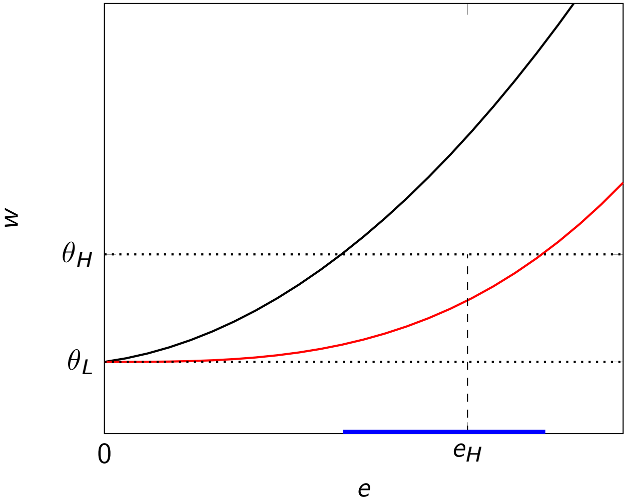
Separating Equilibrium: IC



Separating Equilibrium: IC



Separating Equilibrium: IC



Separating Equilibria

- Previous lemmata describe *necessary* conditions for separating equilibrium.

- These are also *sufficient*: remains to specify out-of-equilibrium beliefs.
  - Suppose any deviation is considered to be by a type L.
  - Then wage would be θ_L for any worker with an education level different than e_H.
  - Any deviation would be unprofitable.

Equilibrium Multiplicity

We have **multiple** separating equilibria.

- These equilibria can be ranked in Pareto sense.
- Best separating equilibrium: the one with lowest education e_H.

$$c(e_H|\theta_L) = \theta_H - \theta_L$$

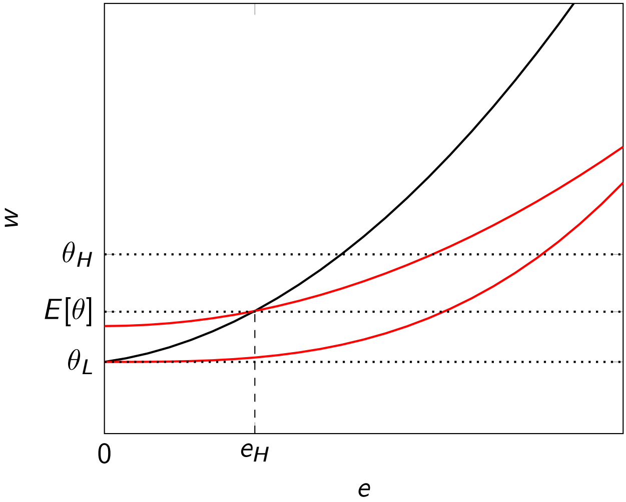
Pooling Equilibria

- Pooling equilibrium: $e_L = e_H = e^*$.

- Bayes' rule where possible: $\mu(e^*) = \Pr(\theta = \theta_H)$
- Competition implies that $w(e^*) = E[\theta]$.

- Out-of-equilibrium beliefs: $\mu(e) = 0$ for $e \neq e^*$.
  - Then $w(e) = \theta_L$ for $e \neq e^*$.

Pooling Equilibria



Multiple Pooling Equilibria

- **Again**: Best pooling equilibrium is the one with the lowest level of education ($e^* = 0$).

- What about the *worst* one?

$$E[\theta] - c(e^*|\theta_L) = \theta_L$$

$$c(e^*|\theta_L) = E[\theta] - \theta_L$$

Comparing Pooling and Separating Equilibra

- The best pooling equilibrium may or may not Pareto dominate the best separating equilibrium.

- The best separating equilibrium *never* Pareto dominates the best pooling equilibrium.
  - The low type is always worse-off.

Reasonable Beliefs (Equilibrium Refinements)

- Forward induction arguments can be used to refine the equilibrium
  - Most uniquely select the least costly separating one.

- Cho and Kreps (1987) 'Intuitive criterion':
  - A PBE *passes the Intuitive Criterion Test (ICT)* if no type θ would be better off deviating to an action $e' \neq e(\theta)$ should the receivers' beliefs following e' assign zero probability to types θ' for whom the deviation is *dominated in equilibrium*.
  - A deviation e' is dominated in equilibrium for type θ if, for any sequentially rational response by the receivers $w' = E_{\mu'}[\theta]$ for some beliefs μ', the resulting payoff $u(e', w', \theta)$ is less than the equilibrium payoff $u(e(\theta), w(e(\theta)), \theta)$.