Strategic Concealment in Innovation Races

Yonggyun Kim and Francisco Poggi

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Introduction

Consider two firms engaging in an innovation race.

- The first firm to have a breakthrough obtains a prize Π.
- Firms pay a flow cost c throughout the race.

- Breakthroughs for firm i arrive at constant rate λ_i.
- Firm A has a piece of knowledge that gives them an advantage: $\lambda_A > \lambda_B$.

Expected Payoff of firm i:

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Suppose Firm A can share knowledge with Firm B, in which case both firms would race with rate λ_A. This would:

1. Reduce race duration.
2. Increase the chance that Firm B wins the race.

Overall, sharing knowledge would be more **efficient**.

Coase Theorem: There exists a price P such that

- Firm B is willing to pay to acquire the knowledge.
- Firm A is willing to accept to share the knowledge with Firm B.

$$P \in \left[\frac{(\lambda_A - \lambda_B)(\lambda_A \Pi - c)}{2\lambda_A(\lambda_A + \lambda_B)}, \frac{(\lambda_A - \lambda_B)(\lambda_A \Pi + c)}{2\lambda_A(\lambda_A + \lambda_B)}\right]$$

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What we do

Most times, knowledge has to be acquired and is private.

We study an innovation race with:

- Unobservable interim breakthroughs (knowledge).
- Firms directing R&D efforts in a flexible, dynamic way.

We characterize the equilibrium behavior of firms

- When they can patent and license interim breakthroughs,
- When they cannot.

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Results in a Nutshell

When interim breakthroughs are **public**, patents work:

- Induce firms to share their breakthroughs.
- Induce more efficient R&D resource allocation.

When interim breakthroughs are **private**, patents might be ineffective.

- firms conceal interim breakthroughs (trade secrets).
- Inefficient allocation of R&D resources.
- Particularly problematic when stakes are high.

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Model

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Two firms $i \in \{A, B\}$ participate in a race.

Time is continuous and infinite $t \in [0, \infty)$.

Two technologies:

- An **incumbent** technology L.
- A **new** technology H (not available at first).

A firm allocates, at each point in time, a unit of resources to:

- **Research**: try to obtain the new technology.
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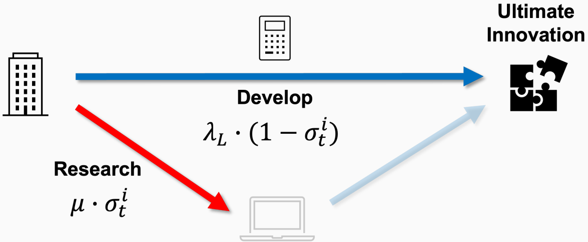

Ultimate Innovation
Develop
$\lambda_L \cdot (1 - \sigma_t^i)$
Research
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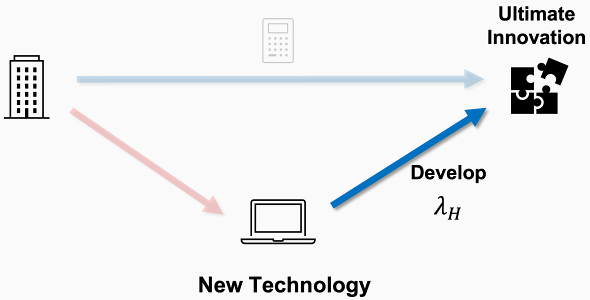
Technology



Payoffs

The race ends when one of the firms develops the innovation.

Payoff of firm i:

$$\Pi \cdot 1_{\{w=i\}} - c \cdot d$$

where

- $\Pi, c > 0$.
- $w \in \{A, B\}$ is the identity of the race winner,
- d is the duration of the race.

Assumption: Incumbent technology is profitable $\Pi > c/\lambda_L$

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- Resource allocation is private information.
- Successful development is public.
- Interim breakthrough (finding of the new technology). Three cases:

  (1) Public (2) Private (3) Patents.

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Observable Interim Breakthroughs

Solution Concept

Markov states: $\Omega = \{\emptyset, \{A\}, \{B\}, \{A, B\}\}$.

Markov strategy: $s : \Omega \to [0, 1]$

Expected payoffs: given a Markov strategy profile (s_A, s_B)

$$U^i_\omega \qquad i \in \{1, 2\} \quad \omega \in \Omega$$

Solution concept: MPE.

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Breakthrough Reaction

In any MPE, the expected payoff when both firms have the new technology:

$$U^i_{\{A,B\}} = \frac{1}{2}\Pi - \frac{c}{2\lambda_H} \tag{1}$$

Suppose only Firm j has the new technology. What should Firm i do?

$$\frac{x \cdot \mu \cdot U^i_{\{A,B\}} + (1-x) \cdot \lambda_L \Pi - c}{x\mu + (1-x)\lambda_L + \lambda_H}$$

Lemma

- If $\mu > \bar{\mu}$ then, in any MPE, $s_A(\{B\}) = s_B(\{A\}) = 1$
- If $\mu < \bar{\mu}$ then, in any MPE, $s_A(\{B\}) = s_B(\{A\}) = 0$

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Expected payoffs

Using the previous lemma, we obtain the payoffs $U^i_{\{i\}}, U^i_{\{j\}}$.

- if $\mu > \bar{\mu}$,

$$U^i_{\{i\}} = \frac{\lambda_H \Pi + \mu V_C - c}{\mu + \lambda_H}, \qquad U^i_{\{j\}} = \frac{\mu V_c - c}{\mu + \lambda_H},$$

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Optimal allocation at state $\emptyset$

Fixing continuation values $U^i_{\{i\}}$ and $U^i_{\{j\}}$, one can define the payoff at state $\emptyset$:

$$u_\emptyset(x,y) := \frac{\mu \cdot x \cdot U^i_{\{i\}} + \mu \cdot y \cdot U^i_{\{j\}} + \lambda_L \cdot (1-x) \cdot \Pi - c}{\mu(x+y) + \lambda_L(2-x-y)}$$

Lemma

Let $\Delta_y = u_\emptyset(1,y) - u_\emptyset(0,y)$.

- If Δ_0, Δ_1 positive (negative), it is best to choose $x = 1$ ($x = 0$) independently of y.
- If only Δ_1 is positive, allocations are strategic complements.
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MPE with Observable Interim Breakthroughs

Proposition

For almost all parameters, there is a unique MPE.

- $\mu > \bar{\mu}$: firms do research until obtaining the *H* technology.
- $\mu < \underline{\mu}$: firms develop with the *L* technology.
- $\mu \in (\underline{\mu}, \bar{\mu})$, firms follow **fall-back strategies**: do research until either of the firms obtains the new technology and develop afterwards.

For the rest of this talk, I'll focus on intermediate μ.

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Unobservable Interim Breakthroughs

Allocation Policy

With *unobservable* interim breakthroughs, firms cannot condition their allocation on the opponents' technology.

An **allocation policy** $\sigma_i(t)$ indicates how much resources Firm i allocates to research at time t, conditional on that

- Firm i doesn't have the new technology.
- the race is still on.

$$\sigma_i : \mathbb{R} \to [0, 1]$$

Solution concept: Pure Symmetric Nash Equilibrium (SNE).

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Evolution of Beliefs

Lemma

- Consider that
  - an opponent follows policy σ.
  - the race is ongoing by time t.
- The probability p_t that the opponent has the new technology evolves according to:

$$
\begin{aligned}
p_0 &= 0 \\
\dot{p}_t &= \underbrace{\mu \cdot \sigma(t) \cdot (1 - p_t)}_{\text{ME}} - \underbrace{[\lambda_H - (1 - \sigma(t))\lambda_L] \cdot p_t \cdot (1 - p_t)}_{\text{BU}}
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\begin{aligned}
p_0 &= 0 \\
\dot{p}_t &= \underbrace{\mu \cdot \sigma(t) \cdot (1 - p_t)}_{\text{ME}} \underbrace{- [\lambda_H - (1 - \sigma(t))\lambda_L] \cdot p_t \cdot (1 - p_t)}_{\text{BU}}
\end{aligned}
$$

Evolution of Beliefs

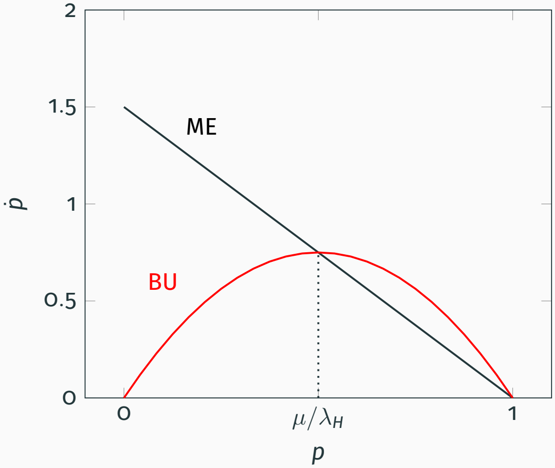

Figure 1: Mechanic and **Bayesian Updating** effects. $\sigma_j = 1$. $\mu = 1.5$, $\lambda_H = 3$, and $\delta = 2/3$.

Steady State

Definition

A *Steady State* (SS) is a pair (p, x) such that

- If $\sigma(t) = x$ then $\dot{p}_t = 0$.
- If opponent develops at constant rate $p\lambda_H + (1-p)(1-x)\lambda_L$, the firm is indifferent between any allocation.

Lemma

If $\mu \in \{\underline{\mu}, \bar{\mu}\}$, there is a unique Steady State $(p^*, x^*) \in (0,1)^2$.

Symmetric Markovian Equilibrium

Proposition

Let $\mu \in (\underline{\mu}, \bar{\mu})$ and (p^*, x^*) is the unique SS. Then (σ^*, σ^*) is a SNE, where

$$\sigma^*(t) = \begin{cases} 1 & t < T^* \\ x^* & t \geq T^* \end{cases}$$

and and the beliefs at $p_{T^*} = p^*$.

Symmetric Markovian Equilibrium

Equilibrium beliefs are strictly increasing until T^* and then constant.

- Unique SNE where p is increasing over time.
- Unique SNE markovian in beliefs.

Comparative statics:

- The effects of λ_L, λ_H and μ on T^* and x^* are the expected ones.
- σ^* does not depend on Π or c.

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Patents

Model with Patents

Same model as before with the following modifications:

- A firm that has the new technology can **apply for a patent**.
  - Patent applications are public.
- **First-to-invent**: The patent is granted if no other firm had the interim breakthrough before.
- If patent is granted, the patent holder makes a TIOLI offer to the opponent.
- If offer is accepted, both firms race with the new technology onward.

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Continuation Payoffs

Suppose firms apply for patents immediately. Then, in equilibrium, patents are granted.

After a patent is granted, the TIOLI offer will capture all the extra surplus and will be accepted.

Then we can use the observable case results for state $\emptyset$, with different continuation values: $\hat{U}^i_{\{i\}}$ and $\hat{U}^i_{\{j\}}$.

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Proposition

If stakes are sufficiently high (Π/c large enough)

- firms do NOT apply for patents in equilibrium.
- Equilibrium allocations and payoffs as in the unobservable case.

Intuition: Coase Theorem fails to hold because patenting changes the outside option of the opponent firm.

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Conclusion

We develop a model of innovation race with interim breakthroughs.

We solve equilibria where these interim breakthrougs are public and private.

We use the results to analyze the effectiveness of intermediate patents.

- Firms might not patent to conceal breakthroughs even when patent holders have all the bargaining power in licensing negotiations.
- Patents for interim breakthroughs are less effective when stakes are high.