Problem Set 6

Spring 2023

Advanced Microeconomics III

Problem 1 Final Exam 2022

Consider a standard symmetric auction setup with a single object to be allocated among $n \geq 3$ bidders that have uniform independent valuations on $[0, 1]$.

a. First, we analyze the *third-price auction*. In this auction, the bidder that submits the highest bid gets the object and pays an amount equal to the third highest bid.

i. Show that for a bidder, bidding their own valuation is **not** a weakly dominant strategy (as it is in the second-price auction).

In the third-price auction there is a symmetric equilibrium in which each bidder bids according to the following bid function:

$$b(v) = \frac{n-2}{n-1} \cdot v$$

ii. Compute the interim expected transfer of player i, as a function of her type, in the previously described equilibrium.

iii. Carefully, apply the result on revenue equivalence to show that the expected revenue from the described equilibrium in the third-price auction is the same as the expected revenue in the second-price auction.

b. Now we analyze an *all-pay auction*: In this auction, each bidder submits a bid. The highest bid gets the object,[1] but **all players pay their bid.**

[1] in case of a tie, the object is allocated randomly among the bidders that submitted the highest bid.

There is a symmetric Bayesian Nash equilibrium of this auction where each bidder bids according to the following bid function:

$$b(v) = \frac{n-1}{n} \cdot v^n$$

i. Prove that this constitutes a Bayesian Nash equilibrium of the game.

ii. What is the interim expected payment that a bidder of value v makes in equilibrium? What is the ex-ante expected payment that a bidder makes in equilibrium?

iii. Carefully, apply the result on revenue equivalence to show that the revenue from the equilibrium described in the all-pay auction is the same as the expected revenue in the second-price auction.

Problem 2 Consider a linear with private values environment for the allocation of a single unit of a good to two agents with type spaces $\Theta_1 = [0, 1]$ and $\Theta_2 = [0, \bar{b}]$ for some $\bar{b} > 1$ and arbitrary densities.

a. Show that, in a first-price auction, there is no equilibrium in which the good is always allocated to the agent that values the good the most (You can assume that ties are broken using a fair coin. **Hint**: proof by contradiction.)

Problem 3 Consider a linear-utility environment for the allocation of a single unit of a private good where one of the buyers is stochastically stronger that the others. Specifically, assume that $\Theta_1 = [0, 3]$ and $\Theta_2 = \ldots = \Theta_N = [0, 1]$. All type distributions are uniform.

Let $\hat{s}$ be a subsidy and $r \geq 0$ be a minimum bid.

Suppose that the following mechanism is played, called *second-price auction with minimum bid and bid subsidy*: each agent's action space is $S_i = [r, \infty) \cup \{\text{abstain}\}$. Any action other than abstain is call a *bid*. The outcome function stipulates that the agent who submits the highest bid obtains the good; if the highest bid is submitted by multiple agents then each of them gets the good with the same probability, if all agents abstain the good is not handed out and no payments are made.

The winner has to pay the highest bid among all the other agents. Unless everybody except the winner has abstained, in which case the winner pays r. In addition, if any of the bidders different than 1 wins, then she obtains a payment of $\hat{s}$.

a. Show that each agent has a weakly dominant strategy.

b. Describe the social choice function that is implemented if each agent plays her weakly dominant strategy.

c. Describe the expected revenue of the seller as a function of $\hat{s}$ and r assuming that agents use their dominant strategies.

d. Shaw that a second-price auction with minimum bid r and subsidy $\hat{s}$ is a revenue-maximizing mechanims for some r and $\hat{s}$ optimally chosen.