The Timing of Complementary Innovations

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Abstract

Socially-valuable technologies sometimes require complementary innovations. This paper studies the development of innovations that exhibit such complementarity. At each point in time, resources are allocated across different innovation projects. The projects are completed stochastically in the form of breakthroughs and the social value of the technology depends on the set of completed projects by the time development ends. In some cases it is optimal to develop the innovations in sequence. In others, it is optimal to develop multiple innovations simultaneously. I provide conditions that determine the efficient timing of development: sequential development is efficient when development costs are high and there is high uncertainty about the innovations' rate of success. I compare the efficient timing of development to the equilibrium outcome with a decentralized industry in which many firms compete on the development of the innovations.

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1 Introduction

Patent laws and innovation policies aim to orient scarce resources toward the most socially valuable R&D projects. The value of an innovation, however, might be tied to the outcome of other developments. A particularly interesting case is complementary innovations, which have been increasingly relevant in industries such as telecommunications and biotechnology. For complementary innovations, the timing is relevant. Consider the case of hardware and software: The first classical computer algorithm was written in 1843,[1] while the first computer capable of running said algorithm wasn't developed until the 1930s. A similar process is taking place for quantum computing: Shor's quantum algorithm, a method for solving integer factorization problems in polynomial time, was written in 1994, four years before the first quantum computer prototype was developed. Today, startups and established companies alike invest resources to develop quantum software that can only be implemented with hardware that does not yet exist,[2] and it is not clear that it ever will. So what determines the timing in which complementary innovations are developed?

For some complementary innovations, the timing is dictated by an exogenous order in which the developments should succeed each other.[3] For other innovations, there is no exogenously imposed order, so the timing is determined endogenously by the allocation of resources to the different projects. My main research question in this paper is, how are resources endogenously assigned to complementary R&D projects? In particular, how does the environment (for example the level of competition or patent rights) affect the allocation of resources to the development of complementary innovations,

[1] By first classical computer algorithm, I mean an algorithm written for a classical computer that has no value given human computing power.

[2] The highest integer that has been factorized using Shor's algorithm is $21 = 7 \times 3$.

[3] For instance, no one was working on the can opener before the invention of the can—the can opener was invented decades after the can became popular. This order is natural, since the problems are very much related: a can opener cannot be invented without the specifications of the can.

and is the allocation efficient?

The key to determining the allocation of resources lies in assessing the prospects of each development. R&D projects carry high levels of uncertainty, both in terms of outcomes—the project may or may not prove successful—and in terms of costs— it is not clear how much time and resources will be needed to complete the development. This paper combines a dynamic of beliefs with the endogenous timing of development by introducing a tractable model that features the main aspects of the R&D process: A unit of attention is allocated over a set of projects at each point in time. A success for project i arrives discretely in the form of a breakthrough. In particular, a success arrives when the total amount of attention paid to a project reaches a certain level τ_i. Successes are observable, but τ_i is unknown.

In the first part of this paper, I study the *efficient* way to sequentially allocate attention to complementary R&D projects given the social value of innovations, which is realized when the development stage ends and is supermodular on the subset of the innovations that have been successfully developed by that time. The cost of development takes the form of a constant flow throughout the development stage.

Consider two complementary projects, A and B. How should society assign resources to them? Should they all be concentrated on A and then be switched to B if and only when A is successfully completed? Or should both A and B be developed in parallel? Moreover, when should a project be abandoned or put on hold? For complements, the success of one project makes it more attractive to keep paying attention to the remaining projects. Proposition 1 shows that this implies for two complements that all that matters for efficiency is how much is invested on each remaining project before abandoning. The intuition is that given the complementarities in payoff, the amount you are willing to work on a project if there is no new success is the minimum you are going to work on that project. Since you are going to do it independently of the outcome of other projects, *when* you do it is not payoff-relevant.

Section 4 considers the case where the rate of success for each project λ_i is constant over time but unknown. The beliefs about λ_i evolve with the outcomes of the development process. In particular, lack of success is evidence in favor of λ_i being relatively low, or in other words "the project i being relatively more challenging." The λs are independent across projects, so working on a project does not affect beliefs about the success rate of the others.

When the rate of success for each project is constant and known, the timing of development is irrelevant. Any project that is worth pursuing is worth completing; therefore, the order of completion is not going to affect the final expected payoff.[4] In contrast, when the rate of success is uncertain, the order of development is relevant, since it affects the arrival of information about the unknown parameters. The failure to develop A not only reduces the prospects of ever completing A, but also decreases the expected returns from completing B. The problem can therefore be thought as a *restless multi-armed bandit*, for which there is no general Gittins-like index rule that governs the optimal dynamic allocation.

Take as an example the case where project A is of uncertain feasibility, that is, the success rate is either zero or λ_A, and project B has a known success rate λ_B. In this case, it is efficient to first work on project A: there is no learning by working on B, so there is no efficiency loss in back-loading all development of B. Front-loading the development of A increases the speed of learning, which is valuable because of the option given by the stopping decision. In the more general setting, the intuition from this example also applies: the efficient allocation of resources reflects the optimal learning process about the potential of the joint project. Proposition 3 claims that for two perfect complements, the nature of the efficient allocation of resources depends on the uncertainty about the projects' difficulty and the flow cost of development. For projects with high uncertainty and high costs, it is efficient to concentrate all resources in one of the projects and thus develope them

[4] A similar logic holds when the completion time for each project is deterministic.

in sequence. The more uncertain the project, the earlier in the sequence it should be developed. For projects with low uncertainty and low costs, it is efficient to spread the resources following a simple "greedy" strategy that maximizes at each point in time the expected increment in value. In the case of symmetric projects, the greedy strategy splits resources equally at all times until either one the projects succeeds or the projects are abandoned.

In the second part of this paper, I study private allocation of the resources. The allocation of private R&D resources depends on several factors: who assigns these resources, the appropriability of the innovations—which is determined by the legal and patent systems—and how informed the agents are about a given project's successes. Section 4.2 analyzes the extreme case of *decentralized* industry.

A *decentralized industry* consists of a continuum of agents, each of whom controls an equal portion of the total unit of resource available at each moment in time. The agents don't consider the informational externalities that their actions generate. With substitute projects, decentralization and competition biases the allocation of resources toward fast, easy projects to the detriment of harder but cost-efficient ones. This *race effect* might be a concern also with complements: if a product requires two components, and it is efficient to start developing the hard one, competition might make it tempting to work on the easy component just to capture a higher share of the value generated. Section 4.2 shows that this is not the case: even if the first agent to succeed appropriates all the surplus from the joint development, the allocation of resources is not biased toward projects just because they are thought to be easier. Competition might introduce new inefficiencies by biasing the allocation toward projects where learning is slower. These inefficiencies disappear, however, if the stakes are sufficiently high.[5]

[5]This contrasts again with the case of substitutes, where higher stakes magnify the race effects. See **?**.