Strategic Concealment in Innovation Races

Yonggyun Kim and Francisco Poggi

August 26, 2023

Introduction

Consider two firms, A and B, engaging in an innovation race and assume A is at the knowledge frontier.

Suppose Firm A can share knowledge with Firm B. This could:

1. Reduce race duration.
2. Increase the chance that Firm B wins the race.

Coase Theorem: if sharing knowledge is **efficient**, there must exists a price P such that

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What we do

Most times, knowledge is endogenous and private.

We study an innovation race with:

- Interim breakthroughs (knowledge).
- Firms directing R&D efforts in a flexible, dynamic way.

We characterize the equilibrium behavior of firms

- When they can patent and license interim breakthroughs,
  - first-to-invent vs first-to-file.
- When they cannot.

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Results in a Nutshell

When interim breakthroughs are **public**, patents work:

- Induce firms to share their breakthroughs.
- Induce more efficient R&D resource allocation.

When interim breakthroughs are **private**, first-to-invent patent systems might be ineffective.

- Firms *conceal* interim breakthroughs, which leads to an inefficient allocation of R&D resources.
- This is particularly problematic when *stakes are high*.

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Related Literature (incomplete)

Innovation Races: Loury ('79); Lee, Wilde ('80);

- **Patent vs. Secrecy**: Horstmann et al. ('85); Denicolo, Franzoni ('04); Anton, Yao ('04); Kultti et al. ('07); Zhang ('12); Kwon ('12)
- **Multiple avenues to innovate**: Akcigit, Liu ('16); Brian, Lemus ('17); Das, Klein ('20); Hopenhayn, Squintani ('21)
- **Multiple-stage innovation**: Scotchmer, Green ('90); Denicolo ('00)
- **Timing of disclosure**: Hopenhayn, Squintani ('16); Bobcheff et al. ('17); Song, Zhao ('21)

Interim R&D Knowledge: Bhattacharya et al. ('86, '92); d'Aspremont et al. ('00); Bhattacharya, Guriev ('06); Spiegel ('07)

Hail-Mary Attempts: Carnehl, Schneider ('22); Kim ('22)

Model

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Two firms $i \in \{A, B\}$ participate in a race.

Time is continuous and infinite $t \in [0, \infty)$.

Firms can race using two technologies:

- An **incumbent** technology L.
- A better **new** technology H (not available at first).

A firm allocates, at each point in time, a unit of resources to:

- **Research**: try to obtain the new technology.
- **Development**: try to win the race with available technology.

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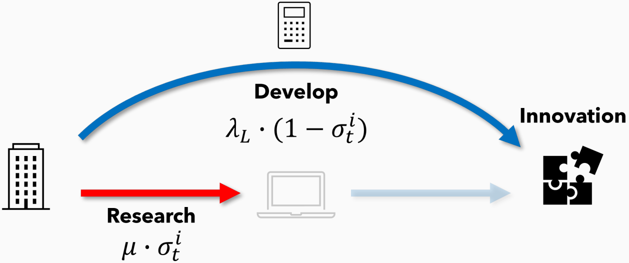
Race before and after interim breakthrough



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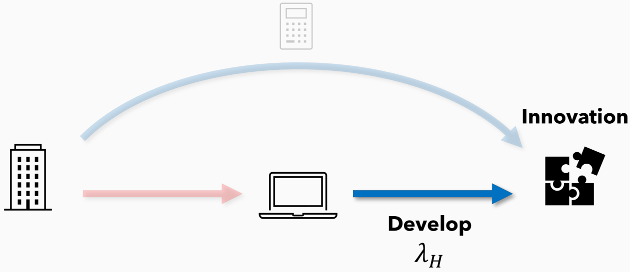

Payoffs

The race ends when one of the firms (winner) produces a *final breakthrough*.

Payoff of firm i:

$$\Pi \cdot 1_{\{w=i\}} - c \cdot d$$

where

- $\Pi, c > 0$.
- $w \in \{A, B\}$ is the identity of the race winner,
- d is the duration of the race.

Assumption: Incumbent technology is viable: $\Pi > c/\lambda_L$

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Information

Resource allocation is private information.

Final breakthroughs are public.

Interim breakthroughs:

- Public (with and without patents)
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Benchmark: Constant hazard opponent

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Suppose that opponent completes the final innovation at a constant rate h.

- What is the best resource allocation for the firm?

Result

There is a threshold rate $\tilde{h}$ such that the firm finds it optimal to develop with the incumbent technology iff $h \geq \tilde{h}$.

- $\tilde{h}$ is constant in Π and c.

Benchmark: Constant hazard opponent

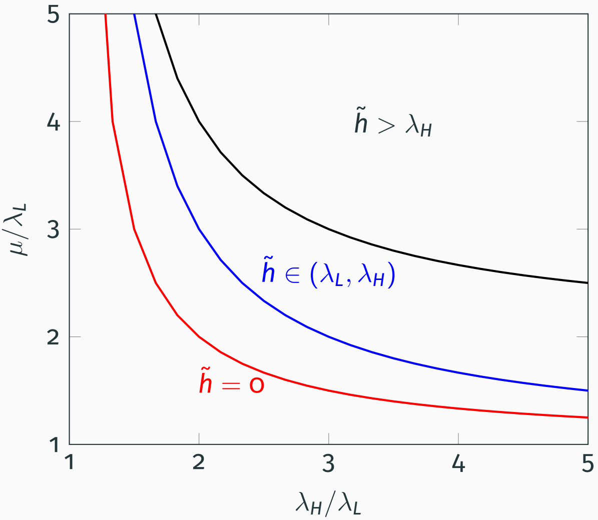
Differential in the probability of winning:

$$\underbrace{\frac{\lambda_L}{\lambda_L + h}}_{\text{incumbent}} - \underbrace{\frac{\mu}{\mu + h} \cdot \frac{\lambda_H}{\lambda_H + h}}_{\text{new}}$$

This differential is positive iff:

$$h \geq \tilde{h} := \lambda_L \left[\frac{\mu}{\lambda_L} \left(\frac{\lambda_H}{\lambda_L} - 1 \right) - \frac{\lambda_H}{\lambda_L} \right]$$

It turns out that what maximizes the winning chances <u>also</u> minimizes the expected duration of the race.



$\tilde{h} > \lambda_H$
$\tilde{h} \in (\lambda_L, \lambda_H)$
$\tilde{h} = 0$
μ/λ_L
λ_H/λ_L
1
2
3
4
5

Observable Interim Breakthroughs (without patents)

Solution Concept

Markov states: $\Omega = \{\emptyset, \{A\}, \{B\}, \{A, B\}\}.$

Markov strategy: $s : \Omega \to [0, 1]$

Expected payoffs: given a Markov strategy profile (s_A, s_B)

$$U^i_\omega \qquad i \in \{1, 2\} \quad \omega \in \Omega$$

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Breakthrough Reaction

In any MPE, the expected payoff when both firms have the new technology:

$$U^i_{\{A,B\}} = \frac{1}{2}\Pi - \frac{c}{2\lambda_H} \tag{1}$$

Suppose only Firm j has the new technology. What should Firm i do?

$$\max_{\sigma\in[0,1]} \quad \frac{\sigma \cdot \mu \cdot U^i_{\{A,B\}} + (1-\sigma)\cdot \lambda_L \Pi - c}{\sigma\mu + (1-\sigma)\lambda_L + \lambda_H}$$

Lemma

- If $\tilde{h} > \lambda_H$ then, in any MPE, $s_A(\{B\}) = s_B(\{A\}) = 1$
- If $\tilde{h} < \lambda_H$ then, in any MPE, $s_A(\{B\}) = s_B(\{A\}) = 0$

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Expected payoffs

Using the previous lemma, we obtain the payoffs $U^i_{\{i\}}, U^i_{\{j\}}$.

- if $\tilde{h} > \lambda_H$,

$$U^i_{\{i\}} = \frac{\lambda_H \Pi + \mu V_C - c}{\mu + \lambda_H}, \qquad U^i_{\{j\}} = \frac{\mu V_c - c}{\mu + \lambda_H},$$

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MPE with Observable Interim Breakthroughs

Proposition

For almost all parameters, there is a unique MPE.

- $\tilde{h} > \lambda_H$: firms do research until obtaining the *H* technology.
- $\tilde{h} < \lambda_L$: firms develop with the *L* technology.
- $\tilde{h} \in (\lambda_L, \lambda_H)$, firms follow **fall-back strategies**: do research until either of the firms obtains the new technology and develop afterwards.

For the rest of this talk, I'll focus mostly on the intermediate case. Why?

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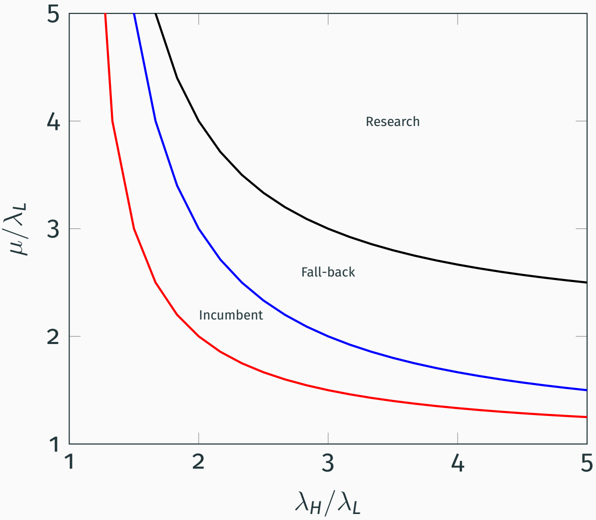

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Unobservable Interim Breakthroughs

Allocation Policy

With *unobservable* interim breakthroughs, firms cannot condition their allocation on the opponents' technology.

An **allocation policy** $\sigma_i(t)$ indicates how much resources Firm i allocates to research at time t, conditional on that

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Evolution of Beliefs

Lemma

- Consider that
  - a firm follows policy σ.
  - the race is ongoing by time t.
- The probability p_t that the firm has the new technology evolves according to:

$$
\begin{aligned}
p_0 &= 0 \\
\dot{p}_t &= \underbrace{\mu \cdot \sigma(t) \cdot (1 - p_t)}_{\text{DE}} - \underbrace{[\lambda_H - (1 - \sigma(t))\lambda_L] \cdot p_t \cdot (1 - p_t)}_{\text{SIR}}
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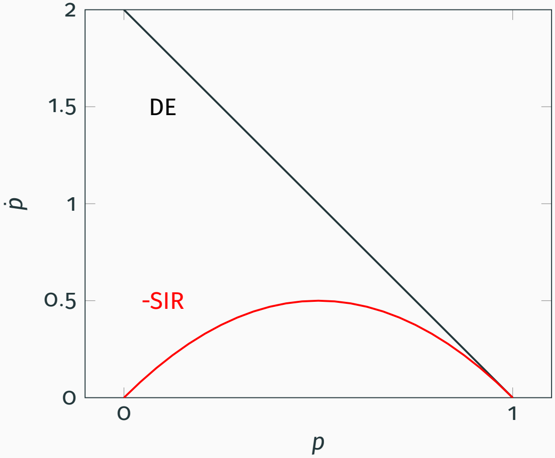

Duration and **Still-in-Race** effects.

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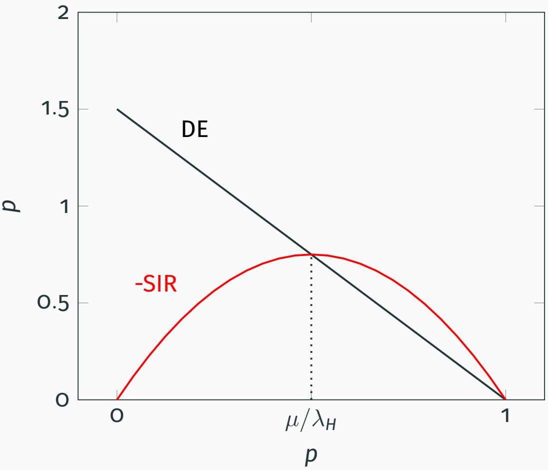

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Useful Rates

Given a policy σ, we can define the following **hazard rate:**

$$h^{\sigma}(t) := \lambda_L(1 - \sigma_t) \cdot (1 - p_t^{\sigma}) + \lambda_H \cdot p_t^{\sigma}$$

We are interested in equilibria with *increasing hazard rate*.

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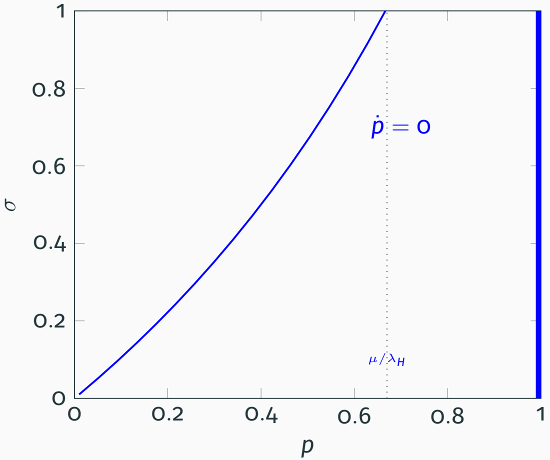

Let $\dot{p}(p,\sigma) = \mu \cdot \sigma \cdot (1-p) - [\lambda_H - (1-\sigma)\lambda_L] \cdot p \cdot (1-p)$

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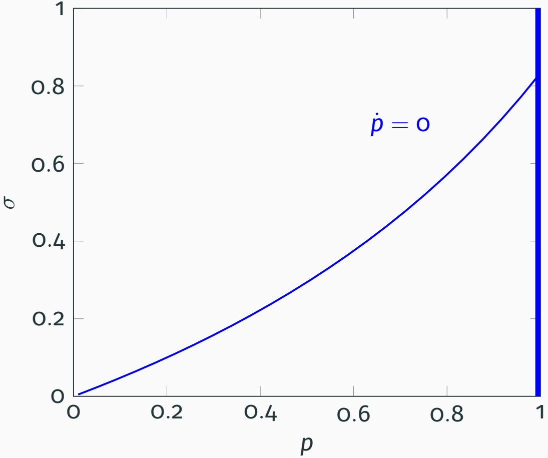
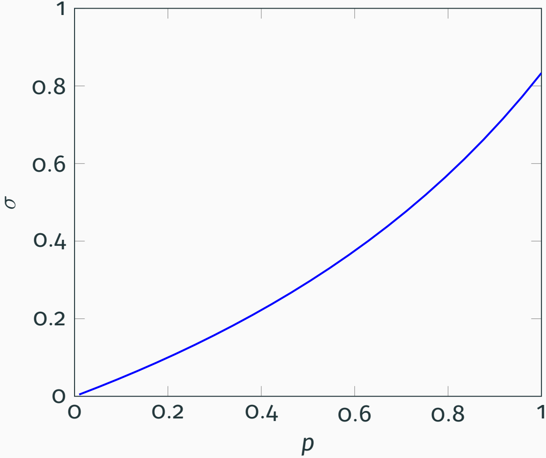


For ease of the exposition, remaining of section $\mu > \lambda_H$.

Iso-hazard rate curves



Let $h(p, \sigma) = p \cdot \lambda_H + (1 - p)(1 - \sigma)\lambda_L$.

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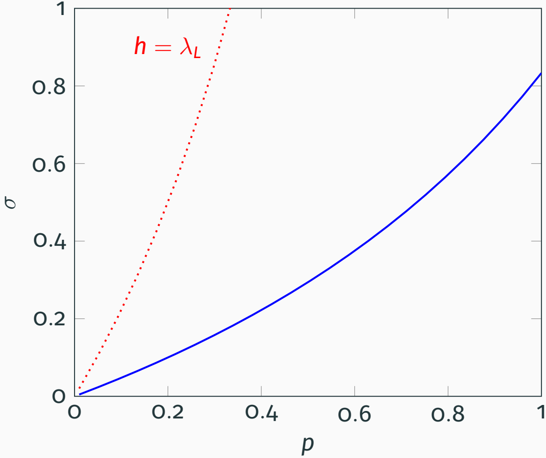

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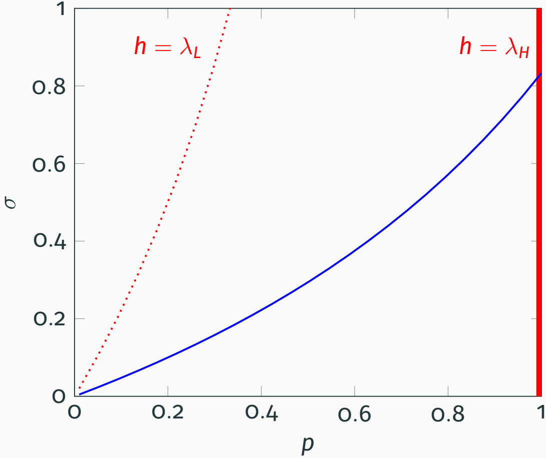

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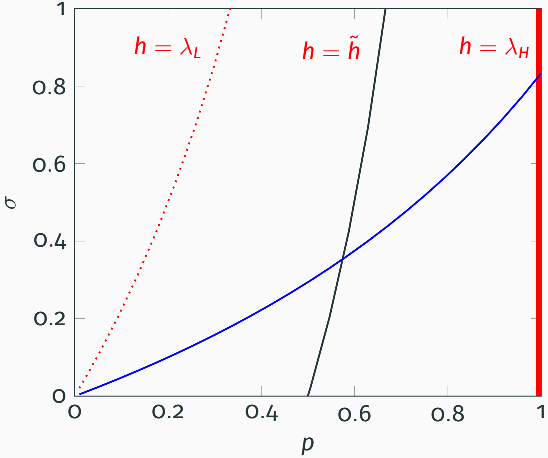

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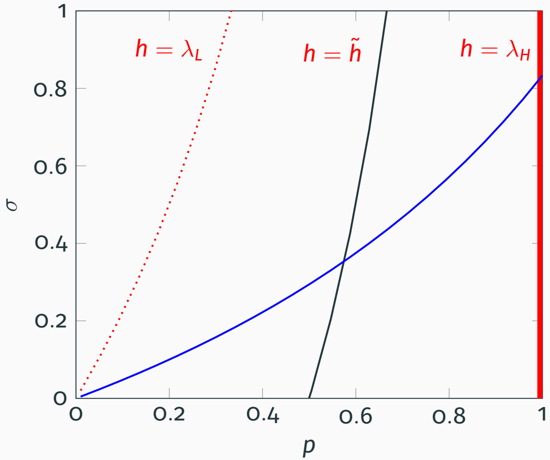

Claim: any isoquant of h is steeper than $\dot{p} = 0$.

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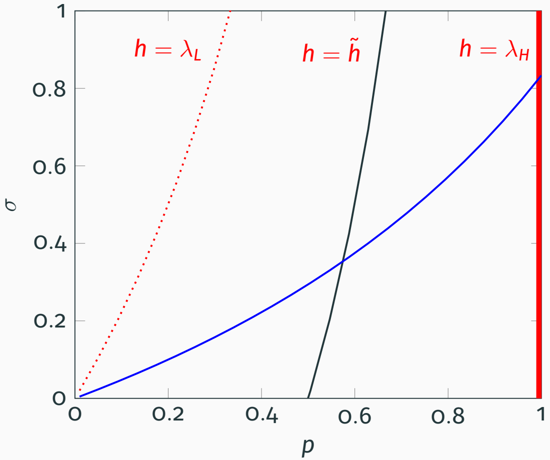

This implies at most one intersection.

Steady State

Definition

We refer to the *Steady State* (p^*, σ^*) as the unique point in both iso-quants $\dot{p}_t = 0$ and $h = \tilde{h}$.

Lemma

If $\tilde{h} \in (\lambda_L, \lambda_H)$, there is a unique steady state $(p^*, \sigma^*) \in (0, 1)^2$.

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Symmetric Equilibrium

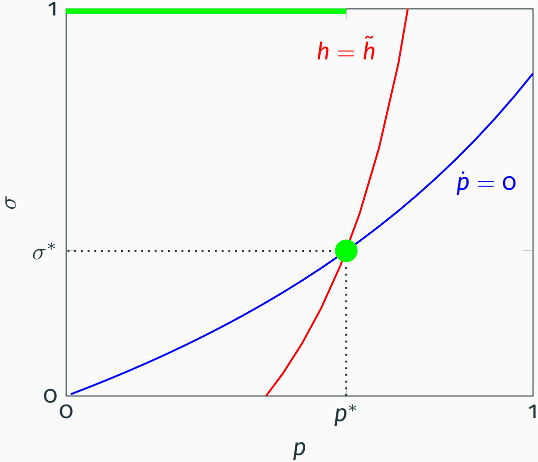
Proposition

If $\tilde{h} \in (\lambda_L, \lambda_H)$, there is a unique equilibrium with increasing hazard rate. This equilibrium entails:

$$\sigma_A(t) = \sigma_B(t) = \begin{cases} 1 & t < T^* \\ \sigma^* & t \geq T^* \end{cases}$$

and T^* is such that $p_{T^*} = p^*$.

Equilibrium Figure



Patents

First-to-Invent Patents

Same model as before with the following modifications:

- A firm that has the new technology can **apply for a patent**.
  - Patent applications are public.
- **First-to-invent**: The patent is granted if no other firm had the interim breakthrough before.
- **Licensing bargaining**: If patent is granted, the patent holder makes a TIOLI license fee offer to the opponent.
  - If offer is accepted, both firms race with the new technology onward.
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Continuation Payoffs

Suppose firms apply for patents immediately. Then, in equilibrium, patents are granted.

After a patent is granted, the TIOLI offer will capture all the extra surplus and will be accepted.

Then we can use the observable case results for state $\emptyset$, with different continuation values: $\hat{U}^i_{\{i\}}$ and $\hat{U}^i_{\{j\}}$.

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Ineffective Patents

Proposition

If stakes are sufficiently high (Π/c large enough)

- firms do NOT apply for patents.
- Equilibrium allocations and payoffs as in the unobservable case.

Intuition: Coase Theorem fails to hold because patenting changes the outside option of the opponent firm.

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First-to-File Patents

Suppose the opponent always patents immediately.

Conditional on no patent application so far, firms assign probability zero to the opponent having the new technology.

A firm with the technology that delays patenting risks the opponent patenting, what would force the firm to

- go back to the old technology.
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There is an equilibrium in which both firms patent immediately. In this equilibrium, the allocations are the same as in the observable case.

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Conclusion

We develop a model of innovation race with interim breakthroughs.

We solved equilibria for the cases in which these interim breakthrougs are public and private.

We use the results to analyze the effectiveness of patent for intermediate breakthroughs.

- Firms might not patent to conceal breakthroughs even when patent holders have all the bargaining power in licensing negotiations.
- First-to-invent rules for interim breakthroughs are less effective, especially when stakes are high.