Problem Set 6

Spring 2022

Advanced Microeconomics III

Problem 1 Consider a linear with private values environment for the allocation of a single unit of a good to two agents with type spaces $\Theta_1 = [0,1]$ and $\Theta_2 = [0,\bar{b}]$ for some $\bar{b} > 1$ and arbitrary densities.

a. Show that in a first-price auction there is no equilibrium in which the agent that values the goods the most obtains the good with probability 1. (Assume that ties are broken using a fair coin. **Hint**: proof by contradiction.)

Problem 2 Consider a linear-utility environment for the allocation of a single unit of a private good where one of the buyers is stochastically stronger that the others. Specifically, assume that $\Theta_1 = [0,3]$ and $\Theta_2 = \ldots = \Theta_N = [0,1]$. All type distributions are uniform.

Let $\hat{s}$ be a subsidy and $r \geq 0$ be a minimum bid.

Suppose that the following mechanism is played: second-price auction with minimum bid and bid subsidy is played: each agent's action space is $S_i = [r,\infty) \cup \{\text{abstain}\}$. Any action other than abstain is call a *bid*. The outcome function stipulates that the agent who submits the highest bid obtains the good; if the highest bid is submitted by multiple agents then each of them gets the good with the same probability, if all agents abstain the good is not handed out and no payments are made.

The winner has to pay the highest bid among all the other agents. Unless everybody except the winner has abstained, in which case the winner pays r. In addition, if any of the bidders different than 1 wins, then she obtains a payment of $\hat{s}$.

a. Show that each agent has a weakly dominant strategy.

b. Describe the social choice function that is implemented if each agent plays her weakly dominant strategy.

c. Describe the expected revenue of the seller as a function of $\hat{s}$ and r assuming that agents use their dominant strategies.

d. Shaw that a second-price auction with minimum bid r and subsidy $\hat{s}$ is a revenue-maximizing mechanims for some r and $\hat{s}$ optimally chosen.