Market-Based Mechanisms

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Motivation

Many market outcomes aggregate dispersed information.

- E.g. prices in financial markets, macro indicators.

Policy makers use markets to inform decisions.

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Policy makers use markets to inform decisions.

Complication: Market participants are forward-looking.

- Behavior conditioned on anticipated action of policy maker.
- Feedback from policy to markets.

Example: regulating carbon emissions

Regulator wants to limit emissions, but doesn't know distribution abatement cost.

- Firms have private information about abatement costs.

Weitzman (1974) "Prices v.s. Quantities"

- Better to set price for emissions, or set quantities?

Example: cap-and-trade

With cap-and-trade policy, regulator sets quantities

- Regulator issues fixed number of credits.
  - 1 credit = 1 ton of carbon
- Credits traded in competitive market.

For fixed issuance, low credit price indicates low abatement cost

- If price lower than expected, regulator will want to lower issuance.
  - Low price creates political pressure to lower issuance (Flachsland et al., 2020).
  - Some systems have price floors, or provisions for adjusting issuance given excess supply (e.g. EU Emissions Trading Scheme).

Example: variable-volume credits

Price can convey information about abatement costs.

- The regulator could explicitly condition issuance on credit price.

Variable-volume credit policy

1. Regulator issues a set number of variable-volume credits.
   - 1 credit = ? tons of carbon
2. Announces rule mapping credit price to per-credit volume.
3. Credits trade in competitive market.
4. Market closes, per-credit volume determined by price via announced rule.

Prices *and* Quantities, not Prices v.s. Quantities

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General market-based policy setting.

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Additional concerns:

- Equilibrium multiplicity
  - Endogeneity of the action can can lead to equilibrium multiplicity.
  - Non-fundamental volatility, resulting from equilibrium multiplicity, is a first-order concern in many settings (Woodford, 1994).
- Market manipulation
  - Market participants may have small but non-zero market power.
  - The market outcome can be manipulated to influence the action.

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How to deal with these concerns?

- What constraints do they impose on the implementable set?

General contribution

1. General framework

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   - Focus: *unique* implementation under robustness to manipulation

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Additional results

3. Unique implementation and robustness to manipulation jointly imply robustness to misspecification/structural uncertainty.

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Some applications

Many policy makers use market outcomes to inform decisions.

With (some) commitment

- Monetary policy (Bernanke and Woodford, 1997).
- Carbon cap-and-trade policies (Flachsland et al., 2020).

Without commitment

- Shareholders replacing firm management (Warner et al., 1988).
- Corporate bailouts (Bond and Goldstein, 2015).

Applications for today

Emissions regulation

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Merger policy

- Given qualitative features of the environment, identify robust features of optimal policy.
    - Policy highly responsive to markets iff regulator's first-best is not implementable.

Related literature

Broadly: two-way feedback, financial markets $\rightleftarrows$ real economy

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Our contribution

1. General framework in a tractable form.
2. Practical issues
   - Equilibrium multiplicity.
   - Manipulation.
   - Structural uncertainty/misspecification.

Outline

- Model
- Market representation
- Implementation
- Robustness
  - Manipulation
  - Multiplicity
- Robust implementation
- Applications
  - Emissions regulation
  - Merger policy

Model

State space $\Theta \subseteq \mathbb{R}^N$, convex.

A compact, convex set $\mathcal{A}$ of principal actions ($\mathcal{A} \subset \mathbb{R}^L$).

A convex set $\mathcal{P} \subseteq \mathbb{R}$ of market outcomes (price).

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Timing:

0. Principal commits to a decision rule $M : \mathcal{P} \to \mathcal{A}$.
1. The price is determined.
2. If the price is p, principal takes action $M(p)$.

Model

Market representation

Implementation

Robustness

- Manipulation
- Multiplicity

Robust implementation

Applications

- Emissions regulation
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Three-step analysis

Start with primitive market model, break design problem into three steps.

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- Model many different types of markets in a unified framework.
- Facilitate a "state-by-state" analysis.

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- Model many different types of markets in a unified framework.
- Facilitate a "state-by-state" analysis.

Step 3. Characterization of implementable outcomes.

Step 1. Outcome space

Principal chooses a decision rule $M : \mathcal{P} \to \mathcal{A}$

In general, principal's ex-ante payoff depends on joint distribution of states, actions, and prices induced in equilibrium.

Describe equilibrium joint distribution via

- *action function* $Q : \Theta \to \mathcal{A}$.
- *price function* $P : \Theta \to \mathcal{P}$.

Principal cares about M only through induced Q and P

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Principal cares about M only through induced Q and P

We want to know the set of *implementable* (Q, P).

- What (Q, P) are equilibrium outcomes given some decision rule?

Step 2. Reduction

Market admits a reduced-form representation: there is a *market-clearing function* $R : \mathcal{A} \times \Theta \mapsto \mathcal{P}$

- Interpretation: when all agents anticipate principal action $a \in \mathcal{A}$ and the state is $\theta \in \Theta$, market-clearing price is $R(a, \theta)$.

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Key feature: R does not depend on decision rule M.

- $Q(\theta)$ uniquely determines $P(\theta)$ via R in *any* equilibrium.

Questions

- Why can a market fail to have a RFR? Decision rule M affects investors in two ways
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- Why can a market fail to have a RFR? Decision rule M affects investors in two ways
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- What markets admit a reduced-form representation?
    - Satisfied in variable-volume credits market (private values).
    - What others? Paper: class of REE models.

- Why is this useful?

Benefits of reduced-form

If market admits reduced-form, can proceed with R as our primitive

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If market admits reduced-form, can proceed with R as our primitive

Other benefits of this approach

- For modeling: Identify qualitative features of R with those of policy. Closed form not needed.
- For practice: Addresses Lucas critique. Aggregate data can be used to estimate $R : \mathcal{A} \times \Theta \to \mathcal{P}$, regardless of past policy. Needn't know past policy or market micro-structure.

Model

Market representation

Implementation

Robustness

- Manipulation
- Multiplicity

Robust implementation

Applications

- Emissions regulation
- Merger policy

Implementation

(Q, P) are implementable if they are equilibrium outcomes given some M.

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Definition

If market admits a reduced form, say (Q, P) is **implementable** if
$\exists\ M : \mathcal{P} \to \mathcal{A}$ such that

1. $Q = M \circ P$ (commitment)
2. $P(\theta) = R(Q(\theta), \theta) \ \ \forall\ \theta \in \Theta$ (market clearing)

Implementation

Lemma

(Q, P) is implementable iff

1. $$Q(\theta) \neq Q(\theta') \quad \Rightarrow \quad P(\theta) \neq P(\theta'). \qquad \text{(measurability)}$$
2. $$P(\theta) = R(Q(\theta), \theta) \quad \forall\ \theta \in \Theta \qquad \text{(market clearing)}$$

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Say that Q is implementable if (Q, P) is, where $P(\theta) := R(Q(\theta), \theta)$.

Illustrative example: merger policy

Three firms A, B, C in a market. A and B announce intention to merge.

Regulator chooses to block or approve merger

- Wants to allow if and only if merger not too anti-competitive.
- Effect on competition is unknown.

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Empirical literature suggests using stock market to identify effect, when investors may have private information. (Duso et al., 2010)

i. Merger is pro-competitive $\Rightarrow$ more competition $\Rightarrow$ Bad for C.
ii. Merger is anti-competitive $\Rightarrow$ less competition $\Rightarrow$ Good for C.

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Regulator can learn from change in C's share price after merger proposal

- $C \nearrow$ = anti-competitive, $C \searrow$ = pro-competitive

Illustrative example: merger policy

$\mathcal{P}$ is change in competitor's share price

Θ is the degree of anti-competitiveness of the merger.

- First-best: approve iff $\theta < \theta^*$

Regulator can randomize

- $\mathcal{A} = [0, 1]$, *a* is probability of blocking.
  - Alternatively, approve with conditions/divestments

Merger policy

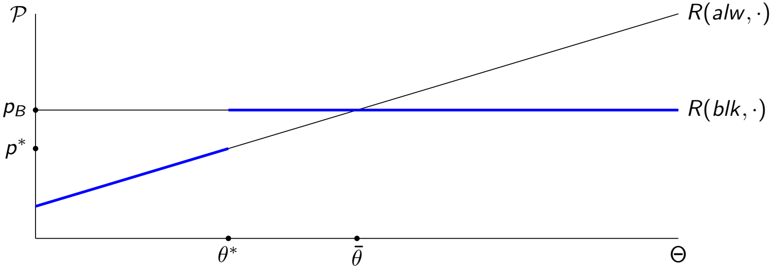


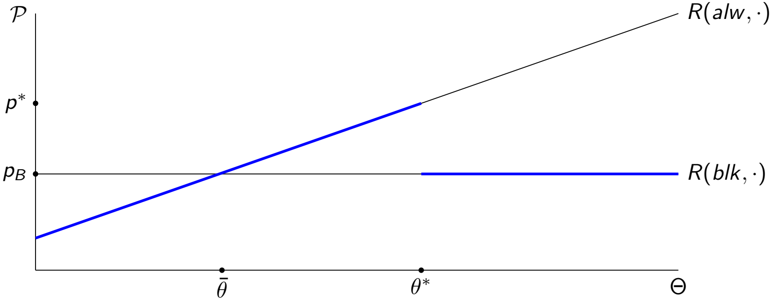
Figure: $\theta^* < \bar{\theta}$

First-best ($Q^*(\theta) = 0$ iff $\theta < \theta^*$) is implementable

Blue line is price: $P(\theta) := R(Q^*(\theta), \theta)$

An implementing decision rule: allow below p^*, block above p^*.

Merger policy



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First-best not implementable, violates measurability at p_B...

Merger policy

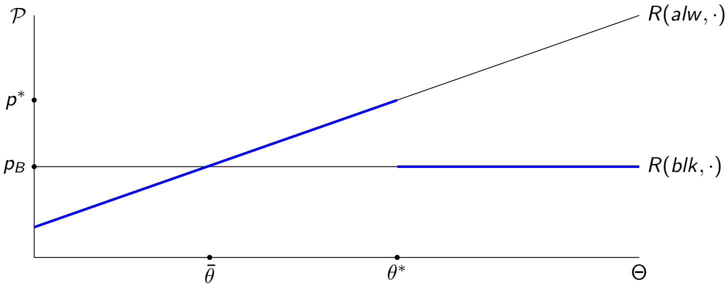

Figure: Implementable

…first-best *almost* implementable: $Q(\theta) = blk$ iff $\theta = \bar{\theta}$ or $\theta \geq \theta^*$

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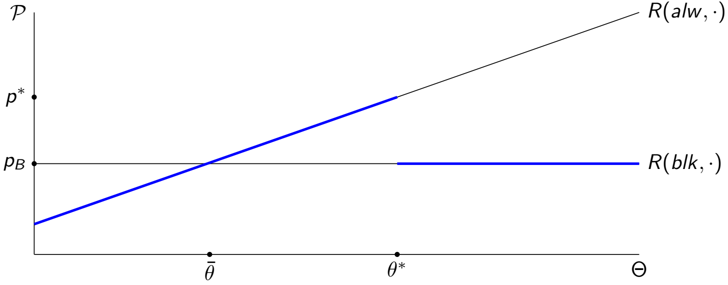

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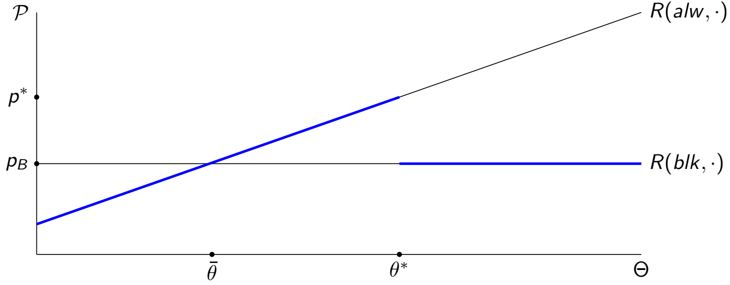
$$M(p) = \begin{cases} blk & \text{if} \quad p = p_B \text{ or } p \geq p^*, \\ alw & \text{if} \quad \text{otherwise} \end{cases}$$

Merger policy



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Problems

Manipulation

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- In reality, agents are small, but not infinitesimal.

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- In reality, agents are small, but not infinitesimal.

Goal: prevent *large* change in principal action from *small* price manipulations.

- Continuity of M.

In fact, continuity only required near possible equilibrium prices

- Discontinuities elsewhere are unreachable via small price changes.
- Imposing continuity everywhere unnecessarily constrains policy.

Manipulation

For any M, let $\bar{P}_M = \cup_{\theta \in \Theta} \{p \in \mathcal{P} : R(M(p), \theta) = p\}$ be the set of market-clearing prices given M, and let $cl(\bar{P}_M)$ be its closure.

Definition

A function $M : \mathcal{P} \to \mathcal{A}$ is **essentially continuous** if it is continuous on an open set containing $cl(\bar{P}_M)$.

$\mathcal{M}$ is the set of essentially continuous decision rules.

Multiplicity

Endogeneity of principal's action can lead to multiple equilibria (Bernanke and Woodford, 1997).

- Agents adopt self-fulfilling beliefs about principal's action

Equilibrium multiplicity and non-fundamental volatility a fundamental concern in many market-based design problems (e.g. monetary policy) (Woodford, 1994)

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Definition

M is **robust to multiplicity** if

$$\{p : p = R(M(p), \theta)\}$$

is singleton for all θ.

Robust implementation

Definition

(Q, P) is **continuously uniquely implementable (CUI)** if it is implementable by an essentially continuous M that is robust to multiplicity.

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- (Q, P) is **virtually CUI** if for any we can approximate them arbitrarily well on and arbitrarily large subset of the state space with a sequence of CUI price and action functions.

Definition

(Q, P) is **virtually CUI** if for any $\varepsilon, \delta > 0$ there exists a CUI $(\hat{Q}, \hat{P})$ such that $\{\theta \in \Theta : |Q(\theta) - \hat{Q}(\theta)| > \delta\}$ has Lebesgue measure less than ε.

Characterizing CUI: one-dimensional Θ

Let Θ be an open interval in $\mathbb{R}$.

Maintained assumption: $R(\cdot, \cdot)$ is continuous.

- Can be derived from conditions on underlying market game

Additional assumption: $R(a, \cdot)$ increasing for all a.

- For any action, state has same qualitative effect on market.
- Satisfied in all applications we've encountered.

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Theorem

Assume $R(a,\cdot)$ is increasing for all a. If Q is virtually CUI then $P(\theta) := R(Q(\theta),\theta)$ is monotone.

Important point

- Not related to monotonicity of allocation in classical mechanism design.
- P can be decreasing.

Characterizing CUI: one-dimensional Θ

When $R(a, \cdot)$ is strictly increasing, the monotonicity of P is 'almost' sufficient:

Theorem

Assume $R(a, \cdot)$ is strictly increasing for all a. Then Q is CUI iff

- *Q is continuous*
- *$P(\theta) := R(Q(\theta), \theta)$ is strictly monotone.*

Minor modifications needed to extend to weakly increasing $R(a, \cdot)$.

Important points

- Continuity of Q not implied by continuity of M.

Tractable characterization, useful in applications.

Proof idea: Q CUI $\Rightarrow$ P monotone

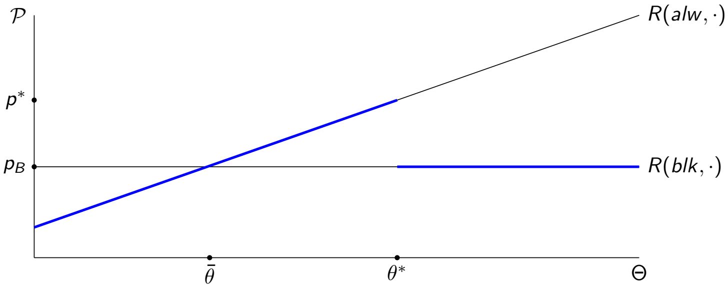
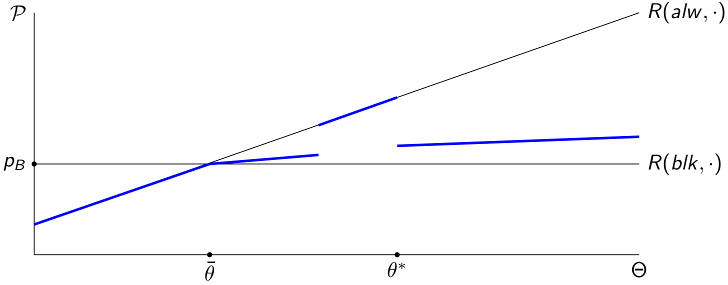


Figure: Implementable, not robustly

First-best almost implementable: $Q(\theta) = blk$ iff $\theta = \bar{\theta}$ or $\theta \geq \theta^*$

- But vulnerable to manipulation and multiplicity.

Proof idea: Q CUI $\Rightarrow$ P monotone



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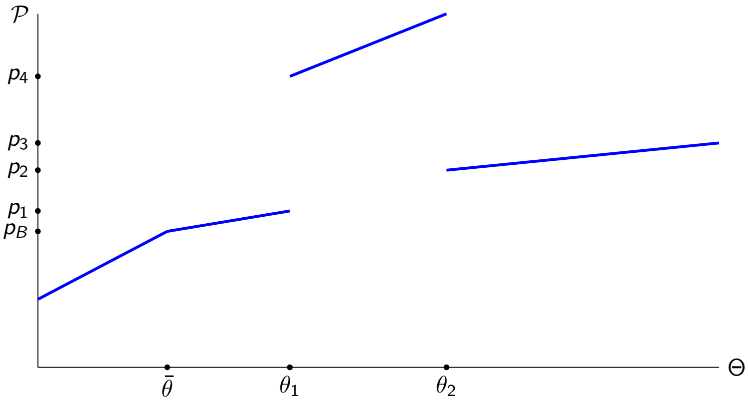
Attempted corrections ...

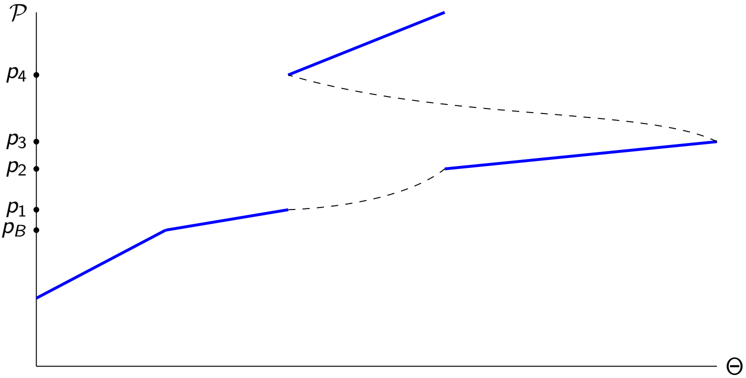
Proof idea: Q CUI $\Rightarrow$ P monotone



...result in non-monotone price.

- Want to show that this cannot be CUI.

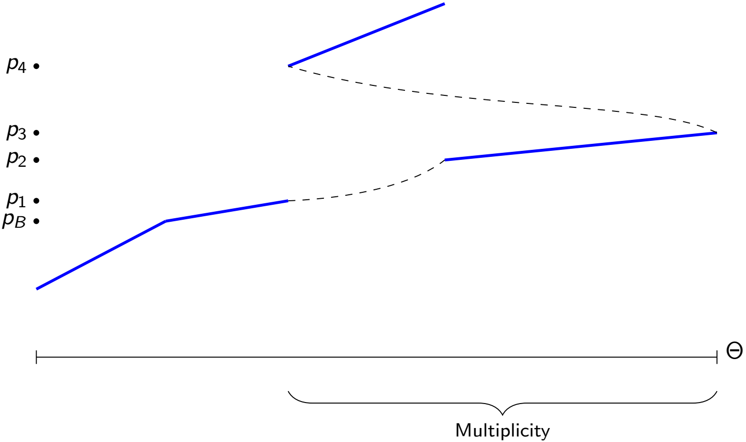
Proof idea: Q CUI $\Rightarrow$ P monotone



$$\theta_M(p) := \{\theta \in \Theta : R(M(p), \theta) = p\}$$

- Graph of P contained in graph of θ_M.
- M continuous $\Rightarrow$ θ_M is convex valued and upper hemicontinuous.

Proof idea: Q CUI $\Rightarrow$ P monotone



$\Rightarrow$ there is multiplicity

- Result extends to essentially continuous M.

Optimal policy

In general, principal solves

$$\max_{Q} \int_{\Theta} U\big(Q(\theta), P(\theta), \theta\big) dF(\theta)$$

subject to Q continuous and $P := R(Q(\theta), \theta)$ monotone.

Standard control problem, existing techniques for solving.

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Emissions regulation   Mergers   Monetary policy

Emissions regulation

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Let q be the quantity of "clean air" produced by society.

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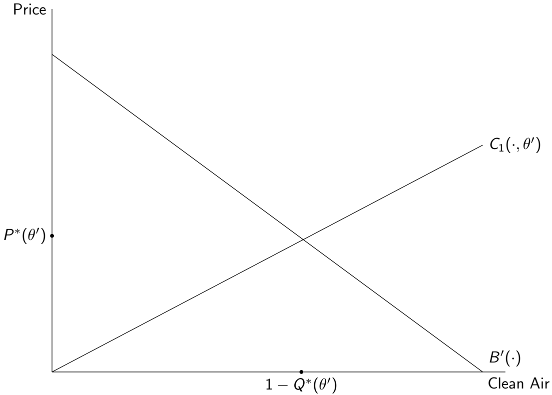
First-best action function

$$Q^*(\theta) = \operatorname*{argmax}_a B(1-a) - C(1-a, \theta)$$

Want $B'(1 - Q^*(\theta)) = C_1(1 - Q^*(\theta), \theta)$

Assume $\theta \mapsto C_1(q, \theta)$ continuous and strictly increasing.

Emissions regulation: First best



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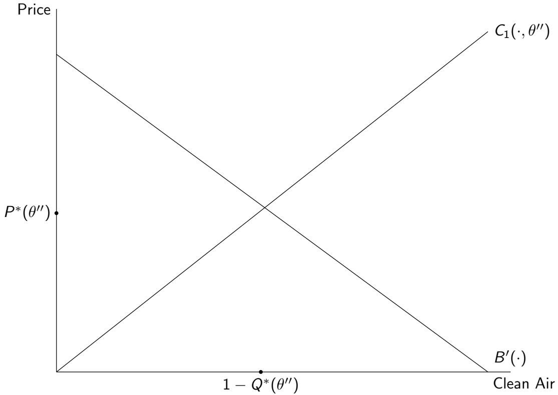

Let $\theta'' > \theta'$

- First-best action function Q^* is continuous and strictly increasing.
- First-best price function P^* is continuous and strictly increasing.
  - $P^*(\theta) = R(Q^*(\theta), \theta) := C_1(1 - Q^*(\theta), \theta)$

Emissions regulation

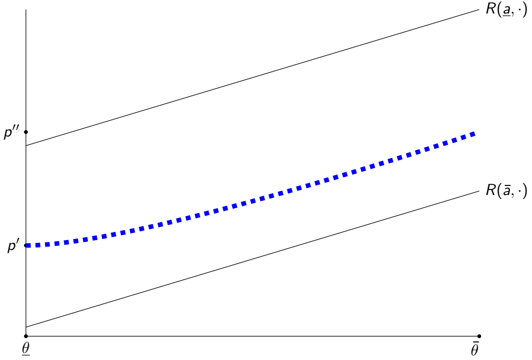

Figure: First-best policy is CUI

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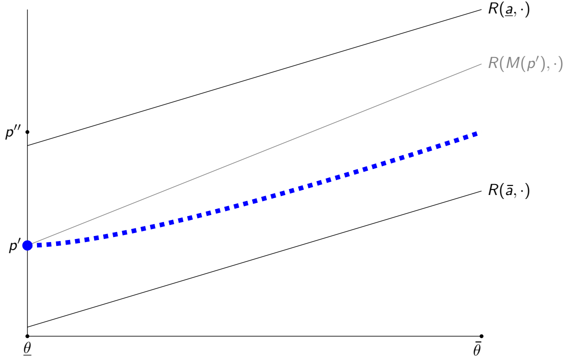

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Implementing M is strictly increasing and continuous.

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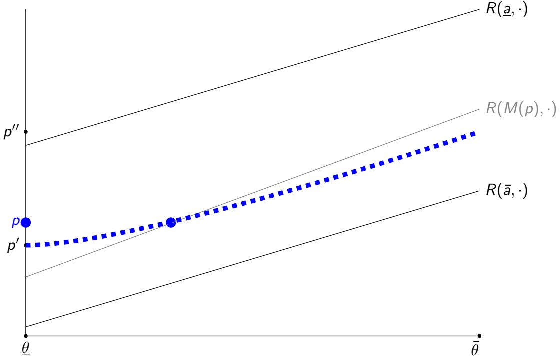

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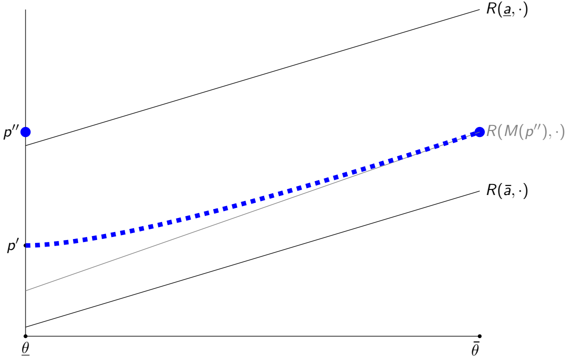

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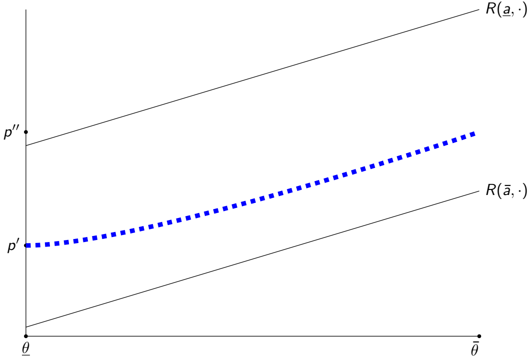


Figure: First-best policy is CUI

Implementing M is strictly increasing and continuous.

- State revealed, first-best implemented $\Rightarrow$ no commitment needed.

Optimal policy: merger decision

skip

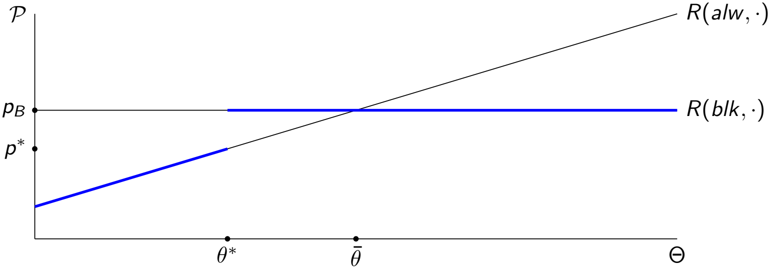
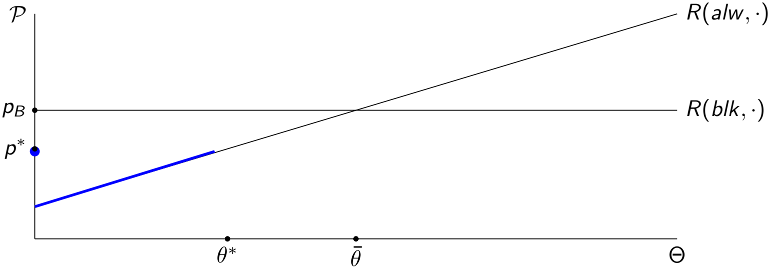


Figure: $\theta^* < \bar{\theta}$

First-best is implementable.

Optimal policy: merger decision



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First-best is *virtually* CUI. Implementing M features

- Certain approval below p^*

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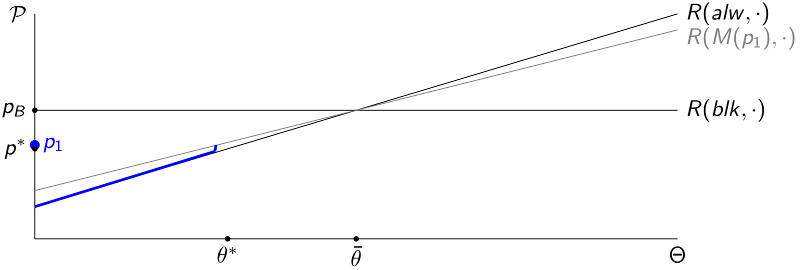

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- Certain approval below p^*
- Gradual increase blocking probability over $(p^*, p_B - \varepsilon)$.

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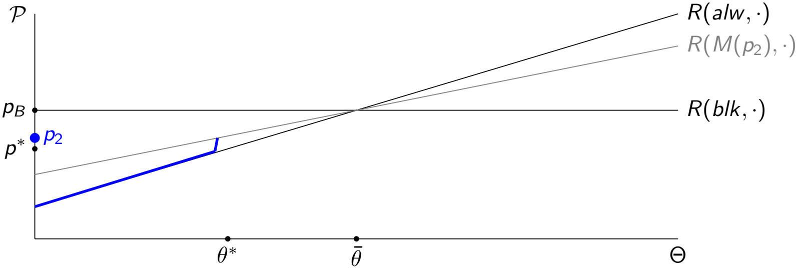

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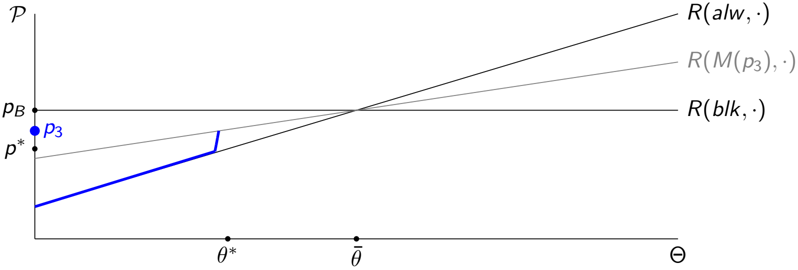

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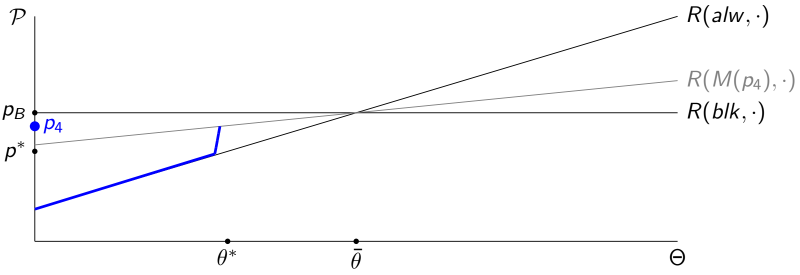

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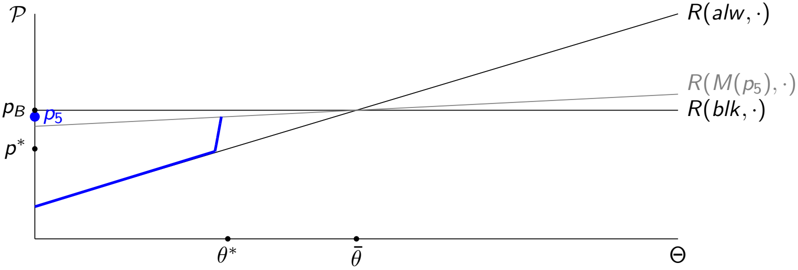

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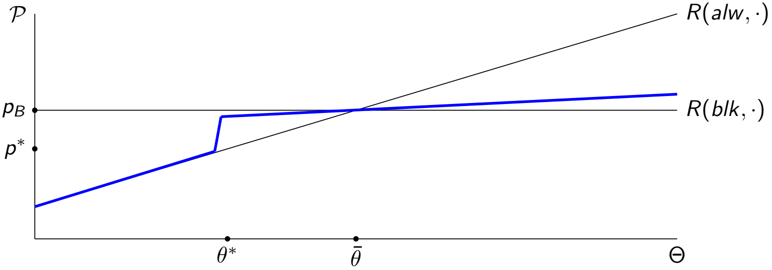

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First-best is *virtually* CUI. Implementing M features

- Certain approval below p^*
- Gradual increase blocking probability over $(p^*, p_B - \varepsilon)$.
- Almost surely block above $p_B - \varepsilon$

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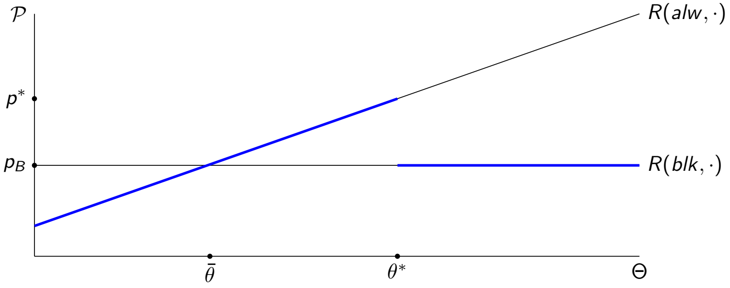


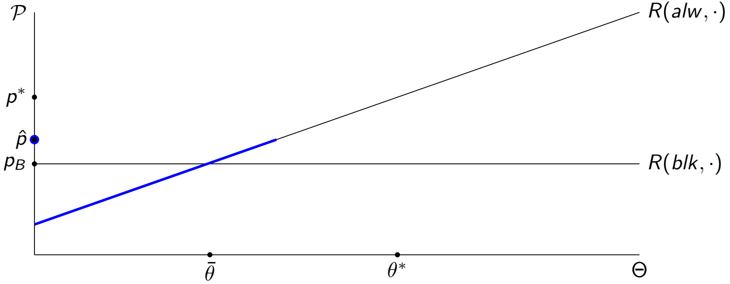
Figure: $\theta^* > \bar{\theta}$

First-best not implementable, but almost:

- Block at $\bar{\theta}$ or above θ^*.
- Allow otherwise.

However non-monotone price $\Rightarrow$ almost-first-best not virtually CUI

Optimal policy: merger decision



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Virtually optimal CUI (Q, P).

Implementing decision rule M

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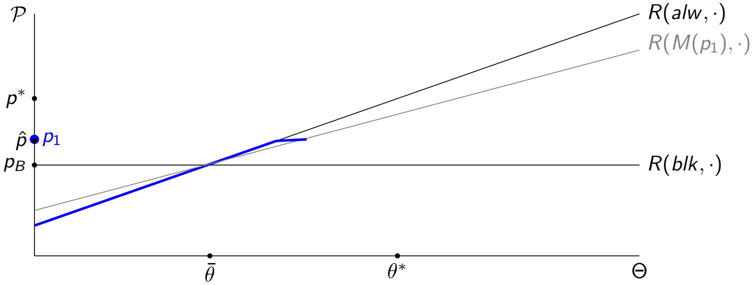

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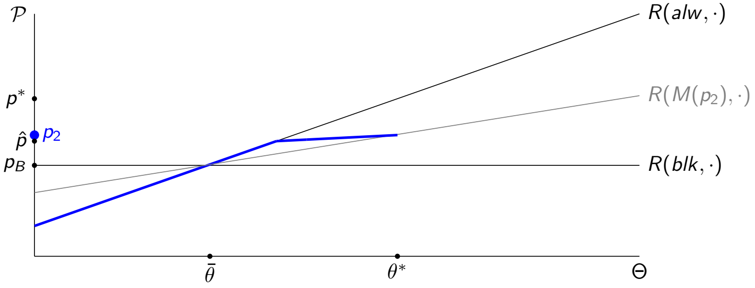

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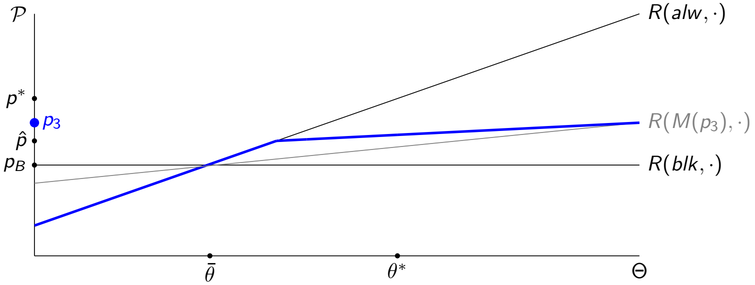

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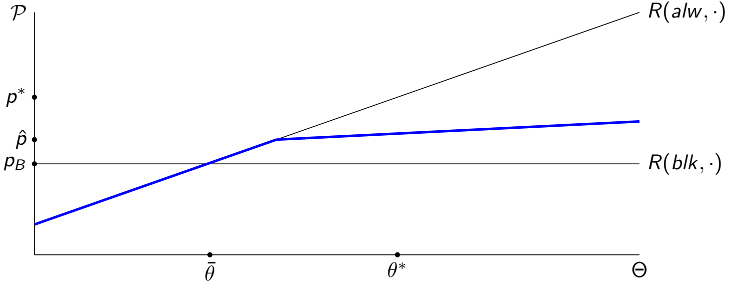

Figure: $\theta^* > \bar{\theta}$

Virtually optimal CUI (Q, P).

Implementing decision rule M

- Certain approval below $\hat{p} \in (p_B, p^*)$.
- Sharp increase in blocking probability above $\hat{p}$.
- Blocking probability bounded away from 1.

Summary

Design/implementation approach to market-based policy

1. General framework
   - Begin with market game $\rightarrow$ reduce to tractable form
2. Characterize feasible set in outcome space
   - Set of implementable maps from states to prices and actions.
   - Focus: *unique* implementation under robustness to manipulation.
   - Simplifies problem of finding optimal policy.
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Extensions

Properties and extensions

- Robustness to manipulation and multiplicity implies robustness to misspecification/structural uncertainty.
- Relaxations of unique implementation requirement.
  - Use characterization results to show that unique implementation is without loss of optimality if principal takes a strict worst-case/adversarial view of multiple equilibria.

Next steps

- Multiple market outcomes
  - E.g. central bank conditions on inflation and unemployment.
- Large identifiable players alongside market
  - E.g. firms in merger example.
- Market design
  - E.g. create derivatives.

Thanks!

Relaxing uniqueness

The principal may tolerate multiple equilibria, provided none are too bad.

Suppose principal takes strict worst-case/adversarial view

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Theorem

Assume the environment is regular. If $M \in \mathcal{M}$ induces multiple equilibria then at least one is virtually CUI.

Regularity guarantees that if $P(\theta) \equiv R(Q(\theta), \theta)$ is increasing then (Q, P) are virtually CUI.

- We can find a continuous Q' that approximates Q and induces a monotone price.

Structural uncertainty

The principal may not know R exactly.

- Misspecification
- Noise in market

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The decision rule should perform well for small perturbations to R.

Structural uncertainty

Let $\tilde{Q}_R(\theta|M) := \{a \in \mathcal{A} : M(R(a,\theta)) = a\}$

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Definition

A decision rule M is **robust to structural uncertainty** if $R \rightrightarrows \tilde{Q}_R(\theta|M)$ is upper and lower hemicontinuous at R, uniformly over Θ.

In other words, the set of equilibrium price and action functions varies continuously around R.

Theorem

If $M \in \mathcal{M}$ is robust to multiplicity then it is robust to structural uncertainty.

Manipulation

Essential continuity characterizes robustness to small manipulations.

"Off path" manipulation: doesn't influence other's behavior.

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Proposition

Assume that M induces a unique equilibrium.

- If $M \in \mathcal{M}$ then the set of equilibria induced by M will be continuous (upper and lower hemicontinuous) in R.
- If M has a jump or removable discontinuity on $\bar{P}_M(R)$ then the set of equilibria induced by M will not be continuous in R.

return

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Class of markets: asset-market REE

Single asset

- Ex-post asset dividend: $\pi(a, \theta)$
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- i observes private signal s_i (think $s_i = \theta + \varepsilon_i$)
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Key feature of REE: investors learn about θ from the price.

Class of markets: asset-market REE

Fix the principal's decision rule $M : \mathcal{P} \to \mathcal{A}$.

Investors are price takers.

REE consists of price function $P_M : \Theta \to \mathcal{P}$ such that

i. Investors optimize, conditioning on signal and price

$$X_i(p, s_i) = \underset{x}{\operatorname{argmax}}\, \mathbb{E}\left[u_i\big(x \cdot (\pi(M(p), \theta) - p)\big) \mid s_i, P_M(\theta) = p\right]$$

ii. Markets clear in all states

$$\int X_i\left(P_M(\theta), s_i\right) di = 0 \quad \forall \quad \theta \in \Theta.$$

(using "continuum law of large numbers" convention)

Challenges

$$X_i(p, s_i) = \underset{x}{\text{argmax}}\, \mathbb{E}\left[u_i\big(x \cdot (\pi(M(p), \theta) - p)\big) \mid s_i, P_M(\theta) = p\right]$$

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1. Forward guidance: anticipated action.
2. Information aggregation: M shapes price function P_M
   - $\{\theta : P_M(\theta) = p\}$ depends on M.
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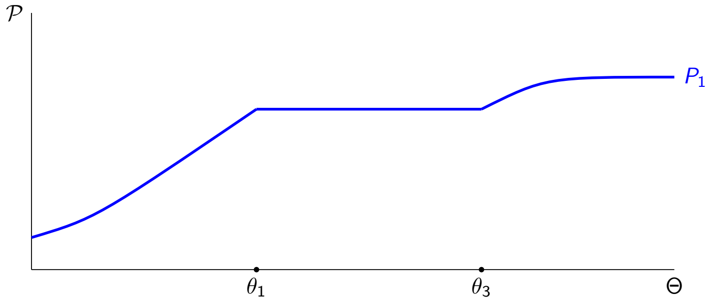
Question. Does equilibrium price in a given state depend on global properties of decision rule and equilibrium price and action functions?

Informational effects & global influence



Difficulty with informational effects

- Let (Q_1, P_1) be implementable.

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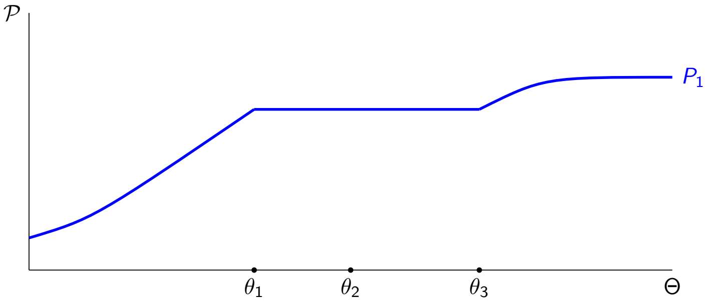

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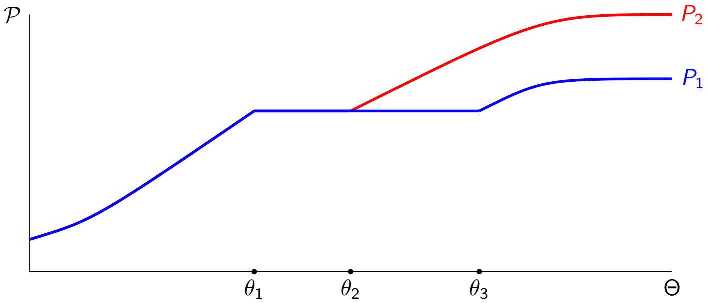

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- Questions:
  - Is there P_2 such that (Q_2, P_2) are implementable?
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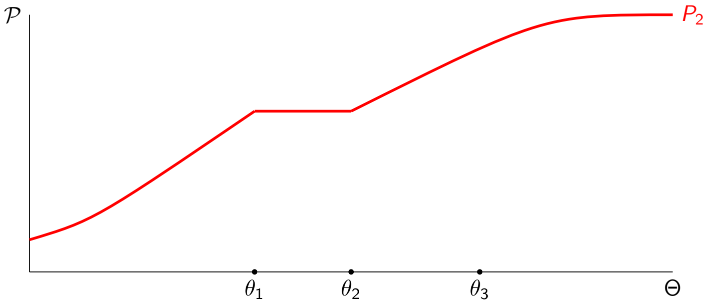

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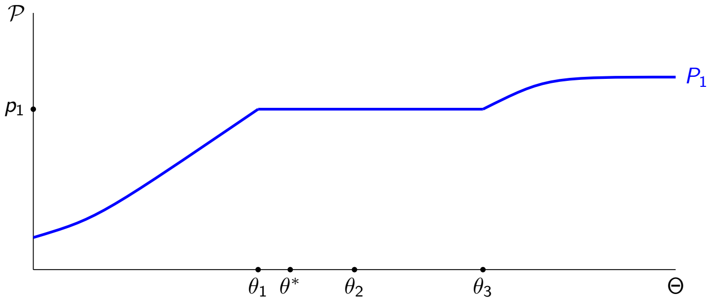

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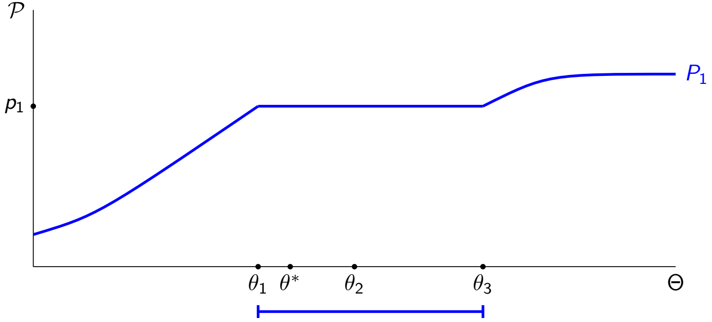

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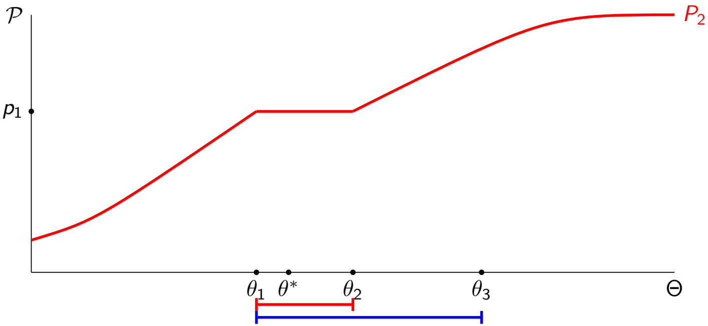

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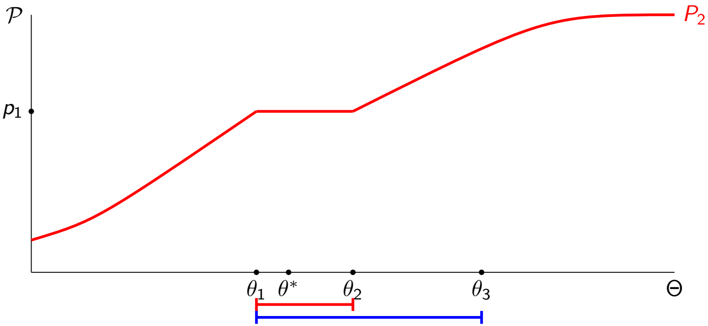

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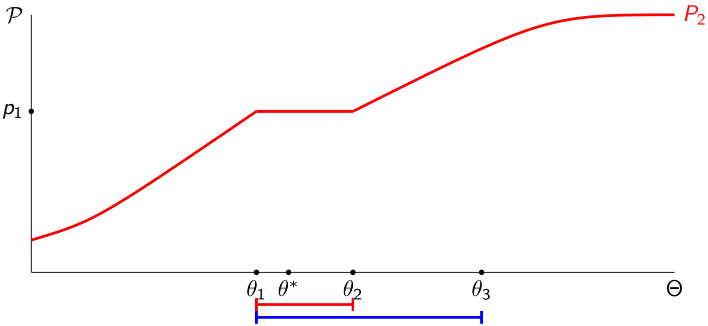

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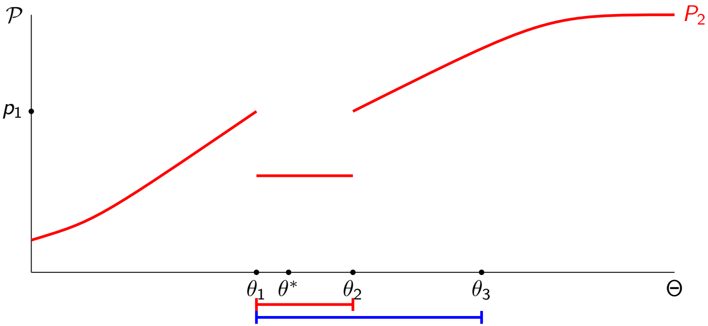
  - In the (Q_1, P_1) equilibrium, price at θ^* reveals $\theta \in [\theta_1, \theta_3]$
- Suppose (Q_2, P_2) implementable, and $P_2(\theta) = P_1(\theta)$ for $\theta \leq \theta_2$
  - In the (Q_2, P_2) equilibrium, price at θ^* reveals $\theta \in [\theta_1, \theta_2]$
- $[\theta_1, \theta_2]$ induces FOSD-lower posteriors than $[\theta_1, \theta_3]$.
  - $\pi(a^*, \cdot)$ strictly inc. $\Rightarrow$ lower demand in state θ^* under Q_2

Informational effects & global influence



Complication: informational effects

- Lower market-clearing price at $\theta \in [\theta_1, \theta_2]$ under Q_2

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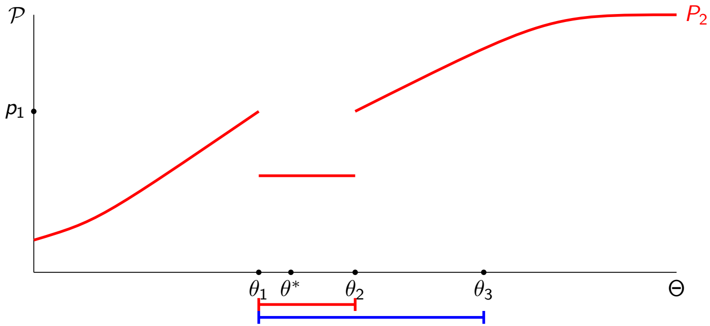

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- Lower market-clearing price at $\theta \in [\theta_1, \theta_2]$ under Q_2
- Price changed in states $\theta < \theta_2$ where action did not.
  - $\Rightarrow$ Global properties of Q determine price at θ^*.

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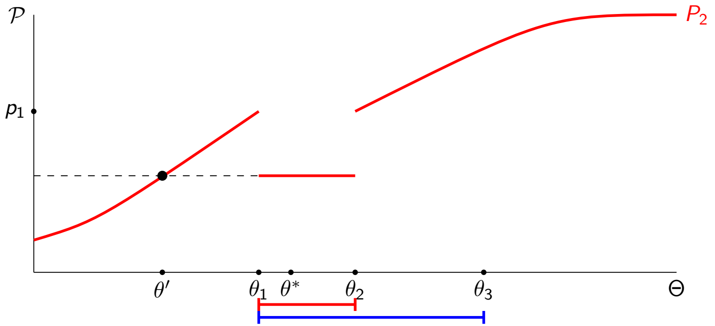

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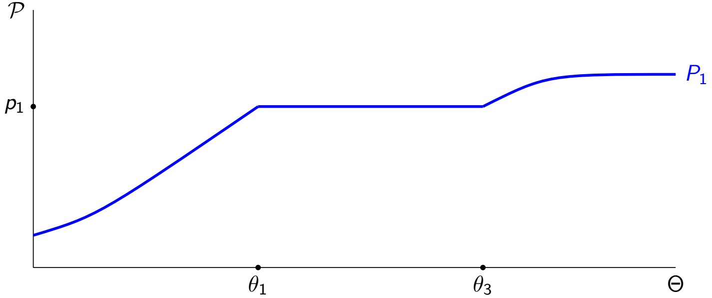
- Lower market-clearing price at $\theta \in [\theta_1, \theta_2]$ under Q_2
- Price changed in states $\theta < \theta_2$ where action did not.
  - $\Rightarrow$ Global properties of Q determine price at θ^*.
- Action not measurable with respect to price if $Q_2(\theta') \neq Q_2(\theta^*)$
  - $\Rightarrow$ (Q_2, P_2) not implementable

Existence of reduced form

If market admits a reduced form representation, $P(\theta)$ depends only on $Q(\theta)$, independent of $Q(\theta')$.

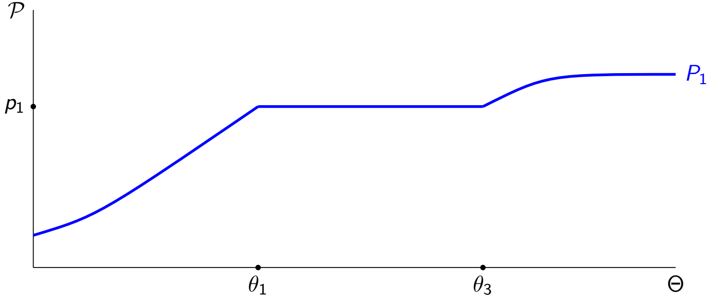
Question: When is this true in REE market?

Existence of reduced form



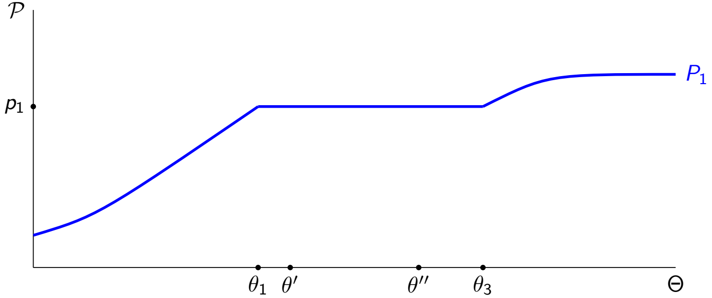
Assume (Q_1, P_1) implementable.

Existence of reduced form



Observation 1. Principal action measurable with respect to price.

Existence of reduced form



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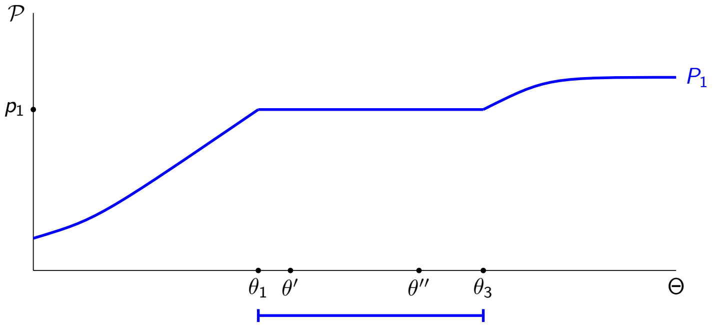

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Observation 2. Price function reveals its level sets.

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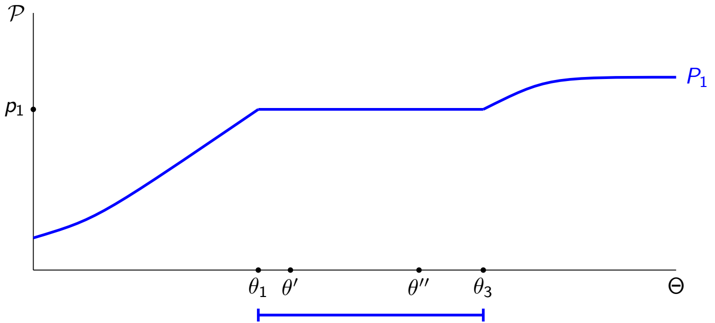

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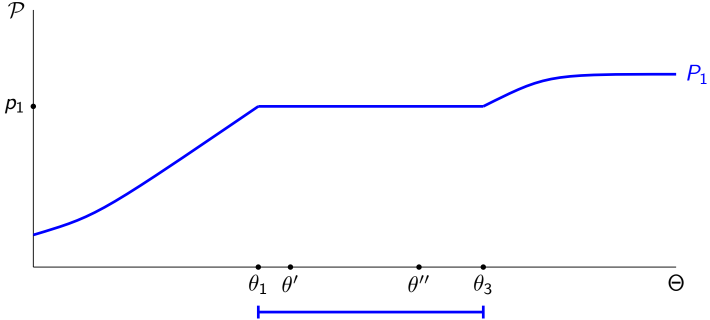

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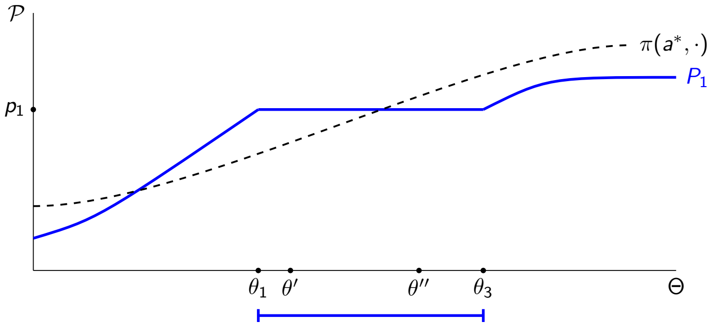

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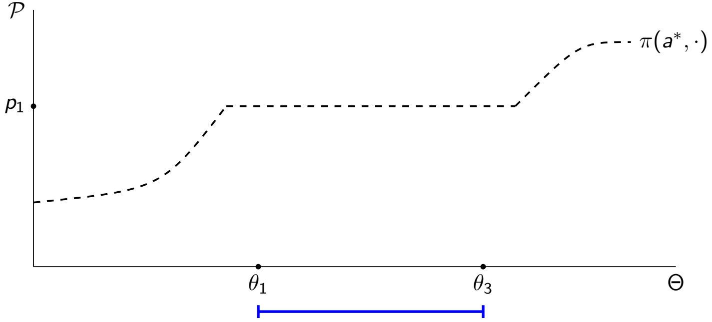
But aggregate *private* information higher at θ'' (think $s_i = \theta + \varepsilon_i$)

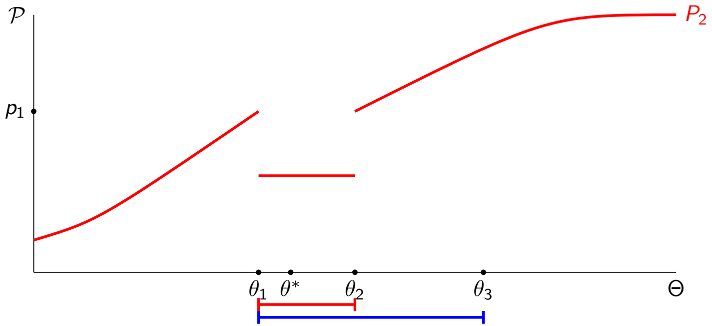
- If $\pi(a^*, \cdot)$ strictly inc. on $[\theta_1, \theta_3]$ then higher demand at θ'' than θ'
- Market can't clear at p_1 in both states.

Existence of reduced form



Assuming $\pi(a^*, \cdot)$ weakly increasing, must be constant on $[\theta_1, \theta_3]$.

Existence of reduced form



Consider (Q_2, P_2) as before, where $Q_2(\theta) = Q_1(\theta)$ for $\theta < \theta_2$

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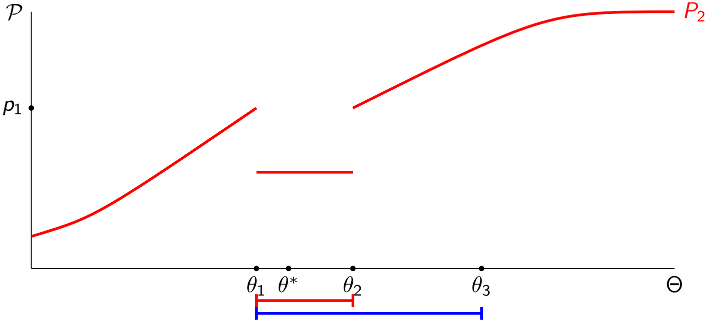

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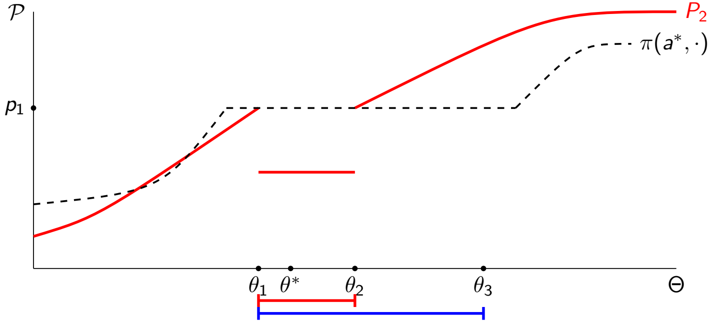

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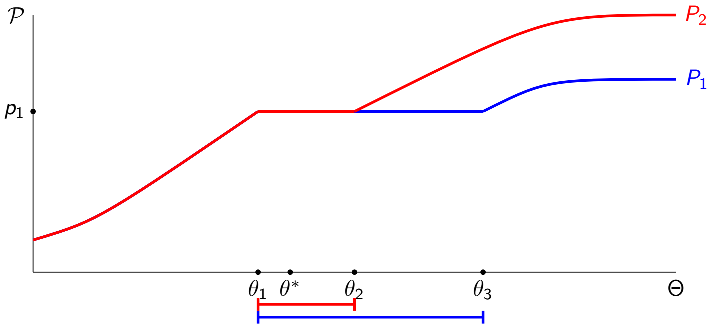

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So $P_1(\theta) = P_2(\theta)$ for all $\theta < \theta_2$, as desired.

Existence of reduced-form

Key observations

1. Principal's action measurable with respect to the price.
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One-dimensional Θ, key assumptions:

i. Monotonicity of demand as function of state for each action
   - e.g. $\pi(a, \cdot)$ weakly increasing for all a.

ii. Monotonicity of aggregate beliefs as function of state
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Proposition

If $\pi(a, \cdot)$ weakly increasing for all a and $s_i = \theta + \varepsilon_i$ then REE asset market admits reduced-form representation.

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Multi-dimensional Θ, complication:

- Generally no complete order on Θ such that *i.* and *ii.* hold.
- E.g. noisy REE model (Grossman and Stiglitz, 1980).
  - $\Theta = \Omega \times \mathcal{Z}$, where dividend is $\pi(a, \omega)$ and aggregate supply is z.

Characterizing CUI: multi-dimensional Θ

Complication relative to one-dimensional Θ:

- No complete order on Θ such that beliefs and agent actions are monotone.
    - Harder to derive reduced-form representation.
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1. Derive reduced form R as before.
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Applications

Reduced form under uniqueness

Definition

The market **admits a reduced-form representation under uniqueness** if $\exists$ a function $R : \mathcal{A} \times \Theta \to \mathcal{P}$ such that for any Q, P, M, the pair (Q, P) are the *unique* equilibrium outcomes given M iff for all θ

i. $Q(\theta) = M(P(\theta))$ (commitment)

ii. $P(\theta) = R(Q(\theta), \theta)$ (market clearing)

iii. $\{p : p = R(M(p), \theta)\}$ is singleton (uniqueness)

Multi-dimensional Θ: noisy REE

As in Grossman and Stiglitz (1980) and Hellwig (1980)

Single asset

- Ex-post dividend: $\pi(a, \omega) = \beta_0^a + \beta_1^a \omega$, with $\beta_1^a > 0$ for all $a \in \mathcal{A}$.
- $z \in \mathcal{Z}$ is stochastic aggregate supply of asset
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Continuum of investors $i \in [0,1]$

- i observes signal $s_i = \omega + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_i^2)$
- Ex-post payoff of purchasing x units at price p: $u_i\big(x \cdot (\pi(a,\omega) - p)\big)$.
- Submit demand schedules to market maker.

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- Submit demand schedules to market maker.

Limited notion of equilibrium uniqueness

- Roughly: want unique equilibrium fixing the inferences investors draw from each price.
- Alternative interpretation: unique market clearing price given investor's demand schedules.

Noisy REE

Theorem

Assume u is CARA, and z has truncated normal distribution. Define $L^* : \Omega \times \mathcal{Z} \times \mathcal{A} \to \mathbb{R}$ by

$$L^*(\omega, z|a) = \left(\frac{1}{\beta_1^a} \int_i \frac{\tau_i}{\sigma_i^2} di \right) \cdot \omega - z.$$

Then for any M such that there is a unique market clearing price in every state, the level sets of the equilibrium price function P_M are given by

$$\{(\omega, z) : P_M(\omega, z) = p\} = \{(\omega, z) : L^*(\omega, z|a) = \ell\}$$

for some ℓ.

Corollary

Assume u is CARA, and z has truncated normal distribution. Then the market admits a reduced-form representation under uniqueness.

R for noisy REE

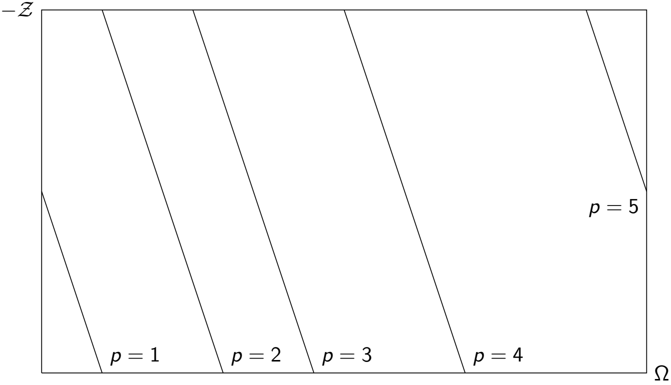


Figure: Level sets, fixed action a

$$\text{Slope } = -\frac{1}{\beta_1^a} \int_i \frac{\tau_i}{\sigma_i^2} di$$

R for noisy REE

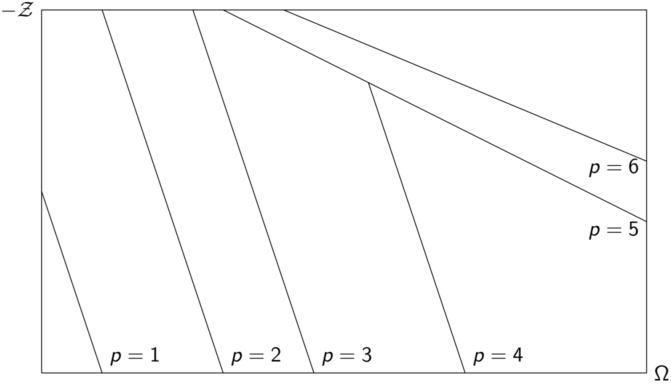


Figure: Level sets, non-trivial M

The theorem rules out intersecting level sets.

R for noisy REE

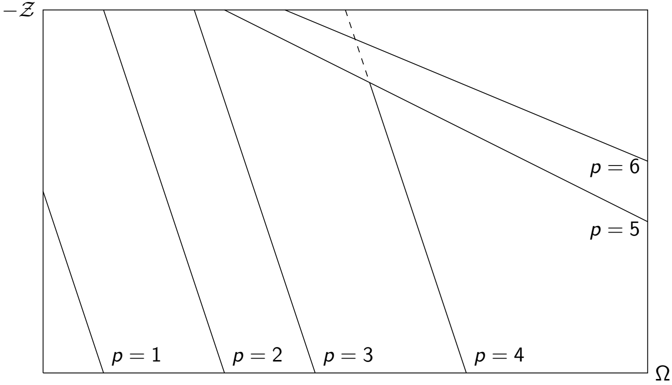
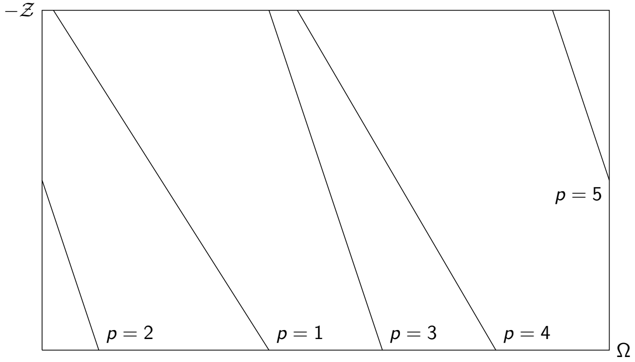


Figure: Level sets, non-trivial M

The theorem rules out intersecting level sets.

- Would cause multiplicity.

R for noisy REE



With CUI Q, level sets must not cross. Implies

1. (ω, z) and a uniquely determine equilibrium price (reduced-form)

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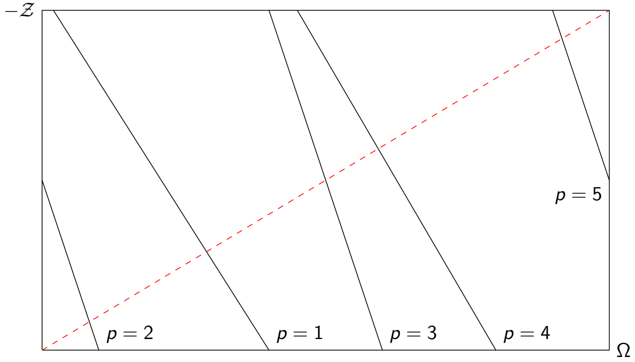

With CUI Q, level sets must not cross. Implies

1. (ω, z) and a uniquely determine equilibrium price (reduced-form)
2. Necessary and sufficient conditions for CUI can be stated for a single chain in $\Omega \times \mathcal{Z}$.