A Taxation Principle with Non-Contractible Events

Francisco Poggi and Bruno Strulovici

Classical Taxation Principle

- Setup:
  - Agent private type $\theta \in \Theta$.
  - Agent takes action $a \in \mathcal{A}$
  - Principal
    - observes action taken.
    - Charges the agent a tax $t \in \mathbb{R}$.
- Agent has a utility function

$$u(\theta, a, t) = v(\theta, a) - t$$

- Principal wants to implement a (deterministic) social choice function $f : \Theta \to \mathcal{A}$. Can commit to any mechanism.

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- **Direct Revelation Mechanisms:**
  - Ask the agent to report their type.
  - Recommend an action $\hat{a} \in A$.
  - Set up taxes so that agent has incentives to
    - Report truthfully.
    - Follow the recommended action.
  - **Revelation Principle**: DRM are without loss.
- **Tariff Mechanism:**
  - Set up a tariff $t : \mathcal{A} \to \mathbb{R}$.
  - Let the agent choose the action.
  - Charge tax according to the tariff.
  - **Taxation Principle**: TM are without loss.

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Extending the Taxation Principle

- Tariff mechanisms are attractive in Law applications.
  - Examples:
    - liability as a function of precaution taken.
    - years of imprisonment as function of crime.
  - Main advantage: no information elicitation.

- The Classical Taxation Principle is ill-suited for many application.

Limitations

- Observability/verifiability of actions.
  - In general, evidence that is partially informative about actions.
- Contractability of outcomes.
  - Injurer can only be liable if damage occurs.
  - Criminal can only be punished if captured.
- Quasilinear preferences.
  - Risk-aversion.
  - Same punishment could be more costly for 'innocent' agents.

Our contribution

- Necessary and sufficient conditions for a Taxation Principle to hold in a more general setup:
  - Moral hazard.
  - Non-contractible outcomes.
  - Non-quasilinear preferences.

- This conditions are strong. Sometimes it is useful to show that a particular family of SCF can be implemented without information elicitation.
  - We show sufficient conditions on f and the environment that guarantee tariff implementation.

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Model Primitives

- Imperfectly verifiable actions.
  - Action $a \in \mathcal{A}$ generates outcome $z \in Z$.
  - Only z is verifiable.
- Non-contractible events.
  - Set of contractible outcomes $Y \subset Z$.
  - Only if $z \in Y$, the principal charges a tax $t \in T(z) \subset \mathbb{R}$.
- Preferences:

$$u(\theta, a, z, t) = v(\theta, a, z) - h(\theta, a) \cdot w(t, z)$$

with $h(\theta, a) > 0$. Set $\mathcal{U}$.

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Our results

Taxation Principle $\Leftrightarrow$

For any $z \in Y$ it is possible to identify the conditional distribution of contractible outcomes.

- Example: A single contractible outcome for each action.

Tariff Implementable SCF

- Taxation Principle guarantees that **all** implementable SCF are tariff implementable.
- Sometimes, the principal is interested in a subset of SCF.
  - $f(\Theta) \subset A$. $f \nearrow$, etc.

f is *observably injective* if there exists a partition $A_1, ..., A_K$ of $\mathcal{A}$ such that:

- Principal identifies which cell the action belongs to.
- If two implemented actions are in the same element, they generate the same distribution of contractible outcomes.

Theorem

If f is implementable and observably injective then f is tariff implementable.

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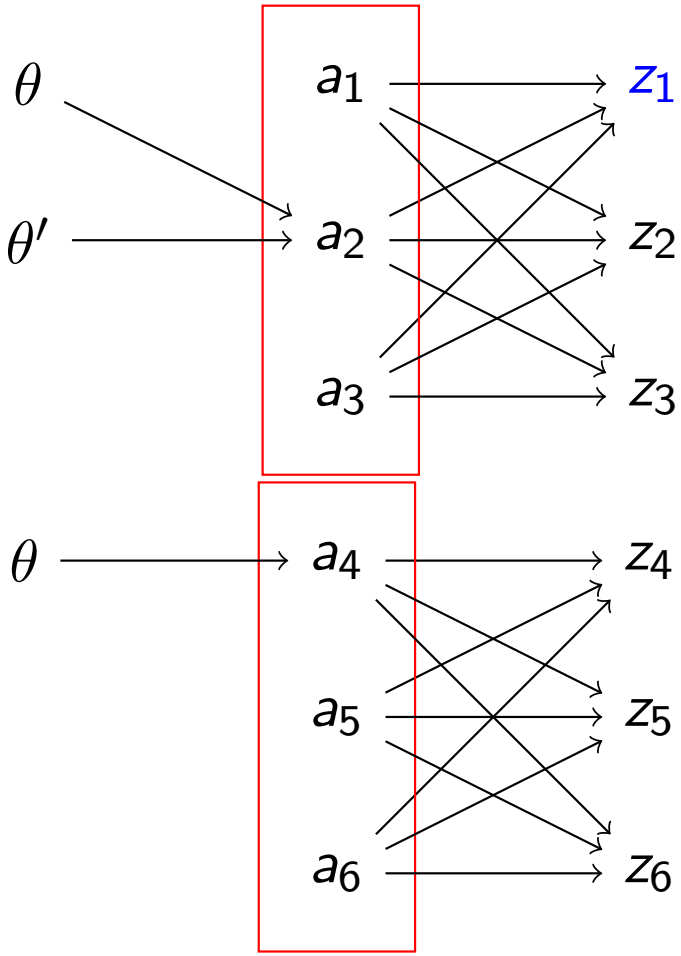
Intuition



Example

- A product can be faulty or safe.
- Firm decides weather to lunch it or not $a \in \{0, 1\}$.
  - has some private information θ about risk,
  - can run an experiment $e \in \mathcal{E}$ before launching it. Cost $C(e)$.
- Z : whether product was launched, faulty, and the likelihood ratio of the evidence obtained in the experiment.
- Contractible outcome only if the product was launched and faulty.
- Action: experiment and map from results to launch decisions.

- The Taxation Principle does not hold.
- However, if we restrict attention to binary experiments such that the agent launches conditional on one of the outcomes, the second result applies.

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