Outsourcing

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Question: Which projects/ideas are pursued, and who bears the development costs?

What we do

- We study a framework where
  - a Seller and a Buyer each own projects with uncertain outcomes.
  - Projects require development, and there is no cost advantage.
  - The seller determines its project's price and whether to develop it before the time of sale.
  - The buyer may explore alternative projects both before and after considering the seller's offer.

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- Seller owns a box (F_0, c_0).
- Buyer owns a (random) collection of boxes $\{(F_i, c_i)\}_{i=1}^N$.
  - Let $\mathcal{B}$ be a set of box collections.
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- For simplicity, assume all boxes have support in $[0, \bar{v}]$.

Model: Timing

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- chooses to open the box ($d = 0$) open or leave it closed ($d = 1$).
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t>1 Buyer observes $B \in \mathcal{B}$ and sequentially decides one of the following.

- Purchase the box from the seller, adding it to its collection.
- Open one of the closed boxes in its collection.
- Stop and adopt an already opened box.
- Stop without adopting any boxes.

Model: Payoffs

Let

- t be the transfer that the buyer pays the seller.
- $d \in \{0, 1\}$ denote whether the seller offers a open or closed box.
- i be the box adopted by the buyer.
- $O \subseteq \{0, 1, ..., N\}$ be the set of boxes that the buyer opens.

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$$\textbf{Seller}: \qquad t - c \cdot d$$

$$\textbf{Buyer}: \qquad v_i - \sum_{j \in O} c_j - t$$

Model: Information

- **Symmetric Information**: When the seller offers an open box, he observes v before choosing the price P.

- **Asymmetric Information**: Only the buyer observes v when the seller offers an open box.

Efficient sequence

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- Inefficiencies:
  - **Inefficient order**: box 0 is opened too late.
  - **Inefficient stopping**: box 0 is open too infrequently.

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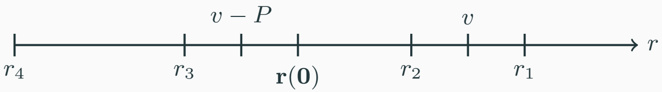

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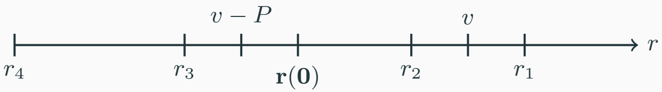

- Let v be the realized value of box 0.

- Inefficiencies:
  - **Inefficient order**: box 0 is opened too early.
  - **Inefficient stopping**. Too many boxes are opened.
  - **Inefficient adoption**. Box 0 is adopted too infrequently.

Benchmark

Proposition: No other boxes

1. If $\mathcal{B} = \{\emptyset\}$ and information is **symmetric**, the seller is indifferent between offering an open and a closed box.
2. If $\mathcal{B} = \{\emptyset\}$ and information is **asymmetric**, the seller prefers to offer a closed box.

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- **Point 1**. The optimal price for a closed box is $E[v] - c_0$.
  - Probability 1 of selling the box.
  - Expected payoff is $E[v] - c_0$.
- The optimal price for an open box of value v is v.
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  - The expected payoff is $E[v] - c_0$.
- **Point 2**. With asymmetric information, the price of the open box must be constant in v.
  - Expected payoff $P[1 - F_0(P)] - c_0 \leq E[v] - c_0$.

Proposition

With symmetric information, the seller offers an open box when c_0 is low: There is a $\bar{c} \in [0, E[v]]$ such that the seller offers an open box if $c_0 < \bar{c}$ and a closed box if $c_0 > \bar{c}$.

Symmetric Information: Proof

Lemma

There exists a function $Q : \mathbb{R} \to [0, 1]$ such that:

- Q is increasing with $Q(r) = 0$ for all $r < 0$.
- The probability of selling a closed box at price P given by $Q(r(P))$.
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- Open box of value v is sold at price P if $z_i < v - P$ for all i. This happens with probability $Q(v - P)$

Symmetric Information: Proof

- Let $V_0(c)$ denote the value of selling an open box when the cost is c.

$$V_0(c) := \max_{P(\cdot)} \int Q(v - P(v)) \cdot P(v) \, dF(v) - c$$

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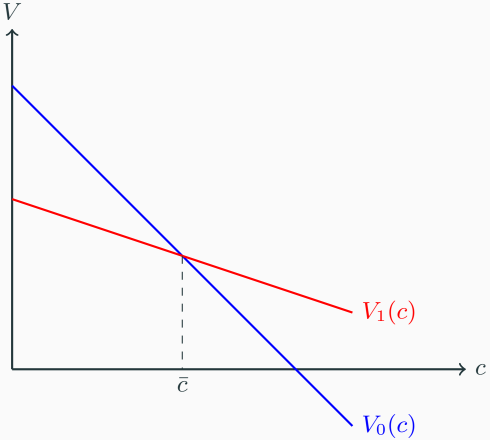
  - Applying the Envelope Theorem, $V_1'(c) = -Q(r^*(c))$.

Symmetric Information: Single Crossing



Symmetric Information: $\bar{c} \geq 0$

Let r^* be the optimal reservation value for a closed box when $c = 0$ and

$$\hat{P}(v) = \begin{cases} v - r^* & v > r^* \\ 2\bar{v} & v < r^* \end{cases}$$

$$\begin{aligned} V_0(0) &\geq \int Q(v - \hat{P}(v))\hat{P}(v)\; dF(v) \\ &= \int_{r^*}^{\bar{v}} Q(r^*)(v - r^*)\; dF(v) \\ &= Q(r^*) \underbrace{\int_{r^*}^{\bar{v}} (v - r^*)\; dF(v)}_{r^{-1}(r^*)} = V_1(0) \end{aligned}$$

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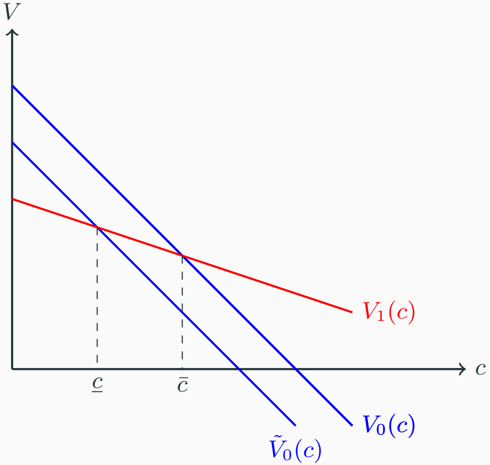
Asymmetric Information



Sufficient conditions

- It could perfectly be that $\underline{c} < 0$.
- **Question:** Under what conditions do we have that $\underline{c} > 0$?

Fail-or-adopt boxes

- **Definition**: A box (F, c) with reservation value r is *fail-or-adopt* iff the support of F is $\{0\} \cup I$ and $r < \inf(I)$.
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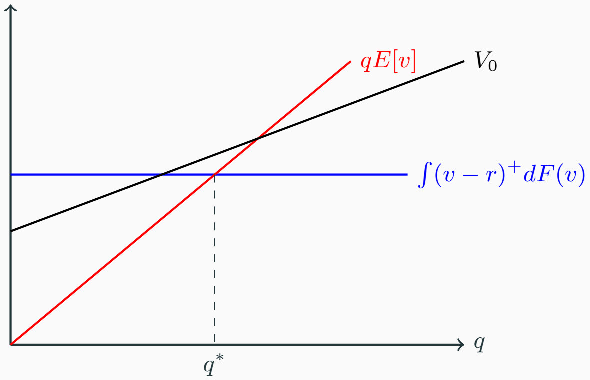
Proposition

Let $\mathcal{B} = \{(F_1, c_1)\}$, where (F_1, c_1) is a fail or adopt box with failure probability q and reservation value r. There are thresholds $q_L \leq \frac{\int_r^\infty (v-r)dF(v)}{E[v]} \leq q_H$ such that the seller strictly prefers to offer an open box iff $q \in (q_L, q_H)$.

Fail-or-adopt boxes: Intuition



Conclusion

- We introduced a model to study inefficiencies arising when selling boxes to buyers with alternative options.
- Even with identical costs, sellers may strictly prefer to open the box for low costs and offer it closed for high costs.
- Being unable to price discriminate reduces the returns to opening the box.
    - The seller may still find it optimal to costly open a box, even if he charges a fixed price. For fail-or-adopt alternatives, this happens for intermediate levels of failure probability.

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