Selling (un)finished products

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Sellers of generic solutions are typically profit maximizers.

- Markups.
- Conceal information before contracting.

Introduction

We study the problem of a buyer and a monopolist seller.

The seller controls:

- price.
- information about value.

The buyer can invest in developing in-house substitutes.

Two type of inefficiencies:

- Sub-optimal adoption.
- Over-investment in in-house substitutes.

What we do

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- The demand only depends on the distribution and price through the *reservation value*.

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- What information the buyer observes at the time of contracting.

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- What information the buyer observes at the time of contracting.

We study how changes in available in-house capabilities affect the timing of contracting.

Benchmark

- Consider a Buyer-Seller problem where the value of the object, v, follows cdf F.
    - Risk-neutrality.
    - Zero production cost.
    - Buyer's outside option is zero.

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- Seller:
    - Sets the price P.
    - Decides whether to allow the buyer to learn v before purchase.

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Two things that the seller can do to increase demand:

- Reduce price.
- Reveal v.

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Claim

The seller doesn't reveal v and charges a price $P = E[v]$.

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Proof.

$$E[v] = \int_0^\infty v \; dF(v) \geq \int_P^\infty v \; dF(v) \geq \int_P^\infty P \; dF(v)$$

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Intuition:

- By revealing information, the seller generates asymmetric information and thus must leave information rents to the agent.
- By setting the correct price, the seller that reveals no information can capture the full surplus.

Outline

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- Seller offers a single product, 0.
- Buyer values v, drawn from commonly known distribution with cdf F.
- The seller chooses a price P and whether to offer a prototype ($d = 0$) or a finished product ($d = 1$).
    - Prototype: the buyer learns v only after purchasing the product.
    - Finished product: the buyer knows v from time zero.

Model: In-house development

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- We assume that c_i and v_i are all independent random variables, with respective cdfs G_i and F_i.

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$$\textbf{Seller's payoff} = b_0 \cdot P$$

$$\textbf{Buyer's payoff} = \max_{i \in \{0,\ldots,N\}} \{b_i \cdot v_i\} - \sum_{i=1}^{N} b_i \cdot c_i - b_0 \cdot P$$

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- Once the buyer stops, payoffs are realized.

General Results

Pandora's Rule

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- Given μ, P and $\{c_i\}$, the problem of the buyer is a Pandora stopping problem.

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Define r_i and r as the unique solution to

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Rule

At time t, let V_t denote the value of the best currently available product.

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Rule

At time t, let V_t denote the value of the best currently available product.

- **Stopping Rule**: stop if reservation value of all remaining options is lower than V_t.
- **Selection Rule**: pursue the remaining option with the highest reservation value.

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$$\mathbf{Pr}(r_i > r) = \mathbf{Pr}\left(c_i < \int_r^\infty (v_i - r)\ dF(v_i)\right) = G_i\left(\int_r^\infty (v_i - r)\ dF(v_i)\right)$$

$$\mathbf{Pr}(v_i < r) = 1 - F_i(r)$$

Example I: Binary in-house project

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Single in-house project with $v_1 \in \{0, \bar{v}\}$. Let $q = Pr(v_1 = \bar{v})$.

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The reservation value of the in-house project is $r_1 = \bar{v} - c_1/q$.

Then,

$$Q(r) = \begin{cases} 1 & \text{if} \quad r \geq r_1 \\ 1 - q & \text{if} \quad r \in [0, r_1) \\ 0 & \text{otherwise} \end{cases}$$

Example II: Uniform in-house project

Uniform outside option

Single in-house project with $v_1 \sim U[0, 1]$.

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$$Q(r) = \begin{cases} 0 & \text{if } r < 0 \\ r & \text{if } r \in [0, r_1] \\ 1 & \text{if } r \geq r_1 \end{cases}$$

Seller's problem equivalence

Seller's problem

Choose $d \in \{0, 1\}$ and a price P to maximize expected profits.

- If $d = 0$, μ corresponds to cdf F.
- If $d = 1$, μ corresponds to δ_v.

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Let

$$\begin{aligned} \pi^{\text{Proto}} &:= \max_P \quad P \cdot Q(r(F, P)) \\ \pi^{\text{Finished}} &:= \max_P \quad E\left[P \cdot Q(r(\delta_v, P))\right] \end{aligned}$$

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Timing

The seller *offers a finished product* iff $\pi^{\text{Finished}} \geq \pi^{\text{Proto}}$.
The seller *offers a prototype* iff $\pi^{\text{Proto}} \geq \pi^{\text{Finished}}$.

Observation

Q is a cdf:

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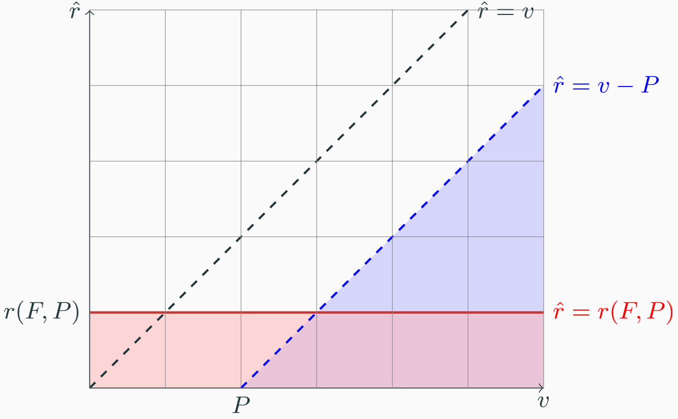
Alternative model

Buyer has a stochastic outside option $\hat{r}$ distributed according to cdf Q. Must decide to purchase or not.

Information Trade-off

Fixing price P.



Optimal Timing

Lemma

The seller offers a finished product if and only if:

$$\max_{x} \int_{x}^{\infty} x \cdot Q(v - x)\ dF(v) \geq \max_{x} \int_{x}^{\infty} Q(x) \cdot (v - x)\ dF(v)$$

Proof

$$\begin{aligned}\pi^{\text{Finished}} &= \max_P \quad P \cdot \int_0^\infty Q(r(\delta_v, P))\ dF(v) \\ &= \max_P \quad P \cdot \int_0^\infty Q(v - P)\ dF(v) \\ &= \max_P \quad \int_P^\infty P \cdot Q(v - P)\ dF(v)\end{aligned}$$

$$\begin{aligned}\pi^{\text{Proto}} &= \max_P \quad P \cdot Q(r(\mu_0, P)) \\ &= \max_P \quad \int_{r(\mu_0,P)}^\infty (v - r(\mu_0, P))\ d\mu_0 \cdot Q(r(\mu_0, P)) \\ &= \max_r \quad \int_r^\infty (v - r)\ d\mu_0 \cdot Q(r) \\ &= \max_r \quad \int_r^\infty Q(r) \cdot (v - r)\ d\mu_0\end{aligned}$$

Sufficient Conditions

Linear Case

Proposition

Consider

$$Q(r) = \begin{cases} 0 & \text{if } r < 0 \\ \alpha \cdot r & \text{if } r \in [0, 1/\alpha] \\ 1 & \text{if } r > 1/\alpha \end{cases}$$

and let $\operatorname{supp}(v) \subseteq [0, 1/\alpha]$. Then, $\pi^{\text{Proto}} = \pi^{\text{Finished}}$.

Linear Case: Intuition

For simplicity, consider $\alpha = 1$. Let $\phi(z) := \int_z^\infty (v - z)\, dF$

Consider offering a finished product at price P.

The probability of selling is $\phi(P)$. The profits are

$$P \cdot \phi(P)$$

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Consider selling a prototype at price $P' = \phi(P)$. Note that $r(F, P') = P$.

- Profits are

$$P' \cdot r(F, P') = \phi(P) \cdot P$$

Binary Case

- Let $v \in \{v_L, v_H\}$ and $v_1 \in \{0, \bar{v}\}$.
- q denotes the probability of the seller strictly prefers to offer a finished product if and only of $v_1 = \bar{v}$.
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Binary case

- Case $r_1 \leq \min\{v_L, v_H - v_L\}$. The seller strictly prefers to offer a finished product if and only if

$$q \in \left(\frac{E[v] - v_L}{\alpha_H v_H} \, , \, \frac{v_L + r_1 - E[v]}{\alpha_L v_L} \right)$$

- Case $v_L < r_1 < v_H - v_L$. The seller strictly prefers to offer a finished product if and only if

$$q \in \left(\frac{E[v] - v_L}{\alpha_H v_H} \, , \, \frac{v_L - \alpha_H (v_H - r_1)}{\alpha_L v_L} \right)$$

Comparative Statics

Improving in-house capabilities

Proposition

Consider adding a project with cost c_k and distribution F_k. If c_k is sufficiently large, the effect is in favor of offering a prototype.

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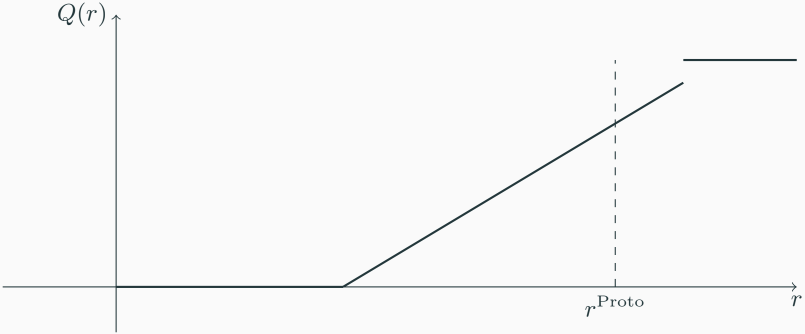
Proof sketch.

Let P^{Proto} and r^{Proto} be the optimal price and respective reservation value for a prototype without m_k. Adding a project k with $r_k < r^{Proto}$ doesn't reduce the demand for prototype at its optimal price, but reduces the demand for a finished product with low value for all prices. $\square$

Adding high-cost project



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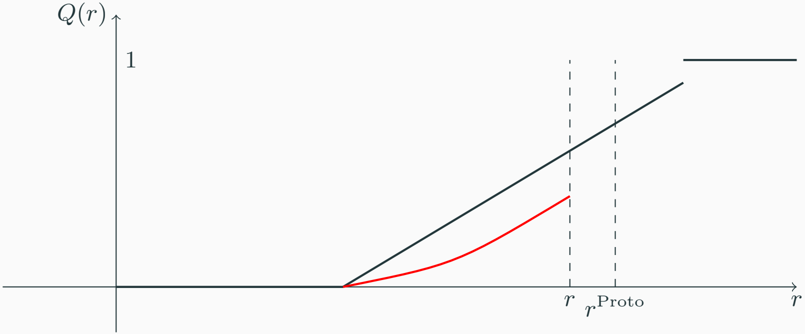

Improving in-house capabilities

Claim

Adding a low-cost in-house project can induce the seller to switch from offering a prototype to offering a finished product.

Example

Adding the binary project in a binary case.

Conclusion

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- We developed a model of outsourcing where
  - Buyer has the possibility of privately developing in-house alternatives.
  - Seller controls price and information.
- We showed that the probability of sale is an increasing function of Weitzman's reservation value.
- In the linear case, the seller is indifferent between contracting when the value is known or not.
- In the binary case, the seller strictly prefers the buyer to know the value for intermediate probabilities of in-house success.
- Adding sufficiently costly in-house projects always induces the Seller to delay contracting, while low-cost in-house projects can induce the Seller to contract earlier.

Literature

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