A Taxation Principle with Moral Hazard

Francisco Poggi and Bruno Strulovici

University of Mannheim and Northwestern University

Origin of the Taxation Principle

- Guesnerie (1981, 1995), Hammond (1979)
- Given the asymmetric information problem, the government could not to do better (in the absense of correlation between the types of the workers) than using taxes (...) an incentive compatible revelation mechanism could not do better than a tax system;

Origin of the Taxation Principle

- Guesnerie (1981, 1995), Hammond (1979)
- Given the asymmetric information problem, the government could not to do better (in the absense of correlation between the types of the workers) than using taxes (...) an incentive compatible revelation mechanism could not do better than a tax system; this is the origin of the term "**taxation principle**"

Linnemer (2019), *Annals of Economics and Statistics*

Taxation Principle: Illustration

Timing:

- Agent of type $\theta \in \Theta$ choses action $a \in A$.

Taxation Principle: Illustration

Timing:

- Agent of type $\theta \in \Theta$ choses action $a \in A$.
- Yields a revenue $z \in Z \subseteq \mathbb{R}$.

Timing:

- Agent of type $\theta \in \Theta$ choses action $a \in A$.
- Yields a revenue $z \in Z \subseteq \mathbb{R}$.
- Tax schedule $t : Z \to \mathbb{R}$.

Taxation Principle: Illustration

Timing:

- Agent of type $\theta \in \Theta$ choses action $a \in A$.
- Yields a revenue $z \in Z \subseteq \mathbb{R}$.
- Tax schedule $t : Z \to \mathbb{R}$.

Taxation Principle: Illustration

Timing:

- Agent of type $\theta \in \Theta$ choses action $a \in A$.
- Yields a revenue $z \in Z \subseteq \mathbb{R}$.
- Tax schedule $t : Z \to \mathbb{R}$.

Utility:

- $v(z - t) - c(a; \theta)$

Taxation Principle: Illustration

Timing:

- Agent of type $\theta \in \Theta$ choses action $a \in A$.
- Yields a revenue $z \in Z \subseteq \mathbb{R}$.
- Tax schedule $t : Z \to \mathbb{R}$.

Utility:

- $v(z - t) - c(a; \theta)$

Goal: to implement

- social choice function (scf) $f : \Theta \to A$
- transfer schedule $t : \Theta \to \mathbb{R}$

Direct Mechanisms and the Revelation Principle

Asymmetric information problem:

- type θ is private and action a is hidden.
- z is what the principal can directly condition on.

Direct Mechanisms and the Revelation Principle

Asymmetric information problem:

- type θ is private and action a is hidden.
- z is what the principal can directly condition on.

Direct Mechanism:

- Agents report $\hat{\theta}$
- Faces tax schedule $t_{\hat{\theta}} : Z \to T$
- Agent chooses action a; z is realized; agent pays $t_{\hat{\theta}}(z)$.

Direct Mechanisms and the Revelation Principle

Asymmetric information problem:

- type θ is private and action a is hidden.
- z is what the principal can directly condition on.

Direct Mechanism:

- Agents report $\hat{\theta}$
- Faces tax schedule $t_{\hat{\theta}} : Z \to T$
- Agent chooses action a; z is realized; agent pays $t_{\hat{\theta}}(z)$.

Revelation Principle: (truthful) Direct Mechanisms are without loss.

Menu Mechanism:

- Planner proposes a menu $M = \{t_j\}_{j \in J}$ of tax schedules.

Menu Taxation Principle

Menu Mechanism:

- Planner proposes a menu $M = \{t_j\}_{j \in J}$ of tax schedules.
- Agent chooses an item $t \in M$.

Menu Mechanism:

- Planner proposes a menu $M = \{t_j\}_{j \in J}$ of tax schedules.
- Agent chooses an item $t \in M$.
- Agent chooses action a; z is realized; agent pays $t(z)$.

Menu Taxation Principle

Menu Mechanism:

- Planner proposes a menu $M = \{t_j\}_{j \in J}$ of tax schedules.
- Agent chooses an item $t \in M$.
- Agent chooses action a; z is realized; agent pays $t(z)$.

Menu Mechanism:

- Planner proposes a menu $M = \{t_j\}_{j \in J}$ of tax schedules.
- Agent chooses an item $t \in M$.
- Agent chooses action a; z is realized; agent pays $t(z)$.

Menu mechanisms are also without loss.

Substantive Taxation Principle

Tax Mechanism:

- Agent faces a tax schedule $t : Z \to \mathbb{R}$, regardless of type.
- Chooses action a; z is realized; pays $t(z)$.

Substantive Taxation Principle

Tax Mechanism:

- Agent faces a tax schedule $t : Z \to \mathbb{R}$, regardless of type.
- Chooses action a; z is realized; pays $t(z)$.

Substantive Taxation Principle: Any scf implemented by a direct mechanism can be implemented by a tax mechanism.

- Arguably the intended principle.

Substantive Taxation Principle

Tax Mechanism:

- Agent faces a tax schedule $t : Z \to \mathbb{R}$, regardless of type.
- Chooses action a; z is realized; pays $t(z)$.

Substantive Taxation Principle: Any scf implemented by a direct mechanism can be implemented by a tax mechanism.

- Arguably the intended principle.
- Requires hypothesies.

Substantive Taxation Principle

Tax Mechanism:

- Agent faces a tax schedule $t : Z \to \mathbb{R}$, regardless of type.
- Chooses action a; z is realized; pays $t(z)$.

Substantive Taxation Principle: Any scf implemented by a direct mechanism can be implemented by a tax mechanism.

- Arguably the intended principle.
- Requires hypothesies.
- Dramatically simplifies the analysis when it holds.

Substantive Taxation Principle

Tax Mechanism:

- Agent faces a tax schedule $t : Z \to \mathbb{R}$, regardless of type.
- Chooses action a; z is realized; pays $t(z)$.

Substantive Taxation Principle: Any scf implemented by a direct mechanism can be implemented by a tax mechanism.

- Arguably the intended principle.
- Requires hypothesies.
- Dramatically simplifies the analysis when it holds.
- Focus of this paper.

Observable Actions

- Suppose actions are observable, e.g. $z = a$.

Observable Actions

- Suppose actions are observable, e.g. $z = a$.
- Given a Menu Mechanism M, let

$$\tilde{t}(a) = \min\{t(a) : t \in M\} \quad \text{for all } a \in A$$

Observable Actions

- Suppose actions are observable, e.g. $z = a$.
- Given a Menu Mechanism M, let

$$\tilde{t}(a) = \min\{t(a) : t \in M\} \quad \text{for all } a \in A$$

- When considering action a, $\tilde{t}(a)$ is the only transfer that matters.

Observable Actions

- Suppose actions are observable, e.g. $z = a$.
- Given a Menu Mechanism M, let

$$\tilde{t}(a) = \min\{t(a) : t \in M\} \quad \text{for all } a \in A$$

- When considering action a, $\tilde{t}(a)$ is the only transfer that matters.
- Proposing a single tax schedule $\tilde{t}$ instead of M doesn't affect incentives and yields same equilibrium transfers.

Moral Hazard

- Outcome z is stochastic function of effort.
- Two type of agents.
  - θ_H: high productivity, risk averse agent.
  - θ_L: low productivity, risk neutral.

Moral Hazard

- Outcome z is stochastic function of effort.
- Two type of agents.
  - θ_H: high productivity, risk averse agent.
  - θ_L: low productivity, risk neutral.

- A Menu Mechanism:
  - t_H insures against shocks at some premium.
  - t_L constant tax (no insurance).

Moral Hazard

- Outcome z is stochastic function of effort.
- Two type of agents.
  - θ_H: high productivity, risk averse agent.
  - θ_L: low productivity, risk neutral.

- A Menu Mechanism:
  - t_H insures against shocks at some premium.
  - t_L constant tax (no insurance).

- Induces high type to reveal his type and allows more rent extraction.

Moral Hazard

- Outcome z is stochastic function of effort.
- Two type of agents.
  - θ_H: high productivity, risk averse agent.
  - θ_L: low productivity, risk neutral.

- A Menu Mechanism:
  - t_H insures against shocks at some premium.
  - t_L constant tax (no insurance).

- Induces high type to reveal his type and allows more rent extraction.
- Not possible to reduce it to a single tax schedule $\Rightarrow$ Substantive TP Fails.

Moral Hazard

- Outcome z is stochastic function of effort.
- Two type of agents.
  - θ_H: high productivity, risk averse agent.
  - θ_L: low productivity, risk neutral.

- A Menu Mechanism:
  - t_H insures against shocks at some premium.
  - t_L constant tax (no insurance).

- Induces high type to reveal his type and allows more rent extraction.
- Not possible to reduce it to a single tax schedule $\Rightarrow$ Substantive TP Fails.
  - What are the *right* conditions for the principle to hold?

General Framework

- **Model elements**:
    - Agent of type $\theta \in \Theta$.
    - Chooses action $a \in A$.
    - Stochastic outcome $z \in Z$.
    - Penalty $t \in T(z) \subseteq \mathbb{R}$

General Framework

- **Model elements**:
  - Agent of type $\theta \in \Theta$.
  - Chooses action $a \in A$.
  - Stochastic outcome $z \in Z$.
  - Penalty $t \in T(z) \subseteq \mathbb{R}$

- **Agents payoff**: $v(\theta, a, z) - q(\theta, a) \cdot d(t, z)$, where q is positive-valued.

General Framework

- **Model elements**:
  - Agent of type $\theta \in \Theta$.
  - Chooses action $a \in A$.
  - Stochastic outcome $z \in Z$.
  - Penalty $t \in T(z) \subseteq \mathbb{R}$

- **Agents payoff**: $v(\theta, a, z) - q(\theta, a) \cdot d(t, z)$, where q is positive-valued.

- **Contractible outcomes** $C \subseteq Z$ such that for all $z \notin C$, we assume w.l.o.g. $T(z) = 0$.

Non Contractible Events

- θ: riskiness of activity

Non Contractible Events

- θ: riskiness of activity
- a: level of care.

Non Contractible Events

- θ: riskiness of activity
- a: level of care.
- $p(\theta, a)$ probability of accident.

Non Contractible Events

- θ: riskiness of activity
- a: level of care.
- $p(\theta, a)$ probability of accident.
- z: outcome (either *accident* or *no accident*).

Non Contractible Events

- θ: riskiness of activity
- a: level of care.
- $p(\theta, a)$ probability of accident.
- z: outcome (either *accident* or *no accident*).
- The agent can only be punished if an accident is caused.

Non Contractible Events

- θ: riskiness of activity
- a: level of care.
- $p(\theta, a)$ probability of accident.
- z: outcome (either *accident* or *no accident*).
- The agent can only be punished if an accident is caused.
  - *'no accident'* is non-contractible.

Independence

Let $\mu_{a,\theta} \in \Delta(C)$ be the *contratible* outcome distribution given a and θ.

Independence

Let $\mu_{a,\theta} \in \Delta(C)$ be the *contratible* outcome distribution given a and θ.

Independence

$\mu_{a,\theta}$ is independent of θ.

Independence

Let $\mu_{a,\theta} \in \Delta(C)$ be the *contratible* outcome distribution given a and θ.

Independence

$\mu_{a,\theta}$ is independent of θ.

- Holds if θ affects the cost of actions, not their consequences.

Independence

Let $\mu_{a,\theta} \in \Delta(C)$ be the *contratible* outcome distribution given a and θ.

Independence

$\mu_{a,\theta}$ is independent of θ.

- Holds if θ affects the cost of actions, not their consequences.
- Holds if θ affects the probability that the outcome is contractible, not the outcome distribution conditional on contractability.

Independence

Let $\mu_{a,\theta} \in \Delta(C)$ be the *contratible* outcome distribution given a and θ.

Independence

$\mu_{a,\theta}$ is independent of θ.

- Holds if θ affects the cost of actions, not their consequences.
- Holds if θ affects the probability that the outcome is contractible, not the outcome distribution conditional on contractability.
- Without it, easy to build examples where TP Fails.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.
  - Agent prefers to missmatch.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.
  - Agent prefers to missmatch.
- Outcome distribution:
  - With probability ϵ, $z = \theta$.
  - With probability $1 - \epsilon$, $z = a$.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.
  - Agent prefers to missmatch.
- Outcome distribution:
  - With probability ϵ, $z = \theta$.
  - With probability $1 - \epsilon$, $z = a$.
- Consider a Direct Mechanism in which the agent is punished if $z \neq \hat{\theta}$.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.
  - Agent prefers to missmatch.
- Outcome distribution:
  - With probability ϵ, $z = \theta$.
  - With probability $1 - \epsilon$, $z = a$.
- Consider a Direct Mechanism in which the agent is punished if $z \neq \hat{\theta}$.
  - Agent can avoid punishment by reporting truthfully and matching the action to the state.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.
  - Agent prefers to missmatch.
- Outcome distribution:
  - With probability ϵ, $z = \theta$.
  - With probability $1 - \epsilon$, $z = a$.
- Consider a Direct Mechanism in which the agent is punished if $z \neq \hat{\theta}$.
  - Agent can avoid punishment by reporting truthfully and matching the action to the state.
  - If sufficiently harsh penalties are available, the planner's optimum can be implemented.

TP Failure without Independence

- Suppose $\Theta = A = \{l, r\}$.
  - Planner wants the action to match the state.
  - Agent prefers to missmatch.
- Outcome distribution:
  - With probability ϵ, $z = \theta$.
  - With probability $1 - \epsilon$, $z = a$.
- Consider a Direct Mechanism in which the agent is punished if $z \neq \hat{\theta}$.
  - Agent can avoid punishment by reporting truthfully and matching the action to the state.
  - If sufficiently harsh penalties are available, the planner's optimum can be implemented.
- With a tax mechanism, transfers are independent of agent's type and the planner's optimum cannot be implemented.

Consider a partition $\mathcal{A}$ of the action space A.

Observability and Invariance

Consider a partition $\mathcal{A}$ of the action space A.

- $\mathcal{A}$ is **observable** if for any a, a' in different cells of $\mathcal{A}$, contractible outcome distributions μ_a^C and $\mu_{a'}^C$ have disjoint supports.

Observability and Invariance

Consider a partition $\mathcal{A}$ of the action space A.

- $\mathcal{A}$ is **observable** if for any a, a' in different cells of $\mathcal{A}$, contractible outcome distributions μ_a^C and $\mu_{a'}^C$ have disjoint supports.
  - Observable partitions are closed under intersection $\Rightarrow$ there exists a finest observable partition $\mathbf{A}$.

Observability and Invariance

Consider a partition $\mathcal{A}$ of the action space A.

- $\mathcal{A}$ is **observable** if for any a, a' in different cells of $\mathcal{A}$, contractible outcome distributions μ_a^C and $\mu_{a'}^C$ have disjoint supports.
  - Observable partitions are closed under intersection $\Rightarrow$ there exists a finest observable partition $\mathbf{A}$.

Observability and Invariance

Consider a partition $\mathcal{A}$ of the action space A.

- $\mathcal{A}$ is **observable** if for any a, a' in different cells of $\mathcal{A}$, contractible outcome distributions μ_a^C and $\mu_{a'}^C$ have disjoint supports.
  - Observable partitions are closed under intersection $\Rightarrow$ there exists a finest observable partition $\mathbf{A}$.

- $\mathcal{A}$ is **invariant** if for any cell $A_i \in \mathcal{A}$, the map μ_a is constant over A_i.

Main Result

Taxation Principle

Suppose independence holds. If the finest observable partition $\mathbf{A}$ is invariant, then any scf that is implementable by a direct mechanism is also implementable by a tax mechanism.

Main Result

Taxation Principle

Suppose independence holds. If the finest observable partition $\mathbf{A}$ is invariant, then any scf that is implementable by a direct mechanism is also implementable by a tax mechanism.

- When $\mathbf{A}$ is invariant, the principal can identify, for each contractible outcome realization, the distribution of contractible outcomes that is associated with the action a.

Main Result

Taxation Principle

Suppose independence holds. If the finest observable partition $\mathbf{A}$ is invariant, then any scf that is implementable by a direct mechanism is also implementable by a tax mechanism.

- When $\mathbf{A}$ is invariant, the principal can identify, for each contractible outcome realization, the distribution of contractible outcomes that is associated with the action a.
- Asking the agent to report his private information becomes redundant.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j \in J}$.
    1. Construct a candidate tax mechanism $\tilde{t}$.
    2. Verify that it induces f.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j \in J}$.
    1. Construct a candidate tax mechanism $\tilde{t}$.
    2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j \in J}$.
    1. Construct a candidate tax mechanism $\tilde{t}$.
    2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.
- $\{C_j\}_{j \in J}$ is a partition of C.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j \in J}$.
    1. Construct a candidate tax mechanism $\tilde{t}$.
    2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.
- $\{C_j\}_{j \in J}$ is a partition of C.
- We will construct a map $\tilde{\theta} : C \to \Theta$ as follows. Let $z \in C_j$, then

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j \in J}$.
    1. Construct a candidate tax mechanism $\tilde{t}$.
    2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.
- $\{C_j\}_{j \in J}$ is a partition of C.
- We will construct a map $\tilde{\theta} : C \to \Theta$ as follows. Let $z \in C_j$, then
    - **Case 1**: $f(\Theta) \cap A_j \neq \emptyset$. Then, $\tilde{\theta}(z)$ such that $f(\theta_j) \in A_j$.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta \in \Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j \in J}$.
  1. Construct a candidate tax mechanism $\tilde{t}$.
  2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.
- $\{C_j\}_{j \in J}$ is a partition of C.
- We will construct a map $\tilde{\theta} : C \to \Theta$ as follows. Let $z \in C_j$, then
  - **Case 1**: $f(\Theta) \cap A_j \neq \emptyset$. Then, $\tilde{\theta}(z)$ such that $f(\theta_j) \in A_j$.
  - **Case 2**: $f(\Theta) \cap A_j = \emptyset$, then $\tilde{\theta}(z) = \theta_\circ$.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta\in\Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j\in J}$.
  1. Construct a candidate tax mechanism $\tilde{t}$.
  2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.
- $\{C_j\}_{j\in J}$ is a partition of C.
- We will construct a map $\tilde{\theta} : C \to \Theta$ as follows. Let $z \in C_j$, then
  - **Case 1**: $f(\Theta) \cap A_j \neq \emptyset$. Then, $\tilde{\theta}(z)$ such that $f(\theta_j) \in A_j$.
  - **Case 2**: $f(\Theta) \cap A_j = \emptyset$, then $\tilde{\theta}(z) = \theta_\circ$.
- We define a tax mechanism $\tilde{t}(z) = t_{\tilde{\theta}(z)}(z)$.

Proof: Construction

- Fix a direct mechanism $\{t_\theta\}_{\theta\in\Theta}$ inducing scf f.
- Suppose there is an observable, invariant partition $\mathcal{A} = \{A_j\}_{j\in J}$.
    1. Construct a candidate tax mechanism $\tilde{t}$.
    2. Verify that it induces f.
- For each cell A_j, consider the set C_j of contractible outcomes generated by actions in A_j.
- $\{C_j\}_{j\in J}$ is a partition of C.
- We will construct a map $\tilde{\theta} : C \to \Theta$ as follows. Let $z \in C_j$, then
    - **Case 1**: $f(\Theta) \cap A_j \neq \emptyset$. Then, $\tilde{\theta}(z)$ such that $f(\theta_j) \in A_j$.
    - **Case 2**: $f(\Theta) \cap A_j = \emptyset$, then $\tilde{\theta}(z) = \theta_\circ$.
- We define a tax mechanism $\tilde{t}(z) = t_{\tilde{\theta}(z)}(z)$.
- It remains to check that $\tilde{t}$ yields same incentives as $\{t_\theta\}_{\theta\in\Theta}$.

Proof: Construction

- Consider any cell A_i such that $A_i \cap f(\Theta) \neq \emptyset$.

Proof: Construction

- Consider any cell A_i such that $A_i \cap f(\Theta) \neq \emptyset$.
    - By Independence and Invariance: each action in A_i yields the same distribution over C_i, independently of θ.

Proof: Construction

- Consider any cell A_i such that $A_i \cap f(\Theta) \neq \emptyset$.
  - By Independence and Invariance: each action in A_i yields the same distribution over C_i, independently of θ.
- From the utility form, all types have the same preference ranking over transfer functions.

Proof: Construction

- Consider any cell A_i such that $A_i \cap f(\Theta) \neq \emptyset$.
    - By Independence and Invariance: each action in A_i yields the same distribution over C_i, independently of θ.
- From the utility form, all types have the same preference ranking over transfer functions.
- By construction, $\tilde{t}$ was preferred by some type chosing an action in A_i, so $\tilde{t}$ is preferred by all such types, and delivers same payoff as the direct mechanism.

Proof: Construction

- Consider any cell A_i such that $A_i \cap f(\Theta) \neq \emptyset$.
    - By Independence and Invariance: each action in A_i yields the same distribution over C_i, independently of θ.
- From the utility form, all types have the same preference ranking over transfer functions.
- By construction, $\tilde{t}$ was preferred by some type chosing an action in A_i, so $\tilde{t}$ is preferred by all such types, and delivers same payoff as the direct mechanism.
- No type gains by deviating from f under $\tilde{t}$ because payoffs from other actions were already available under the direct mechanism.

Weakest Condition

- Note that both conditions (independence and $\mathbf{A}$ invariant) are orthogonal to agents' preferences.

Weakest Condition

- Note that both conditions (independence and **A** invariant) are orthogonal to agents' preferences.
- It turns out that these conditions are the weakest that guarantee T-P for *all* preferences of the agent.

Weakest Condition

- Note that both conditions (independence and **A** invariant) are orthogonal to agents' preferences.
- It turns out that these conditions are the weakest that guarantee T-P for *all* preferences of the agent.

Weakest Condition

- Note that both conditions (independence and $\mathbf{A}$ invariant) are orthogonal to agents' preferences.
- It turns out that these conditions are the weakest that guarantee T-P for *all* preferences of the agent.

Weakest condition

If the finest observable partition $\mathbf{A}$ is not invariant, there is a set of types Θ, a set of feasible penalties $\Gamma : Z \to \mathbb{R}$, a utility function u, and a social choice function f such that f is implementable but not tax implementable.

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).
- Agent choses an experiment e from a set of feasible experiments $\mathcal{E}$.

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).
- Agent choses an experiment e from a set of feasible experiments $\mathcal{E}$.
- Each experiment is a map $e : \{0, 1\} \to \Delta(S)$.

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).
- Agent choses an experiment e from a set of feasible experiments $\mathcal{E}$.
- Each experiment is a map $e : \{0, 1\} \to \Delta(S)$.
- Agent then observes the realized signal $s \in S$ and chooses a level of precaution $x \in \mathbb{R}_+$.

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).
- Agent choses an experiment e from a set of feasible experiments $\mathcal{E}$.
- Each experiment is a map $e : \{0, 1\} \rightarrow \Delta(S)$.
- Agent then observes the realized signal $s \in S$ and chooses a level of precaution $x \in \mathbb{R}_+$.
- If the project is risky, an accident occurs with probability $p(x)$, 0 otherwise.

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).
- Agent choses an experiment e from a set of feasible experiments $\mathcal{E}$.
- Each experiment is a map $e : \{0, 1\} \to \Delta(S)$.
- Agent then observes the realized signal $s \in S$ and chooses a level of precaution $x \in \mathbb{R}_+$.
- If the project is risky, an accident occurs with probability $p(x)$, 0 otherwise.
- If an accident occurs, the principal
  - observes $z = (\text{accident}, e, s)$.
  - Penalizes the agent.

Application: Experiments and Caution

- Project can be risky ($\omega = 1$) or safe ($\omega = 0$).
- Agent choses an experiment e from a set of feasible experiments $\mathcal{E}$.
- Each experiment is a map $e : \{0, 1\} \to \Delta(S)$.
- Agent then observes the realized signal $s \in S$ and chooses a level of precaution $x \in \mathbb{R}_+$.
- If the project is risky, an accident occurs with probability $p(x)$, 0 otherwise.
- If an accident occurs, the principal
  - observes $z = (\text{accident}, e, s)$.
  - Penalizes the agent.
- Private type θ includes the ex-ante probability of $\omega = 1$ and preference parameters (e.g., cost of experiments and care).

Application: Experiments and Caution

- **Independence**: conditional on accident and experiment, the distribution of s is independent of θ.

Application: Experiments and Caution

- **Independence**: conditional on accident and experiment, the distribution of s is independent of θ.
- **Observability**: Partition of A according to the choice of experiment.

Application: Experiments and Caution

- **Independence**: conditional on accident and experiment, the distribution of s is independent of θ.
- **Observability**: Partition of A according to the choice of experiment.
- **Invariance**: conditional on accident, the distribution of signals s is constant in each element of the partition.

Application: Experiments and Caution

- **Independence**: conditional on accident and experiment, the distribution of s is independent of θ.
- **Observability**: Partition of A according to the choice of experiment.
- **Invariance**: conditional on accident, the distribution of signals s is constant in each element of the partition.

Application: Experiments and Caution

- **Independence**: conditional on accident and experiment, the distribution of s is independent of θ.
- **Observability**: Partition of A according to the choice of experiment.
- **Invariance**: conditional on accident, the distribution of signals s is constant in each element of the partition.

Taxation Principle applies!

- Reports are unecessary.
- Penalty as a function of e and s is wlog.

- Taxation Principle Theorem: sufficient condition under which *all* implementable scf can be implemented with a tax mechanism.

Extension: Specific SCF

- Taxation Principle Theorem: sufficient condition under which *all* implementable scf can be implemented with a tax mechanism.
- Sometimes, only a subset of scf is relevant for the principal.

Extension: Specific SCF

- Taxation Principle Theorem: sufficient condition under which *all* implementable scf can be implemented with a tax mechanism.
- Sometimes, only a subset of scf is relevant for the principal.

Extension: Specific SCF

- Taxation Principle Theorem: sufficient condition under which *all* implementable scf can be implemented with a tax mechanism.
- Sometimes, only a subset of scf is relevant for the principal.

Definition

A partition $\mathcal{A}$ of the action space is f-invariant if the partition $\{A_i \cap f(\Theta) : A_i \in \mathcal{A}\}$ is invariant.

Extension: Specific SCF

- Taxation Principle Theorem: sufficient condition under which *all* implementable scf can be implemented with a tax mechanism.
- Sometimes, only a subset of scf is relevant for the principal.

Definition

A partition $\mathcal{A}$ of the action space is f-invariant if the partition $\{A_i \cap f(\Theta) : A_i \in \mathcal{A}\}$ is invariant.

T-P for Specific SCF

Suppose that independence holds and f is implementable. If $\mathbf{A}$ is f-invariant then f is implementable by a tax mechanism.

Extension: Dynamic Taxation Principle

- Two periods: $\tau = 1, 2$.
- Evolving state θ_τ.
- Action $a_\tau \in A_\tau$ at time τ.
- Outcome $z_\tau \in Z_\tau$ at time τ.
- Contractible outcomes $C \subseteq Z_1 \times Z_2$.

Extension: Dynamic Taxation Principle

- Two periods: $\tau = 1, 2$.
- Evolving state θ_τ.
- Action $a_\tau \in A_\tau$ at time τ.
- Outcome $z_\tau \in Z_\tau$ at time τ.
- Contractible outcomes $C \subseteq Z_1 \times Z_2$.

We would like to implement $f = (f_1, f_2)$ where

- $f_1 : \Theta_1 \to A_1$
- $f_2 : \Theta_1 \times Z_1 \times \Theta_2 \to A_2$

Implementable scf

In a direct mechanism, the agent reports $\hat{\theta}_\tau$ at each time before taking the action, and is penalized according to a function

$$t : \Theta_1 \times \Theta_2 \times C \to \mathbb{R}$$

A social choice function f is *implementable* if there is a direct mechanism and an optimal response of the agent that is consistent with f.

Direct Mechanisms

Implementable scf

In a direct mechanism, the agent reports $\hat{\theta}_\tau$ at each time before taking the action, and is penalized according to a function

$$t : \Theta_1 \times \Theta_2 \times C \to \mathbb{R}$$

A social choice function f is *implementable* if there is a direct mechanism and an optimal response of the agent that is consistent with f.

Tax Mechanisms

A *tax mechanism* is a function

$$\tilde{t} : C \to T$$

Extension: Dynamic Taxation Principle

Finest Observable Partitions

Let $\mathbf{A}_\tau$ be the finest observable partition of the action space A_τ at time τ.

Extension: Dynamic Taxation Principle

Finest Observable Partitions

Let $\mathbf{A}_\tau$ be the finest observable partition of the action space A_τ at time τ.

Dynamic Principle

If θ_2 is independent of θ_1 and a_1 conditional on z_1, and independence and invariance hold every period, then any implementable scf f can be implemented by a tax mechanism.

Other Applications and Extensions

- Applications:
  - Liability design.
  - Plea bargaining.
  - Pre-existing conditions and health insurance.
  - Scoring mechanisms.
  - etc.

- Extensions:
  - Multiple agents with independent types.
  - Dynamic contracting.
  - etc.