Liability Design with Information Acquisition

Francisco Poggi and Bruno Strulovici
Northwestern University*

November 13, 2020

1 Introduction

In 2019, a California court sentenced paint maker Sherwin-Williams to pay hundreds of millions of dollars to address the dangers caused by lead paint. The sentence was remarkable because even though lead paint became banned in 1978, the suit concerned damage caused during the decades before the ban and centered on the accusation that paint makers were aware of the dangers caused by lead paint long before the ban.

In essence, the court's argument was that Sherwin-Williams and other paint makers knew or should have known the dangers caused by lead paint.

While it is difficult for a regulator to guess a firm's private information, it is perhaps easier to assess due diligence: did paint makers research the risk of lead paint sufficiently well before marketing it?

Formally, the problem is not just one of private information, but also one of information acquisition: how can a regulator make sure that agents learn sufficiently well before taking actions?

One may model this question as a delegated Wald problem (Wald (1945)): the principal is a regulator, who relies on the agent (the firm) to acquire information before deciding whether to launch a product or abandon it if it is too risky.

*Emails: franciscopoggi2015@u.northwestern.edu and b-strulovici@northwestern.edu. Strulovici gratefully acknowledges financial support from the National Science Foundation (grant 1151410)

If the regulator could unrestrictedly penalize an agent for the damages caused, he could force the agent internalize all damage and implement the socially optimal level of information acquisition and the optimal decision.

For various reasons, liability may be capped, however, which precludes the full transfer of damages to the agent. In this case, the regulator can punish the agent if and only if damage occurs, and charge him a penalty that depends on the information available to the regulator after the damage has occurred.

In this project, we study this problem in a Brownian version of the Wald Problem: the agent observes an arithmetic Brownian whose drift depends on the state of the world, i.e., on the riskiness of the product. Information acquisition is costly. The first-best policy is to search until the riskiness of the product becomes sufficiently clear, and launch the product if this riskiness is low and abandon it if the riskiness is high.

We characterize the optimal liability rule when the agent has private information, liability is capped, and the regulator can penalize the agent only when damage occurs. As part of this characterization, we provide a simple condition under which the identifiability condition of our companion paper (Poggi and Strulovici (2020)) applies, which explains why we can focus without loss of generality on policies that do not extract the agent's initial private information: the Taxation Principle with Non-Contractible Events of that paper applies.

2 Baseline Model

A firm must decide whether to launch a product or abandon its development. If it launched, the product may cause some damage with positive probability. The firm has some private information about the product's riskiness and can acquire additional information before deciding between launching the product and abandoning it.

A regulator wishes to encourage the launch of low-risk products and deter the launch of high-risk ones and to encourage the firm to acquire sufficient information before making its decision.

The regulator faces two constraints. First, the firm has limited liability: the social cost caused by product damage is $L > 0$ and the firm's liability is capped at some lower level $l < L$. Second, the regulator can penalize the firm only if some damage occurs: it cannot penalize firms that acquired too little information and took an overly risky decision unless such risks result in some damage.

The timing of the game is as follows:

1. The firm is endowed with a prior $\theta \in \Theta \subset [0, 1]$ about the product's riskiness $y \in \{0, 1\}$, with $\theta = \Pr(y = 1)$.

2. The firm can acquire additional information about y.

3. The firm adopts or abandons the product.

4. If the firm adopts the product, it causes some damage if the product was risky ($y = 1$) and doesn't if the product was safe ($y = 0$).

5. In case of damage, the firm pays a penalty $\psi \leq l$ set by the regulator.

The assumption that a risky product causes damage with probability 1 is without loss of generality: if this probability were less than 1, the same analysis would apply using expected damage and expected penalties.

Information structure: During the information-acquisition stage, the firms observes a process X given by:

$$X_t = yt + \sigma B_t$$

where B is the standard Brownian motion. The drift of X depends on the product's riskiness y. Therefore, observing X gradually reveals y. This revelation is progressive due to the stochastic component of X.

The firm stops acquiring information at some time τ that is adapted to filtration of X.

The regulator has a prior $\lambda \in \Delta(\Theta)$ about the firm's private information. She observes nothing about X except if some damage occurs, in which case she observes the last value X_τ taken by the process at the time of the firm's decision. X_τ is a measure of the firm's due diligence to assess the product's riskiness before launching it.

In this Brownian model, it is well-known (though not immediate) that for each $t > 0$, the variable X_t is a sufficient statistic for the information about y contained by the path $\{X_s\}_{s \leq t}$ of the process X until time t. Mathematically, the likelihood ratio of y associated with a path of X from time 0 to t is only a function of X's value at time t.

Because the stopping time τ is chosen endogenously by the firm, which has private information about y, X_τ is not a sufficient statistic for y once the firm's strategic timing is taken into account. Our assumption that the regulator observes X_τ instead of the entire path $\{X_t\}_{t\leq\tau}$ captures the idea that the regulator does not perfectly observe all the decisions made by the firm during the information acquisition stage. Intuitively, the regulator observes the most informative signal about y contained by the path of X that is independent of the firm's private information.

Payoffs: The firm incurs a running cost c from acquiring information, and a profit normalized to 1 if it launches the product. Let $d = 1$ if the firm launches the product and $d = 0$ if it abandons it, and τ denote the time spent acquiring information. The firm's realized payoff is

$$u = d(\pi - y\psi) - c\tau$$

where π is the firm's profit from the launch in the absence of damage. The regulator's objective internalizes the entire damage caused by the product:

$$v = d(\beta - yL) - c\tau$$

where β is the social benefit from the launch in the absence of a damage.

To gain some intuition about the role of limited liability, it is sometimes useful to set $\pi = \beta$, which means that the firm extracts the entire social surplus from the product in the absence of damage.

3 Preliminary Analysis

First Best: If θ were publicly known, the regulator's optimal information acquisition strategy would have a simple structure, consisting in abandoning the product as soon as the process X exceeds some upper threshold $\bar{x}^*(\theta)$ and launching it as soon as X drops below some lower threshold $\underline{x}^*(\theta)$.

Moreover, if the liability cap l satisfied $l \geq L$ and $\pi = \beta$, the regulator could achieve the first best by setting $\psi \equiv L$ and align the firm's interest perfectly with the social objective.

Tariffs: Given a tariff $\psi : \mathbb{R} \to \mathbb{R}$, a firm with prior θ chooses a stopping time τ and a launch/abandonment decision $d \in \{0, 1\}$ that maximizes its expected utility

$$E\left[d(\pi - y\psi(X_\tau)) - c\tau \mid \theta\right]. \qquad (1)$$

It is straightforward to check that the solution to this problem consists of cutoffs $\underline{x}_\theta^\psi < \bar{x}_\theta^\psi$ such that the firm acquires information until X reaches either cutoffs, abandons the product if X reaches $\bar{x}_\theta^\psi$ and launches it of X reaches $\underline{x}_\theta^\psi$.

Limited liability affects incentives in two ways. First, since the firm does not fully internalize damages, it is willing to take riskier decisions than is socially optimal for a given belief about the product's safety. Second, the value of information is different. For example, if the tariff is $\psi \equiv 0$, the firm has no incentive to acquire any information. The firm always launches the product immediately.

The regulator could set a uniform tariff at the ceiling: $\psi(x) = l$ for all $x \in \mathbb{R}$. This would incentivize the firm to acquire some information but if $\beta = \pi$, this acquisition would be less than is socially optimal.

Although the uniform tariff $\psi(\cdot) \equiv l$ brings the firm closest to fully internalizing the damage that its product might cause, this tariff is suboptimal: the regulator can increase social welfare by reducing ψ to reward the firm if it acquired more information. The optimal policy is studied next.

4 Optimal Policy Design

Suppose that the regulator can contract with the firm after the firm observes the initial private information and before it takes any action, and the regulator has full commitment.

Definition 1 *A* direct liability mechanism *is a menu $M = (\{\tau_\theta, d_\theta, \psi_\theta\}_{\theta\in\Theta})$ such that for all $\theta \in \Theta$:*

(i) The stopping time τ_θ is measurable with respect to the filtration $\{\mathcal{F}_t^X\}_{t\geq 0}$ generated by X;

(ii) The decision d_θ is measurable with respect to the information at time τ, i.e., to the σ-algebra $\mathcal{F}_{\tau_\theta}^X$;

(iii) The tariff$\psi_\theta(\cdot)$ is uniformly bounded above by l.

Since the regulator has full commitment, the Revelation Principle guarantees that it is without loss of generality to focus on direct liability mechanisms.

Given a liability mechanism, the firm chooses an item $f_{\hat{\theta}} = (\tau_{\hat{\theta}}, d_{\hat{\theta}}, \psi_{\hat{\theta}})$ from the menu. Faced with the tariff $\psi = \psi_{\hat{\theta}}$, the firm chooses a stopping time and a decision to maximizes its expected utility as given by (1).

Definition 2 *A liability mechanism M is* incentive compatible *if for each $\theta \in \Theta$ it is optimal to chooses the item f_θ from M and the strategy (τ_θ, d_θ).*

Because the regulator is often unable to contract with the agent ex ante, we wish to determine when liability mechanism can be implemented by tariffs that are type independent.

Definition 3 *A direct liability mechanism is a* tariff mechanism *if the tariff $\{\psi_\theta\}_{\theta\in\Theta}$ are independent of θ.*

Theorem 1 *Any IC liability mechanism can be implemented by a tariff mechanism. Moreover, the firm's expected payoff conditional on its type is unchanged across both mechanisms.*

Proof. Consider any direct liability mechanism M and let $\underline{x}_\theta = \underline{x}_\theta^{\psi_\theta}$ and $\psi_\theta = \psi_\theta(\underline{x}_\theta)$ denote the firm's adoption threshold and penalty in case of damage that are implemented under mechanism M when the firm has type θ.

We introduce a ceiling mechanism $\tilde{M}$ as follows: for each θ, $\tilde{\psi}_\theta$ gives the maximal penalty l for all x except at $\underline{x}_\theta$, where it gives ψ_θ. The ceiling mechanism $\tilde{M}$ is IC and implements the same thresholds $\underline{x}_\theta$, because under M the firm faces the penalty only when it adopts the product and higher penalties at other levels can only reduce the incentive to deviate.

If M prescribes the same threshold $\underline{x}$ to types $\theta \neq \theta'$, the penalties ψ_θ and ψ'_θ must be identical. Otherwise, one type would want to misreport its type and M would not be incentive compatible.

We define the tariff ψ as follows:

$$\psi(\underline{x}_\theta) = \psi_\theta$$

for all $\theta \in \Theta$ and

$$\psi(x) = l$$

otherwise.

This tariff is type independent. Moreover, it implements the same adoption thresholds as M, as is easily checked. ∎

Theorem 1 shows that any liability mechanism can be implemented by a tariff. The next result shows that the adoption thresholds must be decreasing in θ. We need to show that action is identifiable for the optimal mechanism. We only consider stationary actions (there are no profitable deviations to non-stationary actions since the liability function is stationary too). An action can be characterized by the adoption and abandonment thresholds.

5 Taxation Principle with Identifiable Information Acquisition

When an IC mechanism implements distinct thresholds for distinct types, the conclusion of Theorem 1 is a corollary of the Taxation Principle with Non-Contractible Events of our companion paper (Poggi and Strulovici (2020)).

According to that paper, a mechanism is *identifiable* if satisfies two conditions that we translate into the present setting. Let A denote the set of all possible strategies by the firm. Each element of A consists of a pair (τ, d), where τ is a stopping time adapted to the filtration of X and d is measurable with respect to $\mathcal{F}_\tau^X$. For any subset A' of A, let $X(A')$ denote the set of observable outcomes by the regulator if the firm chooses an action $a \in A'$ and causes some damage.

Definition 4 *An IC mechanism M is identifiable if there exists a partition $\mathcal{A} = \{A_k\}_{k=1}^K$ of A such that*

(i) $X(A_k) \cap X(A_{k'}) = \emptyset$ for all $k \neq k'$.

(ii) All types θ who choose an action in A_k under the mechanism choose the same action of A_k.

Proposition 1 *If M implements distinct adoption thresholds for all types, then it is identifiable.*

Proof. For each θ, let A_θ denote the set of firm strategies that use adoption threshold $\underline{x}_\theta$, and let $A_0 = A \setminus (\cup_{\theta \in \Theta} A_\theta)$. By assumption on M, $\underline{x}_\theta \neq \underline{x}_{\theta'}$ for all $\theta \neq \theta'$. Therefore, A_θ and A'_θ are disjoint for all $\theta \neq \theta'$ and $\mathcal{A} = \{A_0, A_\theta : \theta \in \Theta\}$ forms a partition of A. Condition (ii) is trivially satisfied since for each cell of $\mathcal{A}$ there is at most one type taking action in that cell. Moreover Condition (i) is also satisfied by construction of the partition: $X(A_\theta) = \{\underline{x}_\theta\}$ for all $\theta \in \Theta$ and, hence, $X(A_\theta) \cap X(A_{\theta'}) = \emptyset$ for all $\theta \neq \theta'$. ■

Corollary 1 *If an IC mechanism M implements distinct adoption threshold for all types, it can be implemented by a tariff mechanism.*

Proof. Proposition 1 implies that M is identifiable. The result then immediately follows from Theorem 1 in Poggi and Strulovici (2020) ■