Problem Set 4

Advanced Microeconomics III

Spring 2022

Problem 1 Based on MWG 14.B.3

Consider a variant of the moral hazard model introduced in class. The firm owner is risk neutral. The manager can choose any effort level $e \geq 0$. He has preferences defined over the mean and the variance of his income w and his effort level e as follows: Expected utility $= E[w] - \phi \cdot \text{Var}[w] - g(e)$, where $\phi > 0$ is a fixed parameter. We assume that g is three times continuously differentiable, $g(0) = 0$, $g'(0) = 0$, $g'(e) > 0$, $g''(e) > 0$, and $g'''(e) > 0$ for all $e > 0$.

Conditional on effort e, the profit is distributed with mean e and variance σ^2. Restrict attention to linear compensation schemes $w(\pi) = \alpha + \beta\pi$.

a. Derive a formula for the manager's expected utility from any contract $(e, w(\cdot))$.

b. Derive equations that characterize the contracts on the Pareto frontier (i.e., assume that e is verifiable).

c. Derive equations that characterize the contracts on the constrained Pareto frontier (i.e., assume that e is non-verifiable).

d. Show that, for all contracts on the constrained Pareto frontier, $e > 0$ and $\beta < 1$.

e. Show that, for all contracts on the constrained Pareto frontier, β is strictly decreasing in $\phi\sigma^2$. Interpret.

Problem 2 MWG 14.B.4

Consider the following hidden action model with three possible actions $E = \{e_1, e_2, e_3\}$. There are two possible profit outcomes: $\pi_H = 10$ and $\pi_L = 0$, The probabilities of π_H conditional on the three effort levels are $f(\pi_H|e_1) = 2/3$, $f(\pi_H|e_2) = 1/2$, and $f(\pi_H|e_3) = 1/3$. The agent's cost of effort is $c(e_1) = 5/3$, $c(e_2) = 8/5$, $c(e_3) = 4/3$. Finally, $u(w) = \sqrt{w}$, and the agent's reservation utility is $\bar{u} = 0$.

a. What is the optimal contract when effort is observable?

b. Show that if effort is not observable, then e_2 is not implementable. For what levels of $c(e_2)$ would e_2 be implementable. Hint: focus on the utilitylevels the agent would get with the two outcomes, rather than on the wage payments themselvels.

c. What is the optimal contract when effort is not observable?