Benchmark 2020: Weight minimisation of a speed reducer

Constrained design optimisation of a mechanical part

A. Andrade-Campos, J. Dias-de-Oliveira

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Abstract Today, mechanical design must be performed efficiently. Therefore, optimisation procedures must be applied when designing new mechanical parts. In this challenge, the weight of a speed reducer must be minimised subject to the part's mechanical integrity and reliability. Therefore, constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shafts must be taken into account. This design problem involves seven design variables that characterise the geometry and mechanic characteristics of the part. Integer and continuous variables must be found, making this benchmark a challenge for all optimisation techniques. This design problem is a constrained optimisation problem.

Palavras-Chave Mechanical design · Weight minimisation · Non-linear optimisation · constrained optimisation · Benchmark

1 Introduction

Most engineering design problems are formulated as mathematical programming models. In the last decades, these nonlinear engineering problems have been investigated by different methods. To compare the performance of different optimisation algorithms, several structural engineering applications are often solved to validate or test the suitability of the optimisation algorithms. The speed reducer problem is one of the benchmark problems in structural optimisation. The problem represents the design of a simple gear box used in a light airplane between the engine and propeller to allow each to rotate at its most efficient speed [1].

A speed reducer is simply a gear train between the motor and the machinery that is used to reduce the speed with which power is transmitted. Speed reducers, also called gear reducers, are mechanical gadgets by and large utilised for two purposes. First, they take the torque created by the power source (the input) and multiply it. Second, speed reducers, much as the name implies, reduce the speed of the input so that the output is the correct speed. In other words, gear reducers essential use is to duplicate the measure of torque produced by an information power source to expand the measure of usable work.

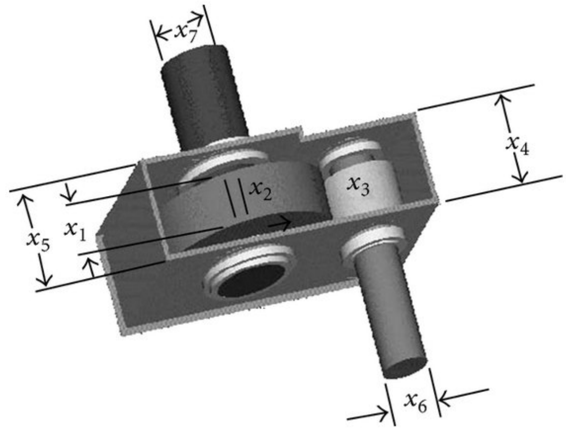

Figure 1. Speed reducer design [1].

The weight of a speed reducer is to be minimised subjected to constraints on levels of bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shafts [2]. This design

A. Andrade-Campos, J. Dias-de-Oliveira
Non-Linear Optimisation in Engineering
Year 2020/21
E-mail: gilac@ua.pt; jalex@ua.pt

problem involves seven design variables, as shown in Figure 2, which are the face width, x_1, module of the teeth, x_2, number of teeth on the pinion, x_3, length of the first shaft between bearings, x_4, length of the second shaft between bearings, x_5, diameter of the first shaft, x_6, and diameter of the second shaft, x_7. The third variable, x_3, is an integer, while the rest are continuous. With eleven constraints, this is a constrained optimisation problem.

2 Formulation

The problem is formulated as a constrained nonlinear mathematical programming to minimise the objective function $f(\mathbf{x})$, subjected to the inequality constraints $g_j(\mathbf{x})$, with $j = 1,2,...,11$, stated as

$$\begin{aligned} \text{minimize} \quad f(\mathbf{x}) &= 0.7854 x_1 x_2^2 \\ &\times \left(3.3333 x_3^2 + 14.9334 x_3 - 43.0934\right) \\ &- 1.508 x_1 \left(x_6^2 + x_7^2\right) \\ &+ 0.7854 \left(x_4 x_6^2 + x_5 x_7^2\right) , \end{aligned} \tag{1}$$

$$\text{subj. to} \quad g_1(\mathbf{x}) = \frac{27}{x_1 x_2^2 x_3} - 1 \le 0 , \tag{2}$$

$$g_2(\mathbf{x}) = \frac{397.5}{x_1 x_2^2 x_3^2} - 1 \le 0 , \tag{3}$$

$$g_3(\mathbf{x}) = \frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \le 0 , \tag{4}$$

$$g_4(\mathbf{x}) = \frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \le 0 , \tag{5}$$

$$g_5(\mathbf{x}) = \frac{\sqrt{\left(\frac{745 x_4}{x_2 x_3}\right)^2 + 16.9\text{E}6}}{110 x_6^3} - 1 \le 0 , \tag{6}$$

$$g_6(\mathbf{x}) = \frac{\sqrt{\left(\frac{745 x_5}{x_2 x_3}\right)^2 + 157.5\text{E}6}}{85 x_7^3} - 1 \le 0 , \tag{7}$$

$$g_7(\mathbf{x}) = \frac{x_2 x_3}{40} - 1 \le 0 , \tag{8}$$

$$g_8(\mathbf{x}) = \frac{5 x_2}{x_1} - 1 \le 0 , \tag{9}$$

$$g_9(\mathbf{x}) = \frac{x_1}{12 x_2} - 1 \le 0 , \tag{10}$$

$$g_{10}(\mathbf{x}) = \frac{1.5 x_6 + 1.9}{x_4} - 1 \le 0 , \tag{11}$$

$$g_{11}(\mathbf{x}) = \frac{1.1 x_7 + 1.9}{x_5} - 1 \le 0 , \tag{12}$$

and

$$\begin{aligned} &2.6 \le x_1 \le 3.6, \\ &0.7 \le x_2 \le 0.8, \\ &17 \le x_3 \le 28.0, \\ &7.3 \le x_4 \le 8.3, \\ &7.8 \le x_5 \le 8.3, \\ &2.9 \le x_6 \le 3.9 \text{ and} \\ &5.0 \le x_7 \le 5.5 . \end{aligned}$$

3 Constraints as external penalties

This constrained problem can be transformed into an unconstrained problem using a penalty function. The method of the exterior penalty function is a simple and common approach to handle constraints. The idea behind this method is to transform a constrained optimisation problem into an unconstrained problem, by adding (or subtracting) a penalty function P to the objective function. Solutions that violate the constraints are penalised and the problem is defined as

$$\text{minimize} \quad F(\mathbf{x},r_\text{h},r_\text{g}) = f(\mathbf{x}) + P(\mathbf{x},r_\text{h},r_\text{g}) , \tag{13}$$

$$\text{subj. to} \quad x_i^{\min} \le x_i \le x_i^{\max} , \quad i = 1,2,...,D , \tag{14}$$

where r_h and r_g are penalty factors and F designates the augmented objective function. A general formulation of the exterior penalty function is defined as

$$\begin{aligned} P(\mathbf{x},r_\text{h},r_\text{g}) = r_\text{h} &\left[\sum_{k=1}^{l} [h_j(\mathbf{x})]^\gamma\right] \\ &+ r_\text{g} \left[\sum_{j=1}^{m} [\max\{0,g_j(\mathbf{x})\}]^\beta\right] . \end{aligned} \tag{15}$$

where, γ and β are positive penalty constants. The penalty function P is non-existent when the constraint functions h and g are not active.

Furthermore, an exterior penalty function can be classified as dynamic when the current iteration number is associated with the corresponding penalty factors, normally defined in such a way that the value of the penalty function increases over time. Joines and Houck [4] proposed a dynamic penalty method in which individuals are evaluated at iteration i as

$$F(\mathbf{x}) = f(\mathbf{x}) + (C\, i)^\alpha \text{SVC}(\mathbf{x},\beta) , \tag{16}$$

where C, α and β are pre-defined constants. $\text{SVC}(\mathbf{x},\beta)$ is defined as

$$\text{SVC}(\mathbf{x},\beta) = \sum_{k=1}^{l} H_k(\mathbf{x}) + \sum_{j=1}^{m} G_j^\beta(\mathbf{x}) , \tag{17}$$

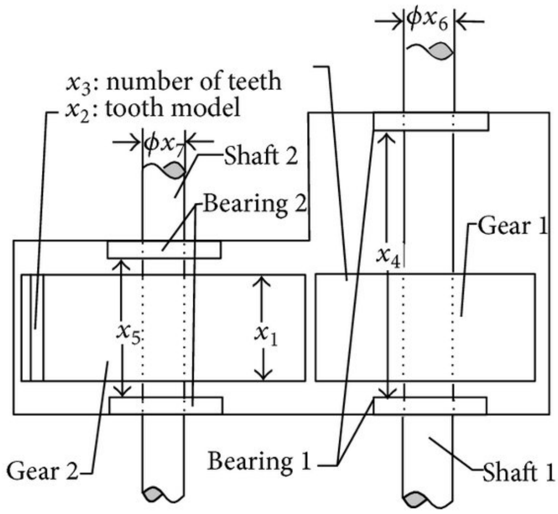

Figure 2. Illustrative representation of the speed reducer design geometry [3].

where functions H_k and G_j are respectively defined as

$$H_k(\mathbf{x}) = \begin{cases} 0 & \text{if } -\epsilon \leq h_k(\mathbf{x}) \leq \epsilon \\ |h_k(\mathbf{x})| & \text{otherwise} \end{cases} \quad \text{and} \tag{18}$$

$$G_j(\mathbf{x}) = \begin{cases} 0 & \text{if } g_j(\mathbf{x}) \leq 0 \\ |g_j(\mathbf{x})| & \text{otherwise} \end{cases}. \tag{19}$$

In this approach, equality constraints are transformed into inequality constraints, where ϵ is the allowed tolerance (normally a very small value). These methods are simple and easy to implement in the advanced optimisation methods. However, a disadvantage is related to the necessity of tuning the parameters depending on the optimisation problem. This problem cannot evaluate and take into account unfeasible solutions (which are not desired). For some optimisation techniques, such as metaheuristics, in order to handle the inequality constraints presented in the design problem, a dynamic penalty function can be used with the parameters $C = 60$, $\alpha = 2$ and $\beta = 1$.

4 Results

As the problem is solved by many authors [5–7] using different algorithms, the best reported result is $f(\mathbf{x}) =$ 2996.348165 located at $\mathbf{x} =$ [3.499999, 0.7, 17, 7.3, 7.8, 3.350215, 5.286683]. The reported result serves as a reference for the global optimum in the analysis of results. The selected number of function evaluations as a stopping criterion is 10^5.

5 Delivery Instructions

Assessment elements for this *benchmark* must follow the format provided in the respective template (English or Portuguese versions).

References

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