Transformada de Laplace.

Exercícios resolvidos.

Exercício 1. Determinar a Transformada de Laplace

(a) $f(t) = 6\mathrm{e}^{-5t} + \mathrm{e}^{3t} + 5t^3 - 9$

(b) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

(c) $h(t) = 3\sinh(2t) + 3\sin(2t)$

(d) $g(t) = \mathrm{e}^{3t} + \cos(6t) - \mathrm{e}^{3t}\cos(6t)$

Resolução (a)

$$\begin{aligned} F(s) &= 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s} \\ &= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s} \end{aligned}$$

Resolução (b)

$$\begin{aligned} G(s) &= 4\frac{s}{s^2+(4)^2} - 9\frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)^2} \\ &= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100} \end{aligned}$$

Resolução (c)

$$\begin{aligned} H(s) &= 3\frac{2}{s^2-(2)^2} + 3\frac{2}{s^2+(2)^2} \\ &= \frac{6}{s^2-4} + \frac{6}{s^2+4} \end{aligned}$$

Resolução (d)

$$\begin{aligned} G(s) &= \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2} \\ &= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36} \end{aligned}$$

Exercício 2. Determinar a Transformada de Laplace

(a) $f(t) = t\cosh(3t)$

(b) $h(t) = t^2\sin(2t)$

Solução (a)

$$F(s) = \frac{s^2+9}{(s^2-9)^2}$$

Solução (b)

$$H(s) = \frac{12s^2-16}{(s^2+4)^3}$$

Exercício 3. Determinar $\mathcal{L}^{-1}$ (inversa)

(a) $F(s) = \dfrac{6}{s} - \dfrac{1}{s-8} + \dfrac{4}{s-3}$

(b) $H(s) = \dfrac{19}{s+2} - \dfrac{1}{3s-5} + \dfrac{7}{s^5}$

(c) $F(s) = \dfrac{6s}{s^2+25} + \dfrac{3}{s^2+25}$

(d) $G(s) = \dfrac{8}{3s^2+12} + \dfrac{3}{s^2-49}$

Resolução (a)

$$\begin{aligned} F(s) &= 6\ \frac{1}{s} - \frac{1}{s-8} + 4\frac{1}{s-3} \\ f(t) &= 6(1) - \mathrm{e}^{8t} + 4\left(\mathrm{e}^{3t}\right) \\ &= 6 - \mathrm{e}^{8t} + 4\mathrm{e}^{3t} \end{aligned}$$

Resolução (b)

$$\begin{aligned} H(s) &= \frac{19}{s-(-2)} - \frac{1}{3\left(s-\frac{5}{3}\right)} + \frac{7\frac{4!}{4!}}{s^{4+1}} \\ &= 19\frac{1}{s-(-2)} - \frac{1}{3}\frac{1}{s-\frac{5}{3}} + \frac{7}{4!}\frac{4!}{s^{4+1}} \end{aligned}$$

$$h(t) = 19\mathrm{e}^{-2t} - \frac{1}{3}\mathrm{e}^{\frac{5t}{3}} + \frac{7}{24}t^4$$

Resolução (c)

$$\begin{aligned} F(s) &= 6\frac{s}{s^2+(5)^2} + \frac{3\frac{5}{5}}{s^2+(5)^2} \\ &= 6\frac{s}{s^2+(5)^2} + \frac{3}{5}\frac{5}{s^2+(5)^2} \end{aligned}$$

$$f(t) = 6\cos(5t) + \frac{3}{5}\sin(5t)$$

Resolução (d)

$$\begin{aligned} G(s) &= \frac{1}{3}\frac{8}{s^2+4} + \frac{3}{s^2-49} \\ &= \frac{1}{3}\frac{(4)(2)}{s^2+(2)^2} + \frac{3\frac{7}{7}}{s^2-(7)^2} \end{aligned}$$

$$g(t) = \frac{4}{3}\sin(2t) + \frac{3}{7}\sinh(7t)$$