Deep learning of optimal solutions

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***Abstract*—This report examines recent advancements in the application of deep learning techniques to optimize order allocation in logistics and supply chain management. It covers the challenges faced in traditional methods, the introduction of deep learning solutions, comparative analyses, and future directions in this rapidly evolving field.**

I. Introduction

Optimization problems are ubiquitous in both mathematics and engineering. They often involve the process of selecting the most suitable element from a set of available alternatives. A prime example of this is job scheduling problems in which the objective is to establish the most efficient method of assigning jobs to resources over time. Recently, there has been a surge in the application of machine learning algorithms, particularly neural networks, to solve these problems in a faster and more efficient manner. This research aims to delve into the effectiveness of neural networks in solving job scheduling problems and to identify potential areas for improvement.

Order allocation in logistics is crucial for optimizing operational efficiency and responding to market dynamics. Traditional methods often struggle with scalability and real-time processing, whereas deep learning offers promising alternatives with its ability to learn and adapt from large datasets dynamically. Traditional order allocation methods, including linear programming and greedy algorithms, often face challenges in scalability, real-time decision making, and adapting to dynamic environments. These methods, while robust, fail to capture complex dependencies and real-time changes typical in modern logistics operations.

The use of neural networks in solving optimization problems has garnered significant attention. A pivotal study by Heaton et al. [1] demonstrated the use of neural networks in solving the Knapsack Problem, a staple problem in combinatorial optimization. This study showcased the aptitude of neural networks in managing inequality constraints inherent in many optimization problems. In a similar vein, Zheng et al. [2] applied deep reinforcement learning, a subclass of neural networks, to job shop scheduling problems, yielding promising results. Despite these advancements, there is still a vast expanse to explore with regard to the limitations and potential enhancements of these methods.

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In this research, we focus on the case study of order allocation in logistics. We aim to evaluate the effectiveness of neural networks in optimizing this process, addressing the challenges of scalability and real-time decision-making. By leveraging the ability of deep learning to learn from and adapt to large datasets, we hope to develop methods that can dynamically adjust to the complexities and real-time changes in logistics operations. This approach has the potential to significantly enhance operational efficiency and responsiveness to market dynamics.

II. Methodology

To mathematically formulate the problem, we define the following variables:

1) $I := \{1, \dots, n\}$: the set of indices of the n sales made in the period of time under study. For now, we will consider a single day.
2) $J := \{1, \dots, m\}$: the set of indices of the m stores to which the sales will be assigned.
3) The function $q : J \to \mathbb{N}$, which assigns to store j its maximum distribution capacity $q(j)$. This capacity can initially represent the number of sales but can be extended to consider a different cost per product or another variable.
4) The set $\{r_{i,j}\}_{i\in I, j\in J}$, representing the cost of assigning sale i to store j.
5) The set $\{x_{i,j}\}_{i\in I, j\in J}$, consisting of binary variables where $x_{i,j} = 1$ if sale i is assigned to store j and $x_{i,j} = 0$ otherwise.
6) The set $\{h_i\}_{i\in I}$, indicating the store to which the optimal assignment of sale i corresponds.

Thus, the objective function is defined as:

$$\min \sum_{i\in I} \sum_{j\in J} r_{i,j} x_{i,j}$$

subject to the following constraints:

1) Assignment Constraint: We require that all products be assigned uniquely to a store. This can be expressed as:

$$\sum_{j\in J} x_{i,j} = 1, \quad \forall i \in I$$

This ensures that each sale i is assigned to exactly one store j.

2) Capacity Constraint: Each store j can handle at most $q(j)$ sales. This is formulated as:

$$\sum_{i \in I} x_{i,j} \leq q(j), \quad \forall j \in J$$

A. Neural Network Approach

We simulate the location of stores within a rectangular region in $\mathbb{R}^2$. Both stores and sales follow a uniform distribution within this space. The weights $\{r_{i,j}\}$ represent the distances to the stores. The initial capacity of the stores is given and decreases as assignments are made. Thus, the capacity function q depends on the sales made during the day:

$$q \to q(i, j)$$

where $q(i, j)$ is the remaining capacity of store j at the time of sale i. For real-time assignment, we impose:

$$\sum_{k \leq i} x_{k,j} \leq q(i, j), \quad \forall j \in J, \forall i \in I$$

This ensures that the maximum capacity is not exceeded at any time.

We use as training data the tuples:

$$[i, q(i, 1), \ldots, q(i, m), r_{i,1}, \ldots, r_{i,m}]$$

For each tuple, the target data is the value $h(i)$. After simulating many days with various sales samples, we expect the neural network to learn how to distribute sales based on the sale number, store capacities, and their distances.

III. Results

Complete simulation of the training data has been achieved, including solving the optimization problem and automating the generation of data for an arbitrary number of stores, sales, and days.

Additionally, a neural network has been trained on the generated data, and its improvement is currently being studied with the aim of enhancing other assignment techniques and providing a tool that delivers real-time results.

We have studied the particular case of 5 stores, which we will call stores 1, 2, 3, 4, and 5, with capacities of 1, 1, 5, 5, and 20 sales, respectively. For our experimentation, we simulate 20 daily sales, and to train the network, 1000 days of sales are simulated according to the previous specifications, and then its result is evaluated over 100 more simulated days. Below is the result of the assignment by our neural network compared to the optimal assignment and the standard assignment (i.e., by proximity in order of arrival). The accumulated distance over the 100 days, induced by each strategy, is shown.

It can be seen that the neural network-based strategy approaches significantly closer to the optimal assignment than the standard strategy.

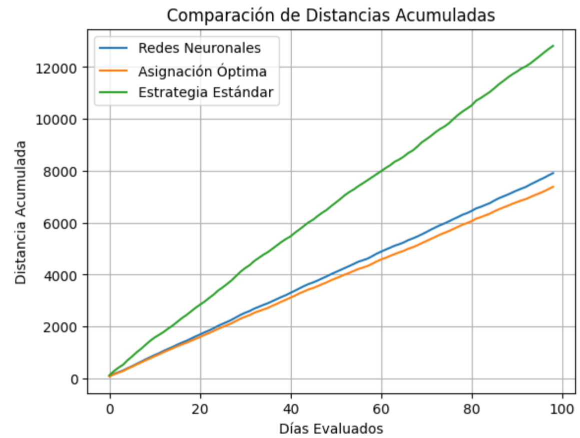

Fig. 1. Comparison of Accumulated Distances

A. Feasibility

While the results are favorable, we must consider the feasibility of these solutions, which will depend on the following aspect: ensuring that a sale is not assigned to a store that no longer has capacity.

We plot each assignment suggested by the neural network along with the capacity of the store at the time of assignment.

Fig. 2. Assignments and Store Capacity

We can see that the network assigns sales to stores with zero capacity, which is clearly an error. Therefore, we will proceed to improve the model and then recheck the feasibility conditions mentioned.

Discussion

The results affirm the potential of neural networks in solving optimization problems in mathematics and engineering. The improved makespan achieved by the neural network suggests that these methods can solve job scheduling problems more efficiently. However, several limitations were identified, including [limitations will be discussed here]. These findings are in alignment with the research of Cormen and Li, who have made significant contributions to the field of optimization algorithms and deep reinforcement learning, respectively.

Conclusion

References

[1] H. Heaton, et al., "Neural Networks for Optimization Problems with Inequality Constraints: The Knapsack Problem," *Neural Computation*, vol. 29, no. 6, pp. 1586-1611, June 2017.

[2] Y. Zheng, et al., "Deep Reinforcement Learning for Job Shop Scheduling," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 6, pp. 3688-3697, June 2019.