Lighting experiment

S1. Simulate lighting data

```
sim.light <- function(T,lambda=0.8,p=0.9){
  wt = NULL
  tau = NULL
  obs = NULL
  ot=0
  i = 1
  j=1
  while(sum(wt)<T){
   wt[i] = rexp(1,rate = lambda)
   tau[i] = rbinom(1,1,prob=p)
   ot = ot+wt[i]
   if(tau[i]==1){
     obs[j] = ot
     ot = 0
     j=j+1
   }
   i=i+1
  }
  d = length(wt)
  wt = wt[-d]
  tau = tau[-d]
  #wt[d] = T-sum(wt)
  return(list(wt=wt,tau=tau,obs=obs))
}


l1 = sim.light(50,p=0.5)
l1
```

```
## $wt
##  [1] 2.206451306 0.367788922 0.101966173 0.482614873 1.785627785
##  [6] 0.222050376 2.570898195 1.170871674 0.429231129 0.244562409
## [11] 0.139204919 0.206429206 1.572231692 0.297160343 0.304670367
## [16] 1.318134605 0.068707094 1.124237005 0.329999247 1.596757624
## [21] 0.828393651 0.668119699 0.810574846 0.404147382 0.020293489
## [26] 3.054059988 0.482124898 0.408303872 1.060689776 1.333827665
## [31] 0.240755125 0.229184814 0.635270178 0.433138168 1.332392277
## [36] 2.108057886 0.399651779 0.121870954 0.002316922 0.692665846
## [41] 1.610636307 0.146995350 1.472126236 0.854715092 1.078472106
## [46] 0.924245517 3.067073936 2.753709260 0.045907864 0.383842516
## [51] 1.379137161 0.392880117 1.460790302 0.547603064 0.611021367
## [56] 0.171383132 1.207085211
##
## $tau
##  [1] 1 1 1 1 1 0 0 0 0 1 1 0 0 0 1 0 0 1 0 0 1 0 0 1 0 1 0 0 0 1 1 1 0 0 1
## [36] 1 0 1 1 1 0 1 1 0 0 0 0 0 0 0 1 1 1 0 1 0 1
##
## $obs
```

```
##  [1]  2.206451306  0.367788922  0.101966173  0.482614873  1.785627785
##  [6]  4.637613784  0.139204919  2.380491608  2.511078704  2.755150522
## [11]  1.882841926  3.074353477  3.284946210  0.240755125  0.229184814
## [16]  2.400800623  2.108057886  0.521522732  0.002316922  0.692665846
## [21]  1.757631658  1.472126236 10.487103453  0.392880117  1.460790302
## [26]  1.158624432  1.378468343
```

S.1.1 Calculate MLE

```
llik.light <- function(light,par){
  lambda = par[1]
  p = par[2]
  exp = dexp(x=light$wt,rate=lambda,log=T)
  bin = dbinom(x=sum(light$tau),size=length(light$tau),prob=p,log=TRUE)
  llik = sum(exp)+bin
  if(p>1 | p<0){
    llik = -Inf
  }
  return(llik)
}
mle.light <- function(light,init_par = c(2,1)){
  p = sum(light$tau)/length(light$tau)
  lambda = 1/mean(light$wt)
  #optim(par=c(0.5,0.5),fn=llik.light,lower=c(0,0),light=light, upper = c(5,1),method="L-BFGS-B")
  return(list(lambda=lambda,p=p))
}
l1
```

```
## $wt
##  [1] 2.206451306 0.367788922 0.101966173 0.482614873 1.785627785
##  [6] 0.222050376 2.570898195 1.170871674 0.429231129 0.244562409
## [11] 0.139204919 0.206429206 1.572231692 0.297160343 0.304670367
## [16] 1.318134605 0.068707094 1.124237005 0.329999247 1.596757624
## [21] 0.828393651 0.668119699 0.810574846 0.404147382 0.020293489
## [26] 3.054059988 0.482124898 0.408303872 1.060689776 1.333827665
## [31] 0.240755125 0.229184814 0.635270178 0.433138168 1.332392277
## [36] 2.108057886 0.399651779 0.121870954 0.002316922 0.692665846
## [41] 1.610636307 0.146995350 1.472126236 0.854715092 1.078472106
## [46] 0.924245517 3.067073936 2.753709260 0.045907864 0.383842516
## [51] 1.379137161 0.392880117 1.460790302 0.547603064 0.611021367
## [56] 0.171383132 1.207085211
##
## $tau
##  [1] 1 1 1 1 1 0 0 0 0 1 1 0 0 0 1 0 0 1 0 0 1 0 0 1 0 1 0 0 0 1 1 1 0 0 1
## [36] 1 0 1 1 1 0 1 1 0 0 0 0 0 0 0 1 1 1 0 1 0 1
##
## $obs
##  [1]  2.206451306  0.367788922  0.101966173  0.482614873  1.785627785
##  [6]  4.637613784  0.139204919  2.380491608  2.511078704  2.755150522
## [11]  1.882841926  3.074353477  3.284946210  0.240755125  0.229184814
## [16]  2.400800623  2.108057886  0.521522732  0.002316922  0.692665846
## [21]  1.757631658  1.472126236 10.487103453  0.392880117  1.460790302
## [26]  1.158624432  1.378468343
```

```
mle.light(l1,init_par=c(1,0.5))
```

```
## $lambda
## [1] 1.141986
##
## $p
## [1] 0.4736842
```

```
n=100
L = vector(mode='numeric',length = n)
P = vector(mode='numeric',length = n)
for(i in 1:n){
  li = sim.light(T=50)
  ml = mle.light(light = li)
  L[i] = ml$lambda
  P[i] = ml$p
}

summary(L)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.5022  0.7373  0.8248  0.8246  0.9236  1.0853
```

```
summary(P)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.7692  0.8684  0.9072  0.8966  0.9290  1.0000
```

Incomplete data

Option 1. The current way

```
rec.light <- function(obslight,lambda,p){
  dt = obslight
  for(i in 1:length(dt)){
    t = dt[i]

  }
}
```

Option 2. topoloy first

```
sim.miss.light <- function(obslight,lambda,p){
  dt = obslight
  j = 1
  wt = NULL
  top = NULL
  error = NULL
  timesuggestion = NULL
  for(i in 1:length(dt)){

    to = m = 0
```

```
    while(to == 0){
      to = rbinom(1,1,p)
      top[j] = to
      j = j+1
      if(to == 0) m <- m+1
    }
    exps = rexp(n=m+1,rate=lambda)
    swt = sum(exps)
    error[i] = abs(obslight[i]-swt)/(m+1)
    timesuggestion[i] = obslight[i]<swt
    wt = c(wt,exps)
  }
  light = list(wt=wt,tau=top,obs=obslight,error=error,errorsum=summary(error),timesuggestion=timesugges
  return(light)
}


s = sim.light(50,p=0.5)
s
```

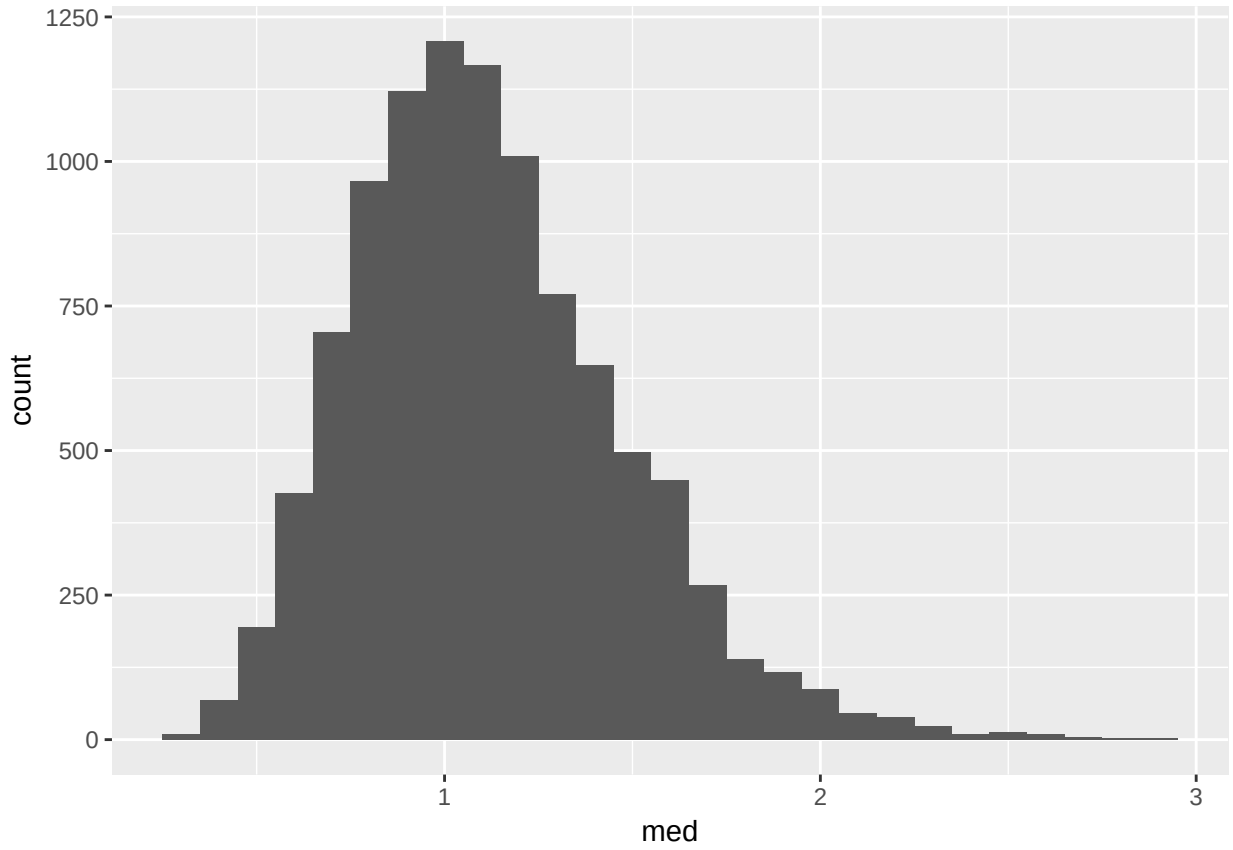
```
## $wt
##  [1] 1.082762355 0.989844159 0.542890438 3.587564807 1.648011603
##  [6] 8.337791541 1.354081789 0.190001657 0.058162162 2.855117826
## [11] 3.366442018 0.006382724 0.801135454 4.032703554 2.224865407
## [16] 0.138800474 0.019147089 1.075203755 0.732778091 4.049817854
## [21] 2.896841375 1.139381702 1.620182303 1.760130394 1.178864938
## [26] 1.223842341
##
## $tau
##  [1] 1 1 0 1 1 0 0 0 0 1 0 1 1 1 1 0 0 1 0 1 1 0 1 1 0 0
##
## $obs
##  [1]  1.0827624  0.9898442  4.1304552  1.6480116 12.7951550  3.3728247
##  [7]  0.8011355  4.0327036  2.2248654  1.2331513  4.7825959  2.8968414
## [13]  2.7595640  1.7601304  7.5334539
```

```
rec = sim.miss.light(obslight = s$obs,lambda=0.8,p=0.5)
rec
```

```
## $wt
##  [1] 1.57779395 0.48661057 2.37632004 1.70219882 0.66449682 0.73007667
##  [7] 0.21325840 1.54800561 0.77047797 0.47965719 0.19668770 0.32088413
## [13] 0.16820128 0.02438104 0.37308712 0.46920808 0.16400124 0.94338449
## [19] 1.46946192 0.29735631 2.06966496 0.50808386 1.47277673 2.84149870
## [25] 2.75180697 0.06576045 0.86129833 2.02023795 2.03045707 3.97196907
## [31] 2.75336689 0.36176215 0.20491602 0.71643363
##
## $tau
##  [1] 1 1 0 0 1 1 0 0 0 1 0 1 1 1 0 0 1 0 0 0 0 0 1 1 0 1 1 0 1 0 0 0 0 1
##
## $obs
##  [1]  1.0827624  0.9898442  4.1304552  1.6480116 12.7951550  3.3728247
##  [7]  0.8011355  4.0327036  2.2248654  1.2331513  4.7825959  2.8968414
## [13]  2.7595640  1.7601304  7.5334539
##
```

```
## $error
##  [1] 0.49503159 0.50323359 0.20418681 0.91793493 2.44593895 1.42762645
##  [7] 0.63293417 4.00832252 0.40618966 0.92126282 1.94109724 0.03963698
## [13] 1.89826567 1.14528231 0.09499876
##
## $errorsum
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.03964 0.45061 0.91793 1.13880 1.66295 4.00832
##
## $timesuggestion
##  [1]  TRUE FALSE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE
## [12] FALSE FALSE  TRUE  TRUE
```

```
n=10000
min = vector(mode='numeric',length=n)
max = vector(mode='numeric',length=n)
med = vector(mode='numeric',length=n)
lambda = vector(mode='numeric',length=n)
p = vector(mode='numeric',length=n)
for(i in 1:n){
  rec = sim.miss.light(obslight = s$obs,lambda=0.8,p=0.5)
  min[i] = rec$errorsum[1]
  med[i] = rec$errorsum[3]
  max[i] = rec$errorsum[6]
  mle = mle.light(light = rec)
  lambda[i] = mle$lambda
  p[i] = mle$p
}
library(ggplot2)
qplot(med,geom='histogram',binwidth=0.1)
```




```
summary(lambda)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.4210  0.7164  0.8102  0.8291  0.9173  1.9815
```

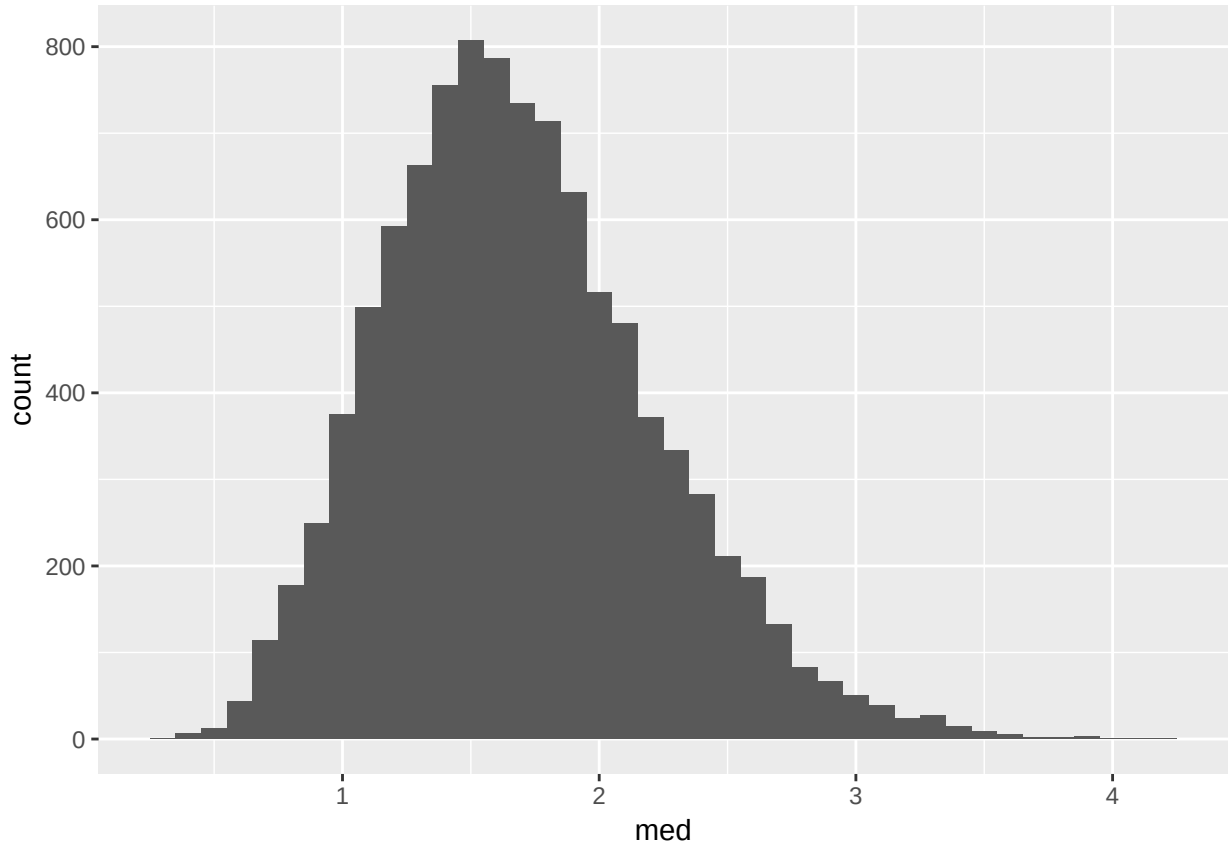
```
summary(p)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.2830  0.4545  0.5172  0.5169  0.5769  0.9375
```

```
###


n=10000
min = vector(mode='numeric',length=n)
max = vector(mode='numeric',length=n)
med = vector(mode='numeric',length=n)
lambda = vector(mode='numeric',length=n)
p = vector(mode='numeric',length=n)
for(i in 1:n){
  rec = sim.miss.light(obslight = s$obs,lambda=0.4,p=0.5)
  min[i] = rec$errorsum[1]
  med[i] = rec$errorsum[3]
  max[i] = rec$errorsum[6]
  mle = mle.light(light = rec)
  lambda[i] = mle$lambda
  p[i] = mle$p
}
```

```
library(ggplot2)
qplot(med,geom='histogram',binwidth=0.1)
```




```
summary(lambda)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.1822  0.3583  0.4044  0.4145  0.4611  0.9925
```

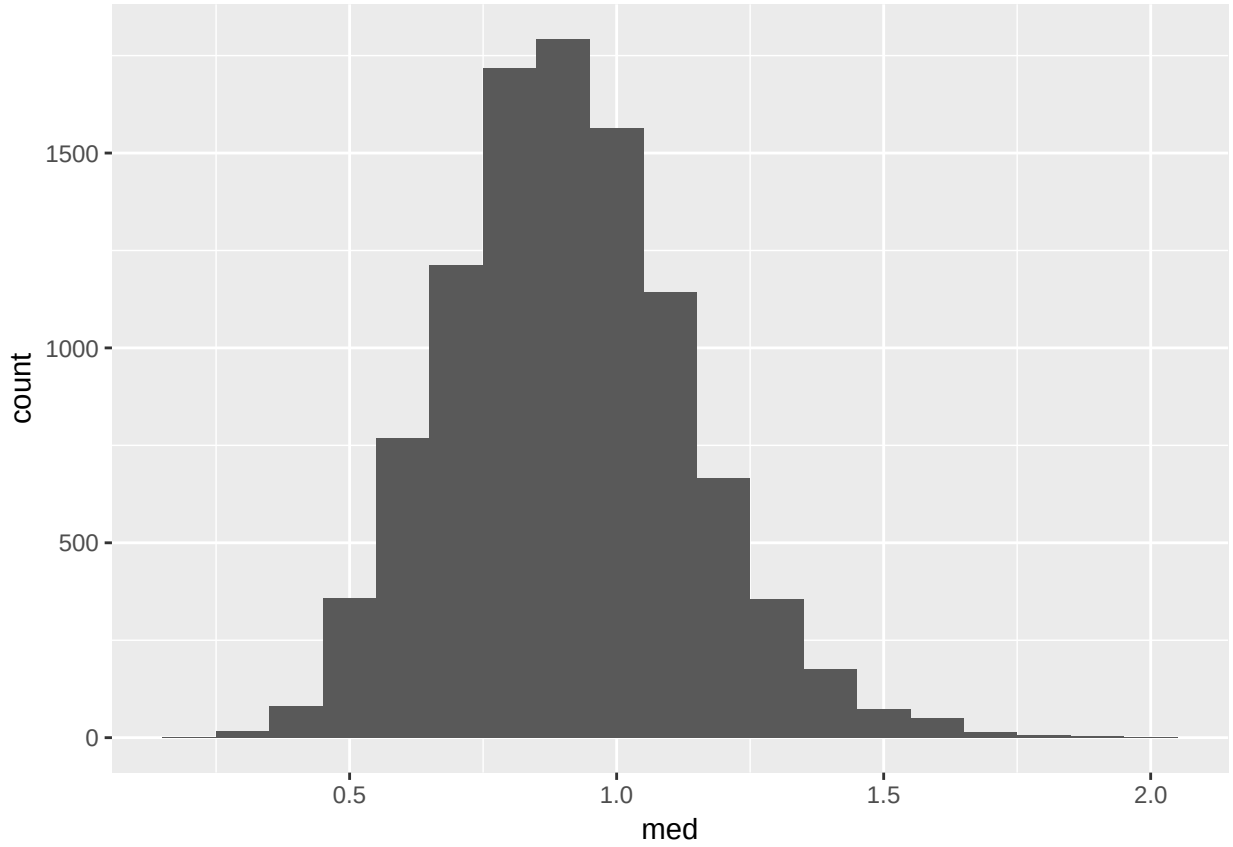
```
summary(p)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.2542  0.4545  0.5172  0.5187  0.5769  1.0000
```

```
###


n=10000
min = vector(mode='numeric',length=n)
max = vector(mode='numeric',length=n)
med = vector(mode='numeric',length=n)
lambda = vector(mode='numeric',length=n)
p = vector(mode='numeric',length=n)
for(i in 1:n){
  rec = sim.miss.light(obslight = s$obs,lambda=0.8,p=0.2)
  min[i] = rec$errorsum[1]
  med[i] = rec$errorsum[3]
  max[i] = rec$errorsum[6]
  mle = mle.light(light = rec)
```

```
  lambda[i] = mle$lambda
  p[i] = mle$p
}
library(ggplot2)
qplot(med,geom='histogram',binwidth=0.1)
```




```
summary(lambda)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.5340  0.7434  0.8021  0.8103  0.8687  1.3015
```

```
summary(p)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.09494 0.17442 0.20270 0.21126 0.23810 0.55556
```

Option 3. Times first

```
sim.miss.light <- function(obslight,lambda,p){
  dt = obslight
  j = 1
  wt = NULL
  top = NULL
  error = NULL
  for(i in 1:length(dt)){
```

```
    to = m = 0
    while(sum(exp)<dt[i]){
      to = rexp
      top[j] = to
      j = j+1
      if(to == 0) m <- m+1
    }
    exps = rexp(n=m+1,rate=lambda)
    swt = sum(exps)
    error[i] = abs(obslight[i]-swt)/(m+1)
    timesuggestion[i] = obslight[i]<swt
    wt = c(wt,exps)
  }
  light = list(wt=wt,tau=top,obs=obslight,error=error,errorsum=summary(error),timesuggestion=timesugges
  return(light)
}


s = sim.light(50,p=0.5)
s
rec = sim.miss.light(obslight = s$obs,lambda=0.8,p=0.5)
rec
```