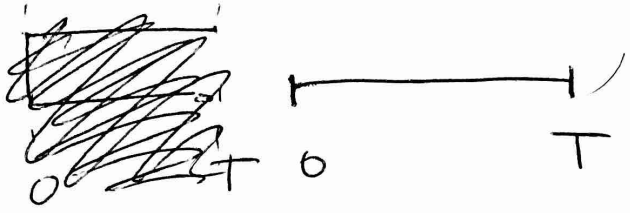
Importance sampling EXAMPLE

At $t=0$ we have 1 branch,

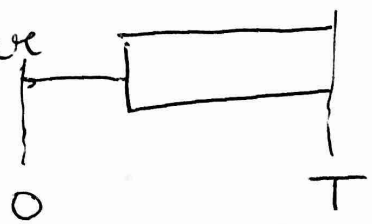
- then a Exponential speciation process starts with rate λ

(A) If nothing happens before T then this is observed

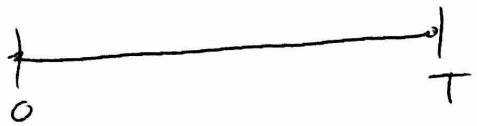


(B) If a speciation happens, then an $\overset{\text{exp}}{\text{extinction}}$ process starts with rate μ

(i) if nothing happens before T, then we observe



(ii) if an extinction happens we observe



(we do not see the extinct species)

t = speciation times $\in$ ~~$\mathbb{R}^+$~~ ~~$[0,T] \cup [e,T]^R$~~ $T \cup ([0,T), T)$

joint

$$f_{joint}(t) = \begin{cases} P(S_p > T) = e^{-\lambda T} & \text{if } t = T \\ \lambda e^{-\lambda t_1} e^{-2\mu(T-t_1)} & \text{if } t = (t_1, T) \\ \lambda e^{-\lambda_1 t_1} e^{-2\mu(t_2-t_1)} \cdot (2\mu) \cdot \frac{1}{2} & \text{if } t = (t_1, t_2\ T) \end{cases}$$

marginal = likelihood of data

$$f_m(t) = \begin{cases} \lambda e^{-\lambda t_1} e^{-2\mu(T-t_1)} & \text{if } t = (t_1, T) \\ e^{-\lambda T} + \frac{1}{2}(1 - e^{-\lambda T}) + \frac{e^{-2\mu T}}{2\mu - \lambda} e\,(1 - e^{(2\mu-\lambda)T}) \cdot \frac{\lambda}{2} & \text{if } t = T \end{cases}$$

$$\int_{t_1}^{T} \lambda e^{-\lambda t_1} e^{-2\mu(s-t_1)} \mu \, ds =$$

$$= \lambda e^{-\lambda t_1} \mu e^{+2\mu t_1} \left[\frac{-1}{2\mu} e^{-2\mu s} \right]_{t_1}^{T}$$

$$= \frac{1}{2} \lambda e^{-\lambda t_1} e^{2\mu t_1} \left(e^{-2\mu t_1} - e^{-2\mu T} \right)$$

$$\frac{1 - e^{(2\mu-\lambda)T}}{2\mu - \lambda} = cT$$

$$\int_0^T \int_{t_1}^T \lambda e^{-\lambda t_1} e^{-2\mu(t_2-t_1)} \mu \, dt_2 \, dt_1$$

$$= \int_0^T \frac{1}{2}\lambda e^{-\lambda t_1} e^{2\mu t_1} \left(e^{-2\mu t_1} - e^{-2\mu T} \right) dt_1$$

$$= \frac{\lambda}{2} \int_0^T e^{t_1(-\lambda+2\mu-2\mu)} - e^{t_1(-\lambda+2\mu)} \cdot e^{-2\mu T} \, dt_1$$

$$= \frac{\lambda}{2} \left\{ \left[\frac{-1}{\lambda} e^{-t_1\lambda} \right]_0^T - e^{-2\mu T} \left[\frac{1}{2\mu-\lambda} e^{t_1(2\mu-\lambda)} \right]_0^T \right\}$$

$$= \frac{1}{2}\left(1 - e^{-T\lambda}\right) + \frac{e^{-2\mu T}}{2\mu-\lambda}\left(1 - e^{T(2\mu-\lambda)}\right) \cdot \frac{\lambda}{2}$$

g. sim

if 2 branches => do nothing.

if 1 branch => flip coin

- H => don't do anything
- T => draw 2 Unif $[0, T]$

add speciation at $\min(u_1, u_2)$

add extinction at $\max(u_1, u_2)$

g

$$g(t_a \mid t_o) = \begin{cases} 1 & \text{if } t_o = \text{2 branches} \\ & \text{if } t_o = \text{1 branch} \\ \frac{1}{2} & \text{if } t_a = \emptyset \\ \frac{1}{T^2} \cdot \frac{1}{2} & \text{if } t_a = (t_1, t_2) \end{cases}$$