Week #10: Stochastic Systems

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This chapter explores the mathematical theory underpinning complex multi-agent interactions and collaborations. By formalizing the core concepts of Agents, Tasks, Tools, Crews, Processes, and Collaboration, we can better understand and optimize their interactions. This theoretical framework provides precise definitions, equations, and illustrative examples to guide the development and assessment of multi-agent systems. While examples may reference specific frameworks like CrewAI, the principles discussed are broadly applicable across various multi-agent environments.

Traditional Software Development	**AI Software Development**
Inputs - text (string) with defined set ("happy", "calm") - numeric (int, float)	**Fuzzy inputs:** Open-ended text - Tabular data - markdown - Text - math operation
Transformations - Math calculation (+, -, *, /) - "Hello {first_name}, how are you?" - if, elif, else - for/while loops	**Fuzzy transformations** - Extract list of key words - Rewrite as a paragraph - Answer a question - Brainstorm new ideas - Perform logic/math reasoning
Outputs - text (string) with defined set ("Positive", "Neutral") - numeric (int, float)	**Fuzzy outputs: text** - Paragraph - Number(s) - JSON - Markdown
Notes - Can be replicated	**Notes** - Can be different every time

1 Agents

Agents are autonomous entities with specific roles, goals, and capabilities. They can operate independently or collaborate with other agents to achieve complex objectives. Each agent can have its own set of tools and can be designed with unique backstories and goals to enhance realism and engagement.

Definition.

An Agent A is defined as a tuple:

$D =$ True. Then, an agent A can be represented as:

$A =$ (Researcher, Identify new AI technologies, $\{\theta_1, \theta_2\}$, Previous research data, PhD in AI with 10 years of ex

The performance of an agent can be evaluated by its ability to achieve its goal given its tools and memory.

$$\text{Goal Achievement: } P(G|T, M) = f(T, M) \tag{1}$$

2 Tasks

Tasks are units of work that agents need to perform. They can range from simple data retrieval to complex problem-solving activities. Tasks are associated with specific agents and include a description, expected output, and any tools required to complete the task.

Definition.

A Task τ is defined as a tuple:

$$\tau = (D, E, T, A)$$

where:

- D is the description of the task.
- E is the expected output of the task.
- T is the set of tools required to complete the task.
- A is the agent assigned to the task.

Let $D =$ Analyze market trends in AI, $E =$ Comprehensive report on AI market trends, $T = \{\theta_2\}$, $A =$ (Researcher, Identify new AI technologies, $\{\theta_1, \theta_2\}$, Previous research data, PhD in AI with 10 y
Then, a task τ can be represented as:

$\tau =$ (Analyze market trends in AI, Comprehensive report on AI market trends, $\{\theta_2\}$, A)

The time required to complete a task depends on its description, the tools available, and the agent assigned to it.

$$\text{Task Completion Time: } T_c(\tau) = g(D, T, A) \tag{2}$$

3 Tools

Tools are functionalities that agents use to perform tasks. They can be internal (provided by CrewAI) or external (such as APIs). Tools extend the capabilities of agents, allowing them to interact with external systems, retrieve information, or perform specialized actions.

Definition.

A Tool θ is defined as a function:

$$\theta : I \to O$$

where:

- I is the set of inputs the tool can take.
- O is the set of outputs the tool can produce.

Let I = Raw data and O = Processed insights. A data analysis tool θ_2 can be represented as:

$$\theta_2(\text{Raw data}) = \text{Processed insights}$$

The effectiveness of a tool can be measured by its ability to produce valuable outputs from given inputs.

$$\text{Tool Effectiveness: } E(\theta) = \sum_{i \in I} P(O|i) \cdot V(O) \tag{3}$$

4 Crews

Crews are groups of agents working together towards common goals. They are structured teams that collaborate on tasks and processes. Crews can be configured with different agents, each bringing their unique skills and tools to the team.

Definition.

A Crew C is defined as a set of agents:

$$C = \{A_1, A_2, \ldots, A_n\}$$

Let $A_1 = (\text{Researcher}, \text{Identify new AI technologies}, \{\theta_1, \theta_2\}, \text{Previous research data}, \text{PhD in AI with 10 ye}$
and $A_2 = (\text{Writer}, \text{Compose articles on AI technologies}, \{\theta_3\}, \text{Extensive writing portfolio}, \text{MA in Journalism}, \text{I}$
Then, a crew C can be represented as:

$$C = \{A_1, A_2\}$$

The efficiency of a crew can be evaluated by averaging the probabilities that each agent achieves its goal.

$$\text{Crew Efficiency: } \eta(C) = \frac{\sum_{i=1}^{n} P(G_i|T_i, M_i)}{n} \tag{4}$$

5 Processes

Processes define how tasks are executed within a crew. They determine the workflow, including the order of task execution and the interaction between agents. CrewAI currently supports sequential processes, with more complex processes like consensual and hierarchical being developed.

Definition.

A Process P is defined as a sequence of tasks:

$$P = (\tau_1, \tau_2, \ldots, \tau_m)$$

Let $\tau_1 = (\text{Collect data}, \text{Raw data collected}, \{\theta_1\}, A_1)$ and $\tau_2 = (\text{Analyze data}, \text{Insights generated}, \{\theta_2\}, A_1)$ Then, a process P can be represented as:

$$P = (\tau_1, \tau_2)$$

The total time to complete a process is the sum of the times required to complete each task in the sequence.

$$\text{Process Completion Time: } T_p(P) = \sum_{i=1}^{m} T_c(\tau_i) \tag{5}$$

6 Collaboration

Collaboration is the mechanism through which agents and crews interact and cooperate. It involves communication, task delegation, and information sharing among agents. Effective collaboration is crucial for achieving complex goals and ensuring smooth operation within crews.

Definition.

Collaboration $\mathcal{C}$ is defined as an interaction function among agents:

$$\mathcal{C} : C \times T \times M \to C$$

where C is the set of agents, T is the set of tools, and M is the set of memory states.

Let $C = \{A_1, A_2\}, T = \{\theta_1, \theta_2, \theta_3\}$, and $M = \{\text{Previous research data}, \text{Extensive writing portfolio}\}$. Then, collaboration $\mathcal{C}$ can be represented as:

$$\mathcal{C}(C, T, M) = \{A_1', A_2'\}$$

where A_1' and A_2' are updated states of agents A_1 and A_2 after collaboration.

The effectiveness of collaboration can be measured by averaging the probabilities that each agent achieves its goal given the tools and memory states after collaboration.

$$\text{Collaboration Effectiveness: } E(\mathcal{C}) = \frac{\sum_{A \in C} P(G|T, M)}{|C|} \tag{6}$$

7 Processes

Processes define how tasks are executed within a crew. They determine the workflow, including the order of task execution and the interaction between agents. CrewAI supports several types of processes, each with its own characteristics and use cases.

7.1 Sequential Process

Definition.

A Sequential Process is one where tasks are executed one after another in a predefined order.

In a research project, data collection must be completed before analysis, which in turn must be completed before writing the final report.

7.2 Parallel Process

Definition.

A Parallel Process is one where multiple tasks are executed simultaneously, without dependencies between them.

In a content creation team, one agent writes an article while another creates graphics.

7.3 Hierarchical Process

Definition.

A Hierarchical Process is one where tasks are organized in a hierarchy, and higher-level tasks can delegate subtasks to other agents.

In a software development project, a project manager delegates specific features to different teams.

7.4 Consensual Process

Definition.

A Consensual Process is one where tasks are executed based on a consensus among agents, often involving decision-making and voting mechanisms.

In a board meeting, members vote on strategic decisions and subsequent actions.

7.5 Iterative Process

Definition.

An Iterative Process is one where tasks are executed in repeated cycles, with each iteration refining or improving the results of the previous one.

In a machine learning model training process, the model is iteratively trained and validated.

7.6 Event-Driven Process

Definition.

An Event-Driven Process is one where tasks are triggered by specific events or conditions, rather than following a predefined order.

In a customer support system, tasks are triggered by incoming support tickets.

8 Propositions and Comments

For an agent A, the probability of achieving its goal $P(G|T, M)$ increases monotonically with the number and effectiveness of tools T and the relevance and accuracy of memory M.

$$P(G|T, M) = f(T, M) \tag{7}$$

The task completion time $T_c(\tau)$ decreases as the effectiveness of the tools T and the proficiency of the agent A increase.

$$T_c(\tau) = g(D, T, A) \tag{8}$$

The overall effectiveness of a set of tools T is maximized when each tool $\theta \in T$ is used for the tasks it is most suited for, based on its input-output efficiency.

$$E(\theta) = \sum_{i \in I} P(O|i) \cdot V(O) \tag{9}$$

The efficiency of a crew $\eta(C)$ is maximized when the agents' roles R, goals G, and tools T are complementary, and their memory states M are aligned towards the crew's collective goal.

$$\eta(C) = \frac{\sum_{i=1}^{n} P(G_i|T_i, M_i)}{n} \tag{10}$$

The total process completion time $T_p(P)$ is minimized when tasks are arranged in a sequence that maximizes parallel execution and minimizes dependencies between tasks.

$$T_p(P) = \sum_{i=1}^{m} T_c(\tau_i) \tag{11}$$

The effectiveness of collaboration $E(\mathcal{C})$ is maximized when communication channels between agents are clear and efficient, and when task delegation is based on agents' strengths and current workload.

$$E(\mathcal{C}) = \frac{\sum_{A \in C} P(G|T, M)}{|C|} \tag{12}$$

9 Randomness in Generated Outputs

The randomness in generated outputs can be modeled using a stochastic process.

Definition.

Let S be the software output, and T be the temperature parameter. The variability V of S can be expressed as:

$$V(S) = \sigma(T) \tag{13}$$

To minimize the variability for consistent outputs, the temperature T should be optimized:

$$\min_{T} \sigma(T) \tag{14}$$

10 Bias in Large Language Models

Bias in language models can be quantified as the deviation from expected problem-solving patterns.

Definition.

Let B be the bias, and E be the expected problem-solving pattern. The deviation D can be expressed as:

$$D(B) = ||B - E|| \tag{15}$$

To minimize the bias and align with real programmers' thinking, the model parameters θ should be optimized:

$$\min_{\theta} D(B) \tag{16}$$

11 Complexity and Task Execution

Task complexity can be modeled as a function of the number of subtasks and dependencies.

Definition.

Let C be the complexity, n be the number of subtasks, and d be the dependencies. The complexity C can be expressed as:

$$C = f(n, d) \tag{17}$$

To optimize task execution and reduce complexity, the number of subtasks n and dependencies d should be minimized:

$$\min_{n,d} C \tag{18}$$

12 Collaboration and Communication

Efficiency of collaboration can be modeled based on communication effectiveness and task delegation.

Definition.

Let E_c be the collaboration efficiency, C_m be the communication effectiveness, and D_t be the task delegation effectiveness. The collaboration efficiency E_c can be expressed as:

$$E_c = g(C_m, D_t) \tag{19}$$

To maximize collaboration efficiency, the communication effectiveness C_m and task delegation effectiveness D_t should be optimized:

$$\max_{C_m, D_t} E_c \tag{20}$$