Optimization and Algorithms
Part 3 of the Project

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Contents

1 Projecting onto a disk

In Task 5, Part 1 of the project, you were asked to find a closed-form expression for the distance from a point p to a disk $D(c,r) = \{x\colon\ \|x-c\|_2 \leq r\}$.

You can solve this problem by first computing the projection of the point onto the disk, that is, by first finding a closed-form solution to the problem

$$\begin{array}{ll} \underset{y\in\mathbf{R}^n}{\text{minimize}} & \|p-y\|_2 \\ \text{subject to} & y \in D(c,r). \end{array} \tag{1}$$

With the solution for problem (1) in hand—call it $y^\star$—the distance from p to $D(c,r)$ is $\|p-y^\star\|_2$.

Task 1. Use the KKT conditions to solve problem (1), that is, to find a closed-form solution for $y^\star$.

2 Solving quickly a trajectory problem

Consider the following three problems:

- Problem A:

$$\begin{array}{ll} \underset{x,u}{\text{minimize}} & \sum_{k=1}^{K} \|Ex(\tau_k) - w_k\|_2^2 + \lambda \sum_{t=1}^{T-1} \|u(t) - u(t-1)\|_2^2 \\ \text{subject to} & x(0) = x_{\text{initial}} \\ & x(T) = x_{\text{final}} \\ & x(t+1) = Ax(t) + Bu(t), \quad \text{for } 0 \le t \le T-1; \end{array} \tag{2}$$

- Problem B:

$$\begin{array}{ll} \underset{x,u}{\text{minimize}} & \sum_{k=1}^{K} \|Ex(\tau_k) - w_k\|_2^2 + \lambda \sum_{t=1}^{T-1} \|u(t) - u(t-1)\|_2 \\ \text{subject to} & x(0) = x_{\text{initial}} \\ & x(T) = x_{\text{final}} \\ & x(t+1) = Ax(t) + Bu(t), \quad \text{for } 0 \le t \le T-1; \end{array} \tag{3}$$

- Problem C:

$$\begin{array}{ll} \underset{x,u}{\text{minimize}} & \sum_{k=1}^{K} \|Ex(\tau_k) - w_k\|_2^2 + \lambda \sum_{t=1}^{T-1} \|u(t) - u(t-1)\|_1 \\ \text{subject to} & x(0) = x_{\text{initial}} \\ & x(T) = x_{\text{final}} \\ & x(t+1) = Ax(t) + Bu(t), \quad \text{for } 0 \le t \le T-1. \end{array} \tag{4}$$

These three problems are problems (2), (6), and (7) from Part 1 of the project, but with the constraint $\|u(t)\|_2 \le U_{\max}$ dropped.

Task 2. Find a closed-form solution to problem A, B, or C. (You have to solve only one of these three problems; you choose which one.)