

Public Policy 529

Quiz #2

Student ID number (8-digits): _____

1. Each year, third graders in Michigan take state-mandated tests for English. The school superintendent cites improvement in these test scores as evidence that the school district's reforms are working. Last year, the mean score was 1,295 ($s=220$; $n=1,190$). This year, the mean score was 1,308 ($s=227$; $n=1,182$). The tests covered the same content.
 - (a) Are these independent or dependent samples? Should we assume equal or unequal variances? Explain.
 - (b) Perform a significance test in which the null hypothesis is that the mean test scores are the same ($\alpha = .05$)? Report the usual items and a p -value.

2. Last year, on the state math tests for third graders, 50% of students were rated as “advanced/proficient” ($n=1,190$). This year, 55% students were in that category ($n=1,182$).

(a) Using the correct formula, calculate the 95% confidence interval for the difference.

(b) Perform the appropriate test of statistical significance versus the null hypothesis of no difference ($\alpha=.05$). Report the usual items.

3. Find the critical value of the test statistic that would be needed to reject the null hypothesis that the two variables below are independent at the .01 level of significance.

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How Fundamental Is R Currently	Favor Or Oppose Gun Permits		Total
	FAVOR	OPPOSE	
FUNDAMENTALIST	229	91	320
MODERATE	387	139	526
LIBERAL	285	91	376
Total	901	321	1,222

4. A voting rights group is comparing the wait times at two polling stations. On average, it took voters 40 minutes to get a ballot at Slauson Middle School ($s=10$; $n=21$), compared to 50 minutes at Scarlett Middle School ($s=15$; $n=19$).
- (a) Perform a test of statistical significance in which the null hypothesis is that average wait times are the same ($\alpha = .01$). Report the usual items and a p -value.

- (b) Would your procedures change if you learned that wait times for a ballot were not normally distributed? Explain.

5. Answer the following questions.

- (a) The t -statistic from a one-sided hypothesis test is 2.59. There are 26 degrees of freedom. What is the p -value? (a range is okay)

- (b) A researcher randomly assigns randomly-selected members of the population into treatment and control groups. These groups are dependent samples. TRUE or FALSE?

- (c) What are the minimum sample size criteria to perform a difference of proportions significance test with a z statistic?

Public Policy 529 Formula Sheet

Descriptive and Distributional Statistics

$$\bar{y} = \frac{\sum y_i}{n}$$

$$s^2 = \frac{\sum (y_i - \bar{y})^2}{n - 1}$$

$$Z = \frac{y - \mu_y}{\sigma}$$

$$IQR = Q_3 - Q_1$$

$$SS = \sum (y_i - \bar{y})^2$$

$$s = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n - 1}}$$

$$\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$$

Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$P(\sim A) = 1 - P(A)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(x) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x}$$

Confidence Intervals and Significance Tests

$$t = \frac{\bar{y} - \mu_0}{\hat{\sigma}_{\bar{y}}}$$

$$\hat{\sigma}_{\hat{\pi}} = \sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}$$

$$Z = \frac{\hat{\pi} - \pi_0}{\hat{\sigma}_{\pi_0}}$$

$$\text{c.i.} = \bar{y} \pm t \cdot \hat{\sigma}_{\bar{y}}$$

$$\hat{\sigma}_{\bar{y}} = \frac{s}{\sqrt{n}}$$

$$\text{c.i.} = \bar{y} \pm Z \cdot \hat{\sigma}_{\bar{y}}$$

$$\hat{\sigma}_{\pi_0} = \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$$

$$\text{c.i.} = \hat{\pi} \pm Z \cdot \hat{\sigma}_{\hat{\pi}}$$

$$se_{\text{diff}} = \sqrt{(se_1)^2 + (se_2)^2}$$

$$z = \frac{(\hat{\pi}_2 - \hat{\pi}_1) - H_0}{se_0}, \text{ where } se_0 = \sqrt{\frac{\hat{\pi}(1 - \hat{\pi})}{n_1} + \frac{\hat{\pi}(1 - \hat{\pi})}{n_2}} \quad c.i. = (\hat{\pi}_2 - \hat{\pi}_1) \pm z \sqrt{\frac{\hat{\pi}_1(1 - \hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1 - \hat{\pi}_2)}{n_2}}$$

$$\hat{\pi} = \frac{\hat{\pi}_1 n_1 + \hat{\pi}_2 n_2}{n_1 + n_2}$$

$$t = \frac{(\bar{y}_2 - \bar{y}_1) - H_0}{se_{\text{diff}}} \quad ci = (\bar{y}_2 - \bar{y}_1) \pm t \cdot se_{\text{diff}}$$

Unequal Variance:

$$se_{\text{diff}} = \hat{\sigma}_{\bar{y}_1 - \bar{y}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \quad \text{approx. } df = \min(n_1 - 1, n_2 - 1)$$

Equal Variance:

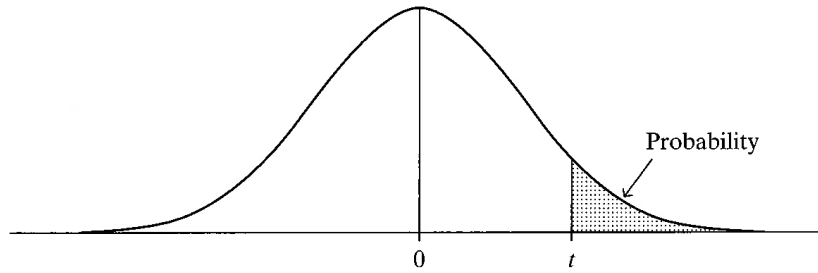
$$se_{\text{diff}} = \hat{\sigma}_{\bar{y}_1 - \bar{y}_2} = \sqrt{\frac{s_{pooled}^2}{n_1} + \frac{s_{pooled}^2}{n_2}} = s_{pooled} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$df = n_1 + n_2 - 2$$

$$t = \frac{\bar{y}_d - H_0}{\hat{\sigma}_{\bar{y}_d}} \quad \hat{\sigma}_{\bar{y}_d} = \frac{s_d}{\sqrt{n}}, \text{ where } s_d = \sqrt{\frac{\sum (y_{di} - \bar{y}_d)^2}{n - 1}}$$

Measures of Association

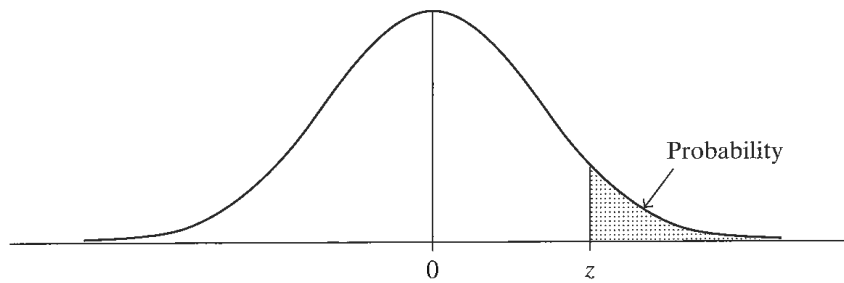
$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}, \text{ where } f_e = \frac{(\text{row total})(\text{column total})}{n}$$

TABLE B: t Distribution Critical Values

df	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.611
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2.000	2.390	2.660	3.232
80	1.292	1.664	1.990	2.374	2.639	3.195
100	1.290	1.660	1.984	2.364	2.626	3.174
∞	1.282	1.645	1.960	2.326	2.576	3.091

Source: "Table of Percentage Points of the t -Distribution." Computed by Maxine Merrington, *Biometrika*, 32 (1941): 300. Reproduced by permission of the *Biometrika* trustees.

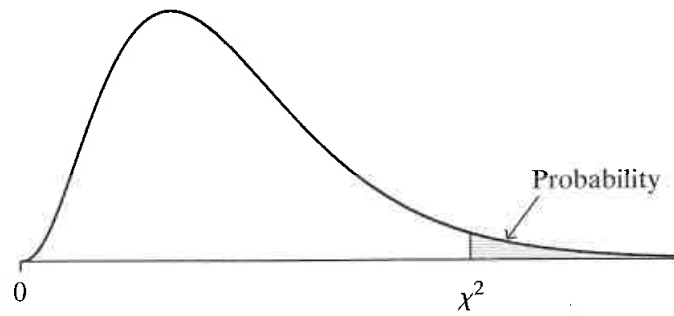
TABLE A: Normal curve tail probabilities. Standard normal probability in right-hand tail (for negative values of z , probabilities are found by symmetry)



z	Second Decimal Place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000233									
4.0	.0000317									
4.5	.00000340									
5.0	.000000287									

Source: R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

TABLE C: Chi-Squared Distribution Values for Various Right-Tail Probabilities



<i>df</i>	Right-Tail Probability						
	0.250	0.100	0.050	0.025	0.010	0.005	0.001
1	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	2.77	4.61	5.99	7.38	9.21	10.60	13.82
3	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	10.22	13.36	15.51	17.53	20.09	21.96	26.12
9	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	23.83	28.41	31.41	34.17	37.57	40.00	45.32
25	29.34	34.38	37.65	40.65	44.31	46.93	52.62
30	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	45.62	51.80	55.76	59.34	63.69	66.77	73.40
50	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	88.13	96.58	101.8	106.6	112.3	116.3	124.8
90	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	109.1	118.5	124.3	129.6	135.8	140.2	149.5

Source: Calculated using *Statable*, software from Cytel Software, Cambridge, MA.