Public Policy 529
Midterm Exam Winter 2016

Student ID number (8-digits): _____

1. In the city school district, the number of absences each day has a normal distribution with a mean of 150 and a standard deviation of 30.

   (a) What is the percentage of days that 190 or *fewer* students are absent? (5 points)

   $$z = \frac{190 - 150}{30} = 1.33$$

   The area in the upper tail is .0918, so the rest of the area is: 1 - .0918 = .9082 or 90.8%. Rounding to nearest percentage is ok.

   (b) What is the percentage of days that 135 students or *fewer* are absent? (5 points)

   $$z = \frac{135 - 150}{30} = -0.50$$

   The area in the upper tail for z=0.50 is .3085, so the below z=-0.50 is 30.9%.

   (c) For what percentage of days is the number of absent students in the range 105 to 195? (5 points)

   $$z = \frac{105 - 150}{30} = -1.50$$

   $$z = \frac{195 - 150}{30} = 1.50$$

   The area in the upper tail is .0668. There is a corresponding area of the same size below z=-1.50. The area in the range 105 to 195 is thus: 100-2*6.68 = 86.6%.

2. Explain the difference between a p-value and α and why we compare these two values in a significance test. (5 points)

   The α is a threshold that reflects our tolerance for error, indicating the probability with which we could make an error when rejecting the null hypothesis. The p-value reflects the estimated probability that we could have obtained our estimate, or something more extreme from the null hypothesis, under the scenario that the null hypothesis is true. We compare p vs. α because it tells us whether our probability of error in rejecting H_0 is less than our tolerance for error.

3. In the 2014 General Social Survey, respondents were asked about their attendance at a political rally. The distribution of the responses is given in the table below.

Attendance at political rally?	Freq.	Percent	Cumulative
Has attended in past year	100	8.1	8.1
Has attended in more distant past	269	21.6	29.7
Has not attended but might attend	479	38.5	68.2
Has not attended and never would	395	31.8	100.0
Total	1,243	100.0	

(a) Calculate the estimated proportion of respondents that have attended a political rally and the 95% confidence interval around this estimate. (6 points)

The estimated proportion is .297, which comes from the cumulative percentage or by adding the first two rows. The formula for the confidence interval is:

$$\begin{aligned} \hat{\pi} \pm z \cdot \sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}} &= .297 \pm 1.96 \cdot \sqrt{\frac{.297(1-.297)}{1,243}} \\ &= .297 \pm 1.96 \cdot .0130 \\ &= .297 \pm .025 \end{aligned}$$

The confidence interval ranges from .272 to .322.

(b) Interpret the confidence interval. (4 points)

In 95% of samples, confidence intervals constructed in this way will include the true population proportion. Or, assuming that the true population proportion is .297, the estimated proportions in 95% of samples of size 1,243 will fall into the range .272 to .322.

4. True or False? When the population distribution of y is highly skewed, it is always inappropriate to use the t distribution to represent the sampling distribution of $\bar{y}$. Explain. (5 points)

False. From the CLT and the Law of Large Numbers, the sampling distribution of the mean is normal when n gets large enough no matter the distribution of the population. A sample of about 30 is sufficient to use the t-distribution. Below that, we cannot use the t when the population is not normally distributed.

5. The Health Monitoring Reform Survey asked respondents whether they had health insurance coverage of some kind (yes/no) and about their employment status. The resulting data are presented in the table below. The cells contain frequencies.

Insured?	Employee	Self-Employed	Retired	Not Working	Total
No	324	97	23	280	724
Yes	4,286	464	547	1,621	6,918
Total	4,610	561	570	1,901	7,642

(a) What are the measurement levels of these two variables? (5 points)
They are both nominal.

(b) What is P(Not Working and Not Insured)?

$$P(\text{Not Working and Not Insured}) = \frac{280}{7642} = .037$$

Rounding to two decimals is okay.

(c) What is P(Insured | Self-Employed)?

$$P(\text{Insured}|\text{Self-Employed}) = \frac{464}{561} = .827$$

(d) Are health insurance coverage and work status independent of each other? Demonstrate mathematically.

No, they are not independent. The conditional probabilities do not match the unconditional probabilities. We found above that P(Insured | Self-Employed) = .827. Since this does not match P(Insured), the two variables are not independent.

$$P(\text{Insured}) = \frac{6918}{7642} = .905$$

Other forms of proof are also fine. They can show any comparison of conditional probabilities to each other or to the unconditional probability.

6. You are asked to evaluate the results of a program to increase the reading test scores of elementary school-aged children over the summer. Among the 780 children who took part in the program, the mean reading test score at the end of summer was 240 with a standard deviation of 70. Perform a test of statistical significance (α=.05) of the assumption that the "true" mean is 230. Report your test statistic, the critical value of the test statistic, and p-value. (10 points)

$$z = \frac{240 - 230}{\frac{70}{\sqrt{780}}} = \frac{10}{2.506} = 3.99$$

This z-score of 3.99 exceeds the critical value of 1.96. Using the z-table, the best we can do is say that the area in the upper tail is between .000233 and .0000317. Doubling these values, we find that $.000643 < p < .000466$. Alternatively, since 3.99 is so close to 4.00, it is okay to say that we see that the area in the upper tail associated z-score is approximately .00003. Doubling .00003 would be .00006, so p is approximately .00006. Since the p value is so small, the approximation is fine, we can reject the null hypothesis with high confidence.

7. Two researchers measure crime rates in the city for various types of crime during the past year. The first researcher obtains police department data for each type of crime reported during this time period. The second researcher conducts a survey of 120 randomly-selected city residents to identify the proportions that were victims of each type of crime during the past year. Evaluating each strategy separately, identify possible sources of measurement error and explain how each

source of error would affect the reliability and/or validity of the measurement strategy. (5 points)

The first strategy should have high reliability. It is just a matter of getting the data from the police department, so all researchers should get the same data (unless poor records are kept and different people get different answers). Some sources of measurement error might be: some crimes are not reported to police (validity) or the police department may have incentives to over or underreport crimes (validity). Credit is given for using sound logic and critical thinking.

For the second strategy, a potential validity problem comes from non-response bias, as some city residents may not want to tell a researcher that they were victim to a violent crime (e.g. a sexual assault). Additionally, the question asks only about whether people were victim of a crime, which fails to measure crimes in which there is not a specific victim (e.g. use of illegal substances). There are some potential reliability problems due to small sample size, which will cause sampling error to be high. Different researchers would get different crime rate measures due to random sampling error. Additionally, people may not recall the time frame in which they were a victim of crime, so they may misreport.

8. Suppose that, in the population, the mean years of education is 14 with a standard deviation of 2.3.

   (a) A researcher is going to take a random sample of size 1,200 from the population described above. In what range will 90% of possible sample means fall? (5 points)

   From the central limit theorem, we can calculate the standard error. The sample means should fall within 1.645 (1.65 is fine) standard errors 90% of the time.

$$\begin{aligned} \bar{y} \pm z \cdot \frac{\sigma}{\sqrt{n}} &= 14 \pm 1.645 \cdot \frac{2.3}{\sqrt{1200}} \\ &= 14 \pm 1.645 \cdot .0664 \\ &= 14 \pm .1092 \end{aligned}$$

   So, 90% of sample means should be in the range from 13.89 to 14.11.

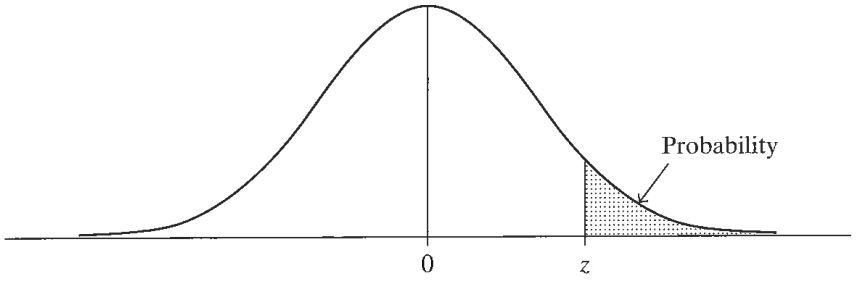
   (b) Continuing with the above scenario, suppose the mean of the sample is 14.2 with a standard deviation of 2. Construct a 90% confidence interval for the estimate and interpret it.

$$\begin{aligned} \bar{y} \pm z \cdot \frac{s}{\sqrt{n}} &= 14.2 \pm 1.645 \cdot \frac{2}{\sqrt{1200}} \\ &= 14.2 \pm 1.645 \cdot .0577 \\ &= 14.2 \pm .0949 \end{aligned}$$

   The 90% confidence interval ranges from 14.105 to 14.295.

Space for Work

TABLE A: Normal curve tail probabilities. Standard normal probability in right-hand tail (for negative values of z, probabilities are found by symmetry)



	Second Decimal Place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000233									
4.0	.0000317									
4.5	.00000340									
5.0	.000000287									

Source: R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

List of Formulas

Descriptive and Distributional Statistics

$$\bar{y} = \frac{\sum y_i}{n} \qquad s^2 = \frac{\sum (y_i - \bar{y})^2}{n-1} \qquad Z = \frac{y - \mu_y}{\sigma}$$

$$IQR = Q_3 - Q_1$$

$$SS = \sum (y_i - \bar{y})^2 \qquad s = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}} \qquad \sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$$

Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \qquad P(A \text{ and } B) = P(A) \times P(B|A)$$

$$P(\sim A) = 1 - P(A)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \qquad P(A \text{ and } B) = P(A) \times P(B)$$

Confidence Intervals and Significance Tests

$$Z \text{ or } t = \frac{\bar{y} - \mu_0}{\hat{\sigma}_{\bar{y}}} \qquad \hat{\sigma}_{\hat{\pi}} = \sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}$$

$$Z = \frac{\hat{\pi} - \pi_0}{\hat{\sigma}_{\pi_0}} \qquad \text{c.i.} = \bar{y} \pm t \cdot \hat{\sigma}_{\bar{y}}$$

$$\hat{\sigma}_{\bar{y}} = \frac{s}{\sqrt{n}} \qquad \text{c.i.} = \bar{y} \pm Z \cdot \hat{\sigma}_{\bar{y}}$$

$$\hat{\sigma}_{\pi_0} = \sqrt{\frac{\pi_0(1-\pi_0)}{n}} \qquad \text{c.i.} = \hat{\pi} \pm Z \cdot \hat{\sigma}_{\hat{\pi}}$$