Public Policy 529
Fall 2023: Problem Set #4

Due Wednesday, October 4

1. Suppose that, in the population, the mean amount of time spent watching television has a normal distribution. The mean is 20 hours per month and the standard deviation is 8 hours.

   (a) What are the z scores associated with the following amounts of time spent watching television: 14, 24, and 33?

   (b) What percentage of people watch *more* than 33 hours of television per month? What percentage of people watch *less* than 14 hours? What percentage of people watch *less* than 24 hours?

   (c) What percentage of people watch between 14 and 33 hours?

   (d) A person's z score for TV watching is -0.25. How many hours of television is that?

2. We learned the definition of percentiles in the Descriptive Statistics lecture. Find the z scores corresponding to the percentiles below. If necessary, provide a range rather than a specific z score.

   (a) The 15th percentile of the normal distribution.

   (b) The 70th percentile of the normal distribution.

   (c) The 95th percentile of the normal distribution.

3. As the previous questions illustrate, a z score is a threshold above which, and below which, is a specific proportion of the area of a normal distribution. These proportions represent the probabilities with which the value of a variable falls above/below the value associated with that z score.

   (a) Suppose that the parameter α (alpha) represents the area in the upper tail of the normal distribution above some threshold z. What is the z score associated with $\alpha = .01$?

   (b) Now suppose α is the area in the lower tail below z. What is the z score associated with $\alpha = .05$?

   (c) Now suppose α is the total area associated with two, equally-sized areas in the upper and lower tails of the normal distribution. In other words, each area is of size $\alpha/2$. If $\alpha = .05$, what are the z scores that define these areas?

4. The distribution of household income in the United States has a strong positive skew. Suppose the mean (μ) is \$97,941, and the standard deviation (σ) is \$76,021. You collect a random sample of size $n = 400$ and measure household income.

   (a) As we have learned, the mean of your sample comes from a probability distribution over all possible sample means (i.e. a sampling distribution). According to the Central Limit Theorem, what value is at the middle of this distribution? Provide a specific number.

   (b) Would this sampling distribution have a positive skew? Why or why not?

   (c) According to the Central Limit Theorem, what is the size of random sampling error for the sample mean? Provide an interpretation of this number.

   (d) Using what you found above, what range includes 95 percent of all sample means?

   (e) What is the probability that your sample mean would be greater than \$101,742?

5. Suppose that, with probability .32, a randomly selected person will agree that astrology has scientific truth. Also, suppose that there is a probability of .89 that a person will agree that science brings opportunities for the next generation. Finally, suppose that there is a probability of .29 that a person will agree with both.

   Using your choice of formulas, a table, a probability tree, or a Venn diagram, find the following.

   (a) What is the probability that a person agrees with neither statement?

   (b) What is the probability that person will agree with the statement that science brings opportunities for the next generation given that they do not agree that astrology has scientific truth?