

Chapter 1

Chapter 29

1.1 Alternating-Current Circuits

1.1.1 Conceptual Problems

1 - An ac generator rotates at 60 Hz and has a coil. Determine the time that elapses between successive peak emf values of the coil?

Successive peaks are one-half period apart. Hence the elapsed time between the peaks is $\frac{1}{2}T = \frac{1}{2f} = \frac{1}{2(60^{-1})} = \boxed{8.33 \text{ ms}}$.

2 - When the rms voltage in an ac circuit doubles, the peak voltage is:
(a) doubled, (b) halved, (c) increased by a factor of $\sqrt{2}$, (d) not changed.
A relationship between V and V_{peak} can be used to determine the effect of doubling the rms voltage on the peak voltage.

First express the initial rms voltage in terms of the peak voltage:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

Then express the doubled rms voltage in terms of the new peak voltage V'_{peak} :

$$2V_{rms} = \frac{V'_{peak}}{\sqrt{2}}$$

After you have done that you have to divide the second of these equations by the first and simplify to obtain:

$$\frac{2V_{rms}}{V_{rms}} = \frac{\frac{V'_{peak}}{\sqrt{2}}}{\frac{V_{peak}}{\sqrt{2}}} \Rightarrow 2 = \frac{V'_{peak}}{V_{peak}}$$

Now you only have to solve for V'_{peak} :

$$V'_{peak} = 2V_{peak} \Rightarrow \boxed{(a)} \text{ is the correct answer.}$$

3 - Suppose the frequency in the circuit shown in figure 29-27 is doubled, then the inductance in the inductor will (a) double, (b) not change, (c) halve, (d) quadruple.

The inductance of an inductor is independent of the frequency of the circuit and is determined only by its construction. The reactance is dependent on the frequency, hence (b) is correct.

4 - Suppose the frequency in the circuit shown in figure 29-27 is doubled, then the inductive reactance of the inductor will (a) double, (b) not change, (c) halve, (d) quadruple.

The inductive reactance of an inductor and the frequency are related by $X_L = \omega L$. Therefore, when ω is doubled, X_L will double as well. (a) is correct.

5 - Suppose the frequency in the circuit shown in figure 29-28 is doubled, then the capacitive reactance of the circuit will (a) double, (b) not change, (c) halve, (d) quadruple.

The capacitive reactance of a capacitor varies with the frequency and are related by $X_C = \frac{1}{\omega C}$. Therefore, if ω change to its double then X_C will halve. (c) is correct.

6 - (a) There is a circuit consisting of an ac generator and an ideal inductor, do exist time intervals when the inductor receives energy from the generator? When this happens? (b) Are there time intervals when the inductor supplies energy back to the generator? When this happens? Explain your answers.

The answer to both questions is yes. (a) During current magnitude increase in the inductor, the inductor absorbs power from the generator. (b) During current decrease in the inductor, the inductor supplies power to the generator.

7 - (a) Imagine a circuit consisting of a generator and a capacitor, are there any time intervals when the capacitor receives energy from the generator? When this happens? (b) Does the capacitor supply power to the generator? When this happens? Explain your answers.

(a) The capacitor absorbs power from the generator while the magnitude of the charge is accumulating on either plate of the capacitor. (b) The capacitor supplies power to the generator whenever the magnitude of the charge on either plate of the capacitor is decreasing.

8 - (a) Demonstrate that the SI unit of inductance multiplied by the SI unit of capacitance is equivalent to seconds squared. (b) Show that the SI unit of inductance divided by the SI unit of resistance is equivalent to seconds.

(a) First substitute the SI units of inductance and capacitance then simplify to obtain:

$$\frac{V \cdot s}{A} \cdot \frac{C}{V} = \frac{s}{\frac{C}{S}} \cdot C = s^2$$

(b) First substitute the SI units of inductance divided by resistance then simplify:

$$\frac{\frac{V \cdot s}{A}}{\Omega} = \frac{\frac{V \cdot s}{A}}{\frac{V}{A}} = \boxed{s}$$

9 - Imagine the rotation rate is increased in the ac circuit in figure 29-29. Then the rms current (a) increases, (b) does not change, (c) may increase or decrease depending on the magnitude of the original frequency, (d) may increase or decrease depending on the magnitude of the resistance, (e) decreases. The rms current through the resistor I_{rms} is directly proportional to ω as shown here:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{\mathcal{E}_{peak}}{\sqrt{2}} = \frac{NBA}{\sqrt{2}} \omega$$

therefore, $\boxed{(a)}$ is correct.

10 - Suppose the inductance is tripled in a circuit consisting solely of a variable inductor and a variable capacitor, how you have to change the capacitance so that the natural frequency of the circuit is unchanged? (a) triple the capacitance, (b) decrease the capacitance to one-third of its original value, (c) you should not change the capacitance, (d) you cannot determine how to change the capacitance from the data given.

The natural frequency of an LC circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

We now express the natural frequencies of the circuit before and after the inductance is tripled:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ and } f'_0 = \frac{1}{2\pi\sqrt{L'C'}}$$

We divide the second equation by the first and simplify:

$$\frac{f'_0}{f_0} = \frac{\frac{1}{2\pi\sqrt{L'C'}}}{\frac{1}{2\pi\sqrt{LC}}} = \sqrt{\frac{LC}{L'C'}}$$

We take into account that the natural frequency is unchanged:

$$1 = \sqrt{\frac{LC}{L'C'}} \Rightarrow \frac{LC}{L'C'} = 1 \Rightarrow C' = \frac{L}{L'}C$$

Therefore when the inductance is tripled:

$$C' = \frac{L}{3L}C = \frac{1}{3}C$$

Hence $\boxed{(b)}$ is the correct answer.

11 - In a circuit consisting only of an ideal inductor and ideal capacitor, how does the maximum energy stored in the capacitor compare to the maximum value stored in the inductor? (a) They are the same and each equal to the total energy stored in the circuit. (b) They are the same and each equal to half of the total energy stored in the circuit. (c) The maximum energy stored in the capacitor is larger than the maximum energy stored in the inductor. (d) The maximum energy stored in the inductor is larger than the maximum energy stored in the capacitor. (e) You cannot compare the maximum energies based on the data given because the ratio of the maximum energies depends on the actual capacitance and inductance values. The maximum energy stored in the electric field of the capacitor is given by

$$U_e = \frac{Q^2}{2C}$$

and the maximum energy stored in the magnetic field of the inductor is given by

$$U_m = \frac{LI^2}{2}.$$

That is because energy is conserved in a LC circuit and oscillates between the inductor and the capacitor,

$$U_e = U_m = U_{total},$$

therefore answer (a) is correct.

12 - Check whether the propositions are true or false.

- (a) A driven series RLC circuit that has a high Q factor has a narrow resonance curve.
- (b) A circuit consists of an inductor, a resistor and a capacitor, all connected in series. If the resistance of the resistor is doubled, the natural frequency of the circuit remains the same.
- (c) At resonance, the impedance of a driven series RLC combination equals the resistance R .
- (d) At resonance, the current in a driven series RLC circuit is in phase with the voltage applied to the combination.

(a) True. The Q factor and the width of the resonance curve at half power are related according to

$$Q = \frac{\omega_0}{\Delta\omega}$$

this means that they are inversely proportional to each other.

(b) True. Circuit's natural frequency depends only on the inductance L of the inductor and the capacitance C of the capacitor and is given by

$$\omega = \frac{1}{\sqrt{LC}}.$$

(c) True. The impedance of an RLC circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

At resonance

$$X_L = X_C \text{ and so } Z = R.$$

(d) True. The phase angle δ is related to X_L and X_C according to

$$\delta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

At resonance $X_L = X_C$ and so $\delta = 0$.

13 - Check whether propositions are true or false.

(a) The power factor of a driven series RLC circuit is close to zero when near resonance.

(b) A driven series RLC circuit's power factor doesn't depend on the value of the resistance.

(c) A driven series RLC circuit's resonance frequency doesn't depend on the value of the resistance.

(d) The peak current of a driven series RLC circuit doesn't depend on the capacitance or the inductance when at resonance.

(e) For frequencies below the resonant frequency, the capacitive reactance of a driven series RLC circuit is larger than the inductive reactance.

(f) For frequencies below the resonant frequency of a driven series RLC circuit, the phase of the current leads ahead the phase of the applied voltage.

(a) False. The power factor given by

$$\cos \delta = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}},$$

is close to 1.

(b) False. The power factor is given by

$$\cos \delta = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}}.$$

(c) True. The resonance frequency for a driven series RLC circuit depends only on L and C is given by

$$\omega_{res} = \frac{1}{\sqrt{LC}}$$

(d) True. At resonance $X_L - X_C = 0$ and so $Z = R$ and the peak current is given by $I_{peak} = V_{app,peak}/R$.

(e) True. The capacitive reactance varies inversely with the driving frequency and the inductive reactance varies directly with the driving frequency, hence at

frequencies well below the resonance frequency the capacitive reactance is larger than the inductive reactance.

(f) True. For frequencies below the resonant frequency, the circuit is more capacitive than inductive and the phase constant ϕ is negative. This means that the current leads the applied voltage.

14 - How can two different radio stations be heard at the same specific frequency when a receiver is tuned it, this situation often occurs while driving between two cities?

The power curves received by the radio have width, hence the two frequencies coming from the radio stations can overlap as a result and you can receive signals from both stations.

15 - Check whether is true or false.

(a) The power factor is close to zero at frequencies much higher than or much lower than the resonant frequency of a driven series RLC circuit.

(b) The larger the resonance width of a driven series RLC circuit is, the larger the Q factor for the circuit becomes.

(c) The larger the resistance of a driven series RLC circuit is, the larger the resonance width for the circuit is.

(a) True. The power factor is given by

$$\cos \delta = \frac{R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}},$$

therefore for values of ω that are much higher or much lower than the resonant frequency, the term in parentheses becomes very large and $\cos \delta$ approaches to zero.

(b) False. When the resonance curve is reasonably narrow, the Q factor can be approximated by

$$Q = \omega_0 / \Delta\omega.$$

Hence a large value for Q corresponds to a narrow resonance curve.

(c) True. See figure 29-21

16 - Suppose an ideal transformer has N_1 turns on its primary and N_2 turn on its secondary. The average power delivered to a load resistance R connected across the secondary is P_2 whilst the primary rms voltage is V_1 . The rms current in the primary windings can then be expressed as (a) P_2/V_1 , (b) $(N_1/N_2)(P_2/V_1)$, (c) $(N_2/N_1)(P_2/V_1)$, (d) $(N_2/N_1)2(P_2/V_1)$.

Subscript 1 and 2 correspond to primary and secondary respectively. We assume no loss of power in the transformer, we can equate the power in the primary circuit to the power in the secondary circuit and solve for the rms current in th

primary windings.

Assuming no power loss in the transformer:

$$P_1 = P_2$$

Substituting for P we obtain:

$$I_{1,rms}V_{1,rms} = I_{2,rms}V_{2,rms}$$

We solve for I and simplify:

$$I_{1,rms} = \frac{I_{2,rms}V_{2,rms}}{V_{1,rms}} = \frac{P_2}{V_{1,rms}}$$

hence answer (a) is correct.

17 - Check whether propositions are true or false: (a) Transformers are used to change frequency.

(b) Transformers are used to change voltage.

(c) If a transformer steps up the current, it must step down the voltage.

(d) A step-up transformer, steps down the current.

(e) The standard household wall-outlet voltage in Europe is 220 V, about twice that used in the United States. If a European traveler wants her hair dryer to work properly in the United States, she should use a transformer that has more windings in its secondary coil than in its primary coil.

(f) The standard household wall-outlet voltage in Europe is 220 V, about twice that used in the United States. If an American traveler wants his electric razor to work properly in Europe, he should use a transformer that steps up the current.

(a) False. A transformer is a device used to raise or lower the voltage in a circuit.

(b) True. A transformer is a device used to raise or lower the voltage in a circuit.

(c) True. If energy is to be conserved, the product of the current and voltage must be constant.

(d) True. Because the product of current and voltage in the primary and secondary circuits is the same, increasing the current in the secondary results in a lowering (or stepping down) of the voltage.

(e) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the voltage in order to make her hair dryer work properly.

(f) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the current (and decrease the voltage) in order to make his razor work properly.

1.1.2 Estimation and Approximation

18 - Resistance and inductive reactance are included in the impedances of motors, electromagnets and transformers. Suppose that phase of the current to

a large industrial plant lags the phase of the applied voltage by 25° when the plant on full operation using 2.3 MW of power. The power is supplied to the plant from a substation 4.5 km from the plant; the 60 Hz rms line voltage at the plant is 40 kV. The resistance of the transmission line from the substation to the plant is 5.2Ω . The cost per kilowatt-hour to the company that owns the plant is \$0.14, and the plant pays only for the actual energy used. (a) Estimate the resistance and inductive reactance of the plant's total load. (b) Estimate the rms current in the power lines and the rms voltage at the substation. (c) How much power is lost in transmission? (d) Suppose that the phase that the current lags the phase of the applied voltage is reduced to 18° by adding a bank of capacitors in series with the load. How much money would be saved by the electric utility during one month of operation, assuming the plant operates at full capacity for 16 h each day? (e) What must be the capacitance of this bank of capacitors to achieve this change in phase angle?

We can find the resistance and inductive reactance of the plant's total load from the impedance of the load and the phase constant. The current in the power lines can be found from the total impedance of the load the potential difference across it and the rms voltage at the substation by applying Kirchhoff's loop rule to the substation-transmission wires-load circuit. The power lost in transmission can be found from $P_{trans} = I_{rms}^2 R_{trans}$. We can find the cost savings by finding the difference in the power lost in transmission when the phase angle is reduced to 18° . Finally, we can find the capacitance that is required to reduce the phase angle to 18° by first finding the capacitive reactance using the definition of $\tan \delta$ and then applying the definition of capacitive reactance to find C.

(a) First we relate the resistance and inductive reactance of the plant's total load to Z and δ :

$$R = Z \cos \delta \text{ and } X_L = Z \sin \delta$$

We then express Z in terms of the rms current I_{rms} in the power lines and the rms voltage \mathcal{E}_{rms} at the plant:

$$Z = \frac{\mathcal{E}_{rms}}{I_{rms}}$$

after express teh power delivered to the plant in terms of \mathcal{E}_{rms} , I_{rms} and δ and solve for I_{rms} :

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos \delta \text{ and } I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms} \cos \delta}$$

then substitute to obtain:

$$Z = \frac{\mathcal{E}_{rms}^2 \cos \delta}{P_{av}}$$

Afterwards substitute numerical values and evaluate Z:

$$Z = \frac{(40kV)^2 \cos 25^\circ}{2.3MW} = 630\Omega$$

Substitute numerical values and evaluate for R and X_L :

$$R = (630\Omega) \cos 25^\circ = 571\Omega = \boxed{0.57k\Omega} \text{ and } X_L = (630\Omega) \sin 25^\circ = 266\Omega = \boxed{0.27k\Omega}$$

(b) Find the current in the power lines:

$$I_{rms} = \frac{2.3MW}{(40kV) \cos 25^\circ} = 63.4A = \boxed{63A}$$

Then apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E}_{sub} - I_{rms}R_{trans} - I_{rms}Z_{tot} = 0$$

Solve for \mathcal{E}_{sub} :

$$\mathcal{E}_{sub} = I_{rms}(R_{trans} + Z_{tot})$$

Substitute numerical values and evaluate \mathcal{E}_{sub} :

$$\mathcal{E}_{sub} = (63.4A)(5.2\Omega + 630\Omega) = \boxed{40.3kV}$$

(c) The power lost in transmission is:

$$P_{trans} = I_{trans}^2 R_{trans} = (63.4A)^2(5.2\Omega) = 20.9kW = \boxed{21kW}$$

(d) Express the cost savings ΔC in terms of the difference in energy consumption $P_{25^\circ} - P_{18^\circ} \Delta t$ and the per-unit cost u of the energy:

$$\Delta C = (P_{25^\circ} - P_{18^\circ}) \Delta t u$$

Express the power lost in transmission when $\delta = 18^\circ$:

$$P_{18^\circ} = I_{18^\circ}^2 R_{trans}$$

Then find the current in the transmission lines when $\delta = 18^\circ$:

$$I_{18^\circ} = \frac{2.3MW}{(40kV) \cos 18^\circ} = 60.5A$$

Evaluate P_{18° :

$$P_{18^\circ} = (60.5A)^2(5.2\Omega) = 19kW$$

Substitute numerical values and evaluate ΔC :

$$\Delta C = (20.9kW - 19kW)(16h/d) \left(30 \frac{d}{\text{month}} \right) \left(\frac{\$0.14}{kW \cdot h} \right) = \boxed{\$128}$$

(e) The required capacitance is given by:

$$C = \frac{1}{2\pi f X_C}$$

We then relate the new phase δ to the inductive reactance X_L , the reactance due to the added capacitance X_C , and the resistance of the load R :

$$\tan \delta = \frac{X_L - X_C}{R} \Rightarrow X_C = X_L - R \tan \delta$$

We substitute for X_C :

$$C = \frac{1}{2\pi f(X_L R \tan \delta)}$$

We then substitute numerical values and evaluate C :

$$C = \frac{1}{2\pi(60s^{-1})(266\Omega - (571\Omega) \tan 18^\circ)} = \boxed{33\mu F}$$

1.2 Alternating Current in Resistors, Inductors, and Capacitors

19 - Suppose we have a 100-W light bulb in a standard 120-V-rms socket. Find (a) the rms current, (b) the peak current, and (c) the peak power.

We use $P_{av} = \mathcal{E}_{rms} I_{rms}$ to find I_{rms} , $I_{peak} = \sqrt{2} I_{rms}$ to find I_{peak} , and $P_{peak} = I_{peak} \mathcal{E}_{peak}$ to find P_{peak} .

(a) We find the relationship of average power delivered by the source to the rms voltage across the bulb and the rms current through it:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \Rightarrow I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms}}$$

We then substitute numerical values and evaluate I_{rms} :

$$I_{rms} = \frac{100W}{120V} = 0.8333A = \boxed{0.833A}$$

(b) We do express I_{peak} in terms of I_{rms} :

$$I_{peak} = \sqrt{2} I_{rms}$$

substitute for I_{rms} and evaluate I_{peak} :

$$I_{peak} = \sqrt{2}(0.8333A) = 1.1785A = \boxed{1.18A}$$

(c) Afterwards we need to express the maximum power in terms of the maximum voltage and maximum current, and substitute numerical values evaluating P_{peak} :

$$P_{peak} = I_{peak} \mathcal{E}_{peak}$$

$$P_{peak} = (1.1785A)\sqrt{2}(120V) = \boxed{200W}$$

20 - Suppose we have a circuit breaker for a current of 15 A rms at a voltage of 120 V rms. (a) What is the largest peak current that the breaker can carry? (b) What is the maximum average power that can be supplied by this circuit?

We can use $I_{peak} = \sqrt{2}I_{rms}$ to find the largest peak current that the breaker can carry and $P_{av} = I_{rms}V_{rms}$ to find the average power supplied by the circuit. (a) We do express I_{peak} in terms of I_{rms} and relate the average power to the rms current and voltage:

$$I_{peak} = \sqrt{2}I_{rms} = \sqrt{2}(15A) = \boxed{21A}$$

$$P_{av} = I_{rms}V_{rms} = (15A)(120V) = \boxed{1.8kW}$$

21 - What is the reactance of a 1.00mH inductor at (a) 60 Hz, (b) 600 Hz, and (c) 6.00 kHz?

Utilize $X_L = \omega L$ to determine the reactance of the inductor at any frequency. We do so by expressing the inductive reactance as a function of f :

$$X_L = \omega L = 2\pi fL$$

(a) At $f = 60Hz$:

$$X_L = 2\pi(60s^{-1})(1mH) = \boxed{0.38\Omega}$$

(b) At $f = 600Hz$:

$$X_L = 2\pi(600s^{-1})(1mH) = \boxed{3.77\Omega}$$

(c) At $f = 6kHz$:

$$X_L = 2\pi(6ks^{-1})(1mH) = \boxed{37.7\Omega}$$

22 - An inductor has a reactance of 100 Ω at 80 Hz. (a) What is its inductance? (b) What is its reactance at 160 Hz?

We can use $X_L = \omega L$ to find the inductance of the inductor at any frequency.

(a) We do relate the reactance or the inductor the its inductance and solve for L doing its evaluation:

$$X_L = \omega L = 2\pi fL \Rightarrow L = \frac{X_L}{2\pi f}$$

$$L = \frac{100\Omega}{2\pi(80s^{-1})} = 0.199H = \boxed{0.20 H}$$

(b) At 160 Hz:

$$X_L = 2\pi(160s^{-1})(0.199H) = \boxed{0.20k\Omega}$$

23 - Consider a $10\mu F$ capacitor, at what frequency would its reactance equal a reactance of an $10mH$ inductor?

If we equate reactances of the capacitor and inductor we then can solve for the frequency.

We do express the reactance of the inductor, then we express the reactance of the capacitor equating these reactances:

$$X_L = \omega L = 2\pi fL$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

$$2\pi fL = \frac{1}{2\pi fC} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

We finally substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{(10\mu F)(1mH)}} = \boxed{1.6kHz}$$

24 - What is the reactance of a $1.00nF$ capacitor at (a) 60.0 Hz, (b) 6.00 kHz, and (c) 6.00 MHz?

We can use $X_C = 1/\omega C$ to find the reactance of the capacitor at any frequency. By expressing the capacitive reactance as a function of f :

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

(a) At $f = 60Hz$:

$$X_C = \frac{1}{2\pi(60s^{-1})(1nF)} = \boxed{2.65M\Omega}$$

(b) At $f = 6.00kHz$:

$$X_C = \frac{1}{2\pi(6.00kHz)(1nF)} = \boxed{26.5k\Omega}$$

(c) At $f = 6.00MHz$:

$$X_C = \frac{1}{2\pi(6.00MHz)(1nF)} = \boxed{26.5\Omega}$$

25 - A 20-Hz ac generator that produces a peak emf of 10 V is connected to a $20 - \mu F$ capacitor. Find (a) the peak current and (b) the rms current.

We can use $I_{peak} = \mathcal{E}_{peak}/X_C$ and $X_C = 1/\omega C$ to express I_{peak} as a function of \mathcal{E}_{peak} , f and C . Once we evaluate I_{peak} , we can use $I_{rms} = I_{peak}/\sqrt{2}$ to find I_{rms} .

We begin by expressing I_{peak} in terms of \mathcal{E}_{peak} and X_C

$$I_{peak} = \frac{\mathcal{E}_{peak}}{X_C}$$

We then express the capacitive reactance and substitute for X_C and simplify:

$$I_{peak} = 2\pi f C \mathcal{E}_{peak}$$

(a) Substitute numerical values and evaluate I_{peak} :

$$I_{peak} = 2\pi(20s)(20\mu F)(10V) = 25.1mA = \boxed{25mA}$$

(b) Express $I_{rms} = \frac{I_{peak}}{\sqrt{2}} = \frac{25.1mA}{\sqrt{2}} = 18mA$

26 - At what frequency is the reactance of a $10 - \mu F$ capacitor (a) 1.00Ω , (b) 100Ω , and (c) $10.0 m\Omega$?

We can use $X_C = 1/\omega C = 1/2\pi f C$ to relate the reactance of the capacitor to the frequency:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi C X_C}$$

(a) Find f when $X_C = 1.00\Omega$:

$$f = \frac{1}{2\pi(10\mu F)(1.00\Omega)} = \boxed{16kHz}$$

(b) Find f when $X_C = 100\Omega$:

$$f = \frac{1}{2\pi(10\mu F)(100\Omega)} = \boxed{0.16kHz}$$

(c) Find f when $X_C = 10.0m\Omega$:

$$f = \frac{1}{2\pi(10\mu F)(10.0m\Omega)} = \boxed{1.6MHz}$$

27 - Suppose a circuit consists of two ideal ac generators and a $25 - \Omega$ resistor, all connected in series. The potential difference across the terminals of one of the generators is given by $V_1 = (5.0V) \cos(\omega t - \alpha)$, and the potential difference across the terminals of the other generator is given by $V_2 = (5.0V) \cos(\omega t + \alpha)$,

where $\alpha = \pi/6$. (a) Use Kirchhoff's loop rule and a trigonometric identity to find the peak current in the circuit. (b) Use a phasor diagram to find the peak current in the circuit. (c) Find the current in the resistor if $\alpha = \pi/4$ and the amplitude of V_2 is increased from 5.0 V to 7.0 V.

We can find the sum of the phasors V_1 and V_2 using this trigonometric identity

$$\cos \theta + \cos \phi = 2 \cos \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi)$$

then we use this sum to express I as a function of time. In (b) we use a phasor diagram to obtain the same result and in (c) we use the phasor diagram appropriate to the given voltages to express the current as a function of time.

(a) Apply the Kirchhoff's loop rule to the circuit and solve for I :

$$V_1 + V_2 - IR = 0$$

$$I = \frac{V_1 + V_2}{R}$$

Then we use the trigonometric identity described above to find $V_1 + V_2$:

$$V_1 + V_2 = (5.0V)[\cos(\omega t - \alpha) + \cos(\omega t + \alpha)] = (5V)[2 \cos \frac{1}{2}(2\omega t) \cos \frac{1}{2}(-2\alpha)] =$$

$$(10V) \cos \frac{\pi}{6} \cos \omega t = (8.66V) \cos \omega t$$

We substitute for $V_1 + V_2$ and R to obtain:

$$I = \frac{(8.66V) \cos \omega t}{25\Omega} = (0.346A) \cos \omega t = (0.35A) \cos \omega t$$

$$I_{peak} = \boxed{0.35A}$$

(b) We express the magnitude of the current in R :

$$|I| = \frac{|\vec{V}|}{R}$$

The phasor diagram for the voltages is shown. We can find \vec{V} using vector addition:

$$|\vec{V}| = 2|\vec{V}_1| \cos 30^\circ = 2(5.0V) \cos 30^\circ = 8.66V$$

We then substitute for $|\vec{V}|$ and R to obtain: $|I| = \frac{8.66V}{25\Omega} = 0.346A$ and $I = (0.35A) \cos \omega t$ where $I_{peak} = \boxed{0.35A}$ (c) The phasor diagram is shown. Note that the phase angle between V_1 and V_2 is now 90° . We use the Pythagorean theorem to find $|\vec{V}|$:

$$|\vec{V}| = \sqrt{|\vec{V}_1|^2 + |\vec{V}_2|^2} = \sqrt{(5.0V)^2 + (7.0V)^2} = 8.60V$$

Then we express I as a function of t :

$$I = \frac{|\vec{V}|}{R} \cos(\omega t + \delta)$$

where $\delta = 45^\circ - (90^\circ - \alpha) = \alpha - 45^\circ = \tan^{-1} \frac{7.0V}{5.0V} - 45^\circ = 9.462^\circ = 0.165rad$
Finally substitute numerical values and evaluate I :

$$I = \frac{8.60V}{25\Omega} \cos(\omega t + 0.165rad) = \boxed{(0.34A) \cos(\omega t + 0.17rad)}$$

1.3 Undriven Circuits Containing Capacitors, Resistors and Inductors

28 - (a) Show that $1/LC$ has units of inverse seconds by substituting SI units for inductance and capacitance into the expression. (b) Show that $\omega L/R$ (the expression for the Q-factor) is dimensionless by substituting SI units for angular frequency, inductance, and resistance into the expression.

We begin by substituting the units of the various physical quantities in $1/\sqrt{LC}$ and $Q = \omega L/R$ to establish their units.

(a) We do substitute the units for ω , L and C in the expression $1/\sqrt{LC}$ and then simplify:

$$\frac{1}{\sqrt{H \cdot F}} = \frac{1}{\sqrt{(\Omega \cdot s)(\frac{s}{\Omega})}} = \frac{1}{\sqrt{s^2}} = \boxed{s^{-1}}$$

(b) We do substitute the units for ω , L and R in the expression $Q = \omega L/R$ and simplify:

$$\frac{\frac{1}{s} \cdot \frac{V \cdot s}{A}}{\frac{V}{A}} = 1 \Rightarrow \boxed{\text{unitless}}$$

29 - (a) What is the period of oscillation of an LC circuit consisting of an ideal 2.0-mH inductor and a 20- μ F capacitor? (b) A circuit that oscillates consists solely of an 80- μ F capacitor and a variable ideal inductor. What inductance is needed in order to tune this circuit to oscillate at 60 Hz?

$T = 2\pi/\omega$ and $\omega = 1/\sqrt{LC}$ can be used to relate T , f to L and C .

(a) We express the period of oscillation:

$$T = \frac{2\pi}{\omega}$$

Taking into account that for a LC circuit $\omega = 1/\sqrt{LC}$, we substitute for ω obtaining:

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{(2.0mH)(20\mu F)} = \boxed{1.3ms}$$

From the equation for ω we solve for L :

$$L = \frac{T^2}{4\pi^2 C} = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (60^{-1})^2 (80\mu F)} = \boxed{88mH}$$

30 - An LC circuit has capacitance C_0 and inductance L . A second LC circuit has capacitance $2C_0$ and inductance $2L$, and a third LC circuit has capacitance $2C_0$ and inductance $12L$. (a) Show that each circuit oscillates with the same frequency. (b) In which circuit would the peak current be greatest if the peak voltage across the capacitor in each circuit was the same?

The expression $f_0 = 1/2\pi\sqrt{LC}$ can be used for the resonance frequency of a LC circuit showing that each circuit oscillates with the same frequency. For (b) we can use $I_{peak} = \omega Q_0$, where Q_0 is the charge of the capacitor at time zero, and the definition of capacitance $Q_0 = CV$ to express I_{peak} in terms of ω, C, V .

We do express the resonance frequency for a LC circuit

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(a) We separate the product of L and C for each circuit:

$$\text{Circuit 1: } L_1 C_1 = L_1 C_0,$$

$$\text{Circuit 2: } L_2 C_2 = (2L_1)\left(\frac{1}{2}C_0\right) = L_1 C_1,$$

$$\text{Circuit 3: } L_3 C_3 = \left(\frac{1}{2}L_1\right)(2C_0) = L_1 C_1,$$

$$L_1 C_1 = L_2 C_2 = L_3 C_3$$

Therefore, the resonance frequencies of the three circuits are equal.

(b) We do express the I peak in terms of the charge stored in the capacitor and Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor, then we substitute:

$$I_{peak} = \omega Q_0, \quad Q_0 = CV \Rightarrow I_{peak} = \omega CV$$

We have I peak directly proportional to C when ω and V are held constant. Hence the circuit with a capacitance of $2C_0$ has the greatest peak current.

31 - A $5.0\text{-}\mu\text{F}$ capacitor is charged to 30 V and is then connected across an ideal 10-mH inductor. (a) How much energy is stored in the system? (b) What is the frequency of oscillation of the circuit? (c) What is the peak current in the circuit?

The energy stored in the electric field of the capacitor can be found using $U = \frac{1}{2}CV^2$, the frequency can be found by these relations $\omega_0 = 2\pi f_0 = 1/\sqrt{LC}$, and I peak can be determined by using these relations $I_{peak} = \omega Q_0$ and $Q_0 = CV$. (a) We express the energy stored in the system as function of C and V , substituting its numerical values:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0\mu\text{F})(30\text{V})^2 = \boxed{2.3\text{mJ}}$$

(b) We express the resonance frequency in terms of L and C, and substitute numerical values:

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10mH)(5.0\mu F)}} = \boxed{0.71kHz}$$

(c) I peak can be expressed in terms of the charge stored in the capacitor and Q_0 can be expressed in terms of the capacitance of the capacitor and the potential difference across the capacitor, using these two, we substitute for Q_0 :

$$I_{peak} = \omega Q_0, \quad Q_0 = CV \Rightarrow I_{peak} = \omega CV$$

$$I_{peak} = 2\pi(712s^{-1})(5.0\mu F)(30V) = \boxed{0.67A}$$

32 - A coil with internal resistance can be modeled as a resistor and an ideal inductor in series. Assume that the coil has an internal resistance of 1.00Ω and an inductance of 400 mH . A $2.00\text{-}\mu\text{ F}$ capacitor is charged to 24.0 V and is then connected across coil. (a) What is the initial voltage across the coil? (b) How much energy is dissipated in the circuit before the oscillations die out? (c) What is the frequency of oscillation the circuit? (Assume the internal resistance is sufficiently small that has no impact on the frequency of the circuit.) (d) What is the quality factor of the circuit?

(a) To find the initial voltage across the coil we can apply the Kirchhoff's loop rule. (b) The total energy lost by the Joule heating is the total energy initially stored in the capacitor. (c) The natural frequency of the circuit is given by $f_0 = 1/2\pi\sqrt{LC}$. (d) The quality factor can be found by its definition.

(a) Applying Kirchhoff's loop rule gives that initial voltage across the coil is $\boxed{24.0V}$

(b) All the energy initially stored in the capacitor will be dissipated as Joule heat in the resistor because the ideal inductor can not dissipate energy as heat:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.00\mu F)(24.0V)^2 = \boxed{0.576mJ}$$

(c) We find the natural frequency of the circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(400mH)(2.00\mu F)}} = \boxed{178Hz}$$

(d) The quality factor definition is $Q = \omega_0 L/R$, we can substitute for ω_0 and simplify, finally we substitute numerical values:

$$Q = \frac{\frac{1}{\sqrt{LC}}L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1.00\Omega}\sqrt{\frac{400mH}{2.00\mu F}} = \boxed{447}$$

33 - An inductor and a capacitor are connected, as shown in Figure 29-30. Initially, the switch is open, the left plate of the capacitor has charge Q_0 . The switch is then closed. (a) Plot both Q versus t and I versus t on the same graph, and explain how it can be seen from these two plots that the current leads the charge by 90° . (b) The expressions for the charge and for the current are given by Equations 29-38 and 29-39, respectively. Use trigonometry and algebra to show that the current leads the charge by 90° .

We can obtain a differential equation for the circuit by applying Kirchhoff's loop rule, letting Q represent the instantaneous charge of the capacitor. Solving this equation we obtain an expression for the charge on the capacitor as a function of time and by differentiating this expression with respect to time, and expression for the current as a function of time.

We apply Kirchhoff's loop rule to a clockwise loop just after the switch is closed:

$$\frac{Q}{C} + L \frac{dI}{dt} = 0$$

Substitute $I = dQ/dt$:

$$L \frac{d^2Q}{dt^2} + \frac{Q}{C} = 0, \text{ hence } \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

The solution to this differential equation is: $Q(t) = Q_0 \cos(\omega t - \delta)$ where $\omega = \sqrt{1/LC}$. Because $Q(0) = Q_0$, and $\delta = 0$ and $Q(t) = Q_0 \cos \omega t$. The current in the circuit is the derivative of Q with respect to t :

$$I = \frac{dQ}{dt} = \frac{d}{dt}[Q_0 \cos \omega t] = -\omega Q_0 \sin \omega t$$

(a) A spreadsheet program was used to plot the following graph showing both the charge on the capacitor and the current in the circuit as functions of time. L , C , and Q_0 were all arbitrarily set equal to one to obtain these graphs. Note that the current leads the charge by one-fourth of a cycle or 90° degrees.

(b) The equation for the current is: $I = -\omega Q_0 \sin \omega t$, we can use the identity between sine and cosine functions: $-\sin \theta = \cos(\theta + \frac{\pi}{2})$. Using this identity we can rewrite the equation for the current:

$$I = -\omega Q_0 \sin \omega t = \boxed{\omega Q_0 \cos\left(\omega t + \frac{\pi}{2}\right)}$$

Therefore, the current leads the charge by 90° degrees.

1.4 Driven RL Circuits

34 - A circuit consists of a resistor, an ideal 1.4-H inductor and an ideal 60-Hz generator, all connected in series. The rms voltage across the resistor is 30 V and the rms voltage across the inductor is 40 V. (a) What is the resistance of

the resistor? (b) What is the peak emf of the generator?

The ratio V_R to V_L can be used to find the resistance of the circuit. For (b) we can use the fact that in a LR circuit, V_L leads V_R by 90 degrees to find the ac input voltage.

(a) We do express the potential differences accross R and L in terms of the common current through these components and express R in thee terms.

$$V_L = IX_L = I\omega L, \quad V_R = IR$$

$$\frac{V_R}{V_L} = \frac{IR}{I\omega L} = \frac{R}{\omega L} \Rightarrow R = \left(\frac{V_R}{V_L}\right) \omega L$$

We substitute numerical values and evaluate R:

$$R = \left(\frac{30V}{40V}\right) 2\pi(60s^{-1})(1.4H) = \boxed{0.40k\Omega}$$

(b) V_R leads V_L by 90 degrees in a LR circuit:

$$V_{peak} = \sqrt{2}V_{rms} = \sqrt{2}\sqrt{V_R^2 + V_L^2}$$

$$V_{peak} = \sqrt{2}\sqrt{(30V)^2 + (40V)^2} = \boxed{71V}$$

35 - A coil that has a resistance of 80.0Ω has an impedance of 200Ω when driven at a frequency of 1.00 kHz . What is the inductance of the coil?

The definition of X_L in a LR circuit can be used to find L .

From the definition of impedance of a coil in terms of its resistance and inductive reactance we can solve for X_L :

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow X_L = \sqrt{Z^2 - R^2}$$

Remember that X_L can be expressed in terms of L , $X_L = 2\pi fL$. If we equate these equations we can solve for L:

$$2\pi fL = \sqrt{Z^2 - R^2} \Rightarrow L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$

Now we only need to substitute numerical values:

$$L = \frac{\sqrt{(200\Omega)^2 - (80.0\Omega)^2}}{2\pi(1.00kHz)} = \boxed{29.2mH}$$

36 - A two conductor transmission line simultaneously carries a superposition of two voltage signals, so the potential difference between the two conductors is given by $V = V_1 + V_2$, where $V_1 = (10.0 \text{ V}) \cos(\omega_1 t)$ and $V_2 = (10.0 \text{ V}) \cos(\omega_2 t)$, where $\omega_1 = 100 \text{ rad/s}$ and $\omega_2 = 10\,000 \text{ rad/s}$. A 1.00 H inductor and a $1.00 \text{ k}\Omega$ shunt resistor are inserted into the transmission line as shown in Figure 29-31. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (a) What is the voltage (V_{out}) at the output of the transmission line? (b) What is the ratio of the low-frequency amplitude to the high-frequency amplitude at the output?

The two output voltage signals can be expressed as the product of the current from each source and $R = 1.00 \text{ k}\Omega$. The impedance definition and given voltage signals can help us for determine the currents due to each source.

(a) We do express the output voltage signals in terms of the potential difference across the resistor:

$$V_{1,\text{out}} = I_1 R \quad V_{2,\text{out}} = I_2 R$$

We need I_1 and I_2 :

$$I_1 = \frac{V_1}{Z_1} = \frac{(10.0 \text{ V}) \cos 100t}{\sqrt{(1.00 \text{ k}\Omega)^2 + [(100 \text{ s}^{-1})(1.00 \text{ H})]^2}} = (9.95 \text{ mA}) \cos 100t$$

$$I_2 = \frac{V_2}{Z_2} = \frac{(10.0 \text{ V}) \cos 10^4 t}{\sqrt{(1.00 \text{ k}\Omega)^2 + [(10^4 \text{ s}^{-1})(1.00 \text{ H})]^2}} = (0.995 \text{ mA}) \cos 10^4 t$$

Substituting these values in voltage expressions:

$$V_{1,\text{out}} = (1.00 \text{ k}\Omega)(9.95 \text{ mA}) \cos 100t = \boxed{(9.95 \text{ V}) \cos 100t}$$

$$V_{2,\text{out}} = (1.00 \text{ k}\Omega)(0.995 \text{ mA}) \cos 10^4 t = \boxed{(0.995 \text{ V}) \cos 10^4 t}$$

where $\omega_1 = 100 \text{ rad/s}$ and $\omega_2 = 10000 \text{ rad/s}$

37 - A coil is connected to a 120-V rms, 60-Hz line. The average power supplied to the coil is 60 W, and the rms current is 1.5 A. Find (a) the power factor, (b) the resistance of the coil, and (c) the inductance of the coil. (d) Does the current lag or lead the voltage? Explain your answer. (e) Support your answer to Part (d) by determining the phase angle.

There is a relationship between the power factor and the average power supplied to the coil $P_{av} = \mathcal{E}_{rms} I_{rms} \cos \delta$. In (b) $P_{av} = I_{rms}^2 R$ this expression gives the needed relationship for R. The resistance of the coil can be found by $X_L = \omega L = R \tan \delta$ this expression relates the resistance, phase angle and inductance. By noting if the circuit is inductive, we can decide if the current leads or lags the voltage.

(a) We express the average power supplied to the coil in terms of the power factor:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos \delta \Rightarrow \cos \delta = \frac{P_{av}}{\mathcal{E}_{rms} I_{rms}}$$

Substituting numerical values:

$$\cos \delta = \frac{60W}{(120V)(1.5A)} = 0.333 = \boxed{0.33}$$

(b) We do express the power supplied by the source in terms of the resistance of the coil and substitute numerical values:

$$P_{av} = I_{rms}^2 R \Rightarrow R = \frac{P_{av}}{I_{rms}^2} = \frac{60W}{(1.5A)^2} = 26.7\Omega = \boxed{27\Omega}$$

(c) We relate the inductive reactance to the resistance and phase angle, solving for L:

$$X_L = \omega L = R \tan \delta \Rightarrow L = \frac{R \tan \delta}{\omega} = \frac{R \tan[\cos^{-1}(0.333)]}{2\pi f}$$

$$L = \frac{(26.7\Omega) \tan(70.5^\circ)}{2\pi(60s^{-1})} = \boxed{0.20H}$$

(d) We just need to evaluate X_L :

$$X_L = (26.7\Omega) \tan(70.5^\circ) = 75.4\Omega$$

The circuite is inductive, hence the current lags the voltage.

(e) From part (a):

$$\delta = \cos^{-1}(0.333) = 72^\circ$$

38 - A 36-mH inductor that has a resistance of 40Ω is connected to an ideal ac voltage source whose output is given by $\epsilon = (345\text{ V}) \cos(150\pi t)$, where t is in seconds. Determine (a) the peak current in the circuit, (b) the peak and rms voltages across the inductor, (c) the average power dissipation, and (d) the peak and average magnetic energy stored in the inductor.

(a) Use $I_{peak} = \mathcal{E}_{peak} / \sqrt{R^2 + (\omega L)^2}$ and $V_{L,peak} = I_{peak} X_L = \omega L I_{peak}$ to find the peak current in the circuit and the peak voltage across the inductor.

(b) Once we've found $V_{L,peak}$ we can find $V_{L,rms}$ using $V_{L,rms} = V_{L,peak} / \sqrt{2}$.

(c) We can use $P_{av} = \frac{1}{2} I_{rms}^2 R$ to find the average power dissipation. (d) $U_{L,peak} = \frac{1}{2} L I_{peak}^2$ to find the peak and average magnetic energy stored in the inductor. The average energy stored in the magnetic field of the inductor can be found using $U_{L,av} = \int P_{av} dt$.

(a) Apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E} - IZ = 0 \Rightarrow I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L)^2}}$$

Substituting numerical values to find for I:

$$I_{peak} = \frac{(345V) \cos(150\pi t)}{\sqrt{(40\Omega)^2 + [(150\pi s^{-1})(36mH)]^2}} = (7.94A) \cos(150\pi t) = \boxed{7.9A}$$

(b) We can find \mathcal{E} as follows:

$$\begin{aligned}\mathcal{E} &= (345V) \cos(150\pi t) \\ V_{L,peak} &= \boxed{345V} \\ V_{L,rms} &= \frac{V_{L,peak}}{\sqrt{2}} = \frac{345V}{\sqrt{2}} = \boxed{244V}\end{aligned}$$

(c) We can relate the average power dissipation to I_{peak} and R :

$$P_{av} = I_{rms}^2 R = \left(\frac{I_{peak}}{\sqrt{2}} \right)^2 R = \frac{1}{2} I_{peak}^2 R$$

Substituting numerical values:

$$P_{av} = \frac{1}{2} (7.94A)^2 (40\Omega) = \boxed{1.3kW}$$

(d) The maximum energy stored in the magnetic field of the inductor can be found as follows:

$$U_{L,peak} = \frac{1}{2} L I_{peak}^2 = \frac{1}{2} (36mH) (7.94A)^2 = \boxed{1.1J}$$

From the definition $U_{L,av} = \frac{1}{T} \int_0^T U(t) dt$ and $U(t) = \frac{1}{2} L [I(t)]^2$. We do substitute $U(t)$ and obtain:

$$U_{L,av} = \frac{L}{2T} \int_0^T [I(t)]^2 dt$$

We evaluate the integral:

$$U_{L,av} = \frac{L}{2T} \left[\frac{1}{2} I_{peak}^2 \right] T = \frac{1}{4} L I_{peak}^2$$

Finally we do substitute numerical values and evaluate:

$$U_{L,av} = \frac{1}{4} (36mH) (7.94A)^2 = \boxed{0.57J}$$

39 - A coil that has a resistance R and an inductance L has a power factor equal to 0.866 when driven at a frequency of 60 Hz. What is the coil's power factor it is driven at 240 Hz?

The relationship between X_L and R define the power factor when the coil is driven at a frequency of 60 Hz and then from the definition of X_L we can relate the inductive reactance at 240 Hz to the inductive reactance at 60 Hz, finally we can use the power factor definition to determine it at 240 Hz.

We use the definition of power factor to relate R and X_L :

$$\cos \delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} \Rightarrow \cos^2 \delta = \frac{R^2}{R^2 + X_L^2}$$

We solve for 60 Hz and substitute for $\cos \delta$:

$$X_L^2(60Hz) = R^2 \left(\frac{1}{\cos^2 \delta} - 1 \right) = R^2 \left(\frac{1}{(0.866)^2} - 1 \right) = \frac{1}{3} R^2$$

We use the definition of X_L :

$$X_L^2(f) = 4\pi f^2 L^2 \text{ and } X_L^2(f') = 4\pi f'^2 L^2$$

Combining these two equations:

$$\frac{X_L^2(f')}{X_L^2(f)} = \frac{4\pi f'^2 L^2}{4\pi f^2 L^2} = \frac{f'^2}{f^2} \Rightarrow X_L^2(f') = \left(\frac{f'}{f} \right)^2 X_L^2(f)$$

Finally we substitute numerical values:

$$X_L^2(240Hz) = \left(\frac{240s^{-1}}{60s^{-1}} \right)^2 X_L^2(60Hz) = 16 \left(\frac{1}{3} R^2 \right) = \frac{16}{3} R^2$$