Chapter 1

Chapter 29

1.1 Alternating-Current Circuits

1.1.1 Conceptual Problems

1 - An ac generator rotates at 60 Hz and has a coil. Determine the time that elapses between successive peak emf values of the coil?
Successive peaks are one-half period apart. Hence the elapsed time between the peaks is $\frac{1}{2}T = \frac{1}{2f} = \frac{1}{2(60^{-1})} = \boxed{8.33 \text{ ms}}$.

2 - When the rms voltage in an ac circuit doubles, the peak voltage is:
(a) doubled, (b) halved, (c) increased by a factor of $\sqrt{2}$, (d) not changed.
A relationship between V and V_{peak} can be used to determine the effect of doubling the rms voltage on the peak voltage.
First express the initial rms voltage in terms of the peak voltage:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

Then expres the doubled rms voltage in terms of the new peak voltage V'_{peak}:

$$2V_{rms} = \frac{V'_{peak}}{\sqrt{2}}$$

After you have done that you have to divide the second of these equations by the first and simplify to obtain:

$$\frac{2V_{rms}}{V_{rms}} = \frac{\frac{V'_{peak}}{\sqrt{2}}}{\frac{V_{peak}}{\sqrt{2}}} \Rightarrow 2 = \frac{V'_{peak}}{V_{peak}}$$

Now you only have to solve for V'_{peak}:

$$V'_{peak} = 2V_{peak} \Rightarrow \boxed{\text{(a)}} \text{ is the correct answer.}$$

3 - Suppose the frequency in the circuit shown in figure 29-27 is doubled, then the inductance in the inductor will (a) double, (b) not change, (c) halve, (d) quadruple.
The inductance of an inductor is independent of the frequency of the circuit and is determined only by its construction. The reactance is dependent on the frequency, hence $\boxed{\text{(b)}}$ is correct.

4 - Suppose the frequency in the circuit shown in figure 29-27 is doubled, then the inductive reactance of the inductor will (a) double, (b) not change, (c) halve, (d) quadruple.
The inductive reactance of an inductor and the frequency are related by $X_L = \omega L$. Therefore, when ω is doubled, X_L will double as well. $\boxed{\text{(a)}}$ is correct.

5 - Suppose the frequency in the circuit shown in figure 29-28 is doubled, then the capacitiive reactance of the circuit will (a) double, (b) not change, (c) halve, (d) quadruple.
The capacitive reactance of a capacitor varies with the frequency and are related by $X_C = \frac{1}{\omega C}$. Therefore, if ω change to its double then X_C will halve. $\boxed{\text{(c)}}$ is correct.

6 - (a) There is a circuit consisting of an ac generator and and ideal inductor, do exist time intervals when the inductor receives energy from the generator? When this happens? (b) Are there time intervals when the inductor supplies energy back to the generator? When this happens? Explain your answers.
The answer to both questions is yes. (a) During current magnitude increase in the inductor, the inductor absorbs power from the generator. (b) During current decrease in the inductor, the inductor supplies power to the generator.

7 - (a) Imagine a circuit consisting of a generator and a capacitor, are there any time intervals when the capacitor receives energy from the generator? When this happens? (b) Does the capacitor supplies power to the generator? When this happens? Explain your answers.
(a) The capacitor absorbs power from the generator while the magnitude of the charge is accumulating on either plate of the capacitor. (b) The capacitor supplies power to the generator whenever the magnitude of the charge on either plate of the capacitor is decreasing.

8 - (a) Demonstrate that the SI unit of inductance multiplied by the SI unit of capacitance is equivalent to seconds squared. (b) Show that the SI unit of inductance divided by the SI unit of resistance is equivalent to seconds.
(a) First substitute the SI units of inductance and capacitance then simplify to obtain:

$$\frac{V \cdot s}{A} \cdot \frac{C}{V} = \frac{s}{\frac{C}{S}} \cdot C = \boxed{s^2}$$

(b) First substitute the SI units of inductance divided by resistance then simplify:

$$\frac{\frac{V\cdot s}{A}}{\Omega} = \frac{\frac{V\cdot s}{A}}{\frac{V}{A}} = \boxed{\text{s}}$$

9 - Imagine the rotation rate is increased in the ac circuit in figure 29-29. Then the rms current (a) increases, (b) does not change, (c) may increase or decrease depending on the magnitude of the original frequency, (d) may increase or decrease depending on the magnitude of the resistance, (e) decreases.
The rms current through the resistor I_{rms} is directly proportional to ω as shown here:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{\mathcal{E}_{peak}}{\sqrt{2}} = \frac{NBA}{\sqrt{2}}\omega$$

therefore, $\boxed{\text{(a)}}$ is correct.

10 - Suppose the inductance is tripled in a circuit consisting solely of a variable inductor and a variable capacitor, how you have to change the capacitance so that the natural frequency of the circuit is unchanged? (a) triple the capacitance, (b) decrease the capacitance to one-third of its original value, (c) you should not change the capacitance, (d) you cannot determine how to change the capacitance from the data given.
The natural frequency of an LC circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

We now express the natural frequencies of the circuit before and after the inductance is tripled:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ and } f_0' = \frac{1}{2\pi\sqrt{L'C'}}$$

We divide the second equation by the first and simplify:

$$\frac{f_0'}{f_0} = \frac{\frac{1}{2\pi\sqrt{L'C'}}}{\frac{1}{2\pi\sqrt{LC}}} = \sqrt{\frac{LC}{L'C'}}$$

We take into account that the natural frequency is unchanged:

$$1 = \sqrt{\frac{LC}{L'C'}} \Rightarrow \frac{LC}{L'C'} = 1 \Rightarrow C' = \frac{L}{L'}C$$

Therefore when the inductance is tripled:

$$C' = \frac{L}{3L}C = \frac{1}{3}C$$

Hence $\boxed{\text{(b)}}$ is the correct answer.

11 - In a circuit consitig only of an ideal inductor and ideal capacitor, how does the maximum energy stored in the capacitor compare to the maximum value stored in the inductor? (a) They are the same and each equal to the total energy stored in the circuit. (b) They are the same and each equal to half of the total energy stored in the circuit. (c) The maximum energy stored in the capacitor is larger than the maximum energy stored in the inductor. (d) The maximum energy stored in the inductor is larger than the maximum energy stored in the capacitor. (e) You cannot compare the maximum energies based on the data given because the ratio of the maximum energies depends on the actual capacitance and inductance values. The maximum energy stored in the electric field ofthe capacitor is given by

$$U_e = \frac{Q^2}{2C}$$

and the maximum energy stored in the magnetic field of the inductor is given by

$$U_m = \frac{LI^2}{2}.$$

That is because energy is conserved in a LC circuit and oscillates between the inductor and the capacitor,

$$U_c = U_m = U_{total},$$

therefore answer $\boxed{\text{(a)}}$ is correct.

12 - Check wheter the propositions are true or false.
(a) A driven series RlC circuit that has a high Q factor has a narrow resonance curve.
(b) A circuit consists of an inductor, a resistor and a capacitor, all connected in series. If the resistance of the resistor is doubled, the natural frequency of the circuit remains the same.
(c) At resonance, the impedance of a driven series RLC combination equals the resistance R.
(d) At resonance, the current in a driven series RLC circuit is in phase with the voltage applied tho the combination.

(a) True. The Q factor and the width of the resonance curve at half power are related according to

$$Q = \frac{\omega_0}{\Delta\omega}$$

this means that they are inversely proportional to each other.
(b) True. Circuit's natural frequency depends only on the inductance L of the inductor and the capacitance C of the capacitor and is given by

$$\omega = \frac{1}{\sqrt{LC}}.$$

(c) True. The impedance of and RLC circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

At resonance

$$X_L = X_C \text{ and so } Z = R.$$

(d) True. The phase angle δ is related to X_L and X_C according to

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

At resonance $X_L = X_C$ and so $\delta = 0$.

13 - Check wether propositions are true or false.
(a) The power factor of a driven series RLC circuit is close to zero when near resonance.
(b) A driven series RLC circuit's power factor doesn't depend on the value of the resistance.
(c) A driven series RLC circuit's resonance frequency doesn't depend on the value of the resistance.
(d) The peak current of a driven series RLC circuit doesn't depend on the capacitance or the inductance when at resonance.
(e) For frequencies below the resonant frequency, the capacitive reactance of a driven series RLC circuit is larger than the inductive reactance.
(f) For frequencies below the resonant frequency of a driven series RLC circuit, the phase of the current leads ahead the phase of the applied voltage.

(a) False. The power factor given by

$$\cos\delta = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}},$$

is close to 1.
(b) False. The power factor is given by

$$\cos\delta = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}}.$$

(c) True. The resonance frequency for a driven series RLC circuit depends only on L and C is given by

$$\omega_{res} = \frac{1}{\sqrt{LC}}$$

(d) True. At resonance $X_L - X_C = 0$ and so $Z = R$ and the peak current is given by $I_{peak} = V_{app,peak}/R$.
(e) True. The capacitive reactance varies inversely with the driving frequency nd the inductive reactance varies directly with the driving frequency, hance at

frequencies well below the resonance frequency the capacitive reactance is larger than the inductive reactance.
(f) True. For frequencies below the resonant frequency, the circuit is more capacitive than inductive and the phase constant *phi* is negative. This means that the current leads the applied voltage.

14 - How can two different radio stations be heard at the same specific frequency when a receiver is tuned it, this situation often occurs while driving between two cities?
The power curves received by the radio have widht, hence the two frequencies coming from the radio stations can overlap as a result and you can receive signals from both stations.

15 - Check wether is true or false.
(a) The power factor is close to zero at frequencies much higher than or much lower than the resonant frequency of a driven series RLC circuit.
(b) The larger the resonance width of a driven series RLC circuit is, the larger the Q factor for the circuit becomes.
(c) The larger the resistance of a driven series RLC circuit is, the larger the resonance width for the circuit is.

(a) True. The power factor is given by

$$\cos\delta = \frac{R}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}},$$

therefore for values of ω that are much higher or much lower than the resonant frequency, the term in parentheses becomes very large and $\cos\delta$ approaches to zero.
(b) False. When the resonance curve is reasonably narrow, the Q factor can be approximated by

$$Q = \omega_0/\Delta\omega.$$

Hence a large value for Q corresponds to a narrow resonance curve.
(c) True. See figure 29-21

16 - Suppose an ideal transformer has N_1 turns on its primary and N_2 turn on its secondary. The average power delivered to a load resistance R connected across the secondary is P_2 whilst the primary rms voltage is V_1. The rms current in the primary windings can then be expressed as (a) P_2/V_1, (b) $(N_1/N_2)(P_2/V_1)$, (c) $(N_2/N_1)(P_2/V_1)$, (d) $(N_2/N_1)2(P_2/V_1)$.
Subscript 1 and 2 correspond to primary and secondary respectively. We assume no loss of power in the transformer, we can equate the power in the primary circuit to the power in the secondary circuit and solve for the rms current in th

primary windings.
Assuming no power loss in the transformer:

$$P_1 = P_2$$

Substituting for P we obtain:

$$I_{1,rms}V_{1,rms} = I_{2,rms}V_{2,rms}$$

We solve for I and simplify:

$$I_{1,rms} = \frac{I_{2,rms}V_{2,rms}}{V_{1,rms}} = \frac{P_2}{V_{1,rms}}$$

hence answer $\boxed{\text{(a)}}$ is correct.

17 - Check wether propositions aret true or false: (a) Transformers are used to change frequency.
(b) Transformers are used to change voltage.
(c) If a transformer steps up the current, it must step down the voltage.
(d) A step-up transformer, steps down the current.
(e) The standard household wall-outlet voltage in Europe is 220 V, about twice that used in the United States. If a European traveler wants her hair dryer to work properly in the United States, she should use a transformer that has more windings in its secondary coil than in its primary coil.
(f) The standard household wall-outlet voltage in Europe is 220 V, about twice that used in the United States. If an American traveler wants his electric razor to work properly in Europe, he should use a transformer that steps up the current.

(a) False. A transformer is a device used to raise or lower the voltage in a circuit.
(b) True. A transformer is a device used to raise or lower the voltage in a circuit.
(c) True. If energy is to be conserved, the product of the current and voltage must be constant.
(d) True. Because the product of current and voltage in the primary and secondary circuits is the same, increasing the current in the secondary results in a lowering (or stepping down) of the voltage.
(e) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the voltage in order to make her hair dryer work properly.
(f) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the current (and decrease the voltage) in order to make his razor work properly.

1.1.2 Estimation and Approximation

18 - Resistance and inductive reactance are included in the impedances of motors, electromagnets and transformers. Suppose that phase of the current to

a large industrial plant lags the phase of the applied voltage by 25° when the plant on full operation using 2.3 MW of power. The power is supplied to the plant from a substation 4.5 km from the plant; the 60 Hz rms line voltage at the plant is 40 kV. The resistance of the transmission line from the substation to the plant is 5.2 Ω. The cost per kilowatt-hour to the company that owns the plant is $0.14, and the plant pays only for the actual energy used. (a) Estimate the resistance and inductive reactance of the plant's total load. (b) Estimate the rms current in the power lines and the rms voltage at the substation. (c) How much power is lost in transmission? (d) Suppose that the phase that the current lags the phase of the applied voltage is reduced to 18° by adding a bank of capacitors in series with the load. How much money would be saved by the electric utility during one month of operation, assuming the plant operates at full capacity for 16 h each day? (e) What must be the capacitance of this bank of capacitors to achieve this change in phase angle?
We can find the resistance and inductive reactance of the plant's total load from the impedance of the load and the phase constant. The current in the power lines can be found from the total impedance of the load the potential difference across it and the rms voltage at the substation by applying Kirchhoff's loop rule to the substation-transmission wires-load circuit. The power lost in transmission can be found from $P_{trans} = I_{rms}^2 R_{trans}$. We can find the cost savings by finding the difference in the power lost in transmission when the phase angle is reduced to 18°. Finally, we can find the capacitance that is required to reduce the phase angle to 18° by first finding the capacitive reactance using the definition of $\tan\delta$ and then applying the definition of capacitive reactance to find C.

(a) First we relate the resistance and inductive reactance of the plant's total load to Z and δ:

$$R = Z\cos\delta \text{ and } X_L = Z\sin\delta$$

We then express Z in terms of the rms current I_{rms} in the power lines and the rms voltage $\mathcal{E}_{rms}$ at the plant:

$$Z = \frac{\mathcal{E}_{rms}}{I_{rms}}$$

after express teh power delivered to the plant in terms of $\mathcal{E}_{rms}, I_{rms}$ and δ and solve for I_{rms} :

$$P_{av} = \mathcal{E}_{rms} I_{rms}\cos\delta \text{ and } I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms}\cos\delta}$$

then substitute to obtain:

$$Z = \frac{\mathcal{E}_{rms}^2\cos\delta}{P_{av}}$$

Afterwards substitute numerical values and evaluate Z:

$$Z = \frac{(40kV)^2\cos 25°}{2.3MW} = 630\Omega$$

Substitute numerical values and evaluate for R and X_L :

$R = (630\Omega)\cos 25^\circ = 571\Omega = \boxed{0.57k\Omega}$ and $X_L = (630\Omega)\sin 25^\circ = 266\Omega = \boxed{0.27k\Omega}$

(b) Find the current in the power lines:

$$I_{rms} = \frac{2.3MW}{(40kV)\cos 25^\circ} = 63.4A = \boxed{63\text{A}}$$

Then apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E}_{sub} - I_{rms}R_{trans} - I_{rms}Z_{tot} = 0$$

Solve for $\mathcal{E}_{sub}$:

$$\mathcal{E}_{sub} = I_{rms}(R_{trans} + Z_{tot})$$

Substitute numerical values and evaluate $\mathcal{E}_{sub}$:

$$\mathcal{E}_{sub} = (63.4A)(5.2\Omega + 630\Omega) = \boxed{40.3\text{kV}}$$

(c) The power lost in transmission is:

$$P_{trans} = I_{trans}^2 R_{trans} = (63.4A)^2(5.2\Omega) = 20.9kW = \boxed{21\text{kW}}$$

(d) Express the cost savings ΔC in terms of the difference in energy consumption $P_{25^\circ} - P_{18^\circ}\Delta t$ and the per-unit cost u of the energy:

$$\Delta C = (P_{25^\circ} - P_{18^\circ})\Delta t u$$

Express the power lost in transmission when $\delta = 18^\circ$:

$$P_{18^\circ} = I_{18^\circ}^2 R_{trans}$$

Then find the current in the transmission lines when $\delta = 18^\circ$:

$$I_{18^\circ} = \frac{2.3MW}{(40kV)\cos 18^\circ} = 60.5A$$

Evaluate P_{18° :

$$P_{18^\circ} = (60.5A)^2(5.2\Omega) = 19kW$$

Substitute numerical values and evaluate ΔC :

$$\Delta C = (20.9kW - 19kW)(16h/d)\left(30\frac{d}{\text{month}}\right)\left(\frac{\$0.14}{kW \cdot h}\right) = \boxed{\$128}$$

(e) The required capacitance is given by:

$$C = \frac{1}{2\pi f X_C}$$

We then relate the new phase δ to the inductive reactance X_L, the reactance due to the added capacitance X_C, and the resistance of the load R:

$$\tan\delta = \frac{X_L - X_C}{R} \Rightarrow X_C = X_L - R\tan\delta$$

We substitute for X_C :

$$C = \frac{1}{2\pi f(X_L R\tan\delta)}$$

We then substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi(60s^{-1})(266\Omega - (571\Omega)\tan 18°)} = \boxed{33\mu F}$$

1.2 Alternating Current in Resistors, Inductors, and Capacitors

19 - Suppose we have a 100-W light bulb in a standard 120-V-rms socket. Find (a) the rms current, (b) the peak current, and (c) the peak power.

We use $P_{av} = \mathcal{E}_{rms} I_{rms}$ to find I_{rms}, $I_{peak} = \sqrt{2} I_{rms}$ to find I_{peak}, and $P_{peak} = I_{peak}\mathcal{E}_{peak}$ to find P_{peak}.
(a) We find the relationship of average power delivered by the source to the rms voltage across the bulb and the rms current through it:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \Rightarrow I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms}}$$

We then substitute numerical values and evaluate I_{rms} :

$$I_{rms} = \frac{100W}{120V} = 0.8333A = \boxed{0.833\text{A}}$$

(b) We do express I_{peak} in terms of I_{rms} :

$$I_{peak} = \sqrt{2} I_{rms}$$

substitute for I_{rms} and evaluate I_{peak} :

$$I_{peak} = \sqrt{2}(0.8333A) = 1.1785A = \boxed{1.18\text{A}}$$

(c) Afterwards we need to express the maximum power in terms of the maximum voltage and maximum current, and substitute numerical values evaluating P_{peak} :

$$P_{peak} = I_{peak}\mathcal{E}_{peak}$$

$$P_{peak} = (1.1785A)\sqrt{2}(120V) = \boxed{200\text{W}}$$

20 - Suppose we have a circuit breaker for a current of 15 A rms at a voltage of 120 V rms. (a) What is the largest peak current that the breaker can carry? (b) What is the maximum average power that can be supplied by this circuit?

We can use $I_{peak} = \sqrt{2} I_{rms}$ to find the largest peak current that the breaker can carry and $P_{av} = I_{rms} V_{rms}$ to find the average power supplied by the circuit. (a) We do express I_{peak} in terms of I_{rms} and relate the average power to the rms current and voltage:

$$I_{peak} = \sqrt{2} I_{rms} = \sqrt{2}(15A) = \boxed{\text{21A}}$$

$$P_{av} = I_{rms} V_{rms} = (15A)(120V) = \boxed{\text{1.8kW}}$$

21 - What is the reactance of a 1.00mH inductor at (a) 60 Hz, (b) 600 Hz, and (c) 6.00 kHz?

Utilize $X_L = \omega L$ to determine the reactance of the inductor at any frequency. We do so by expressing the inductive reactance as a function of f :

$$X_L = \omega L = 2\pi f L$$

(a) At $f = 60Hz$:

$$X_L = 2\pi(60s^{-1})(1mH) = \boxed{0.38\Omega}$$

(b) At $f = 600Hz$:

$$X_L = 2\pi(600s^{-1})(1mH) = \boxed{3.77\Omega}$$

(c) At $f = 6kHz$:

$$X_L = 2\pi(6ks^{-1})(1mH) = \boxed{37.7\Omega}$$

22 - An inductor has a reactance of 100 Ω at 80 Hz. (a) What is its inductance? (b) What is its reactance at 160 Hz?

We can use $X_L = \omega L$ to find the inductance of the inductor at any frequency. (a) We do relate the reactance or the inductor the its inductance and solve for L doing its evaluation:

$$X_L = \omega L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f}$$

$$L = \frac{100\Omega}{2\pi(80s^{-1})} = 0.199H = \boxed{\text{0.20 H}}$$

(b) At 160 Hz:

$$X_L = 2\pi(160s^{-1})(0.199H) = \boxed{0.20k\Omega}$$

23 - Consider a $10\mu F$ capacitor, at what frequency would its reactance equal a reactance of an 10mH inductor?

If we equate reactances of the capacitor and inductor we then can solve for the frequency.
We do express the reactance of the inductor, then we express the reactance of the capacitor equating these reactances:

$$X_L = \omega L = 2\pi f L$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$2\pi f L = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

We finally substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{(10\mu F)(1mH)}} = \boxed{\text{1.6kHz}}$$

24 - What is the reactance of a 1.00nF capacitor at (a) 60.0 Hz, (b) 6.00 kHz, and (c) 6.00 MHz?

We can use $X_C = 1/\omega C$ to find the reactance of the capacitor at any frequency. By expressing the capacitive reactance as a function of f :

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

(a) At $f = 60Hz$:

$$X_C = \frac{1}{2\pi(60s^{-1})(1nF)} = \boxed{2.65M\Omega}$$

(b) At $f = 6.00kHz$:

$$X_C = \frac{1}{2\pi(6.00kHz)(1nF)} = \boxed{26.5k\Omega}$$

(c) At $f = 6.00MHz$:

$$X_C = \frac{1}{2\pi(6.00MHz)(1nF)} = \boxed{26.5\Omega}$$

25 - A 20-Hz ac generator that produces a peak emf of 10 V is connected to a $20 - \mu F$ capacitor. Find (a) the peak current and (b) the rms current.

We can use $I_{peak} = \mathcal{E}_{peak}/X_C$ and $X_C = 1/\omega C$ to express I_{peak} as a function of $\mathcal{E}_{peak}, f$ and C. Once we evaluate I_{peak}, we can use $I_{rms} = I_{peak}/\sqrt{2}$ to find I_{rms}.
We begin by expressing I_{peak} in terms of $\mathcal{E}_{peak}$ and X_C

$$I_{peak} = \frac{\mathcal{E}_{peak}}{X_C}$$

We then express the capacitive reactance and subsitute for X_C and simplify:

$$I_{peak} = 2\pi f C \mathcal{E}_{peak}$$

(a) Substitute numerical values and evaluate I_{peak} :

$$I_{peak} = 2\pi(20s - 1)(20\mu F)(10V) = 25.1mA = \boxed{\text{25mA}}$$

(b) Express $I_{rms} = \frac{I_{peak}}{\sqrt{2}} = \frac{25.1mA}{\sqrt{2}} = 18mA$

26 - At what frequency is the reactance of a 10-μF capacitor (a) 1.00 Ω, (b) 100 Ω, and (c) 10.0 $m\Omega$?

We can use $X_C = 1/\omega C = 1/2\pi f C$ to relate the reactance of the capacitor to the frequency:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi C X_C}$$

(a) Find f when $X_C = 1.00\Omega$:

$$f = \frac{1}{2\pi(10\mu F)(1.00\Omega)} = \boxed{\text{16kHz}}$$

(b) Find f when $X_C = 100\Omega$:

$$f = \frac{1}{2\pi(10\mu F)(100\Omega)} = \boxed{\text{0.16kHz}}$$

(c) Find f when $X_C = 10.0m\Omega$:

$$f = \frac{1}{2\pi(10\mu F)(10.0m\Omega)} = \boxed{\text{1.6MHz}}$$

27 - Suppose a circuit consists of two ideal ac generators and a 25-Ω resistor, all connected in series. The potential difference across the terminals of one of the generators is given by $V_1 = (5.0V)\cos(\omega t - \alpha)$, and the potential difference across the terminals of the other generator is given by $V_2 = (5.0V)\cos(\omega t + \alpha)$,

where $\alpha = \pi/6$. (a) Use Kirchhoff's loop rule and a trigonometric identity to find the peak current in the circuit. (b) Use a phasor diagram to find the peak current in the circuit. (c) Find the current in the resistor if $\alpha = \pi/4$ and the amplitude of V_2 is increased from 5.0 V to 7.0 V.

We can find the sum of the phasors V_1 and V_2 using this trigonometric identity

$$\cos\theta + \cos\phi = 2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$$

then we use this sum to express I as a function of time. In (b) we use a phasor diagram to obtain the same result and in (c) we use the phasor diagram appropriate to the given voltages to express the current as a function of time.
(a) Apply the Kirchhoff's loop rule to the circuit and solve for I:

$$V_1 + V_2 - IR = 0$$

$$I = \frac{V_1 + V_2}{R}$$

Then we use the trigonometric identity described above to find $V_1 + V_2$:

$$V_1 + V_2 = (5.0V)[\cos(\omega t - \alpha) + \cos(\omega t + \alpha)] = (5V)[2\cos\frac{1}{2}(2\omega t)\cos\frac{1}{2}(-2\alpha)] =$$

$$(10V)\cos\frac{\pi}{6}\cos\omega t = (8.66V)\cos\omega t$$

We substitute for $V_1 + V_2$ and R to obtain:

$$I = \frac{(8.66V)\cos\omega t}{25\Omega} = (0.346A)\cos\omega t = (0.35A)\cos\omega t$$

$$I_{peak} = \boxed{0.35\text{A}}$$

(b) We express the magnitude of the current in R :

$$|I| = \frac{|\vec{V}|}{R}$$

The phasor diagram for the voltages is shown. We can find $\vec{V}$ using vector addition:

$$|\vec{V}| = 2|\vec{V}_1|cos30^\circ = 2(5.0V)\cos 30^\circ = 8.66V$$

We then substitute for $|\vec{V}|$ and R to obtain: $|I| = \frac{8.66V}{25\Omega} = 0.346A$ and $I = (0.35A)\cos\omega t$ where $I_{peak} = \boxed{0.35\text{A}}$ (c) The phasor diagram is shown. Note that the phase angle between V_1 and V_2 is now 90°. We use the Pythagorean theorem to find $|\vec{V}|$:

$$|\vec{V}| = \sqrt{|\vec{V}_1|^2 + |\vec{V}_2|^2} = \sqrt{(5.0V)^2 + (7.0V)^2} = 8.60V$$

Then we express I as a function of t:

$$I = \frac{|\vec{V}|}{R}\cos(\omega t + \delta)$$

where $\delta = 45° - (90° - \alpha) = \alpha - 45° = \tan^{-1}\frac{7.0V}{5.0V} - 45° = 9.462° = 0.165rad$
Finally substitute numerical values and evaluate I:

$$I = \frac{8.60V}{25\Omega}\cos(\omega t + 0.165rad) = \boxed{(0.34A)\cos(\omega t + 0.17rad)}$$

1.3 Undriven Circuits Containing Capacitors, Resistors and Inductors

28 - (a) Show that 1 LC has units of inverse seconds by substituting SI units for inductance and capacitance into the expression. (b) Show that ω 0 L/R (the expression for the Q-factor) is dimensionless by substituting SI units for angular frequency, inductance, and resistance into the expression.
We begin by substituting the units of the various physical quantities in $1/\sqrt{LC}$ and $Q = \omega_0 L/R$ to establish their units.
(a) We do substitute the units for ω_0, L and C in the expression $1/\sqrt{LC}$ and then simplify:

$$\frac{1}{\sqrt{H \cdot F}} = \frac{1}{\sqrt{(\Omega \cdot s)(\frac{s}{\Omega})}} = \frac{1}{\sqrt{s^2}} = \boxed{s^{-1}}$$

(b) We do substitute the units for ω_0, L and R in the expression $Q = \omega_0 L/R$ and simplify:

$$\frac{\frac{1}{s} \cdot \frac{V \cdot s}{A}}{\frac{V}{A}} = 1 \Rightarrow \boxed{\text{unitless}}$$

29 - (a) What is the period of oscillation of an LC circuit consisting of an ideal 2.0-mH inductor and a 20- μ F capacitor? (b) A circuit that oscillates consists solely of an 80- μ F capacitor and a variable ideal inductor. What inductance is needed in order to tune this circuit to oscillate at 60 Hz?
$T = 2\pi/\omega$ and $\omega = 1/\sqrt{LC}$ can be used to relate T, f to L and C.
(a) We express the period of oscillation:

$$T = \frac{2\pi}{\omega}$$

Taking into account that for a LC circuit $\omega = 1/\sqrt{LC}$, we substitute for ω obtaining:

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{(2.0mH)(20\mu F)} = \boxed{1.3\text{ms}}$$

From the equation for ω we solve for L:

$$L = \frac{T^2}{4\pi^2 C} = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(60^{-1})(80\mu F)} = \boxed{88\text{mH}}$$

30 - An LC circuit has capacitance C 0 and inductance L. A second LC circuit has capacitance 12 C 0 and inductance 2L, and a third LC circuit has capacitance 2C 0 and inductance 12 L. (a) Show that each circuit oscillates with the same frequency. (b) In which circuit would the peak current be greatest if the peak voltage across the capacitor in each circuit was the same?

The expression $f_0 = 1/2\pi\sqrt{LC}$ can be used for the resonance frequency of a LC circuit showing that each circuit oscillates with the same frequency. For (b) we can use $I_{peak} = \omega Q_0$, where Q_0 is the charge of the capacitor at time zero, and the definition of capacitance $Q_0 = CV$ to express I_{peak} in terms of ω, C, V.
We do express the resonance frequency for a LC circuit

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(a) We separate the product of L and C for each circuit:

$$\text{Circuit 1:} L_1C_1 = L_1C_0,$$

$$\text{Circuit 2:} L_2C_2 = (2L_1)(\frac{1}{2}C_0) = L_1C_1,$$

$$\text{Circuit 3:} L_3C_3 = (\frac{1}{2}L_1)(2C_0) = L_1C_1,$$

$$L_1C_1 = L_2C_2 = L_3C_3$$

Therefore, the resonance frequencies of the three circuits are equal.
(b) We do express the I peak in terms of the charge stored in the capacitor and Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor, then we substitute:

$$I_{peak} = \omega Q_0, \quad Q_0 = CV \Rightarrow I_{peak} = \omega CV$$

We have I peak directly proportional to C when ω and V are held constant. Hence the circuit with a capacitance of $2C_0$ has the greatest peak current.

31 - A 5.0- μ F capacitor is charged to 30 V and is then connected across an ideal 10-mH inductor. (a) How much energy is stored in the system? (b) What is the frequency of oscillation of the circuit? (c) What is the peak current in the circuit?

The energy stored in the electric field of the capacitor can be found using $U = \frac{1}{2}CV^2$, the frecuency can be found by these relations $\omega_0 = 2\pi f_0 = 1/\sqrt{LC}$, and I peak can be determined by using these relations $I_{peak} = \omega Q_0$ and $Q_0 = CV$.
(a) We express the energy stored in the system as function of C and V, substituting its numerical values:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0\mu F)(30V)^2 = \boxed{2.3\text{mJ}}$$

(b) We express the resonance frequency in terms of L and C, and substitute numerical values:

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10mH)(5.0\mu F)}} = \boxed{0.71\text{kHz}}$$

(c) I peak can be expressed in terms of the charge stored in the capacitor and Q_0 can be expressed in terms of the capacitance of the capacitor and the potential difference accross the capacitor, using these two, we substitute for Q_0:

$$I_{peak} = \omega Q_0, \quad Q_0 = CV \Rightarrow I_{peak} = \omega CV$$

$$I_{peak} = 2\pi(712s^{-1})(5.0\mu F)(30V) = \boxed{0.67\text{A}}$$

32 - A coil with internal resistance can be modeled as a resistor and an ideal inductor in series. Assume that the coil has an internal resistance of 1.00 Ω and an inductance of 400 mH. A 2.00- μ F capacitor is charged to 24.0 V and is then connected across coil. (a) What is the initial voltage across the coil? (b) How much energy is dissipated in the circuit before the oscillations die out? (c) What is the frequency of oscillation the circuit? (Assume the internal resistance is sufficiently small that has no impact on the frequency of the circuit.) (d) What is the quality factor of the circuit?

(a) To find the initial voltage across the coil we can apply the Kirchhoff's loop rule. (b) The total energy lost by the Joule heating is the total energy initially stored in the capacitor. (c) The natural frequency of the circuit is given by $f_0 = 1/2\pi\sqrt{LC}$. (d) The quality factor can be found by its definition.
(a) Applying Kirchhoff's loop rule gives that initial voltage across the coil is $\boxed{24.0\text{V}}$
(b) All the energy initially stored in the capacitor will be dissipated as Joule heat in the resistor because the ideal inductor can not dissipate energy as heat:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.00\mu F)(24.0V)^2 = \boxed{0.576\text{mJ}}$$

(c) We find the natural frequency of the circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(400mH)(2.00\mu F)}} = \boxed{178\text{Hz}}$$

(d) The quality factor definition is $Q = \omega_0 L/R$, we can substitute for ω_0 and simpify, finally we substitute numerical values:

$$Q = \frac{\frac{1}{\sqrt{LC}}L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1.00\Omega}\sqrt{\frac{400mH}{2.00\mu F}} = \boxed{447}$$

33 - An inductor and a capacitor are connected, as shown in Figure 29-30. Initially, the switch is open, the left plate of the capacitor has charge Q 0 . The switch is then closed. (a) Plot both Q versus t and I versus t on the same graph, and explain how it can be seen from these two plots that the current leads the charge by 90o. (b) The expressions for the charge and for the current are given by Equations 29-38 and 29-39, respectively. Use trigonometry and algebra to show that the current leads the charge by 90o.

We can obtain a differential equation for the circuit by applying Kirchhoff's loop rule, letting Q represent the instantaneous charge of the capacitor. Solving this equation we obtain an expression for the charge on the capacitor as a function of time and by differentiating this expression with respect to time, and expression for the current as a function of time.
We apply Kirchhoff's loop rule to a clockwise loop just after the switch is closed:

$$\frac{Q}{C} + L\frac{dI}{dt} = 0$$

Substitute $I = dQ/dt$:

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0, \text{ hence } \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

The solution to this differential equation is: $Q(t) = Q_0 \cos(\omega t - \delta)$ where $\omega = \sqrt{1/LC}$. Because $Q(0) = Q_0$, and $\delta = 0$ and $Q(t) = Q_0 \cos \omega t$. The current in the circuit is the derivative of Q with respect to t:

$$I = \frac{dQ}{dt} = \frac{d}{dt}[Q_0 \cos \omega t] = -\omega Q_0 \sin \omega t$$

(a) A spreadsheet program was used to plot the following graph showing both the charge on the capacitor and the current in the circuit as functions of time. L, C, and Q 0 were all arbitrarily set equal to one to obtain these graphs. Note that the current leads the charge by one-fourth of a cycle or 90 degrees.
(b) The equation for the current is: $I = -\omega Q_0 \sin \omega t$, we can use the identity between sine and cosine functions: $-sin\theta = \cos\left(\theta + \frac{\pi}{2}\right)$. Using this identity we can rewrite the equation for the current:

$$I = -\omega Q_0 \sin \omega t = \boxed{\omega Q_0 \cos\left(\omega t + \frac{\pi}{2}\right)}$$

Therefore, the current leads the charge by 90 degrees.

1.4 Driven RL Circuits

34 - A circuit consists of a resistor, an ideal 1.4-H inductor and an ideal 60-Hz generator, all connected in series. The rms voltage across the resistor is 30 V and the rms voltage across the inductor is 40 V. (a) What is the resistance of

the resistor? (b) What is the peak emf of the generator?

The ratio V_R to V_L can be used to find the resistance of the circuit. For (b) we can use the fact that in a LR circuit, V_L leads V_R by 90 degrees to find the ac input voltage.
(a) We do express the potential differences accross R and L in terms of the common current through these components and express R in thee terms.

$$V_L = IX_L = I\omega L, \quad V_R = IR$$

$$\frac{V_R}{V_L} = \frac{IR}{I\omega L} = \frac{R}{\omega L} \Rightarrow R = \left(\frac{V_R}{V_L}\right)\omega L$$

We substitute numerical values and evaluate R:

$$R = \left(\frac{30V}{40V}\right) 2\pi(60s^{-1})(1.4H) = \boxed{0.40k\Omega}$$

(b) V_R leads V_L by 90 degrees in a LR circuit:

$$V_{peak} = \sqrt{2}V_{rms} = \sqrt{2}\sqrt{V_R^2 + V_L^2}$$

$$V_{peak} = \sqrt{2}\sqrt{(30V)^2 + (40V)^2} = \boxed{\text{71V}}$$

35 - A coil that has a resistance of 80.0 Ω has an impedance of 200 Ω when driven at a frequency of 1.00 kHz. What is the inductance of the coil?

The definition of X_L in a LR circuit can be used to find L.
From the definition of impedance of a coil in terms of its resistance and inductive reactance we can solve for X_L:

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow X_L = \sqrt{Z^2 - R^2}$$

Remember that X_L can be expressed in terms of L, $X_L = 2\pi f L$. If we equate these equations we can solve for L:

$$2\pi f L = \sqrt{Z^2 - R^2} \Rightarrow L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$

Now we only need to substitute numerical values:

$$L = \frac{\sqrt{(200\Omega)^2 - (80.0\Omega)^2}}{2\pi(1.00kHz)} = \boxed{\text{29.2mH}}$$

36 - A two conductor transmission line simultaneously carries a superposition of two voltage signals, so the potential difference between the two conductors is given by V = V 1 + V 2 , where V 1 = (10.0 V) cos(ω_1t) and V 2 = (10.0 V) cos(ω_2t), where ω_1 = 100 rad/s and ω_2 = 10 000 rad/s. A 1.00 H inductor and a 1.00 kΩ shunt resistor are inserted into the transmission line as shown in Figure 29-31. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (a) What is the voltage (V out) at the output of the transmission line? (b) What is the ratio of the low-frequency amplitude to the high-frequency amplitude at the output?

The two output voltage signals can be expressed as the product of the current from each source and $R = 1.00k\Omega$. The impedance definition and given voltage signals can help us for determine the currents due to each source.
(a) We do express the output voltage signals in terms of the potential difference across the resistor:

$$V_{1,out} = I_1 R \quad V_{2,out} = I_2 R$$

We need I_1 and I_2:

$$I_1 = \frac{V_1}{Z_1} = \frac{(10.0V)\cos 100t}{\sqrt{(1.00k\Omega)^2 + [(100s^{-1})(1.00H)]^2}} = (9.95mA)\cos 100t$$

$$I_2 = \frac{V_2}{Z_2} = \frac{(10.0V)\cos 10^4 t}{\sqrt{(1.00k\Omega)^2 + [(10^4 s^{-1})(1.00H)]^2}} = (0.995mA)\cos 10^4 t$$

Substituting these values in voltage expressions:

$$V_{1,out} = (1.00k\Omega)(9.95mA)\cos 100t = \boxed{(9.95V)\cos 100t}$$

$$V_{2,out} = (1.00k\Omega)(0.995mA)\cos 10^4 t = \boxed{(0.995V)\cos 10^4 t}$$

where $\omega_1 = 100rad/s$ and $\omega_2 = 10000rad/s$

37 - A coil is connected to a 120-V rms, 60-Hz line. The average power supplied to the coil is 60 W, and the rms current is 1.5 A. Find (a) the power factor, (b) the resistance of the coil, and (c) the inductance of the coil. (d) Does the current lag or lead the voltage? Explain your answer. (e) Support your answer to Part (d) by determining the phase angle.

There is a relationship between the power factor and the average power supplied to the coil $P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta$. In (b) $P_{av} = I_{rms}^2 R$ this expression gives the needed relationship for R. The resistance of the coil can be found by $X_L = \omega L = R\tan\delta$ this expression relates the resistance, phase angle and inductance. By noting if the circuit is inductive, we can decide if the current leads or lags the voltage.
(a) We express the average power supplied to the coil in terms of the power factor:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta \Rightarrow \cos\delta = \frac{P_{av}}{\mathcal{E}_{rms} I_{rms}}$$

Substituting numerical values:

$$\cos\delta = \frac{60W}{(120V)(1.5A)} = 0.333 = \boxed{0.33}$$

(b) We do express the power supplied by the source in terms of the resistance of the coil and substitute numerical values:

$$P_{av} = I_{rms}^2 \Rightarrow R = \frac{P_{av}}{I_{rms}^2} = \frac{60W}{(1.5A)^2} = 26.7\Omega = \boxed{27\Omega}$$

(c) We relate the inductive reactance to the resistance and phase angle, solving for L:

$$X_L = \omega L = R\tan\delta \Rightarrow L = \frac{R\tan\delta}{\omega} = \frac{R\tan[\cos^{-1}(0.333)]}{2\pi f}$$

$$L = \frac{(26.7\Omega)\tan(70.5^\circ)}{2\pi(60s^{-1})} = \boxed{0.20\text{H}}$$

(d) We just need to evaluate X_L:

$$X_L = (26.7\Omega)\tan(70.5^\circ) = 75.4\Omega$$

The circuite is inductive, hence the current lags the voltage.
(e) From part (a):

$$\delta = \cos^{-1}(0.333) = 72^\circ$$

38 - A 36-mH inductor that has a resistance of 40 Ω is connected to an ideal ac voltage source whose output is given by ϵ = (345 V) cos(150 π t), where t is in seconds. Determine (a) the peak current in the circuit, (b) the peak and rms voltages across the inductor, (c) the average power dissipation, and (d) the peak and average magnetic energy stored in the inductor.

(a) Use $I_{peak} = \mathcal{E}_{peak}/\sqrt{R^2 + (\omega L)^2}$ and $V_{L,peak} = I_{peak}X_L = \omega L I_{peak}$ to find the peak current in the circuit and the peak voltage across the inductor. (b) Once we've found $V_{L,peak}$ we can find $V_{L,rms}$ using $V_{L,rms} = V_{L,peak}/\sqrt{2}$. (c) We can use $P_{av} = \frac{1}{2}I_{rms}^2 R$ to find the average power dissipation. (d) $U_{L,peak} = \frac{1}{2}LI_{peak}^2$ to find the peak and average magnetic energy stored in the inductor. The average energy stored in the magnetic field of the inductor can be found using $U_{L,av} = \int P_{av}dt$.
(a) Apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E} - IZ = 0 \Rightarrow I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L)^2}}$$

Substituting numerical values to find for I:

$$I_{peak} = \frac{(345V)\cos(150\pi t)}{\sqrt{(40\Omega)^2 + [(150\pi s^{-1})(36mH)]^2}} = (7.94A)\cos(150\pi t) = \boxed{7.9\text{A}}$$

(b) We can find $\mathcal{E}$ as follows:

$$\mathcal{E} = (345V)\cos(150\pi t)$$

$$V_{L,peak} = \boxed{345\text{V}}$$

$$V_{L,rms} = \frac{V_{L,peak}}{\sqrt{2}} = \frac{345V}{\sqrt{2}} = \boxed{244\text{V}}$$

(c) We can relate the average power dissipation to I_{peak} and R:

$$P_{av} = I_{rms}^2 R = \left(\frac{I_{peak}}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_{peak}^2 R$$

Substituting numerical values:

$$P_{av} = \frac{1}{2}(7.94A)^2(40\Omega) = \boxed{1.3\text{kW}}$$

(d) The maximum energy stored in the magnetic field of the inductor can be found as follows:

$$U_{L,peak} = \frac{1}{2} L I_{peak}^2 = \frac{1}{2}(36mH)(7.94A)^2 = \boxed{1.1\text{J}}$$

From the definition $U_{L,av} = \frac{1}{T}\int_0^T U(t)dt$ and $U(t) = \frac{1}{2}L[I(t)]^2$. We do substitute $U(t)$ and obtain:

$$U_{L,av} = \frac{L}{2T}\int_0^T [I(t)]^2 dt$$

We evaluate the integral:

$$U_{L,av} = \frac{L}{2T}\left[\frac{1}{2} I_{peak}^2\right] T = \frac{1}{4} L I_{peak}^2$$

Finally we do substitute numerical values and evaluate:

$$U_{L,av} = \frac{1}{4}(36mH)(7.94A)^2 = \boxed{0.57\text{J}}$$

39 - A coil that has a resistance R and an inductance L has a power factor equal to 0.866 when driven at a frequency of 60 Hz. What is the coil's power factor it is driven at 240 Hz?

The relationship between X_L and R define the power factor when the coil is driven at a frequency of 60 Hz and then from the definition of X_L we can relate the inductive reactance at 240 Hz to the inductive reactance at 60 Hz, finally we can use the power factor definition to determine it at 240 Hz.
We use the definition of power factor to relate R and X_L:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} \Rightarrow \cos^2\delta = \frac{R^2}{R^2 + X_L^2}$$

We solve for 60 Hz and substitute for $\cos\delta$:

$$X_L^2(60Hz) = R^2\left(\frac{1}{\cos^2\delta} - 1\right) = R^2\left(\frac{1}{(0.866)^2} - 1\right) = \frac{1}{3}R^2$$

We use the definition of X_L:

$$X_L^2(f) = 4\pi f^2 L^2 \text{ and } X_L^2(f') = 4\pi f'^2 L^2$$

Combining these two equations:

$$\frac{X_L^2(f')}{X_L^2(f)} = \frac{4\pi f'^2 L^2}{4\pi f^2 L^2} = \frac{f'^2}{f^2} \Rightarrow X_L^2(f') = \left(\frac{f'}{f}\right)^2 X_L^2(f)$$

Finally we substitute numerical values:

$$X_L^2(240Hz) = \left(\frac{240s^{-1}}{60s^{-1}}\right)^2 X_L^2(60Hz) = 16\left(\frac{1}{3}R^2\right) = \frac{16}{3}R^2$$

We do substitute in the first equation to obtain:

$$(\cos\delta)_{240Hz} = \frac{R}{\sqrt{R^2 + \frac{16}{3}R^2}} = \sqrt{\frac{3}{19}} = \boxed{0.397}$$

40 - A resistor and an inductor are connected in parallel across an ideal ac voltage source whose output is given by E = E peak cos omega t as shown in Figure 29-32. Show that (a) the current in the resistor is given by I R = E peak /R cos omega t, (b) the current in the inductor is given by I L = E peak /X L cos(omega t – 90), and (c) the current in the voltage source is given by I = I R + I L = I peak cos(omega t – delta), where I peak = E max /Z.

The voltage drops across hte inductor and the resistor are equal because they are connected in parallel. The sum of the current through the resistor and through the inductor sum up the total current. The two currents are not in phase, phasors are needed to calculate their sum. The magnitude of the phasors are equal to the amplitudes of the applied voltage and the currents, i.e. $|\vec{\mathcal{E}}| = \mathcal{E}, |\vec{I}| = I_{peak}, |\vec{I}_R| = I_{R,peak}, |\vec{I}_L| = I_{L,peak}$
(a) The voltage given is supplied by the ac source $\mathcal{E} = \mathcal{E}\cos\omega t$. Therefore the voltage drop across the load resistor and the inductor is:

$$I_R = I_{R,peak}\cos\omega t$$

and as $I_{R,peak} = \frac{\mathcal{E}_{peak}}{R}$:

$$I_R = \frac{\mathcal{E}_{peak}}{R}\cos\omega t$$

(b) The current in the inductor lags the applied voltage by 90°:

$$I_L = I_{L,peak}\cos(\omega t - 90°)$$

and as $I_{L,peak} = \frac{\mathcal{E}_{peak}}{X_L}$:

$$I_L = \frac{\mathcal{E}_{peak}}{X_L} \cos(\omega t - 90°)$$

(c) The sum of the currents through the parallel branches sum up the net current I:

$$I = I_R + I_L$$

It helps if we draw the phasor diagram of the circuit. The projections of the phasors onto the horizontal axis are the instantaneous values. The current in the resistor is in phase with the applied voltage, and the current in the inductor lags the applied voltage by 90 degrees. The net current phasor is the sum of the branch current phasors ($\vec{I} = \vec{I}_R + \vec{I}_L$)
The peak current through the parallel combination is equal to $\mathcal{E}_{peak}/Z$, where Z is the impedance of the combination:

$$I = I_{peak} \cos(\omega t - |\delta|) \text{ where } I_{peak} = \frac{\mathcal{E}}{Z}$$

Based on the phasor diagram we have:

$$I^2_{peak} = I^2_{R,peak} + I^2_{L,peak} = \left(\frac{\mathcal{E}_{peak}}{R}\right)^2 + \left(\frac{\mathcal{E}_{peak}}{X_L}\right)^2$$

$$= \mathcal{E}^2_{peak} \left(\frac{1}{R^2} + \frac{1}{X^2_L}\right) = \frac{\mathcal{E}^2_{peak}}{Z^2}$$

where $\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X^2_L}$. We do solve for I_{peak} and yields:

$$I_{peak} = \frac{\mathcal{E}_{peak}}{Z} \text{ where } Z^{-2} = R^{-2} + X_L^{-2}$$

And finally, from the phasor diagram:

$$I = I_{peak} \cos(\omega t - |\delta|)$$

$$\text{where } \tan|\delta| = \frac{I_{L,peak}}{I_{R,peak}} = \frac{\frac{\mathcal{E}_{peak}}{X_L}}{\frac{\mathcal{E}_{peak}}{R}} = \boxed{\frac{R}{X_L}}$$

41 - Figure 29-33 shows a load resistor that has a resistance of R L = 20.0 Omega connected to a high-pass filter consisting of an inductor that has inductance L = 3.20-mH and a resistor that has resistance R = 4.00-Omega. The output of the ideal ac generator is given by epsilon = (100 V) cos(2 pi ft). Find the rms currents in all three branches of the circuit if the driving frequency is (a) 500 Hz and (b) 2000 Hz. Find the fraction of the total average power supplied by the ac generator that is delivered to the load resistor if the frequency is (c) 500 Hz and (d) 2000 Hz.

This equation reflects the voltage drops $\mathcal{E} = V_1 + V_2$, V_1 is the voltage drop across R and V_2 is the voltage drop across the parallel combination of L and R_L, if we take the same vectorial equation we have the relation for the phasors. The current for the parallel combination $\vec{I} = \vec{I}_{R_L} + \vec{I}_L$. Also, V_1 is in phase with I and V_2 is in phase with I_{R_L}. Draw the phasor diagram for the currents in the parallel combination, then add phasors for the voltages to the diagram.
We show the phasor diagram for the currents in the circuit and the modified diagram showing the voltage phasors.
The maximum current in the inductor $I_{2,peak}$ is given by:

$$I_{2,peak} = \frac{V_{2,peak}}{Z_2} \text{ where } Z_2^{-2} = R_L^{-2} + X_L^{-2}$$

the $\tan|\delta|$ is given by:

$$\tan|\delta| = \frac{I_{L,peak}}{I_{R,peak}} = \frac{V_{2,peak}/X_L}{V_{2,peak}/R_L}$$

$$= \frac{R_L}{X_L} = \frac{R_L}{\omega L} = \frac{R_L}{2\pi f L}$$

Solving for $|\delta|$:

$$|\delta| = \tan^{-1}\left(\frac{R_L}{2\pi f L}\right)$$

We apply the law of cosines to the triangle formed by the voltage phasors and obtain:

$$\mathcal{E}_{peak}^2 = V_{1,peak}^2 + V_{2,peak}^2 + 2V_{1,peak}V_{2,peak}\cos|\delta|$$

$$I_{peak}^2 Z^2 = I_{peak}^2 R^2 + I_{peak}^2 Z_2^2 + 2I_{peak} R I_{peak} Z_2 \cos|\delta|$$

Simplyfing for the current and Z:

$$Z^2 = R^2 + Z^2 + 2RZ_2\cos|\delta|$$

$$Z = \sqrt{R^2 + Z_2^2 + 2RZ_2\cos|\delta|}$$

The maximum current I_{peak} in the circuit $I_{peak} = \frac{\mathcal{E}_{peak}}{Z}$
I_{rms} is related to I_{peak} according to: $I_{rms} = \frac{1}{\sqrt{2}} I_{peak}$
(a) We do substitute numerical values to get δ:

$$|\delta| = \tan^{-1}\left(\frac{20.0\Omega}{2\pi(500Hz)(3.20mH)}\right) = \tan^{-1}\left(\frac{20.0\Omega}{10.053\Omega}\right) = 63.31^\circ$$

Solving for Z_2 gives:

$$Z_2^{-2} = R_L^{-2} + X_L^{-2} = \frac{1}{\sqrt{(20.0\Omega)^{-2} + (10.053\Omega)^{-2}}} = 8.982\Omega$$

For Z we do substitute numerical values and evaluate:

$$Z = \sqrt{(4.00\Omega)^2 + (8.982\Omega)^2 + 2(4.00\Omega)(8.982\Omega)\cos 63.31^\circ} = 11.36\Omega$$

We do substitute values and evaluate for I_{peak}:

$$I_{peak} = \frac{100V}{11.36\Omega} = 8.806A$$

Once we have I peak, we do substitute for I rms:

$$I_{rms} = \frac{1}{\sqrt{2}}(8.806A) = \boxed{6.23\text{A}}$$

The maximum and rms values of V_2 are given by:

$$V_{2,peak} = I_{peak}Z_2 = (8.806A)(8.982\Omega) = 79.095V$$

$$V_{2,rms} = \frac{1}{\sqrt{2}}V_{2,peak} = \frac{1}{\sqrt{2}}(79.095V) = 55.929V,$$

The rms values of $I_{R_L,rms}$ and $I_{L,rms}$:

$$I_{R_L,rms} = \frac{V_{2,rms}}{R_L} = \frac{55.929V}{20.0\Omega} = \boxed{2.80\text{A}}$$

$$I_{L,rms} = \frac{V_{2,rms}}{X_L} = \frac{55.929V}{10.053\Omega} = \boxed{5.53\text{A}}$$

(b) We do proceed as in (a) with $f = 2000Hz$ to obtain:

$$X_L = 40.2\Omega, |\delta| = 26.4^\circ, Z_2 = 17.9\Omega, Z = 21.6\Omega, I_{peak} = 4.64A, \text{ and } I_{rms} = \boxed{3.28\text{A}}$$

$$V_{2,max} = 83.0V, V_{2,rms} = 58.7V, I_{R_L,rms} = \boxed{2.94\text{A}}, \text{ and } I_{L,rms} = \boxed{1.46\text{A}}$$

(c) The sum of the power dissipated in the two resistors equals the power delivered by the ac source. The fraction of the total power delivered by the source that is dissipated in load resistor is given by:

$$\frac{P_{R_L}}{P_{R_L} + P_R} = \left(1 + \frac{P_R}{P_{R_L}}\right)^{-1} = \left(1 + \frac{I^2_{rms}R}{I^2_{R_L,rms}R_L}\right)^{-1}$$

We substitute numerical values for $f = 500Hz$:

$$\left.\frac{P_{R_L}}{P_{R_L} + P_R}\right|_{f=500Hz} = \left(1 + \frac{(6.23A)^2(4.00\Omega)}{(2.80A)^2(20.0\Omega)}\right)^{-1} = 0.502 = 50.2\%$$

(d) Substitute numerical values for $f = 2000Hz$ to obtain:

$$\left.\frac{P_{R_L}}{P_{R_L} + P_R}\right|_{f=2000Hz} = \left(1 + \frac{(3.28A)^2(4.00\Omega)}{(2.94A)^2(20.0\Omega)}\right)^{-1} = 0.800 = 80.0\%$$

42 - An ideal ac voltage source whose emf E 1 is given by (20 V) cos(2 pi ft) and an ideal battery whose emf E 2 is 16 V are connected to a combination of two resistors and an inductor (Figure 29-34), where R 1 = 10 Omega, R 2 = 8.0 Omega, and L = 6.0 mH. Find the average power delivered to each resistor if (a) the driving frequency is 100 Hz, (b) the driving frequency is 200 Hz, and (c) the driving frequency is 800 Hz.

Let's treat the ac and dc components separately. L acts like a short circuit for the component. Denote the peak value of the voltage supplied by $\mathcal{E}_{1,peak}$, then use $P = \mathcal{E}_2^2/R$ to find the power dissipated in the resistors by the current from the ideal battery. We apply Kirchhoff's loop including to L, R_1, R_2 to derive an expression for the average power delivered to each resistor by the ac voltage source.
(a) The total power delivered to R_1 and R_2:

$$P_1 = P_{1,dc} + P_{1,ac} \tag{1.1}$$

$$P_2 = P_{2,dc} + P_{2,ac} \tag{1.2}$$

The dc power delivered to the resistors whose resistances are R_1 and R_2:

$$P_{1,dc} = \frac{\mathcal{E}_2^2}{R_1} \text{ and } P_{2,dc} = \frac{\mathcal{E}_2^2}{R_2}$$

We express the average ac power delivered to R_1:

$$P_{1,ac} = \frac{\mathcal{E}_{1,rms}^2}{R_1} = \frac{\mathcal{E}_{1,peak}^2}{2R_1}$$

Applying Kirchhoff's loop rule clockwise to the loop includes R_1, L, R_2:

$$R_1 I_1 - Z_2 I_2 = 0$$

We solve for I_2:

$$I_2 = \frac{R_1}{Z_2} I_1 = \frac{R_1 \mathcal{E}_{1,peak}}{Z_2 R_1} = \frac{\mathcal{E}_{1,peak}}{Z_2}$$

We express teh average ac power delivered to R_2:

$$P_{2,ac} = \frac{1}{2} I_{2,rms}^2 R_2 = \frac{1}{2}\left(\frac{\mathcal{E}_{1,peak}}{Z_2}\right)^2 R_2 = \frac{\mathcal{E}_{1,peak}^2 R_2}{2Z_2^2}$$

Substituting in equations 1.1 and 1.2:

$$P_1 = \frac{\mathcal{E}_2^2}{R_1} + \frac{\mathcal{E}_{1,peak}^2}{2R_1}$$

$$P_2 = \frac{\mathcal{E}_2^2}{R_2} + \frac{\mathcal{E}_{1,peak}^2 R_2}{2Z_2^2}$$

Substituting numerical values and evaluate P:

$$P_1 = \frac{(16V)^2}{10\Omega} + \frac{(20V)^2}{2(10\Omega)} = \boxed{46\text{W}}$$

$$P_2 = \frac{(16V)^2}{8\Omega} + \frac{(20V)^2(8.0\Omega)}{2[(8\Omega)^2 + (2\pi\{100s^{-1}\}\{6.0mH\})^2]} = \boxed{52\text{W}}$$

(b) We proceed as in (a) to evaluate P_1 and P_2 with $f = 200Hz$:

$$P_1 = 25.6W + 20.0W = \boxed{46\text{W}}$$

$$P_2 = 32.0W + 13.2W = \boxed{45\text{W}}$$

(c) We proceed as in (a) to evaluate P_1 and P_2 with $f = 800Hz$:

$$P_1 = 25.6W + 20.0W = \boxed{46\text{W}}$$

$$P_2 = 32.0W + 1.64W = \boxed{34\text{W}}$$

43 - An ac circuit contains a resistor and an ideal inductor connected in series. The voltage rms drop across this series combination is 100-V and the rms voltage drop across the inductor alone is 80 V. What is the rms voltage drop across the resistor?

The voltage across the resistor can be found using a phasor diagram. The phasor diagram is shown. Using the Pythagorean theorem we can express V_R:

$$V_R = \sqrt{\mathcal{E}_{rms}^2 - V_L^2} = \sqrt{(100V)^2 - (80V)^2} = \boxed{60\text{V}}$$

1.5 Filters and Rectifiers

44 - The circuit shown in Figure 29-35 is called an RC high-pass filter because it transmits input voltage signals that have high frequencies with greater amplitude than it transmits input voltage signals that have low frequencies. If the input voltage is given by V in = V in peak cos omega t, show that the output voltage is V out = V H cos(omega t – delta) where V H = V in peak 1 + (omega RC) . (Assume that the output is - 2 connected to a load that draws only an insignificant amount of current.) Show that this result justifies calling this circuit a high-pass filter.

The phasor diagram for the RC high-pass filter is shown. $\vec{V}_{app}$ and $\vec{V}_R$ are the phasors for V_{in} and V_{out} respectively. Note that $\tan\delta = -X_C/R$. That δ is negative follows from the fact that $\vec{V}_{app}$ lags $\vec{V}_R$ by $|\delta|$. The projection of $\vec{V}_{app}$ onto the horizontal axis is $V_{app} = V_{in}$, and the projection of $\vec{V}_R$ onto the horizontal axis is $V_R = V_{out}$. We start by expressing V_{app}:

$$V_{app} = V_{app,peak}\cos\omega t \text{ where } V_{app,peak} = I_{peak}Z$$

$$\text{and } Z^2 = R^2 + X_C^2 \tag{1.3}$$

And as $\delta < 0$:

$$\omega t + |\delta| = \omega t - \delta$$

V_R is given by:

$$V_R = V_{R,peak}\cos(\omega t - \delta) \text{ where } V_{R,peak} = V_H = I_{peak}R$$

Then we solve the equation 1.3 for Z and substitute for X_C:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \tag{1.4}$$

Now, because $V_{out} = V_R$ we can express:

$$V_{out} = V_{R,peak} = \cos(\omega t - delta) = I_{in,peak}R\cos(\omega t - \delta) = \frac{V_{inpeak}}{Z}R\cos(\omega t - \delta)$$

We now use equation 1.4 to substitute for Z:

$$V_{out} = \frac{V_{inpeak}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}R\cos(\omega t - \delta)$$

Simplifying this expression:

$$V_{out} = \frac{V_{inpeak}}{\sqrt{1 + (\omega RC)^{-2}}}\cos(\omega t - \delta) = \boxed{V_H\cos(\omega t - \delta)}$$

$$\text{where } V_H = \frac{V_{inpeak}}{\sqrt{1 + (\omega RC)^{-2}}}$$

As $\omega \to \infty$:

$$V_H \to \frac{V_{inpeak}}{\sqrt{1 + (0)^2}} = V_{inpeak}$$

showing that the result is consistent with the highpass name for this circuit.

45 (a) Find an expression for the phase constant delta in Problem 44 in terms of omega , R and C. (b) What is the value of delta in the limit that omega tends 0? (c) What is the value of delta in the limit that omega tends infinity? (d) Explain your answers to Parts (b) and (c).

The phasor diagram for the RC high-pass filter is shown. $\vec{V}_{app}$ and V_R are the phasors for V_{in} and V_{out}, respectively. The projection of $\vec{V}_{app}$ onto the horizontal axis is $V_{app} = V_{in}$, and the projection of $\vec{V}_R$ onto the horizontal axis is $V_R = V_{out}$.
(a) $\vec{V}_{app}$ lags V_R by δ:

$$\tan\delta = -\frac{V_C}{V_R} = -\frac{IX_C}{IR} = -\frac{X_C}{R}$$

Using the definition of X_C we obtain:

$$\tan\delta = -\frac{\frac{1}{\omega C}}{R} = -\frac{1}{\omega RC}$$

and solving for δ:

$$\delta = \boxed{\tan^{-1}\left[-\frac{1}{\omega RC}\right]}$$

(b) As $\omega \to 0$:

$$\delta \to \boxed{-90^\circ}$$

(c) As $\omega \to \infty$:

$$\delta \to \boxed{0}$$

46 - Assume that in Problem 44, R = 20 kOmega and C = 15 nF. (a) At what frequency is V H = 1/2 V in peak ? This particular frequency is known as the 3 dB frequency, or f 3dB for the circuit. (b) Using a spreadsheet program, make a graph of log 10 (V H) versus log 10 (f), where f is the frequency. Make sure that the scale extends from at least 10% of the 3-dB frequency to ten times the 3-dB frequency. (c) Make a graph of delta versus log 10 (f) for the same range of frequencies as in Part (b). What is the value of the phase constant when the frequency is equal to the 3-dB frequency?

The results found in problems 44 and 45 can be used to find f_{3dB} and to plot graphs of $log(V_{out})$ versus $log(f)$ and δ versus $log(f)$.
(a) Using the result of problem 44 to express the ratio V_{out}/V_{inpeak}:

$$\frac{V_{out}}{V_{inpeak}} = \frac{\frac{V_{inpeak}}{\sqrt{1+(\omega RC)^{-2}}}}{V_{inpeak}} = \frac{1}{\sqrt{1+(\omega RC)^{-2}}}$$

And now when $V_{out} = V_{inpeak}/\sqrt{2}$:

$$\frac{1}{\frac{1}{\sqrt{1+(\omega RC)^{-2}}}} = \frac{1}{\sqrt{2}}$$

Squaring both sides of the equation and solving for ωRC to obtain:

$$\omega RC = 1 \Rightarrow \omega = \frac{1}{RC} \Rightarrow f_{3dB} = \frac{1}{2\pi RC}$$

We now substitute numerical values and evaluate:

$$f_{3dB} = \frac{1}{2\pi(20k\Omega)(15nF)} = \boxed{0.53\text{kHz}}$$

(b) Using result from problem 44 we have:

$$V_{out} = \frac{V_{inpeak}}{\sqrt{1+(\omega RC)^{-2}}}$$

And in problem 45 we showed that:

$$\delta = \tan^{-1}\left[-\frac{1}{\omega RC}\right]$$

We rewrite these expressins in terms of f_{3dB}:

$$V_{out} = \frac{V_{inpeak}}{\sqrt{1+\left(\frac{1}{2\pi fRC}\right)^2}} = \frac{V_{peak}}{\sqrt{1+\left(\frac{f_{3dB}}{f}\right)^2}}$$

and

$$\delta = \tan^{-1}\left[-\frac{1}{2\pi fRC}\right] = \tan^{-1}\left[-\frac{f_{3dB}}{f}\right]$$

A spreadsheet program to generate the data for a graph of V_{out} versus f and δ versus f is shown, including formulas used to calculate the quantities in columns:

Cel	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	1.50E-08	C
B3	1	V_{inpeak}
B4	531	f_{3dB}
A8	53	$0.1f_{3db}$
C8	$B$3/SQRT(1+(1($B$4/A8))ˆ 2)	$\frac{V_{inpeak}}{\sqrt{1+\left(\frac{f_{3dB}}{f}\right)^2}}$
D8	LOG(C8)	$log(V_{out}$
E8	ATAN(-$B$4/A8)	$\tan^{-1}\left[-\frac{f_{3dB}}{f}\right]$
F8	E8*180/PI()	δ in degress

The graph of $log(V_{out}$ versus $log(f)$ is shown for $V_{inpeak} = 1V$.
A graph of δ in degrees as a function of $log(f)$ is shown.
As shown by the spreadsheet program, we can see that when $f = f_{3dB}$, $\delta \approx \boxed{-44.9°}$. This result agrees with its calculated value of $\boxed{-45.0°}$

47 - A slowly varying voltage signal V(t) is applied to the input of the high-pass filter of Problem 44. Slowly varying means that during one time constant (equal to RC) there is no significant change in the voltage signal. Show that under these conditions the output voltage is proportional to the time derivative of V(t). This situation is known as a differentiation circuit.

Using the Kirchhoff's loop rule we can obtain a differential equation relating the input, capacitor, and resistor voltages. Because the voltage drop across the ressistor is small compared to the voltage drop across the capacitor, we can express the voltage drop across the capacitor in terms of the input voltage.
First apply Kirchhoff's loop rule to the input side of the filter:

$$V(t) - V_C - IR = 0$$

where V_C is the potential difference across the capacitor.
The we substitute for $V(t)$ and I to obtain:

$$V_{inpeak} \cos \omega t - V_C - R\frac{dQ}{dt} = 0$$

And because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV_C] = C\frac{dV_C}{dt}$$

Substituting we obtain:

$$V_{peak} \cos \omega t - V_C - RC\frac{dVC}{dt} = 0$$

the differential equation describing the potential difference across the capacitor. There is no significant change in the voltage signal during one time constant, so we can express:

$$\frac{dV_C}{dt} = 0 \Rightarrow RC\frac{dV_C}{dt} = 0$$

Substituting for $RC\frac{dV_C}{dt}$ yields:

$$V_{inpeak} \cos \omega t - V_C = 0$$

and

$$V_C = V_{inpeak} \cos \omega t$$

As a consequence, the potential difference across the resistor is given by:

$$V_R = RC\frac{dV_C}{dt} = \boxed{RC\tfrac{d}{dt}[V_{inpeak} \cos \omega t]}$$

48 - We can describe the output from the high-pass filter from Problem 44 using a decibel scale: beta = (20 dB) log 10 (V H V in peak) , where beta is the output in decibels. Show that for V H = 1 / sqrt2 V in peak , beta = 3.0 dB. The frequency at which VH = 1 / sqrt2 V in peak is known as f 3dB (the 3-dB frequency). Show that for $f << f3dB$, the output beta drops by 6 dB if the frequency f is halved.

We use the expression found for V_H in problem 44 and the definition of β given in the problem statement to show that every time the frequency is halved, the output drops by 6 dB.
This equation we have from problem 44:

$$V_H = \frac{V_{inpeak}}{\sqrt{1 + (\omega RC)^{-2}}}$$

or

$$\frac{V_H}{V_{inpeak}} = \frac{1}{\sqrt{1 + (\omega RC)^{-2}}}$$

We need to express this ratio in terms of f and f_{3dB}, we simplify as well:

$$\frac{V_H}{V_{peak}} = \frac{1}{\sqrt{1+\left(\frac{f_{3dB}}{f}\right)^2}} = \frac{f}{\sqrt{f_{3dB}^2\left(\frac{f^2}{f_{3dB}^2}\right)}}$$

For $f << f_{3dB}$:

$$\frac{V_H}{V_{peak}} \approx \frac{f}{\sqrt{f_{3dB}^2\left(1+\frac{f^2}{f_{3dB}^2}\right)}} = \frac{f}{f_{3dB}}$$

Using the β definition we have:

$$\beta = 20log_{10}\left(\frac{V_H}{V_{peak}}\right)$$

Substituting for V_H/V_{peak} we obtain:

$$\beta = 20log_{10}\left(\frac{f}{f_{3dB}}\right)$$

If we double the frequency we obtain:

$$\beta' = 20log_{10}\left(\frac{\frac{1}{2}f}{f_{3dB}}\right)$$

The decibel level change is given by:

$$\Delta\beta = \beta' - \beta = 20log_{10}\left(\frac{\frac{1}{2}f}{f_{3dB}}\right) - 20log_{10}\left(\frac{f}{f_{3dB}}\right) = 20log_{10}(1/2) \approx \boxed{\text{-6dB}}$$

49 - Show that the average power dissipated in the resistor of the high-pass filter of Problem 44 is given by (formula)

The instantaneous power dissipated in the resistor can be expressed, then using the fact that the average value of the square of the cosine function over one cycle is half to establish the given result.
The instantaneous power P(t) dissipated in the resistor is:

$$P(t) = \frac{V_{out}^2}{R}$$

The output voltage is:

$$V_{out} = V_H \cos(\omega t - \delta)$$

Taking from problem 44:

$$V_H = \frac{V_{inpeak}}{\sqrt{1+(\omega RC)^{-2}}}$$

Substitute in the expression for P(t):

$$P(t) = \frac{V_H^2}{R}\cos(\omega t - \delta) = \frac{V_{inpeak}^2}{R[1 + (\omega RC)^{-2}]}\cos(\omega t - \delta)$$

And taking in consideration that the average value of the square of the cosine function over one cycle is one half:

$$P_{av} = \boxed{\frac{V_{inpeak}^2}{2R[1+(\omega RC)^{-2}]}}$$

50 - One application of the high-pass filter of Problem 44 is a noise filter for electronic circuits (a filter that blocks out low-frequency noise). Using a resistance value of 20 k Omega, find a value for the capacitance for the high-pass filter that attenuates a 60-Hz input voltage signal by a factor of 10. That is, so V H = 1 / 10 V in peak.

We should solve the expression for V_H from problem 44 for the required capacitance of the capacitor.
Taking the expression from problem 44:

$$V_H = \frac{V_{inpeak}}{\sqrt{1 + (\omega RC)^{-2}}}$$

And we do require that:

$$\frac{V_H}{V_{inpeak}} = \frac{1}{\sqrt{1 + (\omega RC)^{-2}}} = \frac{1}{10}$$

or

$$\sqrt{1 + (\omega RC)^{-2}} = 10$$

If we do solve for C yields:

$$C = \frac{1}{\sqrt{99}\omega R} = \frac{1}{2\pi\sqrt{99}Rf}$$

Now we finally substitute numerical values and evaluate:

$$C = \frac{1}{2\pi(20k\Omega)(60Hz)} = \boxed{13\text{nF}}$$

51 - The circuit shown in Figure 29-36 is an example of a low-pass filter. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (a) If the input voltage is given by V in = V in peak cos omega t, show that the output voltage is V out = V L cos(omega t − delta) where V L = V in peak 1 + (omega RC) . (b) Discuss the trend of the output voltage in the limiting cases omega tends to 0 and infinity.

The phasor diagram for the RC low-pass filter shows that $\vec{V}_{app}$ and $\vec{V}_C$ are the phasors for V_{in} and V_{out} respectively. The projection of $\vec{V}_{app}$ onto the horizontal axis is $V_{app} = V_{in}$, the projection of $\vec{V}_C$ onth the horizontal axis is $V_C = V_{out}$, $V_{peak} = |\vec{V}_{app}|$, and ϕ is the angle between $\vec{V}_C$ and the horizontal axis.
(a) We express V_{app}:

$$V_{app} = V_{inpeak} \cos \omega t \text{ where } V_{inpeak} = I_{peak} Z$$
$$\text{and } Z^2 = R^2 + X_C^2 \tag{1.5}$$

$V_{out} = V_C$ is given by:

$$V_{out} = V_{C,peak} = \cos \phi = I_{peak} X_C \cos \phi$$

Now we define δ as shown in the phasor diagram:

$$V_{out} = I_{peak} X_C \cos(\omega t - \delta) = \frac{V_{inpeak}}{Z} X_C \cos(\omega t - \delta)$$

Solving equation 1.5 for Z and substituting X_C:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \tag{1.6}$$

We can use equation 1.8 to substitute for Z and substitute for X_C:

$$V_{out} = \frac{V_{inpeak}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \left(\frac{1}{\omega C}\right) \cos(\omega t - \delta)$$

If we simplify further we can obtain:

$$V_{out} = \frac{V_{inpeak}}{\sqrt{1 + (\omega RC)^2}} \cos(\omega t - \delta)$$

or

$$V_{out} = \boxed{V_L \cos(\omega t - \delta)} \text{ where } V_L = \boxed{\frac{V_{inpeak}}{\sqrt{1+(\omega RC)^2}}}$$

(b) We can note that as $\omega \to 0, V_L \to V_{peak}$. This is physically feasible as for low frequencies X_C is large and hence a larger peak input voltage will appear across it than appears across it for high frequencies.
Yet another note is that as $\omega \to \infty, V_L \to 0$. This is physically feasible since, for high frequencies, X_C is small and, hence, a smaller peak voltage will appear across it than appears it for low frequencies.
Remarks: In Figures 29-19 and 29-20, δ is defined as the phase of the voltage drop across the combination relative to the voltage drop acrossthe resistor.

52 (a) Find an expression for the phase angle delta for the low-pass filter of Problem 51 in terms of omega , R and C. (b) Find the value of delta in the limit that omega tends 0 and in the limit that omega tends infinity. Explain your answer.

The phasor diagram for the RC low-pass filter is shown. $\vec{V}_{app}$ and $\vec{V}_C$ are the phasors for V_{in} and V_{out} respectively. The projection of $\vec{V}_{app}$ onto the horizontal axis is $V_{app} = V_{in}$ and the projection of $\vec{V}_C$ onto the horizontal axis is $V_C = V_{out}$. $V_{peak} = |\vec{V}_{app}|$.
(a) Analyzing the phasor diagram we have:

$$\tan\delta = \frac{V_R}{V_C} = \frac{I_{peak}R}{I_{peak}X_C} = \frac{R}{X_C}$$

and from the X_C definition we obtain:

$$\tan\delta = \frac{R}{\frac{1}{\omega C}} = \omega RC$$

Solving for δ:

$$\delta = \boxed{\tan^{-1}(\omega RC)}$$

(b) As $\omega \to 0, \delta \to \boxed{0°}$. This behaviour makes sense physically in that, at low frequencies, X_C is very large compared to R and, as a consequence, V_C is in phase with V_{in}.
As $\omega \to \infty, \delta \to \boxed{90°}$. This behaviour makes sense physically in that, at high frequencies, X_C is very small compared to R and, as a consequence, V_C is out of phase with V_{in}
Remarks: See the spreadsheet solution in the following problem for additional evidence that our answer for Part (b) is correct.

53 Using a spreadsheet program, make a graph of V L versus input frequency f and a graph of phase angle delta versus input frequency for the low-pass filter of Problems 51 and 52. Use a resistance value of 10 kOmega and a capacitance value of 5.0 nF.

In problems 51 and 52 we derived expressions for V_L and δ to plot graphs of V_L versus f and δ versus f for the low-pass filter of problem 51. We express V_L and δ as functions of f_{3dB} to simplify the spreadsheet program.
From problems 51 and 52 we have:

$$V_L = \frac{V_{inpeak}}{\sqrt{1 + (\omega RC)^2}}$$

and

$$\delta = \tan^{-1}(\omega RC)$$

We express these expressions in terms of f and obtain:

$$V_L = \frac{V_{inpeak}}{\sqrt{1 + (2\pi fRC)^2}}$$

and

$$\delta = \tan^{-1}(2\pi fRC)$$

In the table we show a spreadsheet program to generate the data for graphs of V_L versus f and δ versus f for the low-pass filter. V_{inpeak} has been arbitrarily set equal to 1. The formulas used to calculate the quantities are shown in the columns:

Cel	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	1.50E-09	C
B3	1	V_{inpeak}
B8	$B$3/SQRT(1+((2*PI()*A8*1000*$B$1*$B$2)ˆ2))	$\frac{V_{inpeak}}{\sqrt{1+(2\pi fRC)^2}}$
C8	ATAN(2*PI()*A8*1000*$B$1*$B$2)	$\tan^{-1}(2\pi fRC)$
D8	C8*180/PI()	δ in degrees

A graph of V_L as function of f is shown:
A graph of δ as a function of f is shown:

54 - A rapidly varying voltage signal V(t) is applied to the input of the low-pass filter of Problem 51. Rapidly varying means that during one time constant (equal to RC) there are significant changes in the voltage signal. Show that under these conditions the output voltage is proportional to the integral of V(t) with respect to time. This situation is known as an integration circuit.

The Kirchhoff's loop rule can be used to find a differential equation relating the input, capacitor, and resistor voltages. Then we assume a solution for this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution of these simultaneous equations will yield the amplitude of the output voltage.
First we apply Kirchhoff's loop rule to the input side of the filter to obtain $V(t) - IRV_C = 0$, where V_C is the potential difference across the capacitor.
We substitute for $V(t)$ and I:

$$V_{inpeak}\cos\omega t - R\frac{dQ}{dt} - V_C = 0$$

And because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV_C] = C\frac{dV_C}{dt}$$

Sustituting for dQ/dt:

$$V_{inpeak}\cos\omega t - RC\frac{dV_C}{dt} - V_C = 0$$

the differential equation describing the potential difference across the capacitor. V_C is given by $V_C = IX_C = \frac{1}{\omega C}$. $V(t)$ varies rapidly meaning that $\omega >> 1$, therefore, $V_C \approx 0$, and:

$$V_{peak} \cos \omega t - RC \frac{dV_C}{dt} = 0$$

Separating variables in the differential equation and solve for V_C:

$$V_C = \boxed{\frac{1}{RC} \int V_{peak} \cos \omega t dt}$$

55 - The circuit shown in Figure 29-37 is a trap filter. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (a) Show that the trap filter acts to reject signals in a band of frequencies centered at omega = 1/ LC . (b) How does the width of the frequency band rejected depend on the resistance R?

The phasor diagram for the trap filter is shown. $\vec{V}_{app}$ and $\vec{V}_L + \vec{V}_C$ are the phasors for V_{in} and V_{out} respectively. The projection of $\vec{V}_{app}$ onto the horizontal axis is $V_{app} = V_{in}$, and the projection of $\vec{V}_L + \vec{V}_C$ onto the horizontal axis is $V_L + V_C = V_{out}$. We assume the impedance of the trap to be zero, then the frequency at which the circuits rejects signals will be shown. If we define $\Delta\omega = |\omega - \omega_{trap}|$ and do we require that $|Z_{trap}| = R$ will yield and expression for the bandwidth and reveal its dependence on R.
Expressing V_{app}:

$$V_{app} = V_{app,peak} \cos \omega t$$

where $V_{app,peak} = V_{peak} = I_{peak} Z$ and

$$Z^2 = R^2 + (X_L - X_C)^2 \tag{1.7}$$

On the other hand, V_{out} is given by:

$$V_{out} = V_{out,peak} \cos(\omega t - \delta)$$

where $V_{out,peak} = I_{peak} Z_{trap}$ and $Z_{trap} = X_L - X_C$.
Solving for Z in equation 1.7 yields:

$$Z^2 = \sqrt{R^2 + (X_L - X_C)^2} \tag{1.8}$$

And because $V_{out} = V_L + V_C$:

$$V_{out} = V_{out,peak} \cos(\omega t - \delta) = I_{peak} Z_{trap} \cos(\omega t - \delta)$$

$$= \frac{V_{peak}}{Z} Z_{trap} \cos(\omega t - \delta)$$

We can use equation 1.8 to substitute for Z:

$$V_{out} = \frac{V_{peak}}{\sqrt{R^2 + Z_{trap}^2}} Z_{trap} \cos(\omega t - \delta)$$

Provided that $Z_{trap} = 0$, we note that $V_{out} = 0$, then set $Z_{trap} = 0$ and obtain $Z_{trap} = X_L - X_C = 0$. Substituting for X_L and X_C yields:

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \boxed{\frac{1}{\sqrt{LC}}}$$

(b) We have as bandwith:

$$\Delta\omega = |\omega - \omega_{trap}| \tag{1.9}$$

We use bandwith definition involving the frequency at which $|Z_{trap}| = R$. Then:

$$\omega L - \frac{1}{\omega C} = R \Rightarrow \omega^2 LC - 1 = \omega RC$$

And because $\omega_{trap} = 1/\sqrt{LC}$:

$$\left(\frac{\omega}{\omega_{trap}}\right)^2 - 1 = \omega RC$$

We solve for $\omega^2 - \omega_{trap}^2$:

$$\omega^2 - \omega_{trap}^2 = (\omega - \omega_{trap})(\omega + \omega_{trap})$$

And because $\omega \approx \omega_{trap}, \omega + \omega_{trap} \approx 2\omega_{trap}$:

$$\omega^2 - \omega_{trap}^2 \approx 2\omega_{trap}(\omega - \omega_{trap})$$

Now substitute in equation 1.9 to obtain:

$$\Delta\omega = |\omega - \omega_{trap}| = \frac{RC\omega_{trap}^2}{2} = \boxed{\frac{R}{2L}}$$

56 - The output voltage will mirror input voltage minus a 0.60 V drop for voltages greater than 0.60 V. When the voltage is below 0.60 V, the output voltage will be zero. A spreadsheet program was utilized to plot the following graph. The peak voltage and angular frequency were both arbitrarily set equal to one.

57 - The time constant for the RC circuit and the frequency of the input signal can be related through the use of the potential difference decay across the capacitor. An approximate value for C can be found by expanding the exponential factor in the expression for V_C. The C value will limit the variation in the output voltage by less than 50 percent.
We find the voltage across the capacitor:

$$V_C = V_{in} e^{-t/RC}$$

If we expend the exponential factor:

$$e^{-t/RC} \approx 1 - \frac{1}{RC}t$$

And for a decay of less than 50 percent:

$$1 - \frac{1}{RC}t \leq 0.5 \Rightarrow C \leq \frac{2}{R}t$$

And finally, the voltage is positive at every cycle, $t = 1/60s$:

$$C \leq \frac{2}{1.00k\Omega}\left(\frac{1}{60}s\right) = \boxed{33\mu F}$$

58 - The current lags the voltage across the inductor and leads the voltage across a capacitor. If we use $I_{L,peak} = \mathcal{E}_{peak}/X_L$ and $I_{C,peak} = \mathcal{E}_{peak}X_C$ to find the amplitudes for these currents. The current in the generator is zero under resonance conditions, that is, when $|I_L| = |I_C|$. For finding the currents in the inductor and capacitor at resonance, we can use the shared potential difference across them and their reactances together with our knowledge of the phase relationships just mentioned.
(a) We do express the currents amplitudes through the inductor and capacitor:

$$I_{L_peak} = \frac{\mathcal{E}_{peak}}{X_L} = \frac{\mathcal{E}_{peak}}{2\pi f L}$$

and

$$I_{C_peak} = \frac{\mathcal{E}_{peak}}{X_C} = \frac{\mathcal{E}_{peak}}{\frac{1}{2\pi f C}} = 2\pi f C\mathcal{E}_{peak}$$

Substituting numerical values:

$$I_{L,peak} = \frac{100V}{(4.00H)2\pi f} = \boxed{\frac{25.0V/H}{2\pi f},\ \text{lagging } \mathcal{E} \text{ by } 90^\circ}$$

and

$$I_{C,peak} = (25.0\mu F)(100V)\omega = \boxed{(2.5mV \cdot F)2\pi f,\ \text{leading } \mathcal{E} \text{ by } 90^\circ}$$

(b) We do express the condition that $I = 0$:

$$|I_L| = |I_C| \text{ or } \frac{\mathcal{E}}{\omega L} = \frac{\mathcal{E}}{\frac{1}{\omega C}} = \omega C\mathcal{E} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

Now we substitute numerical values and evaluate ω:

$$\omega = \frac{1}{\sqrt{(4.00H)(25.0\mu F)}} = \boxed{100 \text{ rad/s}}$$

We use numerical values expressing the current in the inductor at $\omega = \omega_0$:

$$I_L = \left(\frac{25.0V/H}{100s^{-1}}\right)\cos\left(\omega t - \frac{\pi}{2}\right) = \boxed{(250mA)\cos\left(\omega t - \frac{\pi}{2}\right)}$$

where $\omega = 100rad/s$ We also express the current in the capacitor at $\omega = \omega_0$:

$$I_C = (2.5mV \cdot F)(100s^{-1})\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$= \boxed{-(250mA)\cos\left(\omega t + \frac{\pi}{2}\right)}$$

where $\omega = 100rad/s$.
(d) We show the phasor diagram for the case where the inductive reactance is larger than the capacitive reactance.

59 - If we differentiate with respect to time we can find I as a function of time. For (b), C can be found using $\omega = 1/\sqrt{LC}$. The energy stored in the magnetic field of the inductor is given by $U_m = \frac{1}{2}LI^2$ and the energy stored in the electric field of the capacitor is given by $U_e = \frac{1}{2}\frac{Q^2}{C}$.
(a) We differentiate the charge with respect to time for obtaining the current:

$$I(t) = \frac{dQ}{dt} = \frac{d}{dt}\left[(15\mu C)\cos\left(\omega t + \frac{\pi}{4}\right)\right] = -(15\mu C)(1250s^{-1})\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$-(18.75mA)\sin\left(\omega t + \frac{\pi}{4}\right) = \boxed{-(19mA)\sin\left(\omega t + \frac{\pi}{4}\right)}$$

where $\omega = 1250rad/s$
(b) We have a relation for C, L and ω, solving for C and evaluating at numerical values:

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(1250s^{-1})(28mH)} = 22.86\mu F = \boxed{23\mu F}$$

(c) For the magnetic energy at time t we have:

$$U_m(t) = \frac{1}{2}LI^2 = \frac{1}{2}(28mH)(18.75mA)^2\sin^2\left(\omega t + \frac{\pi}{4}\right) = \boxed{(4.9\mu J)\sin^2\left(\omega t + \frac{\pi}{4}\right)}$$

where $\omega = 1250rad/s$.
We now find the electrical energy stored in the capacitor $U_e = \frac{1}{2}\frac{Q^2}{C}$ as a function of time:

$$U_e(t) = \frac{1}{2}\frac{(15\mu F)^2}{22.86\mu F}\cos^2\left(\omega t + \frac{\pi}{4}\right)$$

$$= (4.92\mu J)\cos^2\left(\omega t + \frac{\pi}{4}\right)$$

where $\omega = 1250rad/s$.
The total energy sum stored in electric and magnetic field is the sum of $U_e(t)$ and $U_m(t)$:

$$U = (4.92\mu J)\sin^2\left(\omega t + \frac{\pi}{4}\right) + (4.92\mu J)\cos^2\left(\omega t + \frac{\pi}{4}\right) = \boxed{4.9\mu J}$$

60 - The capacitance of a dielectric field capacitor definition and the expression for the resonance frequency of an LC circuit can be used to derive an expression for the fractional change in the thicknes of the dielectric in terms of the resonance frequency and the frequency of the circuit when the dielectric is under copression. Afterwards, we can use this expression for $\Delta t/t$ to calculate the Young's modulus for the dielectric material.
We begin by using th definition for the Young's modulus of the dielectric material:

$$Y = \frac{stress}{strain} = \frac{\Delta P}{\Delta t/t} \tag{1.10}$$

We let t to be the initial thickness of the dielectric, expressing the initial capacitance of the capacitor:

$$C_0 = \frac{\kappa \epsilon_0 A}{t}$$

We express the capacitance of the capacitor when it is under compression:

$$C_C = \frac{\kappa \epsilon_0 A}{t - \Delta t}$$

Now we express the resonance frequency of the capacitor before the dielectric is compressed:

$$\omega_0 = \frac{1}{\sqrt{C_0 L}} = \frac{1}{\sqrt{\frac{\kappa \epsilon_0 A L}{t}}}$$

And now when the dielectric is compressed:

$$\omega_C = \frac{1}{\sqrt{C_C L}} = \frac{1}{\sqrt{\frac{\kappa \epsilon_0 A L}{t - \Delta t}}}$$

Simplifying by expressing the ratio of ω_C and ω_0:

$$\frac{\omega_C}{\omega_0} = \frac{\sqrt{\frac{\kappa \epsilon_0 A L}{t}}}{\sqrt{\frac{\kappa \epsilon_0 A L}{t - \Delta t}}} = \sqrt{1 - \frac{\Delta t}{t}}$$

We can expand the radical binomially to obtain:

$$\frac{\omega_C}{\omega_0} = \left(1 - \frac{\Delta t}{t}\right)^{1/2} \approx 1 - \frac{\Delta t}{2t}$$

provided that $\Delta t << t$.
Solving for $\Delta t/t$:

$$\frac{\Delta t}{t} = 2\left(1 - \frac{\omega_c}{\omega_0}\right)$$

Subsituting in equation 1.10:

$$Y = \frac{\Delta P}{2\left(1 - \frac{\omega_c}{\omega_0}\right)}$$

And finally using the numerical values we evaluate Y:

$$Y = \frac{(800atm)(102.325kPa/atm)}{2\left(1 - \frac{116MHz}{120MHz}\right)} = \boxed{1.22 \times 10^9 N/m^2}$$

61 - The capacitor can be modeled as the equivalent of two capacitors connected in parallel. Let C_1 by the capcacitance of the dielectric-filled capacitor and C_2 be the air-filled capacitor. We will derive expressions for each capacitance and then add them together to obtain $C(x)$. We can then use the given resonance frequency when $x = w/2$ and the given value for L to evaluate C_0. In part (b) we can use our result for $C(x)$ and the relationship between f, L and $C(x)$ at resonance to express $f(x)$.
(a) Let's express the equivalent capacitance of the two capacitors in parallel:

$$C(x) = C_1 + C_2 = \frac{\kappa\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d} \tag{1.11}$$

We need to express A_2 in terms of the total area of a capacitor plate A, w, and the distance x:

$$\frac{A_2}{A} = \frac{x}{w} \Rightarrow A_2 = A\frac{x}{w}$$

We do express A_1 in terms of A and A_2:

$$A_1 = A - A_2 = A\left(1 - \frac{x}{w}\right)$$

Substituting in equation 1.11 and simplifying:

$$C(x) = \frac{\kappa\epsilon_0 A}{d}\left(1 - \frac{x}{w}\right) + \frac{\epsilon_0 Ax}{dw}$$

$$= \frac{\epsilon_0 A}{d}\left[\kappa\left(1 - \frac{x}{w}\right) + \frac{x}{w}\right] = \kappa C_0\left[1 - \frac{\kappa - 1}{kw}x\right]$$

where $C_0 = \frac{\epsilon_0 A}{d}$ We find $C(w/2)$:

$$C\left(\frac{w}{2}\right) = \kappa C_0\left[1 - \frac{(\kappa - 1)w}{2kw}\right] = \kappa C_0\left[1 - \frac{\kappa - 1}{2\kappa}\right] = C_0\frac{\kappa + 1}{2}$$

Now for the resonance frequency of the circuit in terms of L and $C(x)$ we express:

$$f(x) = \frac{1}{2\pi\sqrt{LC(x)}} \tag{1.12}$$

We do evaluate $f(w/2)$:

$$f\left(\frac{w}{2}\right) = \frac{1}{2\pi\sqrt{LC_0\frac{\kappa+1}{2}}} = \frac{1}{2\pi}\sqrt{\frac{2}{(\kappa + 1)LC_0}}$$

Solving for C_0 we obtain:

$$C_0 = \frac{1}{2\pi^2 f^2 \left(\frac{w}{2}\right) L(\kappa+1)}$$

Now we substitute numerical values and evaluate C_0:

$$C_0 = \frac{1}{2\pi^2(90MHz)^2\left(\frac{20cm}{2}\right)(2.0mH)(4.8+1)} = \boxed{5.4\text{fF}}$$

(b) Substitute for $C(x)$ in equation 1.12:

$$f(x) = \frac{1}{2\pi\sqrt{L\kappa C_0\left[1-\frac{\kappa-1}{\kappa w}x\right]}}$$

And finally substitute numerical values and evaluate $f(x)$:

$$f(x) = \frac{1}{2\pi\sqrt{(2.0mH)(4.8)(5.39\times10^{-16}F)\left[1-\frac{4.8-1}{4.8(0.20m)}x\right]}} = \boxed{\frac{70MHz}{\sqrt{1-(4.0m^{-1})x}}}$$

1.6 Driven RLC Circuits

62 - In the diagram is shown the relationship between δ, X_L, X_C, R. This reference triangle can be used to express the power factor for the given circuit. In (b) we can find the rms current from the rms potential difference and the impedance of the circuit. The rms current and the resistance of the resistor can be used to find the average power delivered by the source.
(a) The power factor is defined as:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

We have no inductance in the circuit, therefore $X_L = 0$ and

$$\cos\delta = \frac{R}{\sqrt{R^2+X_C^2}} = \frac{R}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}$$

Substituting numerical values to evaluate $\cos\delta$:

$$\cos\delta = \frac{80\Omega}{\sqrt{(80\Omega)^2+\frac{1}{(400s^{-1})^2(20\mu F)^2}}} = \boxed{0.54}$$

(b) For this exercise, express the rms current in the circuit:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\frac{\mathcal{E}_{max}}{\sqrt{2}}}{\sqrt{R^2+X_C}}$$

$$\frac{\mathcal{E}_{max}}{\sqrt{2}\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

Substituting numerical values and evaluate I_{rms}:

$$I_{rms} = \frac{20V}{\sqrt{2}\sqrt{(80\Omega)^2 + \frac{1}{(400s^{-1})^2(20\mu F)^2}}} = 95.3mA = \boxed{95\text{mA}}$$

(c) The average power that the generator delivers is $P_{av} = I_{rms}^2 R$, substituting numerical values and evaluating P_{av}:

$$P_{av} = (95.3mA)^2(80\Omega) = \boxed{0.73\text{W}}$$

63 - $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives the immpedance of an ac circuit. If we make $X_L = X_C = 0$ and then $R = 0$, we can evaluate the impedance expression for P_{av}.

(a) For $X = 0, Z = R$:

$$P_{av} = \frac{R\mathcal{E}_{rms}^2}{Z^2} = \frac{R\mathcal{E}_{rms}^2}{R^2} = \boxed{\frac{R\mathcal{E}_{rms}^2}{R}}$$

(b) and (c). If $R = 0$, then:

$$P_{av} = \frac{R\mathcal{E}_{rms}^2}{Z^2} = \frac{(0)\mathcal{E}_{rms}^2}{(X_L - X_C)^2} = \boxed{0}$$

Remarks: in an ideal inductor or capacitor there is no energy dissipation.

64 - The resonant frequency of the circuit can be found using $\omega_0 = 1/\sqrt{LC}$, the rms current at resonance can be found using $I_{rms} = \mathcal{E}_{rms}/R$. We can find the reactances at $\omega = 8000rad/s$, by using the definitions of X_C, X_L. The definitions of Z, I_{rms} to find the impedance and rms current at $\omega = 8000rad/s$, and the definition of the phase angle to find δ.

(a) We begin by expressing the resonant frequency ω_0 in terms of L and C and substitute numerical values, evaluating:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10mH)(2.0\mu F)}} = \boxed{7.1 \times 10^3 rad/s}$$

(b) We need to find a relationship between the rms current at resonance and $\mathcal{E}_{rms}$ and the impedance of the circuit at resonance as well:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{\mathcal{E}_{max}}{\sqrt{2}R} = \frac{100V}{\sqrt{2}()5.0\Omega} = \boxed{14\text{A}}$$

(c) We do express and evaluate X_C and X_L at $\omega = 8000rad/s$:

$$X_C = \frac{1}{\omega C} = \frac{1}{(8000s^{-1})(2.0\mu F)} = 62.50\Omega = \boxed{63\Omega}$$

and

$$X_L = \omega L = (8000s^{-1})(10mH) = \boxed{80\Omega}$$

(d) The impedance needs to be expressed in terms of reactances, and substitute results from (c) and evaluate Z:

$$Z^2 = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(5.0\Omega)^2 + (80\Omega - 62.5\Omega)^2} = 18.2\Omega = \boxed{18\Omega}$$

(e) We need to relate the rms current at $\omega = 8000rad/s$ to $\mathcal{E}_{rms}$ and the impedance of the circuit at this same frequency:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{max}}{\sqrt{2}Z} = \frac{100V}{\sqrt{2}(18.2\Omega)} = \boxed{3.9 \text{ A}}$$

(f) We find δ as follows:

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{80\Omega - 62.5\Omega}{5.0\Omega}\right) = \boxed{74°}$$

65 - $Q = \omega L/R$ gives us the Q factor of the circuit, the resonance width by $\Delta f = f_0/Q = \omega_0/2\pi Q$, and the power factor by $\cos\delta = R/Z$. Z is frequency dependent, hence we need to find X_C and X_L at $\omega = 8000rad/s$ to be able to evaluate $\cos\delta$.
Let's use above definitions to express Q factor and resonance width of the circuit:

$$Q = \frac{\omega_0 L}{R} \tag{1.13}$$

and

$$\Delta f = \frac{f_0}{Q} = \frac{\omega_0}{2\pi Q} \tag{1.14}$$

(a) We begin by expressin the resonance frequency for the circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substituting in equation 1.13:

$$Q = \frac{1}{\sqrt{LC}R} = \frac{1}{r}\sqrt{\frac{L}{C}}$$

Substituting numerical values and evaluating:

$$Q = \frac{1}{5.0\Omega}\sqrt{\frac{10mH}{2.0\mu F}} = 14.1$$

Substitute numerical values in equation 1.14 and evaluate:

$$\Delta f = \frac{7.07 \times 10^3 rad/s}{2\pi(14.1)} = \boxed{80 \text{ Hz}}$$

(c) The power factor of the circuit is given by:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Substituting numerical values an evaluating for $\cos\delta$:

$$\cos\delta = \frac{5.0\Omega}{\sqrt{(5.0\Omega)^2 + \left((8000s^{-1})(10mH) - \frac{1}{(8000s^{-1})(2.0\mu F)}\right)^2}} = \boxed{0.27}$$

66 - The Q factor for the circuit can be found from its definition $Q = f_0/\Delta f$. Only substitute the numerical values.

$$Q = \frac{100.1MHz}{0.050MHz} \approx \boxed{2.0 \times 10^3}$$

67 - From the current definition $I_{peak} = \mathcal{E}_{peak}/Z$ to find the current in the coil and the definition of the phase angle to evaluate δ. We can equate X_L and X_C to find the capacitance required so that the current and the voltage are in phase. $V_C = IX_C$ can be used to find the measured voltage across the capacitor.
(a) We express the current in the coil in terms of the potential difference across it and its impedance:

$$I_{peak} = \frac{\mathcal{E}_{peak}}{Z} = \frac{100V}{10\Omega} = \boxed{10 \text{ A}}$$

(b) The phase angle δ is given by the next definition, and substitute numerical values to evaluate:

$$\delta = \cos^{-1}\left(\frac{R}{Z}\right) = \sin^{-1}\left(\frac{X_L}{Z}\right) = \sin^{-1}\left(\frac{8.0\Omega}{10\Omega}\right) = \boxed{53°}$$

(c) Expressing the condition on the on the reactances that must be satisfied if the current and voltage are to be in phase:

$$X_L = X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_L} = \frac{1}{2\pi f X_L}$$

Substituting numerical values:

$$C = \frac{1}{2\pi(60s^{-1})(8.0\Omega)} = 332\mu F = \boxed{0.33 \text{ mF}}$$

(d) Expressing the potential difference across the capacitor:

$$V_C = I_{peak} X_C$$

And relate the peak current in the circuit to the impedance of the circuit when $X_L = X_C$:

$$I_{peak} = \frac{V_{peak}}{R}$$

Substituting for the current:

$$V_C = \frac{V_{peak} X_C}{R} = \frac{V_{peak}}{2\pi f C R}$$

Now we need to relate the impedance of the circuit to the resistance of the coil:

$$Z = \sqrt{R^2 + X^2} \Rightarrow R = \sqrt{Z^2 - X^2}$$

We substitute for R:

$$V_C = \frac{V_{peak}}{2\pi f C \sqrt{Z^2 - X^2}}$$

Finally substitute numerical values and evaluate:

$$V_C = \frac{100V}{2\pi(60s^{-1})(332\mu F)\sqrt{(10\Omega)^2 - (8.0\Omega)^2}} = \boxed{0.13 \text{ kV}}$$

68 - $V_C = I_{rms} X_C$ and I_{rms} from the potential difference across the inductor can serve to find C. If the resistance in the circuit vanishes, the measured rms voltage across both the capacitor and inductor is $V = |V_L - V_C|$.
(a) We find a relationship between the capacitance C and the potential difference across the capacitor:

$$V_C = I_{rms} X_C = \frac{I_{rms}}{2\pi f C} \Rightarrow C = \frac{I_{rms}}{2\pi f V_C}$$

Using the potential difference across the inductor to express the rms current in the circuit:

$$I_{rms} = \frac{V_L}{X_L} = \frac{V_L}{2\pi f L}$$

Substituting for the I_{rms} and using numerical values:

$$C = \frac{V_L}{(2\pi f)^2 L V_C} = \frac{50V}{[2\pi(60s^{-1})]^2(0.25H)(75V)} = \boxed{19\mu F}$$

(b) We need to express the measured rms voltage V across both the capacitor and the inductor when $R = 0$:

$$V = |V_L - V_C|$$

Finally we substitute numerical values and evaluate:

$$V = |50V - 75V| = \boxed{25 \text{ V}}$$

69 - The potential differences across each of the circuit elements can be found by the rms current in the circuit. Phasor diagrams and our knowledge of the phase shifts between the voltages across the three circuit elements to find the voltage diferrences across their combinations.
(a) We do express the potential diffenrence between points A and B in terms of I_{rms} and X_L:

$$V_{AB} = I_{rms} X_L \tag{1.15}$$

Expressing I_{rms} in terms of $\mathcal{E}$ and Z:

$$I_{rms} = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

We evaluate X_L and X_C to obtain:

$$X_L = 2\pi f L = 2\pi(60s^{-1})(137mH) = 51.648\Omega$$

and

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60s^{-1})(25\mu F)} = 106.10\Omega$$

We evaluate for I_{rms}:

$$I_{rms} = \frac{115V}{\sqrt{(50\Omega)^2 + (51.648\Omega - 106.10\Omega)^2}} = 1.5556A$$

Substitute numerical values in equation 1.15 and evaluate:

$$V_{AB} = (1.5556A)(51.648\Omega) = 80.344V = \boxed{\text{80V}}$$

(b) We express the potential difference between points B and C in terms of I_{rms} and R:

$$V_{BC} = I_{rms} R = (1.5556A)(50\Omega) = 77.780V = \boxed{\text{78 V}}$$

(c) Expressing the potential difference between points C and D in terms of I_{rms} and X_C:

$$V_{CD} = I_{rms} X_C = (1.5556A)(106.10\Omega) = 165.05V = \boxed{\text{0.17 kV}}$$

(d) The voltage across the inductor leads the voltage across the resistor as shown in the phasor diagram. We can use the Pythagorean theorem to find V_{AC}

$$V_{AC} = \sqrt{V_{AB}^2 + V_{BC}^2} = \sqrt{(80.0V)^2 + (77.780V)^2} = 111.58V = \boxed{\text{0.11 kV}}$$

(e) The voltage across the capacitor lags the voltage across the resistor as shown in the phasor diagram and use the Pythagorean to find V_{BD}:

$$V_{BD} = \sqrt{V_{CD}^2 + V_{BC}^2}$$

$$= \sqrt{(165.05V)^2 + (77.780V)^2} = \boxed{\text{182.46 V}}$$

70 - We can find the power supplied to the circuit by $P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta$ and the resistance can be found by $P_{av} = I_{rms}^2 R$. In (c) the impedance, inductive reactance, and resistance can be related to the capacitive reactance and solve for the capacitance C. We can use the condition on X_L and X_C at resonance to find the capacitance or inductance you would need to add to the circuit to make the power factor equal to 1.
(a) We begin by expressing the power supplied to the circuit in terms of $\mathcal{E}_{rms}, I_{rms}$ and the power factor $\cos\delta$ and substitute numerical values:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta = (120V)(11A)\cos 45^\circ = 933W$$

(b) We relate the resistance to the power dissipated in the circuit:

$$P_{av} = I_{rms}^2 \Rightarrow R = \frac{P_{av}}{I_{rms}^2}$$

Substituting numerical values and evaluating:

$$R = \frac{933W}{(11A)^2} = 7.71\Omega$$

(c) Expressing the capacitance of the capacitor in terms of its reactance:

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} \tag{1.16}$$

We need to relate the capacitive reactance to the impedance, inductive reactance and resistance of the circuit:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Expressing the impedance of the circuit in terms of the rms emf $\mathcal{E}$ and the rms current:

$$Z^2 = \frac{\mathcal{E}^2}{I_{rms}^2}$$

And equating these expressions yields:

$$\frac{\mathcal{E}^2}{I_{rms}^2} = R^2 + (X_L - X_C)^2$$

We need to solve for $|X_L - X_C|$:

$$|X_L - X_C| = \sqrt{\frac{\mathcal{E}^2}{I_{rms}^2} - R^2}$$

I leads $\mathcal{E}$, then the circuit is capacitive and $X_C > X_L$.

$$|X_L - X_C| = -(X_L - X_C)$$

and

$$X_C = X_L + \sqrt{\frac{\mathcal{E}^2}{I_{rms}^2} - R^2} = 2\pi f L + \sqrt{\frac{\mathcal{E}^2}{I_{rms}^2} - R^2}$$

Substituting numerical values and evaluating:

$$X_C = 2\pi(60s^{-1})(50mH) + \sqrt{\frac{(120V)^2}{(11A)^2} - (7.71\Omega)^2} = 26.6\Omega$$

We substitute in equation 1.16 and evaluate C:

$$C = \frac{1}{2\pi(60s^{-1})(26.6\Omega)} = 99.9\mu F = 0.10mF$$

(d) We'll represent the capacitance required to make $\cos\delta = 1$ as $C_{pf=1}$. The necessary change in capacitance is given by:

$$\Delta C = C_{pf=1} - C = C_{pf=1} - 99.9\mu F$$

We find a relationship between $C_{pf=1}$ and X_L:

$$X_L = \frac{1}{2\pi f C_{pf=1}}$$

Solving for $C_{pf=1}$

$$C_{pf=1} = \frac{1}{2\pi f X_L}$$

Substituting for $C_{pf=1}$ in the expression for ΔC:

$$\Delta C = \frac{1}{2\pi f X_L} - 99.9\mu F$$

Substituting for numefical values and evaluating:

$$\Delta C = \frac{1}{2\pi(60s^{-1})(18.8\Omega)} - 99.9\mu F = 41\mu F$$

(e) We represent the inductance required to make $\cos\delta = 1$ as $L_{pf=1}$. The necessary change in inductance is given by:

$$\Delta L = L_{pf=1} - L = L_{pf=1} - 50mH$$

Relating $L_{pf=1}$ to X_C:

$$X_L = X_C = 2\pi f L_{pf=1}$$

Solving it:

$$L_{pf=1} = \frac{X_C}{2\pi f}$$

Substitute for $L_{pf=1}$ in the expression for ΔL:

$$\Delta L = \frac{X_C}{2\pi f} - 50mH$$

Substituting numerical values and evaluating:

$$\Delta L = \frac{26.6\Omega}{2\pi(60s^{-1})} - 50mH = 20mH$$

71 - The figures for the impedance of the three circuits are shown. Also shown in each figure is the asymptotic approach for large angular frequencies.

72 - The maximum current in the circuit can be found from the maximum voltage across the capacitor and the reactance of the capacitor. To find the range of inductance that is safe to use we can express Z^2 for the circuit in terms of $\mathcal{E}_{peak}^2$ and I_{peak}^2 and solve the resulting quadratic equation for L.
(a) We express the peak current in terms of the maximum potential difference across the capacitor and its resistance:

$$I_{peak} = \frac{V_{C,peak}}{X_C} = \omega C V_{C,peak}$$

Substituting numerical values:

$$I_{peak} = (2500 rad/s)(8.00 \mu F)(150V) = 3.00A$$

(b) We do relate the maximum current in the circuit to the emf of the source and the impedance of the circuit:

$$I_{peak} = \frac{\mathcal{E}_{peak}}{Z} \Rightarrow Z^2 = \frac{\mathcal{E}_{peak}^2}{I_{peak}^2}$$

Expressing Z^2 in terms of R, X_L, X_C

$$Z^2 = R^2 + (X_L - X_C)^2$$

Substituting:

$$\frac{\mathcal{E}_{peak}^2}{I_{peak}^2} = R^2 + (X_L - X_C)^2$$

Evaluating X_C:

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500 rad/s)(8.00 \mu F)} = 50.0\Omega$$

We now substitute numerical values:

$$\frac{(200V)^2}{(3.00A)^2} = (60.0\Omega)^2 + ((2500 rad/s)L - 50.0\Omega)^2$$

Solving for L:

$$L = \frac{50.0\Omega \pm \sqrt{844\Omega^2}}{2500 rad/s}$$

We can denote solutions as L_+, L_-, and find the values for the inductance:

$$L_+ = 31.6mH \text{ and } L_- = 8.38mH$$

The ranges for L:

$$\boxed{8.00mH < L < 8.38mH}$$

and

$$\boxed{31.6mH < L < 40.0mH}$$

73 - The impedance of the circuit can be found from the applied emf and the current drawn by the device. For (b) we can use $P_{av} = I_{rms}^2 R$ to find R and the definition of impedance of a series RLC circuit to find $X = X_L - X_C$.
(a) We express the impedance in terms of the emf provided by the power line and the current, we substitute numerical values:

$$Z = \frac{\mathcal{E}_{rms}}{I_{rms}} = \frac{120V}{10A} = \boxed{12\Omega}$$

(b) We express R using the relationship between the average power supplied to the device and the rms current it draws:

$$P_{av} = I_{rms}^2 R \Rightarrow R = \frac{P_{av}}{I_{rms}^2}$$

Substituting numerical values:

$$R = \frac{720W}{(10A)^2} = \boxed{7.20\Omega}$$

The impedance of a series RLC circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

or

$$Z^2 = R^2 + (X_L - X_C)^2$$

Solving for $X_L - X_C$ yields:

$$X = X_L - X_C = \sqrt{Z^2 - R^2}$$

Substituting numerical values and evaluating:

$$X = \sqrt{(12\Omega)^2 - (7.20\Omega)^2} = \boxed{10\Omega}$$

(c) The reactance is capacitive if the current leads the emf.

74 - (a) The fact that when the current is maximum, $X_L = X_C$, can be used to find the inductance of the circuit. (b) $\mathcal{E}_{peak}$ and the impedance of the circuit at resonance can give $I_{rms,max}$.
(a) Relate X_L and X_C at resonance:

$$X_L = X_C \text{ or } \omega_0 L = \frac{1}{\omega_0 C}$$

Solve for L:

$$L = \frac{1}{\omega_0^2 C}$$

Substituting numerical values and evaluating:

$$L = \frac{1}{(5000s^{-1})^2(10\mu F)} = \boxed{4.0 \text{ mH}}$$

(b) At resonance, $X = 0$, then express $I_{rms,max}$ in terms of the applied emf and the impedance of the circuit at resonance:

$$I_{rms,max} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{max}}{\sqrt{2}Z} = \frac{10V}{\sqrt{2}(100\Omega)} = \boxed{71 \text{ mA}}$$

75 - The capacitor and resistor are connected in parallel, then the voltage drops across then are equal. The total current is the sum of the current through the capacitor and the current through the resistor. These currents are not in phase, we need to calculate phasors to calculate their sum. The amplitudes of the applied voltage and the currents are equal to the magnitude of the phasors, that is $|\vec{\mathcal{E}}| = \mathcal{E}_{peak}$, $|\vec{I}| = I_{peak}$, $|\vec{I}_R| = I_{R,peak}$, and $|\vec{I_C}| = I_{C,peak}$.
(a) $\mathcal{E} = \mathcal{E}_{peak} \cos \omega t$ is the voltage applied by the source. Thus, the voltage drop across both the load resistor and the capacitor is:

$$\mathcal{E}_{peak} \cos \omega t = I_R R$$

The current in the resistor is in phase with the applied voltage:

$$I_R = I_{R,peak} \cos \omega t$$

and because $I_{R,peak} = \frac{\mathcal{E}_{peak}}{R}$:

$$I_R = \frac{\mathcal{E}_{peak}}{R} \cos \omega t$$

(b) The current in the capacitor leads the applied voltage by 90 degrees:

$$I_C = I_{C,peak} \cos(\omega t + 90°)$$

And because $I_{C,peak} = \frac{\mathcal{E}_{peak}}{X_C}$:

$$I_C = \boxed{\frac{\mathcal{E}_{peak}}{X_C} \cos(\omega t + 90°)}$$

(c) The net current I is the sum of the currents through the parallel branches:

$$I = I_R + I_C$$

We need to draw a phasor diagram for the circuit. The projections of the phasors onto the horizontal axis are the instantaneous values. The current in the resistor is in phase with the applied voltage, and hte current in the capacitor leads the applied voltage by 90 degrees. The net current phasor is the sum of the branch current phasors.
The peak current throught the parallel combination is $\mathcal{E}_{peak}/Z$, where Z is the impedance of the combination:

$$I = I_{peak} \cos(\omega t - |\delta|)$$

, where

$$I_{peak} = \frac{\mathcal{E}_{peak}}{Z}$$

Analyzing the phasor diagram we do have:

$$I^2_{peak} = I^2_{R,peak} + I^2_{C,peak} = \left(\frac{\mathcal{E}_{peak}}{R}\right)^2 + \left(\frac{\mathcal{E}_{peak}}{X_C}\right)^2$$

$$= \mathcal{E}^2_{peak}\left(\frac{1}{R^2} + \frac{1}{X_C^2}\right) = \frac{\mathcal{E}^2_{peak}}{Z^2}$$

where

$$\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X_C^2}$$

Solving for I_{peak} yields:

$$I_{peak} = \boxed{\frac{\mathcal{E}_{peak}}{Z}} \text{ where } Z^{-2} = R^{-2} + X_C^{-2}$$

And from the phasor diagram:

$$I = \boxed{I_{peak}\cos(\omega t + \delta)}$$

where

$$\tan\delta = \frac{I_C}{I_R} = \frac{\frac{\mathcal{E}_{peak}}{X_C}}{\frac{\mathcal{E}_{peak}}{R}} = \boxed{\frac{R}{X_C}}$$

76 - Determining the half power points using the condition can be used to tobtain the quadratic equation that we can solve for the frequencies corresponding to the half power points. Let ω_1 be the half power frequency that is less than ω_0 and ω_2 be the half power frequency that is greater than ω_0 will lead us to the result that $\Delta\omega = \omega_2 - \omega_1 \approx R/L$. We then can use the Q definition to complete the proof that $Q \approx \omega_0/\Delta\omega$.
Equation 29-56 is:

$$P_{av} = \frac{V^2_{app,rms}R\omega^2}{L^2(\omega^2 - \omega_0^2) + \omega^2 R^2}$$

When the denominator of equation 29-56 is twice the value near resonance, the half-power points occur:

$$L^2(\omega^2 - \omega_0^2)^2 + \omega^2 R^2 \approx 2\omega_0^2 R^2$$

or

$$L^2[(\omega - \omega_0)(\omega + \omega_0)]^2 + \omega^2 R^2 \approx 2\omega_0^2 R^2$$

For a sharply peaked resonance, $\omega + \omega_0 \approx 2\omega_0$. Hence:

$$L^2[(\omega - \omega_0)(2\omega_0)]^2 + \omega^2 R^2 \approx 2\omega_0^2 R^2$$

or

$$4\omega_0^2 L^2(\omega-\omega_0)^2+\omega^2R^2 \approx 2\omega_0^2R^2$$

We represent with ω_1 a solution to this equation. And we note that for a sharply peaked resonance, $\omega_1 \approx \omega_0$, it follows:

$$4\omega_0^2 L^2(\omega-\omega_0)^2+\omega^2R^2 \approx 2\omega_0^2R^2$$

if we simplify it:

$$(\omega-\omega_0)^2 \approx \frac{R^2}{4L^2}$$

and solving for ω_1:

$$\omega_1 \approx \omega_0 - \frac{R}{2L}$$

where we used the minus sign because $\omega_1 < \omega_0$.
Similarly for ω_2

$$\omega_2 \approx \omega_0 - \frac{R}{2L}$$

where we used the plus sign because $\omega_2 > \omega_0$.
Evaluating $\Delta\omega = \omega_2 - \omega_1$:

$$\Delta\omega \approx \omega_0 + \frac{R}{2L} - \left(\omega_0 - \frac{R}{2L}\right) = \boxed{\frac{R}{L}}$$

Using the Q definition:

$$\frac{R}{L} = \frac{\omega_0}{Q}$$

Substituting in the expression for $\Delta\omega$:

$$\Delta\omega \approx \frac{\omega_0}{Q} \Rightarrow Q \approx \boxed{\frac{\omega_0}{\Delta\omega}}$$

77 - From the equation $Q = Q_0 e^{-t/\tau}\cos\omega' t$ we can start and differentiate twice and then substitute this function and both its derivatives in the differential equation of the circuit. Then we can re-write the resulting equation in the form $A\cos\omega' t + B\sin\omega' t = 0$ will reveal that B is equal to zero. If we require that $A\cos\omega' t = 0$ to hold for all values of t will lead to the fact that $\omega' = \sqrt{1/(LC) - 1/\tau^2}$.
Taking equation 29-43b:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

We do assume a solution of the form:

$$Q = Q_0 e^{-t/\tau}\cos\omega' t$$

Differentiating twice Q(t):

$$\frac{dQ}{dt} = Q_0 \frac{d}{dt}[e^{-t/\tau} \cos\omega' t] = Q_0 \left(e^{-t/\tau} \frac{d}{dt} \cos\omega' t + \cos\omega' t \frac{d}{dt} e^{-t/\tau} \right)$$

$$= Q_0 e^{-t/\tau} \left(-\omega' \sin\omega' t - \frac{1}{\tau} \cos\omega' t \right)$$

and

$$\frac{d^2Q}{dt^2} = Q_0 \frac{d}{dt} \left[e^{-t/\tau} \left(-\omega' \sin\omega' t - \frac{1}{\tau} \cos\omega' t \right) \right]$$

$$= Q_0 e^{-t/\tau} \left[\left(\frac{1}{\tau^2} - \omega'^2 \right) \cos\omega' t + \frac{2\omega'}{\tau} \sin\omega' t \right]$$

We substitute these derivatives in the differential equation and simplify:

$$LQ_0 e^{-t/\tau} \left[\left(\frac{1}{\tau^2} - \omega'^2 \right) \cos\omega' t + \frac{2\omega'}{\tau} \sin\omega' t \right] + RQ_0 e^{-t/\tau} \left(-\omega' \sin\omega' t - \frac{1}{\tau} \cos\omega' t \right)$$

$$+ \frac{1}{C} Q_0 e^{-t/\tau} \cos\omega' t = 0$$

Now, Q_0 and $e^{-t/\tau}$ are never zero, so we can divide them out of the equation and simplify:

$$L \left(\frac{1}{\tau^2} - \omega'^2 \right) \cos\omega' t + \frac{2L\omega'}{\tau} \sin\omega' t - \omega' R \sin\omega' t - \frac{R}{\tau} \cos\omega' t + \frac{1}{C} \cos\omega' t = 0$$

We can re-write this equation in the form $A\cos\omega' t + B\sin\omega' t = 0$:

$$(R\omega' - R\omega') \sin\omega' t + \left[L \left(\frac{1}{\tau^2} - \omega'^2 \right) + \frac{1}{C} - \frac{R}{\tau} \right] \cos\omega' t = 0$$

or

$$\left[L \left(\frac{1}{\tau^2} - \omega'^2 \right) + \frac{1}{C} - \frac{R}{\tau} \right] \cos\omega' t = 0$$

If this equation holds for all values of t, then its coefficient must be zero:

$$L \left(\frac{1}{\tau^2} - \omega'^2 \right) + \frac{1}{C} - \frac{R}{\tau} = 0$$

We now have to solve for ω'

$$\omega' = \boxed{\sqrt{\tfrac{1}{LC} - \left(\tfrac{1}{2L}\right)^2}}$$

the condition that must be satisfied if $Q = Q_0 e^{-t/\tau} \cos\omega' t$ is the solution to equation 29-43b

78 - The inductance of the empty solenoid can be determined by using $L = \mu_0 n^2 A\ell$ and the resonance condition to find the capacitance of the sample-free circuit when the resonance frequency of the circuit is 6.00 MHz. We can evaluate χ from its definition, if we express L as a function of f_0 and then evaluate df_0/dL and approximating the derivative with $\Delta f_0/\Delta L$.
(a) We express the inductance of an air-core solenoid:

$$L = \mu_0 n^2 A\ell$$

Substituting numerical values and evaluating:

$$L = (4\pi \times 10^{-7} N/A^2)\left(\frac{400}{4.00cm}\right)^2 \frac{\pi}{4}(3.00cm)^2(4.00cm) = \boxed{3.553 \text{ mH}}$$

(b) Expressing the condition for resonance in the LC circuit:

$$X_L = X_C \Rightarrow 2\pi f_0 L = \frac{1}{2\pi f_0 C} \tag{1.17}$$

If we solve for C, it yields:

$$C = \frac{1}{4\pi^2 f_0^2 L}$$

Substituting numerical values and evaluating:

$$C = \frac{1}{4\pi^2(6.00MHz)^2(3.553mH)} = 1.9803 \times 10^{-13} F = \boxed{0.198 \text{ pF}}$$

(c) We express the sample's susceptibility in terms of L and ΔL:

$$\chi = \frac{\Delta L}{L} \tag{1.18}$$

Now solve equation 1.17 for f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Differentiating f_0 with respect to L:

$$\frac{df_0}{dL} = \frac{1}{2\pi\sqrt{C}}\frac{d}{dL}L^{-1/2} = -\frac{1}{4\pi\sqrt{C}}L^{-3/2}$$

$$= -\frac{1}{4\pi L\sqrt{LC}} = -\frac{f_0}{2L}$$

We make an approximation of df_0/dL by $\Delta f_0/\Delta L$:

$$\frac{\Delta f_0}{\Delta L} = -\frac{f_0}{2L} \text{ or } \frac{\Delta f_0}{f_0} = -\frac{\Delta L}{2L}$$

Substituting in equation 1.18:

$$\chi = -2\frac{\Delta f_0}{f_0}$$

Finally we substitute numerical values and evaluate:

$$\chi = -2\left(\frac{5.9989MHz - 6.0000MHz}{6.0000MHz}\right) = \boxed{3.7 \times 10^{-4}}$$

1.7 The Transformer

79 - We'll use the subscript 1 to denote the primary and 2 the secondary. These relations can be used to find the turns ratio and the primary current when the transformer connections are reversed $V_2N_1 = V_1N_2$ and $N_1I_1 = N_2I_2$. (a) Find the relationship between the number of primary and secondary turns to the primary and secondary voltages:

$$V_{2,rms}N_1 = V_{1,rms}N_2$$

Solve and evaluate for the ratio N_2/N_1:

$$\frac{N_2}{N_1} = \frac{V_{2,rms}}{V_{1,rms}} = \frac{24V}{120V} = \boxed{\frac{1}{5}}$$

(b) We then relate the current in the primary to the current in the secondary and to the turns ratio:

$$I_{1,rms} = \frac{N_2}{N_1}I_{2,rms}$$

Then we express the current in the primary winding in terms of the voltage across it and its impedance:

$$I_{2,rms} = \frac{V_{2,rms}}{Z_2}$$

Substituting for $I_{2,rms}$ we obtain:

$$I_{1,rms} = \frac{N_2}{N_1}\frac{V_{2,rms}}{Z_2}$$

Finally we substitute numerical values and evaluate:

$$I_1 = \left(\frac{1}{5}\right)\left(\frac{120V}{12\Omega}\right) = \boxed{2\text{ A}}$$

80 - We use subscript 1 for the primary and 2 for the secondary. The transformer is step-up or step-down depending on the examination of the ratio of the number of turns in the secondary to the number of turns in the primary. The open-circuit rms voltage in the secondary can be related to the primary rms voltage and the turns ratio.
(a) There are fewer turns in the secondary than in the primary, hence it is a step-down transformer.
(b) We need to relate open-circuit rms voltages, in the primary and secondary:

$$V_{2,rms} = \frac{N_1}{N_1} V_{1,rms}$$

Substituting numerical values and evaluating:

$$V_{2,rms} = \frac{8}{400}(120V) = \boxed{2.40 \text{ V}}$$

There are no power losses, so we can write:

$$V_{1,rms} I_{1,rms} = V_{2,rms} I_{2,rms}$$

and

$$I_{2,rms} = \frac{V_{1,rms}}{V_{2,rms}} I_{1,rms}$$

Substituting numerical values and evaluating:

$$I_{2,rms} = \frac{120V}{2.40V}(0.100A) = \boxed{5.00 \text{ A}}$$

81 - We use subscript 1 for the primary and 2 for the secondary. $V_{1,rms} I_{1,rms} = V_{2,rms} I_{2,rms}$ can be used to find the current in the primary and $V_{2,rms} N_1 = V_{1,rms} N_2$ to find the number of turns in the secondary.
(a) We have 100 percent efficiency, therefore:

$$V_{1,rms} I_{1,rms} = V_{2,rms} I_{2,rms}$$

and

$$I_{1,rms} = I_{2,rms} \frac{V_{2,rms}}{V_{1,rms}}$$

Substituting numerical values and evaluating:

$$I_{1,rms} = (20A)\frac{9.0V}{120V} = \boxed{1.5 \text{ A}}$$

(b) We need a relationship between the number of primary and secondary turns to the primary and secondary voltages:

$$V_{2,rms} N_1 = V_{1,rms} N_2 \Rightarrow N_2 = \frac{V_{2,rms}}{V_{1,rms}} N_1$$

Substituting numerical values and evaluating:

$$N_2 = \frac{9.0V}{120V}(250) \approx \boxed{19}$$

82 - For a simple circuit, a maximum power transfer from the source requires that the load resistance equal the internal resistance of the source. By Ohm's law and the relationship between the primary and secondary currents and the primary and secondary voltages and the turns ration of the transformer we can derive an expression for the turns ratio as a function of the effective resistance of the circuit and the resistance of the speakers.
(a) We begin by expressing hte effective loudspeaker resistance at the primary of the transformer:

$$R_{eff} = \frac{V_{1,rms}}{I_{1,rms}}$$

Then we relate the $V_{1,rms}$ to $V_{2,rms}, N_1$ and N_2:

$$V_{1,rms} = V_{2,rms}\frac{N_1}{N_2}$$

Expressing the $I_{1,rms}$ in terms of $I_{2,rms}, N_1, N_2$:

$$I_{1,rms} = I_{2,rms}\frac{N_2}{N_1}$$

Substituting for $V_{1,rms}, I_{1,rms}$ and simplifying:

$$R_{eff} = \frac{V_{2,rms}\frac{N_1}{N_2}}{I_{2,rms}\frac{N_2}{N_1}} = \left(\frac{V_{2,rms}}{I_{2,rms}}\right)\left(\frac{N_1}{N_2}\right)^2$$

Solving for N_1/N_2:

$$\frac{N_1}{N_2} = \sqrt{\frac{I_{2,rms}R_{eff}}{V_{2,rms}}} = \sqrt{\frac{R_{eff}}{R_2}} \tag{1.19}$$

Evaluating for $R_{eff} = R_{coil}$:

$$\frac{N_1}{N_2} = \sqrt{\frac{2000\Omega}{8.00\Omega}} = \boxed{15.811}$$

(b) We then express the power delivered to the two speakers connected in parallel:

$$P_{sp} = I_{1,rms}^2 R_{eff} \tag{1.20}$$

Finding the equivalent resistance R_{sp} of the two 8.00Ω speakers in parallel:

$$\frac{1}{R_{sp}} = \frac{1}{8.00\omega} + \frac{1}{8.00\omega} \Rightarrow R_{sp} = 4.00\Omega$$

We now solve equation 1.19:

$$R_{eff} = R_2\left(\frac{N_1}{N_2}\right)^2$$

Substituting numerical values and evaluating:

$$R_{eff} = (4.00\Omega)(15.811)^2 = 1k\Omega$$

We need to find the current supplied by the source:

$$I_{1,rms} = \frac{V_{rms}}{R_{tot}} = \frac{12.0V}{2000V + 1000V} = 4.00mA$$

Finally we substitute numerical values in equation 1.20 and evaluate the power delivered to the parallel speakers:

$$P_{sp} = (4.00mA)^2(1000\Omega) = \boxed{16.0 \text{ mW}}$$

83 - The relationship of number of turns to input and output voltages is what can be used to solve this problem: $V_{2,rms}N_1 = V_{1,rms}N_2$.
We start by relating the output voltage $V_{2,rms}$ to the input voltage $V_{1,rms}$ and the number of turns of the primary and secondary N_1, N_2:

$$V_{2,rms} = \frac{N_2}{N_1}V_{1,rms} \Rightarrow N_1 = N_2\frac{V_{1,rms}}{V_{2,rms}}$$

Finally we substitute numerical values and evaluate:

$$N_1 = (400)\left(\frac{2000V}{240V}\right) = \boxed{3.33 \times 10^3}$$

84 - The definition $I_{rms} = \sqrt{(I^2)_{av}}$ can be used to relate the rms current to the current carried by the resistor and find $(I^2)_{av}$ by integrating I^2.
(a) We do express the rms current in terms of the $(I^2)_{av}$:

$$I_{rms} = \sqrt{(I^2)_{av}}$$

We do evaluate I^2:

$$I^2 = [(5.0A)\sin 2\pi ft + (7.0A)\sin 4\pi ft]^2$$

$$= (25A^2)\sin^2 2\pi ft + (70A^2)\sin 2\pi ft + (49A^2)\sin^2 4\pi ft$$

Now we find $(I^2)_{av}$ by integrating I^2 from $t = 0$ to $t = T = 2\pi/\omega$ and dividing by T:

$$(I^2)_{av} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}[(25A^2)\sin^2 2\pi ft+(70A^2)\sin 2\pi ft \sin 4\pi ft+(49A^2)\sin^2 4\pi ft]dt$$

Using the trigonometric identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ to simplify and evaluate the first and third integrals and recognize that the middle term is of the form $\sin x \sin 2x$ we obtain:

$$(I^2)_{av} = 12.5A^2 + 0 + 24.5A^2 = 37.0A^2$$

We substitute for $(I^2)_{av}$ and evaluate I_{rms}:

$$I_{rms} = \sqrt{37.0A^2} = \boxed{6.1 \text{ A}}$$

(b) The power dissipated in the resistor can be related to its resistance and the rms current in it:

$$P = I_{rms}^2 R$$

Substituting numerical values and evaluating:

$$P = (6.08A)^2(12\Omega) = \boxed{0.44 \text{ kW}}$$

(c) In this exercise we do express the rms voltage across the resistor in terms of R and I_{rms}:

$$V_{rms} = I_{rms}R = (6.08A)(12\Omega) = \boxed{73 \text{ V}}$$

85 - The integral of a quantity over an interval divided by ΔT is the average of that quantity over a time interval ΔT. This definition help us to find both the average of the voltage squared $(V^2)_{av}$ and then use the definition of the rms voltage to finish the analysis.
(a) The V_{rms} definition:

$$V_{rms} = \sqrt{(V_0^2)_{av}}$$

Keep in mind that $-V_0^2 = V_0^2$, and evaluate V_{rms}:

$$V_{rms} = \sqrt{(V^2)_{av}} = V_0 = \boxed{12 \text{ V}}$$

(b) The voltage during the second half of each cycle is zero, we express the voltage during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$V = V_0$$

If we express the square of the voltage during this half cycle:

$$V^2 = V_0^2$$

We need to calculate $(V^2)_{av}$ by integrating V^2 from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by ΔT:

$$(V^2)_{av} = \frac{V_0^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{V_0^2}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = \frac{1}{2}V_0^2$$

Substituting numerical values:

$$V_{rms} = \sqrt{\frac{1}{2}V_0^2} = \frac{V_0}{\sqrt{2}} = \frac{12V}{\sqrt{2}} = \boxed{8.5 \text{ V}}$$

86 - The integral of a quantity over an interval divided by ΔT is the average of that quantity over a time interval ΔT. This definition can help us to find the average current I_{av} and the average of the current squared, $(I^2)_{av}$.
We begin using the definition of I_{av} and I_{rms}:

$$I_{av} = \frac{1}{\Delta T}\int_0^{\Delta T} I dt \text{ and } I_{rms} = \sqrt{(I^2)_{av}}$$

(a) Waveform. We express the current during the first cycle of time interval ΔT:

$$I_a = \frac{4A}{\Delta T}t$$

where I is in A when t and T are in seconds.
Then we evaluate $I_{av,a}$:

$$I_{av,a} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{4.0A}{\Delta T}tdt = \frac{4.0A}{(\Delta T)^2}\int_0^{\Delta T} tdt$$

$$= \frac{4.0A}{(\Delta T)^2}\left[\frac{t^2}{2}\right]_0^{\Delta T} = \boxed{2.0 \text{ A}}$$

Now express the square of the current during this half cycle:

$$I_a^2 = \frac{(4.0A)^2}{(\Delta T)^2}t^2$$

The average value of the current is the same for each time interval ΔT, we calcultate $(I_a^2)_{av}$ by integrating I_a^2 from $t = 0$ to $t = \Delta T$ and divide by ΔT:

$$(I_a^2)_{av} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{(4.0A)^2}{(\Delta T)^3}\left[\frac{t^3}{3}\right]_0^{\Delta T} = \frac{16}{3}A^2$$

Substitute in the expression for $I_{rms,a}$:

$$I_{rms,a} = \sqrt{\frac{16}{3}A^2} = \boxed{2.3 \text{ A}}$$

(b) Waveform. The current during the second half of each cycle is zero, therefore we can express the current during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$I_b = 4.0A$$

Evaluate $I_{av,b}$:

$$I_{av,b} = \frac{4.0A}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{4.0A}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = \boxed{2.0 \text{ A}}$$

Expressing the square of the current during this half cycle:

$$I_b^2 = (4.0A)^2$$

Now calculate $(I_b^2)_{av}$ by integrating (I_b^2) from $t = 0$ to $t = \frac{1}{2}\Delta T$ and divide by ΔT:

$$(I_b^2)_{av} = \frac{(4.0A)^2}{\Delta T} \int_0^{\frac{1}{2}\Delta T} dt$$

$$= \frac{(4.0A)^2}{\Delta T} [t]_0^{\frac{1}{2}\Delta T} = 8.0A^2$$

Finally we substitute in the expression for $I_{rms,b}$:

$$I_{rms,b} = \sqrt{8.0A^2} = \boxed{2.8 \text{ A}}$$

87 - The current in the circuit in terms of the emf of the sources and the resistance of the resistor can be expressed if we utilize Kirchhoff's loop rule. Afterwards, we can find the minimum and maximum currents by considering the conditions under which the time-dependent factor in I will be maximum or minimum. Finally, we can use $I_{rms} = \sqrt{(I^2)_{av}}$ to derive an expression for I_{rms} that we can use to determine its value.
First we begin by applying Kirchhoff's loop rule:

$$\mathcal{E}_{1,peak} \cos \omega t + \mathcal{E}_2 - IR = 0$$

We do solve for I:

$$I = \frac{\mathcal{E}_{1,peak}}{R} \cos \omega t + \frac{\mathcal{E}_2}{R}$$

or

$$I = A_1 \cos \omega t + A_2$$

where

$$A_1 = \frac{\mathcal{E}_{1,peak}}{R} \text{ and } A_2 = \frac{\mathcal{E}_2}{R}$$

Substituting numerical values:

$$I = \left(\frac{20V}{36\Omega}\right) \cos(2\pi(180s^{-1})t) + \frac{18V}{36\Omega}$$

$$= (0.556A) \cos(1131s^{-1})t + 0.50A$$

The current is a maximum when $\cos(1131s^{-1})t = 1$. Hence:

$$I_{max} = 0.50A + 0.556A = \boxed{1.06 \text{ A}}$$

We now evaluate I_{min}:

$$I_{min} = 0.50A - 0 - 556A = \boxed{\text{-0.06 A}}$$

And because the average value of $\cos \omega t = 0$:

$$I_{av} = \boxed{0.50 \text{ A}}$$

The rms current is the square root of the average of the squared current:

$$I_{rms} = \sqrt{[I^2]_{av}} \tag{1.21}$$

where $[I^2]_{av}$ is given by:

$$[I^2]_{av} = [(A_1 \cos \omega t + A_2)^2]_{av}$$

$$= [A_1^2 \cos^2 \omega t + 2A_1 A_2 \cos \omega t + A_2^2]_{av}$$

$$[A_1^2 \cos^2 \omega t]_{av} + [2A_1 A_2 \cos \omega t]_{av} + [A_2^2]_{av}$$

$$A_1^2 [\cos^2 \omega t]_{av} + 2A_1 A_2 [\cos \omega t]_{av} + A_2^2$$

And because $[\cos^2 \omega t]_{av} = \frac{1}{2}$ and $[\cos \omega t]_{av} = 0$:

$$[I^2]_{av} = \frac{1}{2} A_1^2 + A_2^2$$

Substituting in equation 1.21:

$$I_{rms} = \sqrt{\frac{1}{2} A_1^2 + A_2^2}$$

$$I_{rms} = \sqrt{\frac{1}{2} \left(\frac{\mathcal{E}_1}{R}\right)^2 + \left(\frac{\mathcal{E}_2}{R}\right)^2}$$

Finally we substitute numerical values and evaluate:

$$I_{rms} = \sqrt{\frac{1}{2} \left(\frac{20V}{36\Omega}\right)^2 + \left(\frac{18V}{36\Omega}\right)^2} = \boxed{0.64 \text{ A}}$$

88 - We need to obtain an expression for charge on the capacitor as a function of time and this can be obtained using Kirchhoff's loop rule. If we differentiate this expression with respect to time will give us the current in the circuit. Then we can find I_{max} and I_{min} by considering the conditions under which the time dependent factor in I will be a maximum or a minimum. Finally, we can use $I_{rms} = \sqrt{(I^2)_{av}}$ to derive an expression for I_{rms} that we can use to determine its value.

We begin by applying the Kirchhoff's loop rule:

$$\mathcal{E}_{1,peak} \cos \omega t + \mathcal{E}_2 - \frac{q(t)}{C} = 0$$

Solving for q(t):

$$q(t) = C(\mathcal{E}_{1,peak} \cos \omega t + \mathcal{E}_2) = A_1 \cos \omega t + A_2$$

where

$$A_1 = C\mathcal{E}_{1,peak} \text{ and } A_2 = C\mathcal{E}_2$$

We differentiate this expression with respect to time to obtain the current as a function of time:

$$I = \frac{dq}{dt} = \frac{d}{dt}(A_1 \cos \omega t + A_2) = -\omega A_1 \sin \omega t$$

We now substitute numerical values:

$$I = -2\pi(180Hz)(2.0\mu F)\sin(2\pi(180Hz)t) = (-2.26mA)\sin(1131s^{-1})t$$

Then we analyze that the current is at minimum when $\sin(1131s^{-1} = 1)$. Hence:

$$I_{min} = \boxed{\text{-2.3 mA}}$$

And the current is at maximum when $\sin(1131s^{-1} = -1)$, then:

$$I_{max} = \boxed{\text{2.3 mA}}$$

The dc source senses the capacitor as an open circuit and the average value fo the sine function over a period is zero, hence:

$$I_{av} = \boxed{0}$$

The rms current is the square root f the average of the squared current:

$$I_{rms} = \sqrt{[I^2]_{av}} \tag{1.22}$$

where $[I^2]_{av}$ is given by:

$$\begin{aligned}
[I^2]_{av} &= [(A_1 \cos \omega t + A_2)^2]_{av} \\
&= [A_1^2 \cos^2 \omega t + 2A_1 A_2 \cos \omega t + A_2^2]_{av} \\
&[A_1^2 \cos^2 \omega t]_{av} + [2A_1 A_2 \cos \omega t]_{av} + [A_2^2]_{av} \\
&A_1^2[\cos^2 \omega t]_{av} + 2A_1 A_2[\cos \omega t]_{av} + A_2^2
\end{aligned}$$

And because $[\cos^2 \omega t]_{av} = \frac{1}{2}$ and $[\cos \omega t]_{av} = 0$:

$$[I^2]_{av} = \frac{1}{2}A_1^2 + A_2^2$$

Substituting in equation 1.21:

$$I_{rms} = \sqrt{\frac{1}{2}A_1^2 + A_2^2}$$

Substituting for A_1, A_2:

$$\begin{aligned}
I_{rms} &= \sqrt{\frac{1}{2}(C\mathcal{E}_1)^2 + (C\mathcal{E}_2)^2} \\
&= C\sqrt{\frac{1}{2}(\mathcal{E}_1)^2 + (\mathcal{E}_2)^2}
\end{aligned}$$

Substituting numerical values and evaluating:

$$I_{rms} = (2.0\mu F)\sqrt{\frac{1}{2}(20V)^2 + (18V)^2} = \boxed{46\mu A}$$

89 - In part (a) we can obtain the second order differential equation relating the charge on the capacitor to the time by using the Kirchhoff's loop rule. In part (b) we take a solution of the form $Q = Q_{peak}\cos\omega t$, differentiate it two times, and substitute for d^2Q/dt^2 and Q to show that the assumed solution satisfies the differential equation provided $Q_{peak} = -\frac{\mathcal{E}_{peak}}{L(\omega^2-\omega_0^2)}$. In part (c) we use the results from (a) and (b) to find a result for I_{peak} given the problem statement.
(a) We begin by applying the Kirchhoff's loop rule:

$$\mathcal{E} - \frac{Q}{C} - L\frac{dI}{dt} = 0$$

Substituting for $\mathcal{E}$ and rearranging the diffenrential equation:

$$L\frac{dI}{dt} + \frac{Q}{C} = \mathcal{E}_{max}\cos\omega t$$

And taking into account that $I = dQ/dt$:

$$\boxed{L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \mathcal{E}_{max}\cos\omega t}$$

(b) We assume a solution of the form:

$$Q = Q_{peak}\cos\omega t$$

Differentiating twice:

$$\frac{dQ}{dt} = -\omega Q_{peak}\sin\omega t$$

and

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_{peak}\cos\omega t$$

Then substituting $\frac{dQ}{dt}$ and $\frac{d^2Q}{dt^2}$ in the differential equation:

$$-\omega^2 LQ_{peak}\cos\omega t + \frac{Q_{peak}}{C}\cos\omega t = \mathcal{E}_{peak}\cos\omega t$$

Taking $\cos\omega t$ as factor:

$$\left(-\omega^2 LQ_{peak} + \frac{Q_{peak}}{C}\right)\cos\omega t = \mathcal{E}_{peak}\cos\omega t$$

For this equation to hold for all values of t, we need:

$$\left(-\omega^2 LQ_{peak} + \frac{Q_{peak}}{C}\right) = \mathcal{E}_{peak}$$

Solving for Q_{peak}:

$$Q_{peak} = \frac{\mathcal{E}_{peak}}{-\omega^2 L + \frac{1}{C}}$$

We now take L as factor and substitute for $1/LC$:

$$Q_{peak} = \frac{\mathcal{E}_{peak}}{L\left(-\omega^2 + \frac{1}{LC}\right)} = \boxed{-\frac{\mathcal{E}_{peak}}{L(\omega^2 - \omega_0^2)}}$$

From (a) and (b) we have:

$$I = \frac{dQ}{dt} = -\omega Q_{peak} \sin \omega t = \frac{\omega \mathcal{E}_{peak}}{L(\omega^2 - \omega_0^2)} \sin \omega t$$

$$= I_{peak} \sin \omega t = \boxed{I_{peak} \cos(\omega t - \delta)}$$

where

$$I_{peak} = \boxed{\frac{\omega \mathcal{E}_{peak}}{L|\omega^2 - \omega_0^2|}} = \frac{\mathcal{E}_{peak}}{\frac{L}{\omega}|\omega^2 - \omega_0^2|}$$

$$= \frac{\mathcal{E}_{peak}}{|\omega L - \frac{1}{\omega C}|} = \boxed{\frac{\mathcal{E}_{peak}}{|X_L - X_C|}}$$

If $\omega > \omega_0, X_L > X_C$ and the current lags the voltage by $90°(\delta = 90°)$ If $\omega < \omega_0, X_L < X_C$ and the current lags the voltage by $90°(\delta = -90°)$