Chapter 29
Alternating-Current Circuits

Conceptual Problems

1 • A coil in an ac generator rotates at 60 Hz. How much time elapses between successive peak emf values of the coil?

Determine the Concept Successive peaks are one-half period apart. Hence the elapsed time between the peaks is $\frac{1}{2}T = \frac{1}{2f} = \frac{1}{2\left(60\ \text{s}^{-1}\right)} = \boxed{8.33\ \text{ms}}$.

2 • If the rms voltage in an ac circuit is doubled, the peak voltage is (*a*) doubled, (*b*) halved, (*c*) increased by a factor of $\sqrt{2}$, (*d*) not changed.

Picture the Problem We can use the relationship between V and V_{peak} to decide the effect of doubling the rms voltage on the peak voltage.

Express the initial rms voltage in terms of the peak voltage:

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}$$

Express the doubled rms voltage in terms of the new peak voltage V'_{peak}:

$$2V_{\text{rms}} = \frac{V'_{\text{peak}}}{\sqrt{2}}$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{2V_{\text{rms}}}{V_{\text{rms}}} = \frac{\frac{V'_{\text{peak}}}{\sqrt{2}}}{\frac{V_{\text{peak}}}{\sqrt{2}}} \Rightarrow 2 = \frac{V'_{\text{peak}}}{V_{\text{peak}}}$$

Solving for V'_{peak} yields:

$$V'_{\text{peak}} = 2V_{\text{peak}} \Rightarrow \boxed{(a)} \text{ is correct.}$$

3 • **[SSM]** If the frequency in the circuit shown in Figure 29-27 is doubled, the inductance of the inductor will (*a*) double, (*b*) not change, (*c*) halve, (*d*) quadruple.

Determine the Concept The inductance of an inductor is determined by the details of its construction and is independent of the frequency of the circuit. The inductive reactance, on the other hand, is frequency dependent. $\boxed{(b)}$ is correct.

4 • If the frequency in the circuit shown in Figure 29-27 is doubled, the inductive reactance of the inductor will (*a*) double, (*b*) not change, (*c*) halve, (*d*) quadruple.

Determine the Concept The inductive reactance of an inductor varies with the frequency according to $X_L = \omega L$. Hence, doubling ω will double X_L. $\boxed{(a)}$ is correct.

5 • If the frequency in the circuit in Figure 29-28 is doubled, the capacitive reactance of the circuit will (*a*) double, (*b*) not change, (*c*) halve, (*d*) quadruple.

Determine the Concept The capacitive reactance of an capacitor varies with the frequency according to $X_C = 1/\omega C$. Hence, doubling ω will halve X_C. $\boxed{(c)}$ is correct.

6 • (*a*) In a circuit consisting solely of a ac generator and an ideal inductor, are there any time intervals when the inductor receives energy from the generator? If so, when? Explain your answer. (*b*) Are there any time intervals when the inductor supplies energy back to the generator? If so when? Explain your answer.

Determine the Concept Yes to both questions. (*a*) While the magnitude of the current in the inductor is increasing, the inductor absorbs power from the generator. (*b*) When the magnitude of the current in the inductor decreases, the inductor supplies power to the generator.

7 • **[SSM]** (*a*) In a circuit consisting of a generator and a capacitor, are there any time intervals when the capacitor receives energy from the generator? If so, when? Explain your answer. (*b*) Are there any time intervals when the capacitor supplies power to the generator? If so, when? Explain your answer.

Determine the Concept Yes to both questions. (*a*) While the magnitude of the charge is accumulating on either plate of the capacitor, the capacitor absorbs power from the generator. (*b*) When the magnitude of the charge is on either plate of the capacitor is decreasing, it supplies power to the generator.

8 • (*a*) Show that the SI unit of inductance multiplied by the SI unit of capacitance is equivalent to seconds squared. (*b*) Show that the SI unit of inductance divided by the SI unit of resistance is equivalent to seconds.

Determine the Concept

(*a*) Substitute the SI units of inductance and capacitance and simplify to obtain:

$$\frac{\mathrm{V\cdot s}}{\mathrm{A}}\cdot\frac{\mathrm{C}}{\mathrm{V}}=\frac{\mathrm{s}}{\frac{\mathrm{C}}{\mathrm{s}}}\cdot\mathrm{C}=\boxed{\mathrm{s}^2}$$

(*b*) Substitute the SI units of inductance divided by resistance and simplify to obtain:

$$\frac{\frac{\text{V}\cdot\text{s}}{\text{A}}}{\Omega}=\frac{\frac{\text{V}\cdot\text{s}}{\text{A}}}{\frac{\text{V}}{\text{A}}}=\boxed{\text{s}}$$

9 • **[SSM]** Suppose you increase the rotation rate of the coil in the generator shown in the simple ac circuit in Figure 29-29. Then the rms current (*a*) increases, (*b*) does not change, (*c*) may increase or decrease depending on the magnitude of the original frequency, (*d*) may increase or decrease depending on the magnitude of the resistance, (*e*) decreases.

Determine the Concept Because the rms current through the resistor is given by $I_{\text{rms}}=\frac{\mathcal{E}_{\text{rms}}}{R}=\frac{\mathcal{E}_{\text{peak}}}{\sqrt{2}}=\frac{NBA}{\sqrt{2}}\omega$, I_{rms} is directly proportional to *ω*. $\boxed{(a)}$ is correct.

10 • If the inductance value is tripled in a circuit consisting solely of a variable inductor and a variable capacitor, how would you have to change the capacitance so that the natural frequency of the circuit is unchanged? (*a*) triple the capacitance, (*b*) decrease the capacitance to one-third of its original value, (*c*) You should not change the capacitance.(*d*) You cannot determine how to change the capacitance from the data given.

Determine the Concept The natural frequency of an *LC* circuit is given by $f_0=1/2\pi\sqrt{LC}$.

Express the natural frequencies of the circuit before and after the inductance is tripled:

$$f_0=\frac{1}{2\pi\sqrt{LC}}\text{ and }f_0^{'}=\frac{1}{2\pi\sqrt{L'C'}}$$

Divide the second of the these equations by the first and simplify to obtain:

$$\frac{f_0^{'}}{f_0}=\frac{\frac{1}{2\pi\sqrt{L'C'}}}{\frac{1}{2\pi\sqrt{LC}}}=\sqrt{\frac{LC}{L'C'}}$$

Because the natural frequency is unchanged:

$$1=\sqrt{\frac{LC}{L'C'}}\Rightarrow\frac{LC}{L'C'}=1\Rightarrow C'=\frac{L}{L'}C$$

When the inductance is tripled:

$$C'=\frac{L}{3L}C=\frac{1}{3}C\Rightarrow\boxed{(b)}\text{ is correct.}$$

11 • **[SSM]** Consider a circuit consisting solely of an ideal inductor and an ideal capacitor. How does the maximum energy stored in the capacitor compare to the maximum value stored in the inductor? (*a*) They are the same and each equal to the total energy stored in the circuit. (*b*) They are the same and each equal to half of the total energy stored in the circuit. (*c*) The maximum energy stored in the capacitor is larger than the maximum energy stored in the inductor. (*d*) The maximum energy stored in the inductor is larger than the maximum energy stored in the capacitor. (e) You cannot compare the maximum energies based on the data given because the ratio of the maximum energies depends on the actual capacitance and inductance values.

Determine the Concept The maximum energy stored in the electric field of the capacitor is given by $U_e = \frac{1}{2}\frac{Q^2}{C}$ and the maximum energy stored in the magnetic field of the inductor is given by $U_m = \frac{1}{2}LI^2$. Because energy is conserved in an *LC* circuit and oscillates between the inductor and the capacitor, $U_e = U_m = U_{total}$. $\boxed{(a)}$ is correct.

12 • True or false:

(*a*) A driven series *RLC* circuit that has a high *Q* factor has a narrow resonance curve.
(*b*) A circuit consists solely of a resistor, an inductor and a capacitor, all connected in series. If the resistance of the resistor is doubled, the natural frequency of the circuit remains the same.
(*c*) At resonance, the impedance of a driven series *RLC* combination equals the resistance *R*.
(*d*) At resonance, the current in a driven series *RLC* circuit is in phase with the voltage applied to the combination.

(*a*) True. The *Q* factor and the width of the resonance curve at half power are related according to $Q = \omega_0/\Delta\omega$; i.e., they are inversely proportional to each other.

(*b*) True. The natural frequency of the circuit depends only on the inductance *L* of the inductor and the capacitance *C* of the capacitor and is given by $\omega = 1/\sqrt{LC}$.

(*c*) True. The impedance of an *RLC* circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance $X_L = X_C$ and so $Z = R$.

(*d*) True. The phase angle δ is related to X_L and X_C according to $\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$. At resonance $X_L = X_C$ and so $\delta = 0$.

13 • True or false (all questions related to a driven series *RLC* circuit):

(*a*) Near resonance, the power factor of a driven series *RLC* circuit is close to zero.
(*b*) The power factor of a driven series *RLC* circuit does not depend on the value of the resistance.
(*c*) The resonance frequency of a driven series *RLC* circuit does not depend on the value of the resistance.
(*d*) At resonance, the peak current of a driven series *RLC* circuit does not depend on the capacitance or the inductance.
(*e*) For frequencies below the resonant frequency, the capacitive reactance of a driven series *RLC* circuit is larger than the inductive reactance.
(*f*) For frequencies below the resonant frequency of a driven series *RLC* circuit, the phase of the current leads (is ahead of) the phase of the applied voltage.

(*a*) False. Near resonance, the power factor, given by $\cos\delta = \dfrac{R}{\sqrt{(X_L - X_C)^2 + R^2}}$, is close to 1.

(*b*) False. The power factor is given by $\cos\delta = \dfrac{R}{\sqrt{(X_L - X_C)^2 + R^2}}$.

(*c*) True. The resonance frequency for a driven series *RLC* circuit depends only on L and C and is given by $\omega_{\text{res}} = 1/\sqrt{LC}$

(*d*) True. At resonance $X_L - X_C = 0$ and so $Z = R$ and the peak current is given by $I_{\text{peak}} = V_{\text{app, peak}}/R$.

(*e*) True. Because the capacitive reactance varies inversely with the driving frequency and the inductive reactance varies directly with the driving frequency, at frequencies well below the resonance frequency the capacitive reactance is larger than the inductive reactance.

(*f*) True. For frequencies below the resonant frequency, the circuit is more capacitive than inductive and the phase constant ϕ is negative. This means that the current leads the applied voltage.

14 • You may have noticed that sometimes two radio stations can be heard when your receiver is tuned to a specific frequency. This situation often occurs when you are driving and are between two cities. Explain how this situation can occur.

Determine the Concept Because the power curves received by your radio from two stations have width, you could have two frequencies overlapping as a result of receiving signals from both stations.

15 • True or false (all questions related to a driven series *RLC* circuit):

(*a*) At frequencies much higher than or much lower than the resonant frequency of a driven series *RLC* circuit, the power factor is close to zero.
(*b*) The larger the resonance width of a driven series *RLC* circuit is, the larger the *Q* factor for the circuit is.
(*c*) The larger the resistance of a driven series *RLC* circuit is, the larger the resonance width for the circuit is.

(*a*) True. Because the power factor is given by $\cos\delta = \dfrac{R}{\sqrt{\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2}}$, for values of ω that are much higher or much lower than the resonant frequency, the term in parentheses becomes very large and $\cos\delta$ approaches zero.

(*b*) False. When the resonance curve is reasonably narrow, the *Q* factor can be approximated by $Q = \omega_0/\Delta\omega$. Hence a large value for *Q* corresponds to a narrow resonance curve.

(*c*) True. See Figure 29-21.

16 • An ideal transformer has N_1 turns on its primary and N_2 turns on its secondary. The average power delivered to a load resistance *R* connected across the secondary is P_2 when the primary rms voltage is V_1. The rms current in the primary windings can then be expressed as (*a*) P_2/V_1, (*b*) $(N_1/N_2)(P_2/V_1)$, (*c*) $(N_2/N_1)(P_2/V_1)$, (*d*) $(N_2/N_1)2(P_2/V_1)$.

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. Assuming no loss of power in the transformer, we can equate the power in the primary circuit to the power in the secondary circuit and solve for the rms current in the primary windings.

Assuming no loss of power in the transformer: $P_1 = P_2$

Substitute for P_1 and P_2 to obtain: $I_{1,\,\text{rms}}V_{1,\,\text{rms}} = I_{2,\,\text{rms}}V_{2,\,\text{rms}}$

Solving for $I_{1,\text{rms}}$ and simplifying yields:

$$I_{1,\,\text{rms}}=\frac{I_{2,\,\text{rms}}V_{2,\,\text{rms}}}{V_{1,\,\text{rms}}}=\frac{P_2}{V_{1,\,\text{rms}}}$$

$\boxed{(a)}$ is correct.

17 • **[SSM]** True or false:

(*a*) A transformer is used to change frequency.
(*b*) A transformer is used to change voltage.
(*c*) If a transformer steps up the current, it must step down the voltage.
(*d*) A step-up transformer, steps down the current.
(*e*) The standard household wall-outlet voltage in Europe is 220 V, about twice that used in the United States. If a European traveler wants her hair dryer to work properly in the United States, she should use a transformer that has more windings in its secondary coil than in its primary coil.
(*f*) The standard household wall-outlet voltage in Europe is 220 V, about twice that used in the United States. If an American traveler wants his electric razor to work properly in Europe, he should use a transformer that steps up the current.

(*a*) False. A transformer is a device used to raise or lower the voltage in a circuit.

(*b*) True. A transformer is a device used to raise or lower the voltage in a circuit.

(*c*) True. If energy is to be conserved, the product of the current and voltage must be constant.

(*d*) True. Because the product of current and voltage in the primary and secondary circuits is the same, increasing the current in the secondary results in a lowering (or stepping down) of the voltage.

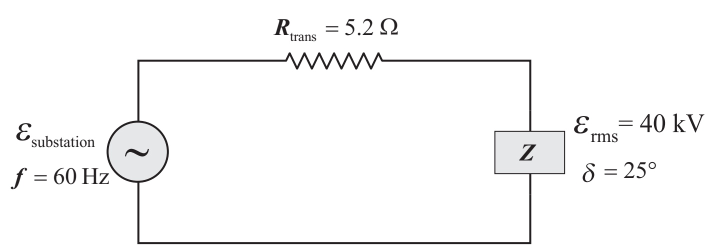
(*e*) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the voltage in order to make her hair dryer work properly.

(*f*) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the current (and decrease the voltage) in order to make his razor work properly.

Estimation and Approximation

18 •• The impedances of motors, transformers, and electromagnets include both resistance and inductive reactance. Suppose that phase of the current to a large industrial plant lags the phase of the applied voltage by 25° when the plant is under full operation and using 2.3 MW of power. The power is supplied to the plant from a substation 4.5 km from the plant; the 60 Hz rms line voltage at the plant is 40 kV. The resistance of the transmission line from the substation to the plant is 5.2 Ω. The cost per kilowatt-hour to the company that owns the plant is \$0.14, and the plant pays only for the actual energy used. (*a*) Estimate the resistance and inductive reactance of the plant's total load. (*b*) Estimate the rms current in the power lines and the rms voltage at the substation. (*c*) How much power is lost in transmission? (*d*) Suppose that the phase that the current lags the phase of the applied voltage is reduced to 18º by adding a bank of capacitors in series with the load. How much money would be saved by the electric utility during one month of operation, assuming the plant operates at full capacity for 16 h each day? (*e*) What must be the capacitance of this bank of capacitors to achieve this change in phase angle?

Picture the Problem We can find the resistance and inductive reactance of the plant's total load from the impedance of the load and the phase constant. The current in the power lines can be found from the total impedance of the load the potential difference across it and the rms voltage at the substation by applying Kirchhoff's loop rule to the substation-transmission wires-load circuit. The power lost in transmission can be found from $P_{\text{trans}} = I_{\text{rms}}^2 R_{\text{trans}}$. We can find the cost savings by finding the difference in the power lost in transmission when the phase angle is reduced to 18°. Finally, we can find the capacitance that is required to reduce the phase angle to 18° by first finding the capacitive reactance using the definition of $\tan\delta$ and then applying the definition of capacitive reactance to find C.



(*a*) Relate the resistance and inductive reactance of the plant's total load to Z and δ:

$$R = Z\cos\delta$$
and
$$X_L = Z\sin\delta$$

Express Z in terms of the rms current I_{rms} in the power lines and the rms voltage $\mathcal{E}_{\text{rms}}$ at the plant:

$$Z = \frac{\mathcal{E}_{\text{rms}}}{I_{\text{rms}}}$$

Express the power delivered to the plant in terms of $\mathcal{E}_{rms}$, I_{rms}, and δ and solve for I_{rms}:

$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta$

and

$$I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms}\cos\delta} \qquad (1)$$

Substitute to obtain:

$$Z = \frac{\varepsilon_{rms}^2 \cos\delta}{P_{av}}$$

Substitute numerical values and evaluate Z:

$$Z = \frac{(40\,\text{kV})^2 \cos 25°}{2.3\,\text{MW}} = 630\,\Omega$$

Substitute numerical values and evaluate R and X_L:

$$R = (630\,\Omega)\cos 25° = 571\,\Omega = \boxed{0.57\,\text{k}\Omega}$$

and

$$X_L = (630\,\Omega)\sin 25° = 266\,\Omega = \boxed{0.27\,\text{k}\Omega}$$

(*b*) Use equation (1) to find the current in the power lines:

$$I_{rms} = \frac{2.3\,\text{MW}}{(40\,\text{kV})\cos 25°} = 63.4\,\text{A} = \boxed{63\,\text{A}}$$

Apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E}_{sub} - I_{rms}R_{trans} - I_{rms}Z_{tot} = 0$$

Solve for $\mathcal{E}_{sub}$:

$$\mathcal{E}_{sub} = I_{rms}(R_{trans} + Z_{tot})$$

Substitute numerical values and evaluate $\mathcal{E}_{sub}$:

$$\mathcal{E}_{sub} = (63.4\,\text{A})(5.2\,\Omega + 630\,\Omega) = \boxed{40.3\,\text{kV}}$$

(*c*) The power lost in transmission is:

$$P_{trans} = I_{rms}^2 R_{trans} = (63.4\,\text{A})^2 (5.2\,\Omega) = 20.9\,\text{kW} = \boxed{21\text{kW}}$$

(*d*) Express the cost savings ΔC in terms of the difference in energy consumption $(P_{25°} - P_{18°})\Delta t$ and the per-unit cost u of the energy:

$$\Delta C = (P_{25°} - P_{18°})\Delta t u$$

Express the power lost in transmission when $\delta = 18°$:

$$P_{18°} = I_{18°}^2 R_{\text{trans}}$$

Find the current in the transmission lines when $\delta = 18°$:

$$I_{18°} = \frac{2.3\,\text{MW}}{(40\,\text{kV})\cos 18°} = 60.5\,\text{A}$$

Evaluate $P_{18°}$:

$$P_{18°} = (60.5\,\text{A})^2(5.2\,\Omega) = 19.0\,\text{kW}$$

Substitute numerical values and evaluate ΔC:

$$\Delta C = (20.9\,\text{kW} - 19.0\,\text{kW})\left(16\frac{\text{h}}{\text{d}}\right)\left(30\frac{\text{d}}{\text{month}}\right)\left(\frac{\$0.14}{\text{kW}\cdot\text{h}}\right) = \boxed{\$128}$$

(*e*) The required capacitance is given by:

$$C = \frac{1}{2\pi f X_C}$$

Relate the new phase angle δ to the inductive reactance X_L, the reactance due to the added capacitance X_C, and the resistance of the load R:

$$\tan\delta = \frac{X_L - X_C}{R} \Rightarrow X_C = X_L - R\tan\delta$$

Substituting for X_C yields:

$$C = \frac{1}{2\pi f\left(X_L - R\tan\delta\right)}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(266\,\Omega - (571\,\Omega)\tan 18°\right)} = \boxed{33\,\mu\text{F}}$$

Alternating Current in Resistors, Inductors, and Capacitors

19 • **[SSM]** A 100-W light bulb is screwed into a standard 120-V-rms socket. Find (*a*) the rms current, (*b*) the peak current, and (*c*) the peak power.

Picture the Problem We can use $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}$ to find I_{rms}, $I_{\text{peak}} = \sqrt{2} I_{\text{rms}}$ to find I_{peak}, and $P_{\text{peak}} = I_{\text{peak}} \mathcal{E}_{\text{peak}}$ to find P_{peak}.

(*a*) Relate the average power delivered by the source to the rms voltage across the bulb and the rms current through it:

$$P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \Rightarrow I_{\text{rms}} = \frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}}$$

Substitute numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{100\,\text{W}}{120\,\text{V}} = 0.8333\ \text{A} = \boxed{0.833\,\text{A}}$$

(*b*) Express I_{peak} in terms of I_{rms}:

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}}$$

Substitute for I_{rms} and evaluate I_{peak}:

$$I_{\text{peak}} = \sqrt{2}(0.8333\,\text{A}) = 1.1785\ \text{A} = \boxed{1.18\,\text{A}}$$

(*c*) Express the maximum power in terms of the maximum voltage and maximum current:

$$P_{\text{peak}} = I_{\text{peak}}\mathcal{E}_{\text{peak}}$$

Substitute numerical values and evaluate P_{peak}:

$$P_{\text{peak}} = (1.1785\,\text{A})\sqrt{2}(120\,\text{V}) = \boxed{200\,\text{W}}$$

20 • A circuit breaker is rated for a current of 15 A rms at a voltage of 120 V rms. (*a*) What is the largest value of the peak current that the breaker can carry? (*b*) What is the maximum average power that can be supplied by this circuit?

Picture the Problem We can $I_{\text{peak}} = \sqrt{2}I_{\text{rms}}$ to find the largest peak current the breaker can carry and $P_{\text{av}} = I_{\text{rms}}V_{\text{rms}}$ to find the average power supplied by this circuit.

(*a*) Express I_{peak} in terms of I_{rms}:

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}(15\ \text{A}) = \boxed{21\,\text{A}}$$

(*b*) Relate the average power to the rms current and voltage:

$$P_{\text{av}} = I_{\text{rms}}V_{\text{rms}} = (15\,\text{A})(120\,\text{V}) = \boxed{1.8\,\text{kW}}$$

21 • **[SSM]** What is the reactance of a 1.00-μH inductor at (*a*) 60 Hz, (*b*) 600 Hz, and (*c*) 6.00 kHz?

Picture the Problem We can use $X_L = \omega L$ to find the reactance of the inductor at any frequency.

Express the inductive reactance as a function of *f*:

$$X_L = \omega L = 2\pi f L$$

(*a*) At *f* = 60 Hz:

$$\boldsymbol{X_L} = 2\boldsymbol{\pi}(60\,\text{s}^{-1})(1.00\,\text{mH}) = \boxed{0.38\,\Omega}$$

(*b*) At $f = 600$ Hz:

$$X_L = 2\pi\left(600\,\text{s}^{-1}\right)(1.00\,\text{mH}) = \boxed{3.77\,\Omega}$$

(*c*) At $f = 6.00$ kHz:

$$X_L = 2\pi(6.00\,\text{kHz})(1.00\,\text{mH}) = \boxed{37.7\,\Omega}$$

22 • An inductor has a reactance of 100 Ω at 80 Hz. (*a*) What is its inductance? (*b*) What is its reactance at 160 Hz?

Picture the Problem We can use $X_L = \omega L$ to find the inductance of the inductor at any frequency.

(*a*) Relate the reactance of the inductor to its inductance:

$$X_L = \omega L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f}$$

Solve for and evaluate L:

$$L = \frac{100\,\Omega}{2\pi\left(80\,\text{s}^{-1}\right)} = 0.199\,\text{H} = \boxed{0.20\,\text{H}}$$

(*b*) At 160 Hz:

$$X_L = 2\pi\left(160\,\text{s}^{-1}\right)(0.199\,\text{H}) = \boxed{0.20\,\text{k}\Omega}$$

23 • At what frequency would the reactance of a 10-μF capacitor equal the reactance of a 1.0-μH inductor?

Picture the Problem We can equate the reactances of the capacitor and the inductor and then solve for the frequency.

Express the reactance of the inductor:

$$X_L = \omega L = 2\pi f L$$

Express the reactance of the capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Equate these reactances to obtain:

$$2\pi f L = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{(10\,\mu\text{F})(1.0\,\text{mH})}} = \boxed{1.6\,\text{kHz}}$$

24 • What is the reactance of a 1.00-nF capacitor at (*a*) 60.0 Hz, (*b*) 6.00 kHz, and (*c*) 6.00 MHz?

Picture the Problem We can use $X_C = 1/\omega C$ to find the reactance of the capacitor at any frequency.

Express the capacitive reactance as a function of f:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

(a) At $f = 60.0$ Hz:

$$X_C = \frac{1}{2\pi\left(60.0\,\text{s}^{-1}\right)\left(1.00\,\text{nF}\right)} = \boxed{2.65\,\text{M}\Omega}$$

(b) At $f = 6.00$ kHz:

$$X_C = \frac{1}{2\pi\left(6.00\,\text{kHz}\right)\left(1.00\,\text{nF}\right)} = \boxed{26.5\,\text{k}\Omega}$$

(c) At $f = 6.00$ MHz:

$$X_C = \frac{1}{2\pi\left(6.00\,\text{MHz}\right)\left(1.00\,\text{nF}\right)} = \boxed{26.5\,\Omega}$$

25 • **[SSM]** A 20-Hz ac generator that produces a peak emf of 10 V is connected to a 20-μF capacitor. Find (a) the peak current and (b) the rms current.

Picture the Problem We can use $I_{\text{peak}} = \mathcal{E}_{\text{peak}}/X_C$ and $X_C = 1/\omega C$ to express I_{peak} as a function of $\mathcal{E}_{\text{peak}}$, f, and C. Once we've evaluate I_{peak}, we can use $I_{\text{rms}} = I_{\text{peak}}/\sqrt{2}$ to find I_{rms}.

Express I_{peak} in terms of $\mathcal{E}_{\text{peak}}$ and X_C:

$$I_{\text{peak}} = \frac{\mathcal{E}_{\text{peak}}}{X_C}$$

Express the capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Substitute for X_C and simplify to obtain:

$$I_{\text{peak}} = 2\pi f C \mathcal{E}_{\text{peak}}$$

(a) Substitute numerical values and evaluate I_{peak}:

$$I_{\text{peak}} = 2\pi\left(20\,\text{s}^{-1}\right)\left(20\,\mu\text{F}\right)\left(10\,\text{V}\right) = 25.1\,\text{mA} = \boxed{25\,\text{mA}}$$

(b) Express I_{rms} in terms of I_{peak}:

$$I_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} = \frac{25.1\,\text{mA}}{\sqrt{2}} = \boxed{18\,\text{mA}}$$

26 • At what frequency is the reactance of a 10-μF capacitor (a) 1.00 Ω, (b) 100 Ω, and (c) 10.0 mΩ?

Picture the Problem We can use $X_C = 1/\omega C = 1/2\pi fC$ to relate the reactance of the capacitor to the frequency.

The reactance of the capacitor is given by:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \Rightarrow f = \frac{1}{2\pi CX_C}$$

(*a*) Find *f* when $X_C = 1.00\ \Omega$:

$$f = \frac{1}{2\pi(10\ \mu\text{F})(1.00\ \Omega)} = \boxed{16\ \text{kHz}}$$

(*b*) Find *f* when $X_C = 100\ \Omega$:

$$f = \frac{1}{2\pi(10\ \mu\text{F})(100\ \Omega)} = \boxed{0.16\ \text{kHz}}$$

(*c*) Find *f* when $X_C = 10.0\ \text{m}\Omega$:

$$f = \frac{1}{2\pi(10\ \mu\text{F})(10.0\ \text{m}\Omega)} = \boxed{1.6\ \text{MHz}}$$

27 •• A circuit consists of two ideal ac generators and a 25-Ω resistor, all connected in series. The potential difference across the terminals of one of the generators is given by $V_1 = (5.0\ \text{V})\cos(\omega t - \alpha)$, and the potential difference across the terminals of the other generator is given by $V_2 = (5.0\ \text{V})\cos(\omega t + \alpha)$, where $\alpha = \pi/6$. (*a*) Use Kirchhoff's loop rule and a trigonometric identity to find the peak current in the circuit. (*b*) Use a phasor diagram to find the peak current in the circuit. (*c*) Find the current in the resistor if $\alpha = \pi/4$ and the amplitude of V_2 is increased from 5.0 V to 7.0 V.

Picture the Problem We can use the trigonometric identity

$$\cos\theta + \cos\phi = 2\cos\tfrac{1}{2}(\theta+\phi)\cos\tfrac{1}{2}(\theta-\phi)$$

to find the sum of the phasors V_1 and V_2 and then use this sum to express *I* as a function of time. In (*b*) we'll use a phasor diagram to obtain the same result and in (*c*) we'll use the phasor diagram appropriate to the given voltages to express the current as a function of time.

(*a*) Applying Kirchhoff's loop rule to the circuit yields:

$$V_1 + V_2 - IR = 0$$

Solve for *I* to obtain:

$$I = \frac{V_1 + V_2}{R}$$

Use the trigonometric identity $\cos\theta + \cos\phi = 2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$ to find $V_1 + V_2$:

$$V_1 + V_2 = (5.0\,\mathrm{V})[\cos(\omega t - \alpha) + \cos(\omega t + \alpha)] = (5\,\mathrm{V})[2\cos\tfrac{1}{2}(2\omega t)\cos\tfrac{1}{2}(-2\alpha)]$$
$$= (10\,\mathrm{V})\cos\frac{\pi}{6}\cos\omega t = (8.66\,\mathrm{V})\cos\omega t$$

Substitute for $V_1 + V_2$ and R to obtain:

$$I = \frac{(8.66\,\mathrm{V})\cos\omega t}{25\,\Omega} = (0.346\,\mathrm{A})\cos\omega t$$
$$= (0.35\,\mathrm{A})\cos\omega t$$

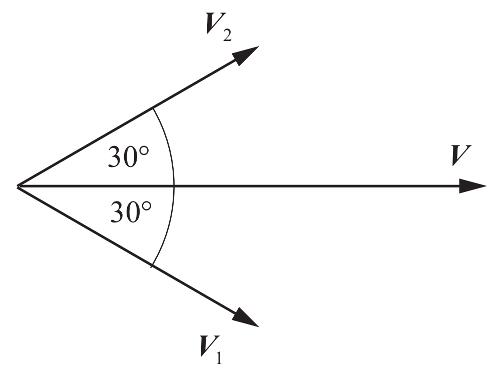
where

$$I_{\mathrm{peak}} = \boxed{0.35\,\mathrm{A}}$$

(*b*) Express the magnitude of the current in R:

$$|I| = \frac{|\vec{V}|}{R}$$

The phasor diagram for the voltages is shown to the right.



Use vector addition to find $|\vec{V}|$:

$$|\vec{V}| = 2|\vec{V}_1|\cos 30° = 2(5.0\,\mathrm{V})\cos 30°$$
$$= 8.66\,\mathrm{V}$$

Substitute for $|\vec{V}|$ and R to obtain:

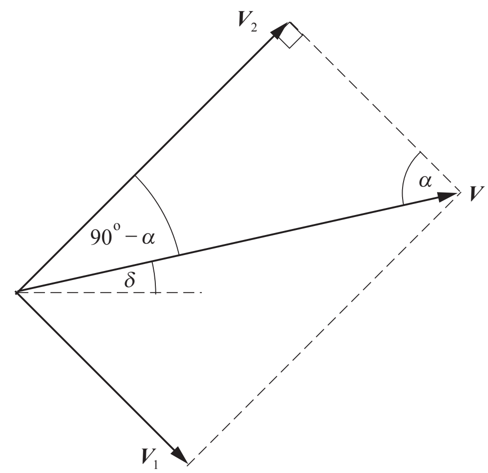
$$|I| = \frac{8.66\,\mathrm{V}}{25\,\Omega} = 0.346\,\mathrm{A}$$

and $I = (0.35\,\mathrm{A})\cos\omega t$

where

$$I_{\mathrm{peak}} = \boxed{0.35\,\mathrm{A}}$$

(*c*) The phasor diagram is shown to the right. Note that the phase angle between $\vec{V}_1$ and $\vec{V}_2$ is now 90°.



Use the Pythagorean theorem to find $|\vec{V}|$:

$$|\vec{V}| = \sqrt{|\vec{V}_1|^2 + |\vec{V}_2|^2} = \sqrt{(5.0\text{ V})^2 + (7.0\text{ V})^2}$$
$$= 8.60\text{ V}$$

Express I as a function of t:

$$I = \frac{|\vec{V}|}{R}\cos(\omega t + \delta)$$

where

$$\delta = 45^\circ - (90^\circ - \alpha) = \alpha - 45^\circ$$
$$= \tan^{-1}\left(\frac{7.0\text{ V}}{5.0\text{ V}}\right) - 45^\circ$$
$$= 9.462^\circ = 0.165\text{ rad}$$

Substitute numerical values and evaluate I:

$$I = \frac{8.60\text{ V}}{25\,\Omega}\cos(\omega t + 0.165\text{ rad})$$
$$= \boxed{(0.34\text{ A})\cos(\omega t + 0.17\text{ rad})}$$

Undriven Circuits Containing Capacitors, Resistors and Inductors

28 • (*a*) Show that $1/\sqrt{LC}$ has units of inverse seconds by substituting SI units for inductance and capacitance into the expression. (*b*) Show that $\omega_0 L/R$ (the expression for the *Q*-factor) is dimensionless by substituting SI units for angular frequency, inductance, and resistance into the expression.

Picture the Problem We can substitute the units of the various physical quantitities in $1/\sqrt{LC}$ and $Q = \omega_0 L/R$ to establish their units.

(*a*) Substitute the units for *L* and *C* in the expression $1/\sqrt{LC}$ and simplify to obtain:

$$\frac{1}{\sqrt{\mathrm{H}\cdot\mathrm{F}}}=\frac{1}{\sqrt{(\Omega\cdot\mathrm{s})\left(\frac{\mathrm{s}}{\Omega}\right)}}=\frac{1}{\sqrt{\mathrm{s}^2}}=\boxed{\mathrm{s}^{-1}}$$

(*b*) Substitute the units for ω_0, *L*, and *R* in the expression $Q=\omega_0 L/R$ and simplify to obtain:

$$\frac{\frac{1}{\mathrm{s}}\cdot\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}}{\frac{\mathrm{V}}{\mathrm{A}}}=\frac{\frac{1}{\mathrm{s}}\cdot\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}}{\frac{\mathrm{V}}{\mathrm{A}}}=1\Rightarrow\boxed{\text{no units}}$$

29 • **[SSM]** (*a*) What is the period of oscillation of an *LC* circuit consisting of an ideal 2.0-mH inductor and a 20-μF capacitor? (*b*) A circuit that oscillates consists solely of an 80-μF capacitor and a variable ideal inductor. What inductance is needed in order to tune this circuit to oscillate at 60 Hz?

Picture the Problem We can use $T=2\pi/\omega$ and $\omega=1/\sqrt{LC}$ to relate *T* (and hence *f*) to *L* and *C*.

(*a*) Express the period of oscillation of the *LC* circuit:

$$T=\frac{2\pi}{\omega}$$

For an *LC* circuit:

$$\omega=\frac{1}{\sqrt{LC}}$$

Substitute for ω to obtain:

$$T=2\pi\sqrt{LC} \qquad (1)$$

Substitute numerical values and evaluate *T*:

$$T=2\pi\sqrt{(2.0\,\mathrm{mH})(20\,\mu\mathrm{F})}=\boxed{1.3\,\mathrm{ms}}$$

(*b*) Solve equation (1) for *L* to obtain:

$$L=\frac{T^2}{4\pi^2 C}=\frac{1}{4\pi^2 f^2 C}$$

Substitute numerical values and evaluate *L*:

$$L=\frac{1}{4\pi^2\left(60\,\mathrm{s}^{-1}\right)^2(80\,\mu\mathrm{F})}=\boxed{88\,\mathrm{mH}}$$

30 • An *LC* circuit has capacitance C_0 and inductance *L*. A second *LC* circuit has capacitance $\frac{1}{2}C_0$ and inductance 2*L*, and a third *LC* circuit has capacitance $2C_0$ and inductance $\frac{1}{2}L$. (*a*) Show that each circuit oscillates with the same frequency. (*b*) In which circuit would the peak current be greatest if the peak voltage across the capacitor in each circuit was the same?

Picture the Problem We can use the expression $f_0 = 1/2\pi\sqrt{LC}$ for the resonance frequency of an LC circuit to show that each circuit oscillates with the same frequency. In (*b*) we can use $I_{\text{peak}} = \omega Q_0$, where Q_0 is the charge of the capacitor at time zero, and the definition of capacitance $Q_0 = CV$ to express I_{peak} in terms of ω, C and V.

Express the resonance frequency for an LC circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(*a*) Express the product of L and C_0 for each circuit:

Circuit 1: $L_1C_1 = L_1C_0$,
Circuit 2: $L_2C_2 = (2L_1)(\frac{1}{2}C_0) = L_1C_1$,
and
Circuit 3: $L_3C_3 = (\frac{1}{2}L_1)(2C_0) = L_1C_1$

Because $L_1C_1 = L_2C_2 = L_3C_3$, the resonance frequencies of the three circuits are the same.

(*b*) Express I_{peak} in terms of the charge stored in the capacitor:

$$I_{\text{peak}} = \omega Q_0$$

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substituting for Q_0 yields:

$$I_{\text{peak}} = \omega CV$$

or, for ω and V constant, $I_{\text{peak}} \propto C$.
Hence, the circuit with capacitance $2C_0$ has the greatest peak current.

31 •• A 5.0-μF capacitor is charged to 30 V and is then connected across an ideal 10-mH inductor. (*a*) How much energy is stored in the system? (*b*) What is the frequency of oscillation of the circuit? (*c*) What is the peak current in the circuit?

Picture the Problem We can use $U = \frac{1}{2}CV^2$ to find the energy stored in the electric field of the capacitor, $\omega_0 = 2\pi f_0 = 1/\sqrt{LC}$ to find f_0, and $I_{\text{peak}} = \omega Q_0$ and $Q_0 = CV$ to find I_{peak}.

(*a*) Express the energy stored in the system as a function of *C* and *V*:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate *U*:

$$U = \tfrac{1}{2}(5.0\,\mu\text{F})(30\,\text{V})^2 = \boxed{2.3\,\text{mJ}}$$

(*b*) Express the resonance frequency of the circuit in terms of *L* and *C*:

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{(10\,\text{mH})(5.0\,\mu\text{F})}} = 712\,\text{Hz}$$
$$= \boxed{0.71\,\text{kHz}}$$

(*c*) Express I_{peak} in terms of the charge stored in the capacitor:

$$I_{\text{peak}} = \omega Q_0$$

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substituting for Q_0 yields:

$$I_{\text{peak}} = \omega CV$$

Substitute numerical values and evaluate I_{peak}:

$$I_{\text{peak}} = 2\pi\left(712\,\text{s}^{-1}\right)(5.0\,\mu\text{F})(30\,\text{V})$$
$$= \boxed{0.67\,\text{A}}$$

32 •• A coil with internal resistance can be modeled as a resistor and an ideal inductor in series. Assume that the coil has an internal resistance of 1.00 Ω and an inductance of 400 mH. A 2.00-μF capacitor is charged to 24.0 V and is then connected across coil. (*a*) What is the initial voltage across the coil? (*b*) How much energy is dissipated in the circuit before the oscillations die out? (*c*) What is the frequency of oscillation the circuit? (Assume the internal resistance is sufficiently small that has no impact on the frequency of the circuit.) (*d*) What is the quality factor of the circuit?

Picture the Problem In Part (*a*) we can apply Kirchhoff's loop rule to find the initial voltage across the coil. (*b*) The total energy lost via joule heating is the total energy initially stored in the capacitor. (*c*) The natural frequency of the circuit is given by $f_0 = 1/2\pi\sqrt{LC}$. In Part (*d*) we can use its definition to find the quality factor of the circuit.

(*a*) Application of Kirchhoff's loop rule leads us to conclude that the initial voltage across the coil is $\boxed{24.0\ \text{V}}$.

(*b*) Because the ideal inductor can not dissipate energy as heat, all of the energy initially stored in the capacitor will be dissipated as joule heat in the resistor:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.00\ \mu\text{F})(24.0\ \text{V})^2 = \boxed{0.576\ \text{mJ}}$$

(*c*) The natural frequency of the circuit is:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(400\ \text{mH})(2.00\ \mu\text{F})}} = \boxed{178\ \text{Hz}}$$

(*d*) The quality factor of the circuit is given by:

$$Q = \frac{\omega_0 L}{R}$$

Substituting for ω_0 and simplifying yields:

$$Q = \frac{\frac{1}{\sqrt{LC}}L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{1}{1.00\ \Omega}\sqrt{\frac{400\ \text{mH}}{2.00\ \mu\text{F}}} = \boxed{447}$$

33 ••• [SSM] An inductor and a capacitor are connected, as shown in Figure 29-30. Initially, the switch is open, the left plate of the capacitor has charge Q_0. The switch is then closed. (*a*) Plot both Q versus t and I versus t on the same graph, and explain how it can be seen from these two plots that the current leads the charge by 90º. (*b*) The expressions for the charge and for the current are given by Equations 29-38 and 29-39, respectively. Use trigonometry and algebra to show that the current leads the charge by 90º.

Picture the Problem Let Q represent the instantaneous charge on the capacitor and apply Kirchhoff's loop rule to obtain the differential equation for the circuit. We can then solve this equation to obtain an expression for the charge on the capacitor as a function of time and, by differentiating this expression with respect to time, an expression for the current as a function of time. We'll use a spreadsheet program to plot the graphs.

Apply Kirchhoff's loop rule to a clockwise loop just after the switch is closed:

$$\frac{Q}{C}+L\frac{dI}{dt}=0$$

Because $I=dQ/dt$:

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0 \text{ or } \frac{d^2Q}{dt^2}+\frac{1}{LC}Q=0$$

The solution to this equation is:

$$Q(t)=Q_0\cos(\omega t-\delta)$$

where $\omega=\sqrt{\frac{1}{LC}}$

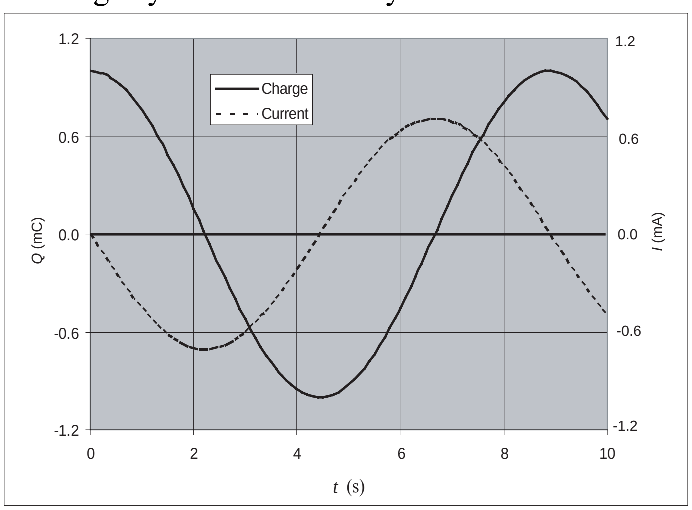
Because $Q(0) = Q_0$, $\delta = 0$ and:

$$Q(t)=Q_0\cos\omega t$$

The current in the circuit is the derivative of Q with respect to t:

$$I=\frac{dQ}{dt}=\frac{d}{dt}[Q_0\cos\omega t]=-\omega Q_0\sin\omega t$$

(*a*) A spreadsheet program was used to plot the following graph showing both the charge on the capacitor and the current in the circuit as functions of time. L, C, and Q_0 were all arbitrarily set equal to one to obtain these graphs. Note that the current leads the charge by one-fourth of a cycle or 90°.



(*b*) The equation for the current is:

$$I=-\omega Q_0\sin\omega t \qquad (1)$$

The sine and cosine functions are related through the identity:

$$-\sin\theta=\cos\left(\theta+\frac{\pi}{2}\right)$$

Use this identity to rewrite equation (1):

$$I = -\omega Q_0 \sin\omega t = \boxed{\omega Q_0 \cos\left(\omega t + \frac{\pi}{2}\right)}$$

Thus, the current leads the charge by 90°.

Driven *RL* Circuits

34 •• A circuit consists of a resistor, an ideal 1.4-H inductor and an ideal 60-Hz generator, all connected in series. The rms voltage across the resistor is 30 V and the rms voltage across the inductor is 40 V. (*a*) What is the resistance of the resistor? (*b*) What is the peak emf of the generator?

Picture the Problem We can express the ratio of V_R to V_L and solve this expression for the resistance *R* of the circuit. In (*b*) we can use the fact that, in an *LR* circuit, V_L leads V_R by 90° to find the ac input voltage.

(*a*) Express the potential differences across *R* and *L* in terms of the common current through these components:

$$V_L = IX_L = I\omega L$$

and

$$V_R = IR$$

Divide the second of these equations by the first to obtain:

$$\frac{V_R}{V_L} = \frac{IR}{I\omega L} = \frac{R}{\omega L} \Rightarrow R = \left(\frac{V_R}{V_L}\right)\omega L$$

Substitute numerical values and evaluate *R*:

$$R = \left(\frac{30\,\text{V}}{40\,\text{V}}\right)2\pi\left(60\,\text{s}^{-1}\right)\left(1.4\,\text{H}\right) = \boxed{0.40\,\text{k}\Omega}$$

(*b*) Because V_R leads V_L by 90° in an *LR* circuit:

$$V_{\text{peak}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}\sqrt{V_R^2 + V_L^2}$$

Substitute numerical values and evaluate V_{peak}:

$$V_{\text{peak}} = \sqrt{2}\sqrt{\left(30\,\text{V}\right)^2 + \left(40\,\text{V}\right)^2} = \boxed{71\,\text{V}}$$

35 •• **[SSM]** A coil that has a resistance of 80.0 Ω has an impedance of 200 Ω when driven at a frequency of 1.00 kHz. What is the inductance of the coil?

Picture the Problem We can solve the expression for the impedance in an *LR* circuit for the inductive reactance and then use the definition of X_L to find *L*.

Express the impedance of the coil in terms of its resistance and inductive reactance:

$$Z = \sqrt{R^2 + X_L^2}$$

Solve for X_L to obtain:

$$X_L = \sqrt{Z^2 - R^2}$$

Express X_L in terms of L:

$$X_L = 2\pi fL$$

Equate these two expressions to obtain:

$$2\pi fL = \sqrt{Z^2 - R^2} \Rightarrow L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$

Substitute numerical values and evaluate L:

$$L = \frac{\sqrt{(200\,\Omega)^2 - (80.0\,\Omega)^2}}{2\pi(1.00\,\text{kHz})} = \boxed{29.2\,\text{mH}}$$

36 •• A two conductor transmission line simultaneously carries a superposition of two voltage signals, so the potential difference between the two conductors is given by $V = V_1 + V_2$, where $V_1 = (10.0\text{ V})\cos(\omega_1 t)$ and $V_2 = (10.0\text{ V})\cos(\omega_2 t)$, where $\omega_1 = 100$ rad/s and $\omega_2 = 10\,000$ rad/s. A 1.00 H inductor and a 1.00 kΩ shunt resistor are inserted into the transmission line as shown in Figure 29-31. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (*a*) What is the voltage (V_{out}) at the output of the transmission line? (*b*) What is the ratio of the low-frequency amplitude to the high-frequency amplitude at the output?

Picture the Problem We can express the two output voltage signals as the product of the current from each source and R = 1.00 kΩ. We can find the currents due to each source using the given voltage signals and the definition of the impedance for each of them.

(*a*) Express the voltage signals observed at the output side of the transmission line in terms of the potential difference across the resistor:

$$V_{1,\text{out}} = I_1 R$$

and

$$V_{2,\text{out}} = I_2 R$$

Evaluate I_1 and I_2:

$$I_1 = \frac{V_1}{Z_1} = \frac{(10.0\,\text{V})\cos 100t}{\sqrt{(1.00\,\text{k}\Omega)^2 + \left[(100\,\text{s}^{-1})(1.00\,\text{H})\right]^2}} = (9.95\,\text{mA})\cos 100t$$

and

$$I_2 = \frac{V_2}{Z_2} = \frac{(10.0\,\text{V})\cos 10^4 t}{\sqrt{(1.00\,\text{k}\Omega)^2 + \left[(10^4\,\text{s}^{-1})(1.00\,\text{H})\right]^2}} = (0.995\,\text{mA})\cos 10^4 t$$

Substitute for I_1 and I_2 to obtain:

$$V_{1,\,\text{out}} = (1.00\,\text{k}\Omega)(9.95\,\text{mA})\cos 100\boldsymbol{t} = \boxed{(9.95\,\text{V})\cos 100\boldsymbol{t}}$$

where ω_1 = 100 rad/s and ω_2 = 10 000 rad/s.

and

$$V_{2,\,\text{out}} = (1.00\,\text{k}\Omega)(0.995\,\text{mA})\cos 10^4\boldsymbol{t} = \boxed{(0.995\,\text{V})\cos 10^4\boldsymbol{t}}$$

(*b*) Express the ratio of $V_{1,\text{out}}$ to $V_{2,\text{out}}$:

$$\frac{V_{1,\,\text{out}}}{V_{2,\,\text{out}}} = \frac{9.95\,\text{V}}{0.995\,\text{V}} = \boxed{10:1}$$

37 •• A coil is connected to a 120-V rms, 60-Hz line. The average power supplied to the coil is 60 W, and the rms current is 1.5 A. Find (*a*) the power factor, (*b*) the resistance of the coil, and (*c*) the inductance of the coil. (*d*) Does the current lag or lead the voltage? Explain your answer. (*e*) Support your answer to Part (*d*) by determining the phase angle.

Picture the Problem The average power supplied to coil is related to the power factor by $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta$. In (*b*) we can use $P_{\text{av}} = I_{\text{rms}}^2 R$ to find *R*. Because the inductance *L* is related to the resistance *R* and the phase angle δ according to $X_L = \omega L = R\tan\delta$, we can use this relationship to find the resistance of the coil. Finally, we can decide whether the current leads or lags the voltage by noting that the circuit is inductive.

(*a*) Express the average power supplied to the coil in terms of the power factor of the circuit:

$$P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta \Rightarrow \cos\delta = \frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}} I_{\text{rms}}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\boldsymbol{\delta} = \frac{60\,\text{W}}{(120\,\text{V})(1.5\,\text{A})} = 0.333 = \boxed{0.33}$$

(*b*) Express the power supplied by the source in terms of the resistance of the coil:

$$P_{av} = I_{rms}^2 R \Rightarrow R = \frac{P_{av}}{I_{rms}^2}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{60\,\text{W}}{(1.5\,\text{A})^2} = 26.7\,\Omega = \boxed{27\,\Omega}$$

(*c*) Relate the inductive reactance to the resistance and phase angle:

$$X_L = \omega L = R\tan\delta$$

Solving for *L* yields:

$$L = \frac{R\tan\delta}{\omega} = \frac{R\tan\left[\cos^{-1}(0.333)\right]}{2\pi f}$$

Substitute numerical values and evaluate *L*:

$$L = \frac{(26.7\,\Omega)\tan(70.5°)}{2\pi\left(60\,\text{s}^{-1}\right)} = \boxed{0.20\,\text{H}}$$

(*d*) Evaluate X_L:

$$X_L = (26.7\,\Omega)\tan(70.5°) = 75.4\,\Omega$$

Because the circuit is inductive, the current lags the voltage.

(*e*) From Part (*a*):

$$\delta = \cos^{-1}(0.333) = \boxed{71°}$$

38 •• A 36-mH inductor that has a resistance of 40 Ω is connected to an ideal ac voltage source whose output is given by $\mathcal{E}$ = (345 V) cos(150πt), where *t* is in seconds. Determine (*a*) the peak current in the circuit, (*b*) the peak and rms voltages across the inductor, (*c*) the average power dissipation, and
(*d*) the peak and average magnetic energy stored in the inductor.

Picture the Problem (*a*) We can use $I_{peak} = \mathcal{E}_{peak}\Big/\sqrt{R^2 + (\omega L)^2}$ and $V_{L,\,peak} = I_{peak}X_L = \omega L I_{peak}$ to find the peak current in the circuit and the peak voltage across the inductor. (*b*) Once we've found $V_{L,\,peak}$ we can find $V_{L,\,rms}$ using $V_{L,\,rms} = V_{L,\,peak}\big/\sqrt{2}$. (*c*) We can use $P_{av} = \frac{1}{2}I_{rms}^2 R$ to find the average power dissipation, and (*d*) $U_{L,\,peak} = \frac{1}{2}LI_{peak}^2$ to find the peak and average magnetic energy stored in the inductor. The average energy stored in the magnetic field of the inductor can be found using $U_{L,av} = \int P_{av}dt$.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - IZ = 0 \Rightarrow I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L)^2}}$$

Substitute numerical values and evaluate I:

$$I = \frac{(345\text{ V})\cos(150\pi t)}{\sqrt{(40\,\Omega)^2 + \left[(150\pi\text{ s}^{-1})(36\text{ mH})\right]^2}} = (7.94\text{ A})\cos(150\pi t)$$

and $I_{\text{peak}} = \boxed{7.9\text{ A}}$.

(*b*) Because $\mathcal{E}$ = (345 V) cos(150πt) :

$$V_{L,\text{peak}} = \boxed{345\text{ V}}$$

Find $V_{L,\text{rms}}$ from $V_{L,\text{peak}}$:

$$V_{L,\text{rms}} = \frac{V_{L,\text{peak}}}{\sqrt{2}} = \frac{345\text{ V}}{\sqrt{2}} = \boxed{244\text{ V}}$$

(*c*) Relate the average power dissipation to I_{peak} and R:

$$P_{\text{av}} = I_{\text{rms}}^2 R = \left(\frac{I_{\text{peak}}}{\sqrt{2}}\right)^2 R = \tfrac{1}{2} I_{\text{peak}}^2 R$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = \tfrac{1}{2}(7.94\text{ A})^2(40\,\Omega) = \boxed{1.3\text{ kW}}$$

(*d*) The maximum energy stored in the magnetic field of the inductor is:

$$U_{L,\text{peak}} = \tfrac{1}{2} L I_{\text{peak}}^2 = \tfrac{1}{2}(36\text{ mH})(7.94\text{ A})^2 = \boxed{1.1\text{ J}}$$

The definition of $U_{L,\text{av}}$ is:

$$U_{L,\text{av}} = \frac{1}{T}\int_0^T U(t)\,dt$$

$U(t)$ is given by:

$$U(t) = \frac{1}{2} L[I(t)]^2$$

Substitute for $U(t)$ to obtain:

$$U_{L,\text{av}} = \frac{L}{2T}\int_0^T [I(t)]^2\,dt$$

Evaluating the integral yields:

$$U_{L,\text{av}} = \frac{L}{2T}\left[\frac{1}{2} I_{\text{peak}}^2\right]T = \frac{1}{4} L I_{\text{peak}}^2$$

Substitute numerical values and evaluate $U_{L,\text{av}}$:

$$U_{L,\text{av}} = \frac{1}{4}(36\text{ mH})(7.94\text{ A})^2 = \boxed{0.57\text{ J}}$$

39 •• [SSM] A coil that has a resistance R and an inductance L has a power factor equal to 0.866 when driven at a frequency of 60 Hz. What is the coil's power factor it is driven at 240 Hz?

Picture the Problem We can use the definition of the power factor to find the relationship between X_L and R when the coil is driven at a frequency of 60 Hz and then use the definition of X_L to relate the inductive reactance at 240 Hz to the inductive reactance at 60 Hz. We can then use the definition of the power factor to determine its value at 240 Hz.

Using the definition of the power factor, relate R and X_L:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} \qquad (1)$$

Square both sides of the equation to obtain:

$$\cos^2\delta = \frac{R^2}{R^2 + X_L^2}$$

Solve for $X_L^2(60\,\text{Hz})$:

$$X_L^2(60\,\text{Hz}) = R^2\left(\frac{1}{\cos^2\delta} - 1\right)$$

Substitute for $\cos\delta$ and simplify to obtain:

$$X_L^2(60\,\text{Hz}) = R^2\left(\frac{1}{(0.866)^2} - 1\right) = \tfrac{1}{3}R^2$$

Use the definition of X_L to obtain:

$$X_L^2(f) = 4\pi f^2 L^2 \text{ and } X_L^2(f') = 4\pi f'^2 L^2$$

Dividing the second of these equations by the first and simplifying yields:

$$\frac{X_L^2(f')}{X_L^2(f)} = \frac{4\pi f'^2 L^2}{4\pi f^2 L^2} = \frac{f'^2}{f^2}$$

or

$$X_L^2(f') = \left(\frac{f'}{f}\right)^2 X_L^2(f)$$

Substitute numerical values to obtain:

$$X_L^2(240\,\text{Hz}) = \left(\frac{240\,\text{s}^{-1}}{60\,\text{s}^{-1}}\right)^2 X_L^2(60\,\text{Hz})$$

$$= 16\left(\frac{1}{3}R^2\right) = \frac{16}{3}R^2$$

Substitute in equation (1) to obtain:

$$(\cos\delta)_{240\,\text{Hz}} = \frac{R}{\sqrt{R^2 + \frac{16}{3}R^2}} = \sqrt{\frac{3}{19}}$$
$$= \boxed{0.397}$$

40 •• A resistor and an inductor are connected in parallel across an ideal ac voltage source whose output is given by $\mathcal{E} = \mathcal{E}_{\text{peak}}\cos\omega t$ as shown in Figure 29-32. Show that (*a*) the current in the resistor is given by $I_R = \mathcal{E}_{\text{peak}}/R\cos\omega t$, (*b*) the current in the inductor is given by $I_L = \mathcal{E}_{\text{peak}}/X_L\cos(\omega t - 90°)$, and (*c*) the current in the voltage source is given by $I = I_R + I_L = I_{\text{peak}}\cos(\omega t - \delta)$, where $I_{\text{peak}} = \mathcal{E}_{\text{max}}/Z$.

Picture the Problem Because the resistor and the inductor are connected in parallel, the voltage drops across them are equal. Also, the total current is the sum of the current through the resistor and the current through the inductor. Because these two currents are not in phase, we'll need to use phasors to calculate their sum. The amplitudes of the applied voltage and the currents are equal to the magnitude of the phasors. That is $\left|\vec{\boldsymbol{\mathcal{E}}}\right| = \mathcal{E}_{\text{peak}}$, $\left|\vec{\boldsymbol{I}}\right| = I_{\text{peak}}$, $\left|\vec{\boldsymbol{I}}_R\right| = I_{R,\,\text{peak}}$, and $\left|\vec{\boldsymbol{I}}_L\right| = I_{L,\,\text{peak}}$.

(*a*) The ac source applies a voltage given by $\mathcal{E} = \mathcal{E}_{\text{peak}}\cos\omega t$. Thus, the voltage drop across both the load resistor and the inductor is:

$$\mathcal{E}_{\text{peak}}\cos\omega t = I_R R$$

The current in the resistor is in phase with the applied voltage:

$$I_R = I_{R,\,\text{peak}}\cos\omega t$$

Because $I_{R,\,\text{peak}} = \dfrac{\mathcal{E}_{\text{peak}}}{R}$:

$$I_R = \boxed{\frac{\mathcal{E}_{\text{peak}}}{R}\cos\omega t}$$

(*b*) The current in the inductor lags the applied voltage by 90°:

$$I_L = I_{L,\,\text{peak}}\cos(\omega t - 90°)$$

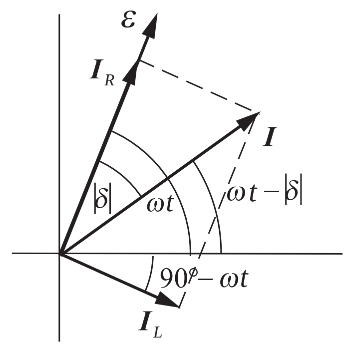
Because $I_{L,\,\text{peak}} = \dfrac{\mathcal{E}_{\text{peak}}}{X_{\text{L}}}$:

$$I_L = \boxed{\frac{\mathcal{E}_{\text{peak}}}{X_{\text{L}}}\cos(\omega t - 90°)}$$

(*c*) The net current I is the sum of the currents through the parallel branches:

$$I = I_R + I_L$$

Draw the phasor diagram for the circuit. The projections of the phasors onto the horizontal axis are the instantaneous values. The current in the resistor is in phase with the applied voltage, and the current in the inductor lags the applied voltage by 90°. The net current phasor is the sum of the branch current phasors $\left(\vec{I} = \vec{I}_L + \vec{I}_R\right)$.



The peak current through the parallel combination is equal to $\mathcal{E}_{\text{peak}}/Z$, where Z is the impedance of the combination:

$$I = I_{\text{peak}}\cos(\omega t - |\delta|),$$

where $I_{\text{peak}} = \dfrac{\mathcal{E}_{\text{peak}}}{Z}$

From the phasor diagram we have:

$$I_{\text{peak}}^2 = I_{R,\,\text{peak}}^2 + I_{L,\,\text{peak}}^2$$

$$= \left(\frac{\mathcal{E}_{\text{peak}}}{R}\right)^2 + \left(\frac{\mathcal{E}_{\text{peak}}}{X_{\text{L}}}\right)^2$$

$$= \mathcal{E}_{\text{peak}}^2\left(\frac{1}{R^2} + \frac{1}{X_{\text{L}}^2}\right) = \frac{\mathcal{E}_{\text{peak}}^2}{Z^2}$$

where $\dfrac{1}{Z^2} = \dfrac{1}{R^2} + \dfrac{1}{X_L^2}$

.

Solving for I_{peak} yields:

$$I_{\text{peak}} = \boxed{\frac{\mathcal{E}_{\text{peak}}}{Z}} \text{ where } Z^{-2} = R^{-2} + X_L^{-2}$$

From the phasor diagram:

$$I = \boxed{I_{\text{peak}}\cos(\omega t - |\delta|)}$$

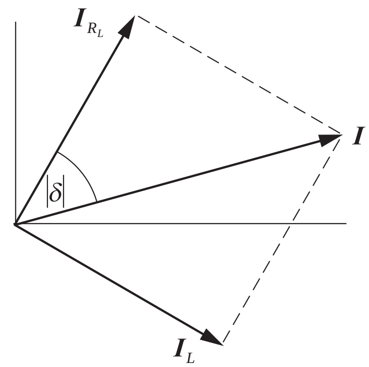
where

$$\tan|\delta| = \frac{I_{L,\,\text{peak}}}{I_{R,\,\text{peak}}} = \frac{\dfrac{\mathcal{E}_{\text{peak}}}{X_L}}{\dfrac{\mathcal{E}_{\text{peak}}}{R}} = \boxed{\frac{R}{X_L}}$$

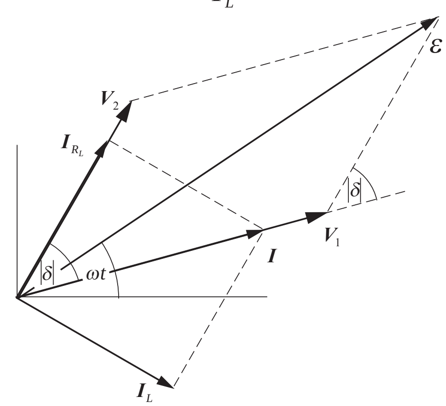
41 •• **[SSM]** Figure 29-33 shows a load resistor that has a resistance of $R_L = 20.0\ \Omega$ connected to a high-pass filter consisting of an inductor that has inductance $L = 3.20\text{-mH}$ and a resistor that has resistance $R = 4.00\text{-}\Omega$. The output of the ideal ac generator is given by $\mathcal{E} = (100\text{ V})\cos(2\pi ft)$. Find the rms currents in all three branches of the circuit if the driving frequency is (*a*) 500 Hz and (*b*) 2000 Hz. Find the fraction of the total average power supplied by the ac generator that is delivered to the load resistor if the frequency is (*c*) 500 Hz and (*d*) 2000 Hz.

Picture the Problem $\mathcal{E} = V_1 + V_2$, where V_1 is the voltage drop across R and V_2 is the voltage drop across the parallel combination of L and R_L. $\vec{\boldsymbol{\mathcal{E}}} = \vec{\boldsymbol{V}}_1 + \vec{\boldsymbol{V}}_2$ is the relation for the phasors. For the parallel combination $\vec{\boldsymbol{I}} = \vec{\boldsymbol{I}}_{R_L} + \vec{\boldsymbol{I}}_L$. Also, V_1 is in phase with I and V_2 is in phase with I_{R_L}. First draw the phasor diagram for the currents in the parallel combination, then add the phasors for the voltages to the diagram.

The phasor diagram for the currents in the circuit is:



Adding the voltage phasors to the diagram gives:



The maximum current in the inductor, $I_{2,\,\text{peak}}$, is given by:

$$I_{2,\,\text{peak}} = \frac{V_{2,\,\text{peak}}}{Z_2} \qquad (1)$$

where $Z_2^{-2} = R_L^{-2} + X_L^{-2} \qquad (2)$

$\tan|\delta|$ is given by:

$$\tan|\delta| = \frac{I_{L,\,\text{peak}}}{I_{R,\,\text{peak}}} = \frac{V_{2,\,\text{peak}}/X_\text{L}}{V_{2,\,\text{peak}}/R_\text{L}} = \frac{R_L}{X_L} = \frac{R_L}{\omega L} = \frac{R_L}{2\pi f L}$$

Solve for $|\delta|$ to obtain:

$$|\delta| = \tan^{-1}\left(\frac{R_\text{L}}{2\pi f L}\right) \qquad (3)$$

Apply the law of cosines to the triangle formed by the voltage phasors to obtain:

$$\mathcal{E}_{\text{peak}}^2 = V_{1,\,\text{peak}}^2 + V_{2,\,\text{peak}}^2 + 2V_{1,\,\text{peak}}V_{2,\,\text{peak}}\cos|\delta|$$

or

$$I_{\text{peak}}^2 Z^2 = I_{\text{peak}}^2 R^2 + I_{\text{peak}}^2 Z_2^2 + 2I_{\text{peak}} R I_{\text{peak}} Z_2 \cos|\delta|$$

Dividing out the current squared yields:

$$Z^2 = R^2 + Z_2^2 + 2RZ_2\cos|\delta|$$

Solving for Z yields:

$$Z = \sqrt{R^2 + Z_2^2 + 2RZ_2\cos|\delta|} \quad (4)$$

The maximum current I_{peak} in the circuit is given by:

$$I_{\text{peak}} = \frac{\mathcal{E}_{\text{peak}}}{Z} \quad (5)$$

I_{rms} is related to I_{peak} according to:

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} I_{\text{peak}} \quad (6)$$

(*a*) Substitute numerical values in equation (3) and evaluate $|\delta|$:

$$|\delta| = \tan^{-1}\left(\frac{20.0\,\Omega}{2\pi(500\,\text{Hz})(3.20\,\text{mH})}\right) = \tan^{-1}\left(\frac{20.0\,\Omega}{10.053\,\Omega}\right) = 63.31°$$

Solving equation (2) for Z_2 yields:

$$Z_2 = \frac{1}{\sqrt{R_L^{-2} + X_L^{-2}}}$$

Substitute numerical values and evaluate Z_2:

$$Z_2 = \frac{1}{\sqrt{(20.0\,\Omega)^{-2} + (10.053\,\Omega)^{-2}}} = 8.982\,\Omega$$

Substitute numerical values and evaluate Z:

$$Z = \sqrt{(4.00\,\Omega)^2 + (8.982\,\Omega)^2 + 2(4.00\,\Omega)(8.982\,\Omega)\cos 63.31°} = 11.36\,\Omega$$

Substitute numerical values in equation (5) and evaluate I_{peak}:

$$I_{\text{peak}} = \frac{100\,\text{V}}{11.36\,\Omega} = 8.806\,\text{A}$$

Substitute for I_{peak} in equation (6) and evaluate I_{rms}:

$$I_{rms} = \frac{1}{\sqrt{2}}(8.806\,\text{A}) = \boxed{6.23\,\text{A}}$$

The maximum and rms values of V_2 are given by:

$$V_{2,\,peak} = I_{peak}Z_2 = (8.806\,\text{A})(8.982\,\Omega) = 79.095\,\text{V}$$

and

$$V_{2,rms} = \frac{1}{\sqrt{2}}V_{2,\,peak} = \frac{1}{\sqrt{2}}(79.095\,\text{V}) = 55.929\,\text{V}$$

The rms values of $I_{R_L,rms}$ and $I_{L,rms}$ are:

$$I_{R_L,rms} = \frac{V_{2,rms}}{R_L} = \frac{55.929\,\text{V}}{20.0\,\Omega} = \boxed{2.80\,\text{A}}$$

and

$$I_{L,rms} = \frac{V_{2,rms}}{X_L} = \frac{55.929\,\text{V}}{10.053\,\Omega} = \boxed{5.53\,\text{A}}$$

(*b*) Proceed as in (*a*) with $f = 2000$ Hz to obtain:

$X_L = 40.2\,\Omega, |\delta| = 26.4°, Z_2 = 17.9\,\Omega$, $Z = 21.6\,\Omega$, $I_{peak} = 4.64\,\text{A}$, and $I_{rms} = \boxed{3.28\,\text{A}}$, $V_{2,max} = 83.0\,\text{V}, V_{2,rms} = 58.7\,\text{V}$, $I_{R_L,rms} = \boxed{2.94\,\text{A}}$, and $\boldsymbol{I}_{L,rms} = \boxed{1.46\,\text{A}}$

(*c*) The power delivered by the ac source equals the sum of the power dissipated in the two resistors. The fraction of the total power delivered by the source that is dissipated in load resistor is given by:

$$\frac{P_{R_L}}{P_{R_L} + P_R} = \left(1 + \frac{P_R}{P_{R_L}}\right)^{-1} = \left(1 + \frac{I_{rms}^2 R}{I_{R_L,rms}^2 R_L}\right)^{-1}$$

Substitute numerical values for $f = 500$ Hz to obtain:

$$\left.\frac{P_{R_L}}{P_{R_L} + P_R}\right|_{f=500\,\text{Hz}} = \left(1 + \frac{(6.23\,\text{A})^2(4.00\,\Omega)}{(2.80\,\text{A})^2(20.0\,\Omega)}\right)^{-1} = 0.502 = \boxed{50.2\%}$$

(*d*) Substitute numerical values for f = 2000 Hz to obtain:

$$\left.\frac{P_{R_L}}{P_{R_L}+P_R}\right|_{f=2000\,\text{Hz}}=\left(1+\frac{(3.28\,\text{A})^2(4.00\,\Omega)}{(2.94\,\text{A})^2(20.0\,\Omega)}\right)^{-1}=0.800=\boxed{80.0\%}$$

42 •• An ideal ac voltage source whose emf $\mathcal{E}_1$ is given by (20 V) cos($2\pi ft$) and an ideal battery whose emf $\mathcal{E}_2$ is 16 V are connected to a combination of two resistors and an inductor (Figure 29-34), where R_1 = 10 Ω, R_2 = 8.0 Ω, and L = 6.0 mH. Find the average power delivered to each resistor if (*a*) the driving frequency is 100 Hz, (*b*) the driving frequency is 200 Hz, and (*c*) the driving frequency is 800 Hz.

Picture the Problem We can treat the ac and dc components separately. For the dc component, L acts like a short circuit. Let $\mathcal{E}_{1,\,\text{peak}}$ denote the peak value of the voltage supplied by the ac voltage source. We can use $P=\mathcal{E}_2^2/R$ to find the power dissipated in the resistors by the current from the ideal battery. We'll apply Kirchhoff's loop rule to the loop including L, R_1, and R_2 to derive an expression for the average power delivered to each resistor by the ac voltage source.

(*a*) The total power delivered to R_1 and R_2 is:

$$P_1=P_{1,\,\text{dc}}+P_{1,\,\text{ac}} \quad (1)$$

and

$$P_2=P_{2,\,\text{dc}}+P_{2,\,\text{ac}} \quad (2)$$

The dc power delivered to the resistors whose resistances are R_1 and R_2 is:

$$P_{1,\text{dc}}=\frac{\mathcal{E}_2^2}{R_1}\text{ and }P_{2,\text{dc}}=\frac{\mathcal{E}_2^2}{R_2}$$

Express the average ac power delivered to R_1:

$$P_{1,\text{ac}}=\frac{\mathcal{E}_{1,\,\text{rms}}^2}{R_1}=\frac{\mathcal{E}_{1,\,\text{peak}}^2}{2R_1}$$

Apply Kirchhoff's loop rule to a clockwise loop that includes R_1, L, and R_2:

$$R_1I_1-Z_2I_2=0$$

Solving for I_2 yields:

$$I_2=\frac{R_1}{Z_2}I_1=\frac{R_1}{Z_2}\frac{\mathcal{E}_{1,\,\text{peak}}}{R_1}=\frac{\mathcal{E}_{1,\,\text{peak}}}{Z_2}$$

Express the average ac power delivered to R_2:

$$P_{2,\,\text{ac}} = \tfrac{1}{2} I_{2,\,\text{rms}}^2 R_2 = \tfrac{1}{2}\left(\frac{\mathcal{E}_{1,\,\text{peak}}}{Z_2}\right)^2 R_2 = \frac{\mathcal{E}_{1,\,\text{peak}}^2 R_2}{2Z_2^2}$$

Substituting in equations (1) and (2) yields:

$$P_1 = \frac{\mathcal{E}_2^2}{R_1} + \frac{\mathcal{E}_{1,\,\text{peak}}^2}{2R_1}$$

and

$$P_2 = \frac{\mathcal{E}_2^2}{R_2} + \frac{\mathcal{E}_{1,\,\text{peak}}^2 R_2}{2Z_2^2}$$

Substitute numerical values and evaluate P_1:

$$P_1 = \frac{(16\,\text{V})^2}{10\,\Omega} + \frac{(20\,\text{V})^2}{2(10\,\Omega)} = \boxed{46\text{ W}}$$

Substitute numerical values and evaluate P_2:

$$P_2 = \frac{(16\,\text{V})^2}{8.0\,\Omega} + \frac{(20\,\text{V})^2(8.0\,\Omega)}{2\left[(8.0\,\Omega)^2 + \left(2\pi\{100\,\text{s}^{-1}\}\{6.0\,\text{mH}\}\right)^2\right]} = \boxed{52\text{ W}}$$

(*b*) Proceed as in (*a*) to evaluate P_1 and P_2 with f = 200 Hz:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{46\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 13.2\,\text{W} = \boxed{45\,\text{W}}$$

(*c*) Proceed as in (*a*) to evaluate P_1 and P_2 with f = 800 Hz:

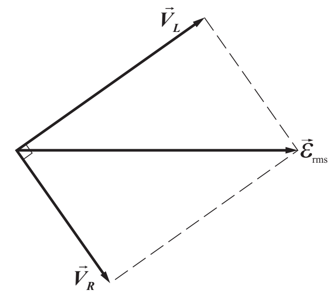
$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{46\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 1.64\,\text{W} = \boxed{34\,\text{W}}$$

43 •• An ac circuit contains a resistor and an ideal inductor connected in series. The voltage rms drop across this series combination is 100-V and the rms voltage drop across the inductor alone is 80 V. What is the rms voltage drop across the resistor?

Picture the Problem We can use the phasor diagram for an *RL* circuit to find the voltage across the resistor.

The phasor diagram for the voltages in the circuit is shown to the right:



Use the Pythagorean theorem to express V_R:

$$V_R = \sqrt{\mathcal{E}_{\text{rms}}^2 - V_L^2}$$

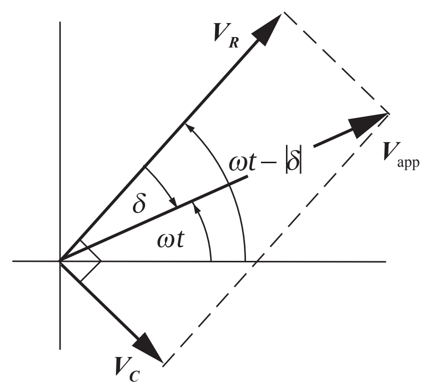
Substitute numerical values and evaluate V_R:

$$V_R = \sqrt{(100\text{ V})^2 - (80\text{ V})^2} = \boxed{60\text{ V}}$$

Filters and Rectifiers

44 •• The circuit shown in Figure 29-35 is called an *RC high-pass filter* because it transmits input voltage signals that have high frequencies with greater amplitude than it transmits input voltage signals that have low frequencies. If the input voltage is given by $V_{\text{in}} = V_{\text{in peak}} \cos \omega t$, show that the output voltage is $V_{\text{out}} = V_{\text{H}} \cos(\omega t - \delta)$ where $V_{\text{H}} = V_{\text{in peak}} \Big/ \sqrt{1 + (\omega RC)^{-2}}$. (Assume that the output is connected to a load that draws only an insignificant amount of current.) Show that this result justifies calling this circuit a high-pass filter.

Picture the Problem The phasor diagram for the *RC* high-pass filter is shown to the right. $\vec{V}_{\text{app}}$ and $\vec{V}_R$ are the phasors for V_{in} and V_{out}, respectively. Note that $\tan\delta = -X_C/R$. That δ is negative follows from the fact that $\vec{V}_{\text{app}}$ lags $\vec{V}_R$ by $|\delta|$. The projection of $\vec{V}_{\text{app}}$ onto the horizontal axis is $V_{\text{app}} = V_{\text{in}}$, and the projection of $\vec{V}_R$ onto the horizontal axis is $V_R = V_{\text{out}}$.



Express V_{app}:

$$V_{\text{app}} = V_{\text{app,peak}} \cos \omega t$$

where $V_{\text{app,peak}} = V_{\text{peak}} = I_{\text{peak}} Z$

and $Z^2 = R^2 + X_C^2$ (1)

Because $\delta < 0$:

$$\omega t + |\delta| = \omega t - \delta$$

V_R is given by:

$$V_R = V_{R,\,\text{peak}}\cos(\omega t - \delta)$$
where $V_{R,\,\text{peak}} = V_{\text{H}} = I_{\text{peak}}R$

Solving equation (1) for Z and substituting for X_C yields:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad (2)$$

Because $V_{\text{out}} = V_R$:

$$\begin{aligned} V_{\text{out}} &= V_{R,\,\text{peak}}\cos(\omega t - \delta) \\ &= I_{\text{in peak}}R\cos(\omega t - \delta) \\ &= \frac{V_{\text{in peak}}}{Z}R\cos(\omega t - \delta) \end{aligned}$$

Using equation (2) to substitute for Z yields:

$$V_{\text{out}} = \frac{V_{\text{in peak}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}R\cos(\omega t - \delta)$$

Simplify further to obtain:

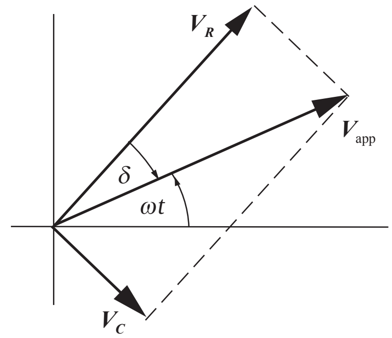
$$V_{\text{out}} = \frac{V_{\text{in peak}}}{\sqrt{1 + (\omega RC)^{-2}}}\cos(\omega t - \delta)$$
or
$$V_{\text{out}} = \boxed{V_{\text{H}}\cos(\omega t - \delta)}$$
where
$$V_{\text{H}} = \boxed{\frac{V_{\text{in peak}}}{\sqrt{1 + (\omega RC)^{-2}}}}$$

As $\omega \to \infty$:

$$V_{\text{H}} \to \frac{V_{\text{in peak}}}{\sqrt{1 + (0)^2}} = V_{\text{in peak}}$$ showing that the result is consistent with the high-pass name for this circuit.

45 •• (*a*) Find an expression for the phase constant δ in Problem 44 in terms of ω, R and C. (*b*) What is the value of δ in the limit that $\omega \to 0$? (*c*) What is the value of δ in the limit that $\omega \to \infty$? (*d*) Explain your answers to Parts (*b*) and (*c*).

Picture the Problem The phasor diagram for the *RC* high-pass filter is shown below. $\vec{V}_{\text{app}}$ and $\vec{V}_R$ are the phasors for V_{in} and V_{out}, respectively. The projection of $\vec{V}_{\text{app}}$ onto the horizontal axis is $V_{\text{app}} = V_{\text{in}}$, and the projection of $\vec{V}_{\text{R}}$ onto the horizontal axis is $V_R = V_{\text{out}}$.



(*a*) Because $\vec{V}_{\text{app}}$ lags $\vec{V}_{\text{R}}$ by $|\delta|$.

$$\tan\delta = -\frac{V_C}{V_R} = -\frac{IX_C}{IR} = -\frac{X_C}{R}$$

Use the definition of X_C to obtain:

$$\tan\delta = -\frac{\frac{1}{\omega C}}{R} = -\frac{1}{\omega RC}$$

Solving for δ yields:

$$\delta = \boxed{\tan^{-1}\left[-\frac{1}{\omega RC}\right]}$$

(*b*) As $\omega \to 0$:

$$\delta \to \boxed{-90°}$$

(*c*) As $\omega \to \infty$:

$$\delta \to \boxed{0}$$

(*d*) For very low driving frequencies, $\boldsymbol{X_C} >> \boldsymbol{R}$ and so $\vec{V}_C$ effectively lags $\vec{V}_{\text{in}}$ by 90°. For very high driving frequencies, $\boldsymbol{X_C} << \boldsymbol{R}$ and so $\vec{V}_R$ is effectively in phase with $\vec{V}_{\text{in}}$.

46 •• Assume that in Problem 44, $R = 20\ \text{k}\Omega$ and $C = 15$ nF. (*a*) At what frequency is $V_{\text{H}} = \frac{1}{\sqrt{2}} V_{\text{in peak}}$? This particular frequency is known as the 3 dB frequency, or $f_{3\text{dB}}$ for the circuit. (*b*) Using a **spreadsheet** program, make a graph of $\log_{10}(V_{\text{H}})$ versus $\log_{10}(f)$, where f is the frequency. Make sure that the scale extends from at least 10% of the 3-dB frequency to ten times the 3-dB frequency. (*c*) Make a graph of δ versus $\log_{10}(f)$ for the same range of frequencies as in Part (*b*). What is the value of the phase constant when the frequency is equal to the 3-dB frequency?

Picture the Problem We can use the results obtained in Problems 44 and 45 to find $f_{3\text{ dB}}$ and to plot graphs of $\log(V_{\text{out}})$ versus $\log(f)$ and δ versus $\log(f)$.

(a) Use the result of Problem 44 to express the ratio $V_{out}/V_{in\,peak}$:

$$\frac{V_{out}}{V_{in\,peak}}=\frac{\dfrac{V_{in\,peak}}{\sqrt{1+(\omega RC)^{-2}}}}{V_{in\,peak}}=\frac{1}{\sqrt{1+(\omega RC)^{-2}}}$$

When $V_{out}=V_{in\,peak}/\sqrt{2}$:

$$\frac{1}{\sqrt{1+(\omega RC)^{-2}}}=\frac{1}{\sqrt{2}}$$

Square both sides of the equation and solve for ωRC to obtain:

$$\omega RC=1\Rightarrow\ \omega=\frac{1}{RC}\Rightarrow\ f_{3\,dB}=\frac{1}{2\pi RC}$$

Substitute numerical values and evaluate $f_{3\,dB}$:

$$f_{3\,dB}=\frac{1}{2\pi(20\,k\Omega)(15\,nF)}=\boxed{0.53\,kHz}$$

(b) From Problem 44 we have:

$$V_{out}=\frac{V_{in\,peak}}{\sqrt{1+(\omega RC)^{-2}}}$$

In Problem 45 it was shown that:

$$\delta=\tan^{-1}\left[-\frac{1}{\omega RC}\right]$$

Rewrite these expressions in terms of $f_{3\,dB}$ to obtain:

$$V_{out}=\frac{V_{in\,peak}}{\sqrt{1+\left(\dfrac{1}{2\pi fRC}\right)^2}}=\frac{V_{peak}}{\sqrt{1+\left(\dfrac{f_{3\,dB}}{f}\right)^2}}$$

and

$$\delta=\tan^{-1}\left[-\frac{1}{2\pi fRC}\right]=\tan^{-1}\left[-\frac{f_{3\,dB}}{f}\right]$$

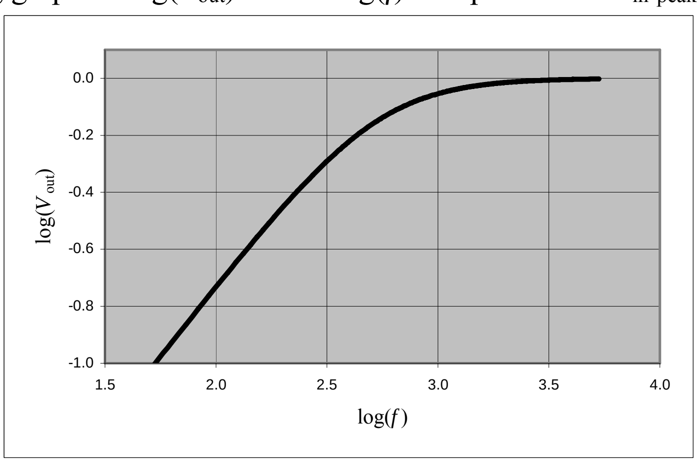
A spreadsheet program to generate the data for a graph of V_{out} versus f and δ versus f is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	1.50E−08	C
B3	1	$V_{in\,peak}$
B4	531	$f_{3\,dB}$
A8	53	$0.1f_{3\,dB}$

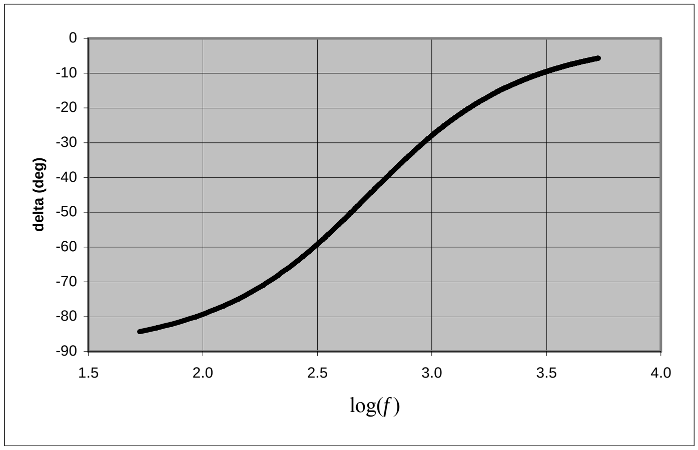
C8	$B$3/SQRT(1+(1($B$4/A8))^2)	$\dfrac{V_{\text{in peak}}}{\sqrt{1+\left(\dfrac{f_{3\,\text{dB}}}{f}\right)^2}}$
D8	LOG(C8)	$\log(V_{\text{out}})$
E8	ATAN(−$B$4/A8)	$\tan^{-1}\left[-\dfrac{f_{3\,\text{dB}}}{f}\right]$
F8	E8*180/PI()	δ in degrees

	A	B	C	D	E	F
1	$R=$	2.00E+04	ohms			
2	$C=$	1.50E−08	F			
3	$V_{\text{in peak}}=$	1	V			
4	$f_{3\text{ dB}}=$	531	Hz			
5						
6						
7	f	log(f)	V_{out}	log(V_{out})	delta(rad)	delta(deg)
8	53	1.72	0.099	−1.003	−1.471	−84.3
9	63	1.80	0.118	−0.928	−1.453	−83.2
10	73	1.86	0.136	−0.865	−1.434	−82.2
11	83	1.92	0.155	−0.811	−1.416	−81.1
55	523	2.72	0.702	−0.154	−0.793	−45.4
56	533	2.73	0.709	−0.150	−0.783	−44.9
57	543	2.73	0.715	−0.146	−0.774	−44.3
531	5283	3.72	0.995	−0.002	−0.100	−5.7
532	5293	3.72	0.995	−0.002	−0.100	−5.7
533	5303	3.72	0.995	−0.002	−0.100	−5.7
534	5313	3.73	0.995	−0.002	−0.100	−5.7

The following graph of $\log(V_{\text{out}})$ versus $\log(f)$ was plotted for $V_{\text{in peak}} = 1$ V.



A graph of δ (in degrees) as a function of log(f) follows.



Referring to the spreadsheet program, we see that when $f = f_{3\text{ dB}}$, $\delta \approx \boxed{-44.9°.}$

This result is in good agreement with its calculated value of $-45.0°$.

47 •• [SSM] A slowly varying voltage signal $V(t)$ is applied to the input of the high-pass filter of Problem 44. Slowly varying means that during one time constant (equal to RC) there is no significant change in the voltage signal. Show that under these conditions the output voltage is proportional to the time derivative of $V(t)$. This situation is known as a *differentiation circuit.*

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. Because the voltage drop across the resistor is small compared to the voltage drop across the capacitor, we can express the voltage drop across the capacitor in terms of the input voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V(t) - V_C - IR = 0$$

where V_C is the potential difference across the capacitor.

Substitute for $V(t)$ and I to obtain:

$$V_{\text{in peak}} \cos \omega t - V_c - R\frac{dQ}{dt} = 0$$

Because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV_C] = C\frac{dV_C}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}} \cos \omega t - V_C - RC\frac{dV_C}{dt} = 0$$

the differential equation describing the potential difference across the capacitor.

Because there is no significant change in the voltage signal during one time constant:

$$\frac{dV_C}{dt}=0 \Rightarrow RC\frac{dV_C}{dt}=0$$

Substituting for $RC\frac{dV_C}{dt}$ yields:

$$V_{\text{in peak}}\cos\omega t - V_C = 0$$

and

$$V_C = V_{\text{in peak}}\cos\omega t$$

Consequently, the potential difference across the resistor is given by:

$$V_R = RC\frac{dV_C}{dt} = \boxed{RC\frac{d}{dt}\left[V_{\text{in peak}}\cos\omega t\right]}$$

48 •• We can describe the output from the high-pass filter from Problem 44 using a decibel scale: $\beta=(20\text{ dB})\log_{10}\left(V_{\text{H}}/V_{\text{in peak}}\right)$, where β is the output in decibels. Show that for $V_{\text{H}}=\frac{1}{\sqrt{2}}V_{\text{in peak}}$, β = 3.0 dB. The frequency at which $V_{\text{H}}=\frac{1}{\sqrt{2}}V_{\text{in peak}}$ is known as $f_{3\text{dB}}$ (the 3-dB frequency). Show that for $f << f_{3\text{dB}}$, the output β drops by 6 dB if the frequency f is halved.

Picture the Problem We can use the expression for V_{H} from Problem 44 and the definition of β given in the problem to show that every time the frequency is halved, the output drops by 6 dB.

From Problem 44:

$$V_{\text{H}} = \frac{V_{\text{in peak}}}{\sqrt{1+(\omega RC)^{-2}}}$$

or

$$\frac{V_{\text{H}}}{V_{\text{in peak}}} = \frac{1}{\sqrt{1+(\omega RC)^{-2}}}$$

Express this ratio in terms of f and $f_{3\text{ dB}}$ and simplify to obtain:

$$\frac{V_{\text{H}}}{V_{\text{peak}}} = \frac{1}{\sqrt{1+\left(\frac{f_{3\text{ dB}}}{f}\right)^2}} = \frac{f}{\sqrt{f_{3\text{ dB}}^2\left(1+\frac{f^2}{f_{3\text{ dB}}^2}\right)}}$$

For $f << f_{3\text{dB}}$:

$$\frac{V_{\text{H}}}{V_{\text{peak}}} \approx \frac{f}{\sqrt{f_{3\text{ dB}}^2\left(1+\frac{f^2}{f_{3\text{ dB}}^2}\right)}} = \frac{f}{f_{3\text{ dB}}}$$

From the definition of β we have:

$$\beta = 20\log_{10}\left(\frac{V_{\text{H}}}{V_{\text{peak}}}\right)$$

Substitute for V_H/V_{peak} to obtain:

$$\beta = 20\log_{10}\left(\frac{f}{f_{3\,dB}}\right)$$

Doubling the frequency yields:

$$\beta' = 20\log_{10}\left(\frac{\frac{1}{2}f}{f_{3\,dB}}\right)$$

The change in decibel level is:

$$\begin{aligned}\Delta\beta &= \beta' - \beta \\ &= 20\log_{10}\left(\frac{\frac{1}{2}f}{f_{3\,dB}}\right) - 20\log_{10}\left(\frac{f}{f_{3\,dB}}\right) \\ &= 20\log_{10}\left(\tfrac{1}{2}\right) \approx \boxed{-6\,dB}\end{aligned}$$

49 •• [SSM] Show that the average power dissipated in the resistor of the high-pass filter of Problem 44 is given by $P_{ave} = \dfrac{V_{in\,peak}^2}{2R\left[1+(\omega RC)^{-2}\right]}$.

Picture the Problem We can express the instantaneous power dissipated in the resistor and then use the fact that the average value of the square of the cosine function over one cycle is ½ to establish the given result.

The instantaneous power $P(t)$ dissipated in the resistor is:

$$P(t) = \frac{V_{out}^2}{R}$$

The output voltage V_{out} is:

$$V_{out} = V_H\cos(\omega t - \delta)$$

From Problem 44:

$$V_H = \frac{V_{in\,peak}}{\sqrt{1+(\omega RC)^{-2}}}$$

Substitute in the expression for $P(t)$ to obtain:

$$\begin{aligned}P(t) &= \frac{V_H^2}{R}\cos^2(\omega t - \delta) \\ &= \frac{V_{in\,peak}^2}{R\left[1+(\omega RC)^{-2}\right]}\cos^2(\omega t - \delta)\end{aligned}$$

Because the average value of the square of the cosine function over one cycle is ½:

$$P_{ave} = \boxed{\frac{V_{in\,peak}^2}{2R\left[1+(\omega RC)^{-2}\right]}}$$

50 •• One application of the high-pass filter of Problem 44 is a noise filter for electronic circuits (a filter that blocks out low-frequency noise). Using a resistance value of 20 kΩ, find a value for the capacitance for the high-pass filter that attenuates a 60-Hz input voltage signal by a factor of 10. That is, so $V_{\mathrm{H}} = \frac{1}{10} V_{\mathrm{in\ peak}}$.

Picture the Problem We can solve the expression for V_{H} from Problem 44 for the required capacitance of the capacitor.

From Problem 44:

$$V_{\mathrm{H}} = \frac{V_{\mathrm{in\ peak}}}{\sqrt{1+(\omega RC)^{-2}}}$$

We require that:

$$\frac{V_{\mathrm{H}}}{V_{\mathrm{in\ peak}}} = \frac{1}{\sqrt{1+(\omega RC)^{-2}}} = \frac{1}{10}$$

or

$$\sqrt{1+(\omega RC)^{-2}} = 10$$

Solving for C yields:

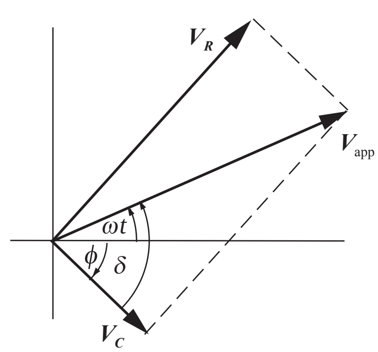
$$C = \frac{1}{\sqrt{99}\,\omega R} = \frac{1}{2\pi\sqrt{99}\,Rf}$$

Substitute numerical values and evaluate C:

$$\boldsymbol{C} = \frac{1}{2\boldsymbol{\pi}\sqrt{99}(20\,\mathrm{k\Omega})(60\,\mathrm{Hz})} = \boxed{13\,\mathrm{nF}}$$

51 •• [SSM] The circuit shown in Figure 29-36 is an example of a low-pass filter. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (*a*) If the input voltage is given by $V_{\mathrm{in}} = V_{\mathrm{in\ peak}} \cos \omega t$, show that the output voltage is $V_{\mathrm{out}} = V_{\mathrm{L}} \cos(\omega t - \delta)$ where $V_{\mathrm{L}} = V_{\mathrm{in\ peak}} \Big/ \sqrt{1+(\omega RC)^2}$. (*b*) Discuss the trend of the output voltage in the limiting cases $\omega \to 0$ and $\omega \to \infty$.

Picture the Problem In the phasor diagram for the *RC* low-pass filter, $\vec{V}_{\mathrm{app}}$ and $\vec{V}_C$ are the phasors for V_{in} and V_{out}, respectively. The projection of $\vec{V}_{\mathrm{app}}$ onto the horizontal axis is $V_{\mathrm{app}} = V_{\mathrm{in}}$, the projection of $\vec{V}_C$ onto the horizontal axis is $V_C = V_{\mathrm{out}}$, $V_{\mathrm{peak}} = \left|\vec{V}_{\mathrm{app}}\right|$, and ϕ is the angle between $\vec{V}_C$ and the horizontal axis.



(*a*) Express V_{app}:

$$V_{\text{app}} = V_{\text{in peak}} \cos\omega t$$
where $V_{\text{in peak}} = I_{\text{peak}} Z$
and $Z^2 = R^2 + X_{\text{C}}^2 \qquad (1)$

$V_{\text{out}} = V_C$ is given by:

$$V_{\text{out}} = V_{C,\text{peak}} \cos\phi = I_{\text{peak}} X_C \cos\phi$$

If we define δ as shown in the phasor diagram, then:

$$V_{\text{out}} = I_{\text{peak}} X_C \cos(\omega t - \delta) = \frac{V_{\text{in peak}}}{Z} X_C \cos(\omega t - \delta)$$

Solving equation (1) for Z and substituting for X_C yields:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad (2)$$

Using equation (2) to substitute for Z and substituting for X_C yields:

$$V_{\text{out}} = \frac{V_{\text{in peak}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \frac{1}{\omega C} \cos(\omega t - \delta)$$

Simplify further to obtain:

$$V_{\text{out}} = \frac{V_{\text{in peak}}}{\sqrt{1 + (\omega RC)^2}} \cos(\omega t - \delta)$$
or
$$V_{\text{out}} = \boxed{V_{\text{L}} \cos(\omega t - \delta)}$$
where
$$V_L = \boxed{\frac{V_{\text{in peak}}}{\sqrt{1 + (\omega RC)^2}}}$$

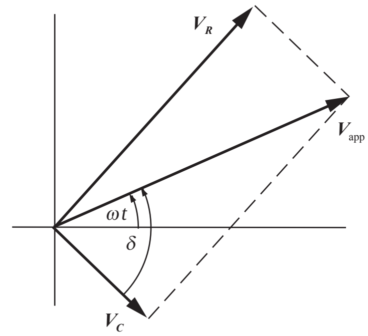
(*b*) Note that, as $\omega \to 0$, $V_L \to V_{\text{peak}}$. This makes sense physically in that, for low frequencies, X_C is large and, therefore, a larger peak input voltage will appear across it than appears across it for high frequencies.

Note further that, as $\omega \to \infty$, $V_L \to 0$. This makes sense physically in that, for high frequencies, X_C is small and, therefore, a smaller peak voltage will appear across it than appears across it for low frequencies.

Remarks: In Figures 29-19 and 29-20, δ is defined as the phase of the voltage drop across the combination relative to the voltage drop across the resistor.

52 •• (*a*) Find an expression for the phase angle δ for the low-pass filter of Problem 51 in terms of ω, R and C. (*b*) Find the value of δ in the limit that $\omega \to 0$ and in the limit that $\omega \to \infty$. Explain your answer.

Picture the Problem The phasor diagram for the *RC* low-pass filter is shown to the right. $\vec{V}_{\text{app}}$ and $\vec{V}_C$ are the phasors for V_{in} and V_{out}, respectively. The projection of $\vec{V}_{\text{app}}$ onto the horizontal axis is $V_{\text{app}} = V_{\text{in}}$ and the projection of $\vec{V}_C$ onto the horizontal axis is $V_C = V_{\text{out}}$. $V_{\text{peak}} = \left|\vec{V}_{\text{app}}\right|$.



(*a*) From the phasor diagram we have:

$$\tan\delta = \frac{V_R}{V_C} = \frac{I_{\text{peak}}R}{I_{\text{peak}}X_C} = \frac{R}{X_C}$$

Use the definition of X_C to obtain:

$$\tan\delta = \frac{R}{\dfrac{1}{\omega C}} = \omega RC$$

Solving for δ yields:

$$\delta = \boxed{\tan^{-1}(\omega RC)}$$

(*b*) As $\omega \to 0$, $\delta \to \boxed{0°}$. This behavior makes sense physically in that, at low frequencies, X_C is very large compared to R and, as a consequence, V_C is in phase with V_{in}.

As $\omega \to \infty$, $\delta \to \boxed{90°}$. This behavior makes sense physically in that, at high frequencies, X_C is very small compared to R and, as a consequence, V_C is out of phase with V_{in}.

Remarks: See the spreadsheet solution in the following problem for additional evidence that our answer for Part (*b*) is correct.

53 •• Using a **spreadsheet** program, make a graph of V_L versus input frequency f and a graph of phase angle δ versus input frequency for the low-pass filter of Problems 51 and 52. Use a resistance value of 10 kΩ and a capacitance value of 5.0 nF.

Picture the Problem We can use the expressions for V_L and δ derived in Problems 51 and 52 to plot the graphs of V_L versus f and δ versus f for the low-pass filter of Problem 51. We'll simplify the spreadsheet program by expressing both V_L and δ as functions of $f_{3\text{ dB}}$.

From Problems 51 and 52 we have:

$$V_L = \frac{V_{\text{in peak}}}{\sqrt{1+(\omega RC)^2}}$$

and

$$\delta = \tan^{-1}(\omega RC)$$

Rewrite each of these expressions in terms of f to obtain:

$$V_L = \frac{V_{\text{in peak}}}{\sqrt{1+(2\pi fRC)^2}}$$

and

$$\delta = \tan^{-1}(2\pi fRC)$$

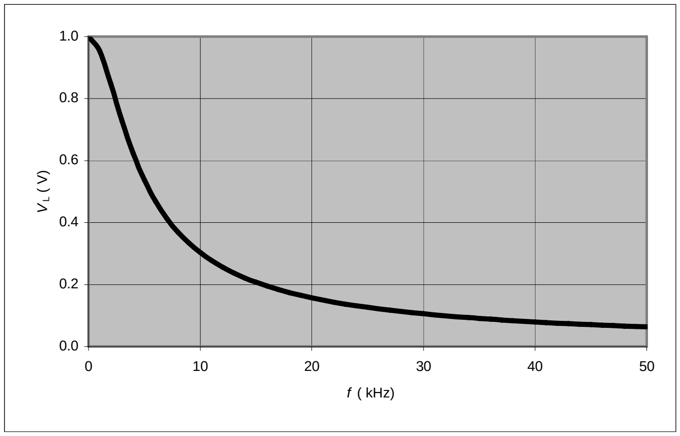
A spreadsheet program to generate the data for graphs of V_L versus f and δ versus f for the low-pass filter is shown below. Note that $V_{\text{in peak}}$ has been arbitrarily set equal to 1 V. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	5.00E−09	C
B3	1	$V_{\text{in peak}}$
B8	$B$3/SQRT(1+((2*PI()*A8*1000*$B$1*$B$2)^2))	$\frac{V_{\text{in peak}}}{\sqrt{1+(2\pi fRC)^2}}$
C8	ATAN(2*PI()*A8*1000*$B$1*$B$2)	$\tan^{-1}(2\pi fRC)$
D8	C8*180/PI()	δ in degrees

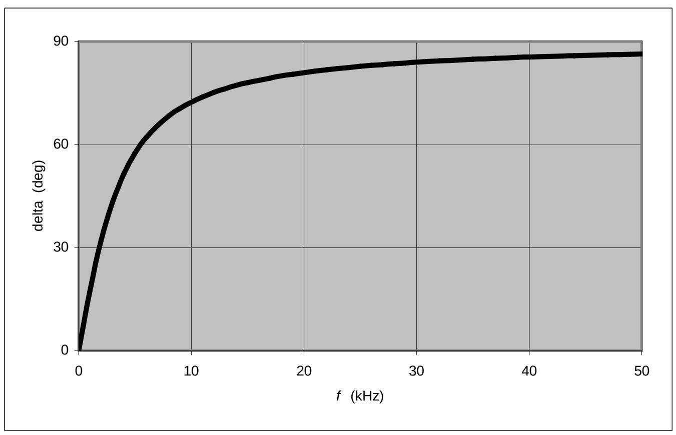
	A	B	C	D
1	R=	1.00E+04	ohms	
2	C=	5.00E−09	F	
3	$V_{\text{in peak}}$=	1	V	
4				
5				
6	f(kHz)	V_{out}	δ(rad)	δ(deg)
7	0	1.000	0.000	0.0
8	1	0.954	0.304	17.4
9	2	0.847	0.561	32.1
10	3	0.728	0.756	43.3

54	47	0.068	1.503	86.1
55	48	0.066	1.505	86.2
56	49	0.065	1.506	86.3
57	50	0.064	1.507	86.4

A graph of V_L as a function of f follows:



A graph of δ as a function of f follows:



54 ••• A rapidly varying voltage signal $V(t)$ is applied to the input of the low-pass filter of Problem 51. Rapidly varying means that during one time constant (equal to RC) there are significant changes in the voltage signal. Show that under these conditions the output voltage is proportional to the integral of $V(t)$ with respect to time. This situation is known as an *integration circuit*.

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution to these simultaneous equations will yield the amplitude of the output voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V(t) - IR - V_C = 0$$

where V_C is the potential difference across the capacitor.

Substitute for $V(t)$ and I to obtain:

$$V_{\text{in peak}}\cos\omega t - R\frac{dQ}{dt} - V_C = 0$$

Because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV_C] = C\frac{dV_C}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{in peak}}\cos\omega t - RC\frac{dV_C}{dt} - V_C = 0$$

the differential equation describing the potential difference across the capacitor.

V_C is given by:

$$V_C = IX_c = \frac{I}{\omega C}$$

The fact that $V(t)$ varies rapidly means that $\omega >> 1$ and so:

$$V_C \approx 0$$

and

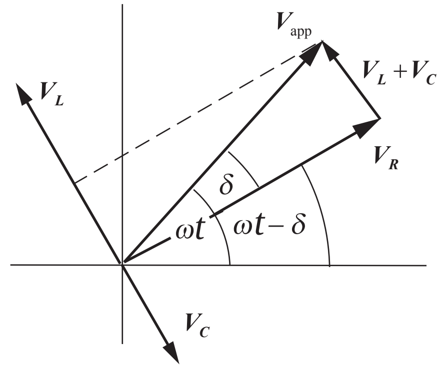
$$V_{\text{peak}}\cos\omega t - RC\frac{dV_C}{dt} = 0$$

Separating the variables in this differential equation and solving for V_C yields:

$$V_C = \boxed{\frac{1}{RC}\int V_{\text{peak}}\cos\omega t dt}$$

55 ••• **[SSM]** The circuit shown in Figure 29-37 is a *trap filter*. (Assume that the output is connected to a load that draws only an insignificant amount of current.) (*a*) Show that the *trap filter* acts to reject signals in a band of frequencies centered at $\omega = 1/\sqrt{LC}$. (*b*) How does the width of the frequency band rejected depend on the resistance R?

Picture the Problem The phasor diagram for the *trap* filter is shown below. $\vec{V}_{\text{app}}$ and $\vec{V}_L + \vec{V}_C$ are the phasors for V_{in} and V_{out}, respectively. The projection of $\vec{V}_{\text{app}}$ onto the horizontal axis is $V_{\text{app}} = V_{\text{in}}$, and the projection of $\vec{V}_L + \vec{V}_C$ onto the horizontal axis is $V_L + V_C = V_{\text{out}}$. Requiring that the impedance of the trap be zero will yield the frequency at which the circuit rejects signals. Defining the bandwidth as $\Delta\omega = |\omega - \omega_{\text{trap}}|$ and requiring that $|Z_{\text{trap}}| = R$ will yield an expression for the bandwidth and reveal its dependence on R.



(*a*) Express V_{app}:

$$V_{app} = V_{app,\,peak}\cos\omega t$$
where $V_{app,\,peak} = V_{peak} = I_{peak}Z$
and $Z^2 = R^2 + (X_L - X_C)^2$ (1)

V_{out} is given by:

$$V_{out} = V_{out,\,peak}\cos(\omega t - \delta)$$
where $V_{out,\,peak} = I_{peak}Z_{trap}$
and $Z_{trap} = X_L - X_C$

Solving equation (1) for Z yields:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (2)$$

Because $V_{out} = V_L + V_C$:

$$\begin{aligned} V_{out} &= V_{out,\,peak}\cos(\omega t - \delta) \\ &= I_{peak}Z_{trap}\cos(\omega t - \delta) \\ &= \frac{V_{peak}}{Z}Z_{trap}\cos(\omega t - \delta) \end{aligned}$$

Using equation (2) to substitute for Z yields:

$$V_{out} = \frac{V_{peak}}{\sqrt{R^2 + Z_{trap}^2}}Z_{trap}\cos(\omega t - \delta)$$

Noting that $V_{out} = 0$ provided $Z_{trap} = 0$, set $Z_{trap} = 0$ to obtain:

$$Z_{trap} = X_L - X_C = 0$$

Substituting for X_L and X_C yields:

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \boxed{\frac{1}{\sqrt{LC}}}$$

(*b*) Let the bandwidth $\Delta\omega$ be:

$$\Delta\omega = |\omega - \omega_{trap}| \qquad (3)$$

Let the frequency bandwidth be defined by the frequency at which $\left|Z_{\text{trap}}\right| = R$. Then:

$$\omega L - \frac{1}{\omega C} = R \Rightarrow \omega^2 LC - 1 = \omega RC$$

Because $\omega_{\text{trap}} = \frac{1}{\sqrt{LC}}$:

$$\left(\frac{\omega}{\omega_{\text{trap}}}\right)^2 - 1 = \omega RC$$

For $\omega \approx \omega_{\text{trap}}$:

$$\left(\frac{\omega^2 - \omega_{\text{trap}}^2}{\omega_{\text{trap}}}\right) \approx \omega_{\text{trap}} RC$$

Solve for $\omega^2 - \omega_{\text{trap}}^2$:

$$\omega^2 - \omega_{\text{trap}}^2 = \left(\omega - \omega_{\text{trap}}\right)\left(\omega + \omega_{\text{trap}}\right)$$

Because $\omega \approx \omega_{\text{trap}}$, $\omega + \omega_{\text{trap}} \approx 2\omega_{\text{trap}}$:

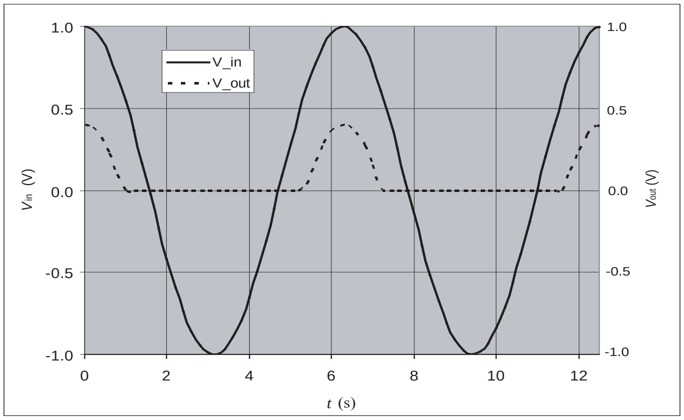
$$\omega^2 - \omega_{\text{trap}}^2 \approx 2\omega_{\text{trap}}\left(\omega - \omega_{\text{trap}}\right)$$

Substitute in equation (3) to obtain:

$$\Delta\omega = \left|\omega - \omega_{\text{trap}}\right| = \frac{RC\omega_{\text{trap}}^2}{2} = \boxed{\frac{R}{2L}}$$

56 ••• A half-wave rectifier for transforming an ac voltage into a dc voltage is shown in Figure 29-38. The diode in the figure can be thought of as a one-way valve for current. It allows current to pass in the forward direction (the direction of the arrowhead) only when V_{in} is at a higher electric potential than V_{out} by 0.60 V (that is, whenever $V_{\text{in}} - V_{\text{out}} \geq +0.60\text{ V}$). The resistance of the diode is effectively infinite when $V_{\text{in}} - V_{\text{out}}$ is less than +0.60 V. Plot two cycles of both input and output voltages as a function of time, on the same graph, assuming the input voltage is given by $V_{\text{in}} = V_{\text{in peak}} \cos \omega t$.

Picture the Problem For voltages greater than 0.60 V, the output voltage will mirror the input voltage minus a 0.60 V drop. But when the voltage swings below 0.60 V, the output voltage will be 0. A spreadsheet program was used to plot the following graph. The angular frequency and the peak voltage were both arbitrarily set equal to one.



57 ••• The output of the rectifier of Problem 56, can be further filtered by putting its output through a low-pass filter as shown in Figure 29-39*a*. The resulting output is a dc voltage with a small ac component (ripple) shown in Figure 29-39*b*. If the input frequency is 60 Hz and the load resistance is 1.00 kΩ, find the value for the capacitance so that the output voltage varies by less than 50 percent of the mean value over one cycle.

Picture the Problem We can use the decay of the potential difference across the capacitor to relate the time constant for the *RC* circuit to the frequency of the input signal. Expanding the exponential factor in the expression for V_C will allow us to find the approximate value for *C* that will limit the variation in the output voltage by less than 50 percent (or any other percentage).

The voltage across the capacitor is given by:

$$V_C = V_{\text{in}} e^{-t/RC}$$

Expand the exponential factor to obtain:

$$e^{-t/RC} \approx 1 - \frac{1}{RC} t$$

For a decay of less than 50 percent:

$$1 - \frac{1}{RC} t \le 0.5 \Rightarrow C \le \frac{2}{R} t$$

Because the voltage goes positive every cycle, $t = 1/60$ s and:

$$C \le \frac{2}{1.00\,\text{k}\Omega}\left(\frac{1}{60}\,\text{s}\right) = \boxed{33\,\mu\text{F}}$$

Driven *LC* Circuits

58 •• The generator voltage in Figure 29-40, is given by

$\mathcal{E} = (100\text{ V})\cos(2\pi f t)$. (*a*) For each branch, what is the peak current and what is the phase of the current relative to the phase of the generator voltage? (*b*) At the resonance frequency there is no current in the generator. What is the angular frequency at resonance? (*c*) At the resonance frequency, find the current in the inductor and what is the current in the capacitor. Express your results as functions of time. (*d*) Draw a phasor diagram showing the phasors for the applied voltage, the generator current, the capacitor current, and the inductor current for the case where frequency is higher than the resonance frequency.

Picture the Problem We know that the current leads the voltage across and capacitor and lags the voltage across an inductor. We can use $I_{L,\,\text{peak}} = \mathcal{E}_{\text{peak}}/X_L$ and $I_{C,\,\text{peak}} = \mathcal{E}_{\text{peak}}/X_C$ to find the amplitudes of these currents. The current in the generator will vanish under resonance conditions, i.e., when $|I_L| = |I_C|$. To find the currents in the inductor and capacitor at resonance, we can use the common potential difference across them and their reactances together with our knowledge of the phase relationships mentioned above.

(*a*) Express the amplitudes of the currents through the inductor and the capacitor:

$$I_{L,\,\text{peak}} = \frac{\mathcal{E}_{\text{peak}}}{X_L} = \frac{\mathcal{E}_{\text{peak}}}{2\pi fL}$$

and

$$I_{C,\,\text{peak}} = \frac{\mathcal{E}_{\text{peak}}}{X_C} = \frac{\mathcal{E}_{\text{peak}}}{\dfrac{1}{2\pi fC}} = 2\pi fC\mathcal{E}_{\text{peak}}$$

Substitute numerical values to obtain:

$$I_{L,\,\text{peak}} = \frac{100\,\text{V}}{(4.00\,\text{H})2\pi f} = \boxed{\frac{25.0\,\text{V/H}}{2\pi f}\text{, lagging } \mathcal{E} \text{ by } 90°}$$

and

$$I_{C,\,\text{peak}} = (25.0\,\mu\text{F})(100\,\text{V})\omega = \boxed{(2.50\,\text{mV}\cdot\text{F})2\pi f\text{, leading } \mathcal{E} \text{ by } 90°}$$

(*b*) Express the condition that $I = 0$:

$$|I_L| = |I_C| \text{ or } \frac{\mathcal{E}}{\omega L} = \frac{\mathcal{E}}{\dfrac{1}{\omega C}} = \omega C\mathcal{E}$$

Solve for ω to obtain:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{1}{\sqrt{(4.00\,\text{H})(25.0\,\mu\text{F})}} = \boxed{100\,\text{rad/s}}$$

(*c*) Express the current in the inductor at $\omega = \omega_0$:

$$I_L = \left(\frac{25.0\,\text{V/H}}{100\,\text{s}^{-1}}\right)\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$= \boxed{(250\,\text{mA})\cos\left(\omega t - \frac{\pi}{2}\right)}$$

where $\omega = 100$ rad/s.

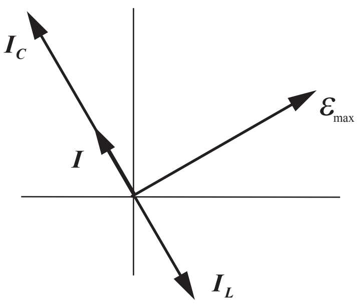
Express the current in the capacitor at $\omega = \omega_0$:

$$I_C = (2.50\,\text{mV}\cdot\text{F})(100\,\text{s}^{-1})\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$= \boxed{-(250\,\text{mA})\cos\left(\omega t + \frac{\pi}{2}\right)}$$

where $\omega = 100$ rad/s.

(*d*) The phasor diagram for the case where the inductive reactance is larger than the capacitive reactance is shown to the right.



59 •• A circuit consist of an ideal ac generator, a capacitor and an ideal inductor, all connected in series. The charge on the capacitor is given by $Q = (15\ \mu\text{C})\cos(\omega t + \frac{\pi}{4})$, where $\omega = 1250$ rad/s. (*a*) Find the current in the circuit as a function of time. (*b*) Find the capacitance if the inductance is 28 mH. (*c*) Write expressions for the electrical energy U_e, the magnetic energy U_m, and the total energy U as functions of time.

Picture the Problem We can differentiate Q with respect to time to find I as a function of time. In (*b*) we can find C by using $\omega = 1/\sqrt{LC}$. The energy stored in the magnetic field of the inductor is given by $U_m = \frac{1}{2}LI^2$ and the energy stored in the electric field of the capacitor by $U_e = \frac{1}{2}\frac{Q^2}{C}$.

(*a*) Differentiate the charge with respect to time to obtain the current:

$$I(t)=\frac{dQ}{dt}=\frac{d}{dt}\left[(15\,\mu\text{C})\cos\left(\omega t+\frac{\pi}{4}\right)\right]=-(15\,\mu\text{C})(1250\,\text{s}^{-1})\sin\left(\omega t+\frac{\pi}{4}\right)$$

$$=-(18.75\,\text{mA})\sin\left(\omega t+\frac{\pi}{4}\right)=\boxed{-(19\,\text{mA})\sin\left(\omega t+\frac{\pi}{4}\right)}$$

where ω = 1250 rad/s

(*b*) Relate C to L and ω:

$$\omega=\frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C=\frac{1}{\omega^2 L}$$

Substitute numerical values and evaluate C:

$$C=\frac{1}{(1250\,\text{s}^{-1})^2(28\,\text{mH})}=22.86\,\mu\text{F}$$

$$=\boxed{23\,\mu\text{F}}$$

(*c*) Express and evaluate the magnetic energy $U_{\text{m}}(t)$:

$$U_{\text{m}}(t)=\tfrac{1}{2}LI^2=\tfrac{1}{2}(28\,\text{mH})(18.75\,\text{mA})^2\sin^2\left(\omega t+\frac{\pi}{4}\right)=\boxed{(4.9\,\mu\text{J})\sin^2\left(\omega t+\frac{\pi}{4}\right)}$$

where ω = 1250 rad/s

Use $U_{\text{e}}=\tfrac{1}{2}\frac{Q^2}{C}$ to find the electrical energy stored in the capacitor as a function of time:

$$U_{\text{e}}(t)=\tfrac{1}{2}\frac{(15\,\mu\text{F})^2}{22.86\,\mu\text{F}}\cos^2\left(\omega t+\frac{\pi}{4}\right)$$

$$=(4.92\,\mu\text{J})\cos^2\left(\omega t+\frac{\pi}{4}\right)$$

$$=\boxed{(4.9\,\mu\text{J})\cos^2\left(\omega t+\frac{\pi}{4}\right)}$$

where ω = 1250 rad/s

The total energy stored in the electric and magnetic fields is the sum of $U_{\text{m}}(t)$ and $U_{\text{e}}(t)$:

$$U=(4.92\,\mu\text{J})\sin^2\left(\omega t+\frac{\pi}{4}\right)+(4.92\,\mu\text{J})\cos^2\left(\omega t+\frac{\pi}{4}\right)=\boxed{4.9\,\mu\text{J}}$$

where ω = 1250 rad/s.

60 ••• One method for determining the compressibility of a dielectric material uses a driven *LC* circuit that has a parallel-plate capacitor. The dielectric is inserted between the plates and the change in resonance frequency is determined as the capacitor plates are subjected to a compressive stress. In one such arrangement, the resonance frequency is 120 MHz when a dielectric of thickness 0.100 cm and dielectric constant $\kappa = 6.80$ is placed between the plates. Under a compressive stress of 800 atm, the resonance frequency decreases to 116 MHz. Find the Young's modulus of the dielectric material. (Assume that the dielectric constant does not change with pressure.)

Picture the Problem We can use the definition of the capacitance of a dielectric-filled capacitor and the expression for the resonance frequency of an *LC* circuit to derive an expression for the fractional change in the thickness of the dielectric in terms of the resonance frequency and the frequency of the circuit when the dielectric is under compression. We can then use this expression for $\Delta t/t$ to calculate the value of Young's modulus for the dielectric material.

Use its definition to express Young's modulus of the dielectric material:

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\Delta P}{\Delta t/t} \qquad (1)$$

Letting t be the initial thickness of the dielectric, express the initial capacitance of the capacitor:

$$C_0 = \frac{\kappa \in_0 A}{t}$$

Express the capacitance of the capacitor when it is under compression:

$$C_c = \frac{\kappa \in_0 A}{t - \Delta t}$$

Express the resonance frequency of the capacitor before the dielectric is compressed:

$$\omega_0 = \frac{1}{\sqrt{C_0 L}} = \frac{1}{\sqrt{\dfrac{\kappa \in_0 AL}{t}}}$$

When the dielectric is compressed:

$$\omega_c = \frac{1}{\sqrt{C_c L}} = \frac{1}{\sqrt{\dfrac{\kappa \in_0 AL}{t - \Delta t}}}$$

Express the ratio of ω_c to ω_0 and simplify to obtain:

$$\frac{\omega_c}{\omega_0} = \frac{\sqrt{\dfrac{\kappa \in_0 AL}{t}}}{\sqrt{\dfrac{\kappa \in_0 AL}{t - \Delta t}}} = \sqrt{1 - \frac{\Delta t}{t}}$$

Expand the radical binomially to obtain:

$$\frac{\omega_c}{\omega_0} = \left(1 - \frac{\Delta t}{t}\right)^{1/2} \approx 1 - \frac{\Delta t}{2t}$$

provided $\Delta t << t$.

Solve for $\Delta t/t$:

$$\frac{\Delta t}{t} = 2\left(1 - \frac{\omega_c}{\omega_0}\right)$$

Substitute in equation (1) to obtain:

$$Y = \frac{\Delta P}{2\left(1 - \frac{\omega_c}{\omega_0}\right)}$$

Substitute numerical values and evaluate Y:

$$Y = \frac{(800\,\text{atm})(101.325\,\text{kPa/atm})}{2\left(1 - \frac{116\,\text{MHz}}{120\,\text{MHz}}\right)} = \boxed{1.22 \times 10^9\ \text{N/m}^2}$$

61 ••• Figure 29-41 shows an inductor in series with a parallel plate capacitor. The capacitor has a width w of 20 cm and a gap of 2.0 mm. A dielectric that has a dielectric constant of 4.8 can be slid in and out of the gap. The inductor has an inductance of 2.0 mH. When half the dielectric is between the capacitor plates (when $x = \frac{1}{2}w$), the resonant frequency of this combination is 90 MHz.
(a) What is the capacitance of the capacitor without the dielectric?
(b) Find the resonance frequency as a function of x for $0 \le x \le w$.

Picture the Problem We can model this capacitor as the equivalent of two capacitors connected in parallel. Let C_1 be the capacitance of the dielectric-filled capacitor and C_2 be the capacitance of the air-filled capacitor. We'll derive expressions for the capacitances of the parallel capacitors and add these expressions to obtain $C(x)$. We can then use the given resonance frequency when $x = w/2$ and the given value for L to evaluate C_0. In Part (b) we can use our result for $C(x)$ and the relationship between f, L, and $C(x)$ at resonance to express $f(x)$.

(a) Express the equivalent capacitance of the two capacitors in parallel:

$$C(x) = C_1 + C_2 = \frac{\kappa \in_0 A_1}{d} + \frac{\in_0 A_2}{d} \quad (1)$$

Express A_2 in terms of the total area of a capacitor plate A, w, and the distance x:

$$\frac{A_2}{A} = \frac{x}{w} \Rightarrow A_2 = A\frac{x}{w}$$

Express A_1 in terms of A and A_2:

$$A_1 = A - A_2 = A\left(1 - \frac{x}{w}\right)$$

Substitute in equation (1) and simplify to obtain:

$$C(x) = \frac{\kappa \in_0 A}{d}\left(1 - \frac{x}{w}\right) + \frac{\in_0 A}{d}\frac{x}{w}$$
$$= \frac{\in_0 A}{d}\left[\kappa\left(1 - \frac{x}{w}\right) + \frac{x}{w}\right]$$
$$= \kappa C_0\left[1 - \frac{\kappa - 1}{\kappa w}x\right]$$

where $C_0 = \dfrac{\in_0 A}{d}$

Find $C(w/2)$:

$$C\left(\frac{w}{2}\right) = \kappa C_0\left[1 - \frac{\kappa - 1}{\kappa w}\frac{w}{2}\right]$$
$$= \kappa C_0\left[1 - \frac{\kappa - 1}{2\kappa}\right]$$
$$= C_0\frac{\kappa + 1}{2}$$

Express the resonance frequency of the circuit in terms of L and $C(x)$:

$$f(x) = \frac{1}{2\pi\sqrt{LC(x)}} \qquad (2)$$

Evaluate $f(w/2)$:

$$f\left(\frac{w}{2}\right) = \frac{1}{2\pi\sqrt{LC_0\dfrac{\kappa + 1}{2}}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{2}{(\kappa + 1)LC_0}}$$

Solve for C_0 to obtain:

$$C_0 = \frac{1}{2\pi^2 f^2\left(\dfrac{w}{2}\right)L(\kappa + 1)}$$

Substitute numerical values and evaluate C_0:

$$C_0 = \frac{1}{2\pi^2(90\ \text{MHz})^2\left(\dfrac{20\ \text{cm}}{2}\right)(2.0\ \text{mH})(4.8 + 1)} = \boxed{5.4\ \text{fF}}$$

(*b*) Substitute for *C*(*x*) in equation (2) to obtain:

$$f(x)=\frac{1}{2\pi\sqrt{L\kappa C_0\left[1-\frac{\kappa-1}{\kappa w}x\right]}}$$

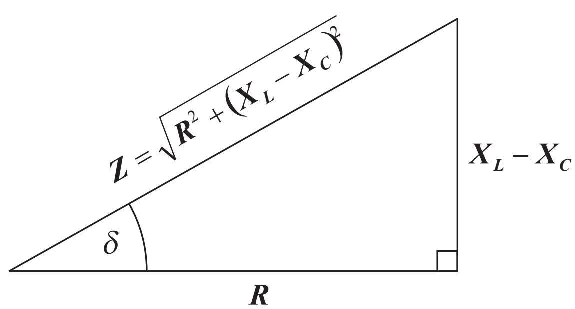
Substitute numerical values and evaluate *f*(*x*):

$$\boldsymbol{f(x)}=\frac{1}{2\boldsymbol{\pi}\sqrt{(2.0\,\text{mH})(4.8)(5.39\times10^{-16}\,\text{F})\left[1-\frac{4.8-1}{4.8(0.20\,\text{m})}\boldsymbol{x}\right]}}=\boxed{\frac{70\,\text{MHz}}{\sqrt{1-(4.0\,\text{m}^{-1})\boldsymbol{x}}}}$$

Driven *RLC* Circuits

62 • A circuit consists of an ideal ac generator, a 20-μF capacitor and an 80-Ω resistor, all connected in series. The output of the generator has a peak emf of 20-V, and the armature of the generator rotates at 400 rad/s. Find (*a*) the power factor, (*b*) the rms current, and (*c*) the average power supplied by the generator.

Picture the Problem The diagram shows the relationship between δ, X_L, X_C, and R. We can use this reference triangle to express the power factor for the given circuit. In (*b*) we can find the rms current from the rms potential difference and the impedance of the circuit. We can find the average power delivered by the source from the rms current and the resistance of the resistor.



(*a*) The power factor is defined to be:

$$\cos\delta=\frac{R}{Z}=\frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

With no inductance in the circuit:

$$X_L=0$$

and

$$\cos\delta=\frac{R}{\sqrt{R^2+X_C^2}}=\frac{R}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{80\,\Omega}{\sqrt{(80\,\Omega)^2 + \dfrac{1}{(400\,\text{s}^{-1})^2(20\,\mu\text{F})^2}}} = \boxed{0.54}$$

(*b*) Express the rms current in the circuit:

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\dfrac{\mathcal{E}_{\max}}{\sqrt{2}}}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_{\max}}{\sqrt{2}\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$$

Substitute numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{20\,\text{V}}{\sqrt{2}\sqrt{(80\,\Omega)^2 + \dfrac{1}{(400\,\text{s}^{-1})^2(20\,\mu\text{F})^2}}} = 95.3\,\text{mA} = \boxed{95\,\text{mA}}$$

(*c*) The average power delivered by the generator is given by:

$$P_{\text{av}} = I_{\text{rms}}^2 R$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = (95.3\,\text{mA})^2(80\,\Omega) = \boxed{0.73\,\text{W}}$$

63 •• [SSM] Show that the expression $P_{\text{av}} = R\mathcal{E}_{\text{rms}}^2/Z^2$ gives the correct result for a circuit containing only an ideal ac generator and (*a*) a resistor, (*b*) a capacitor, and (*c*) an inductor. In the expression $P_{\text{av}} = R\mathcal{E}_{\text{rms}}^2/Z^2$, P_{av} is the average power supplied by the generator, $\mathcal{E}_{\text{rms}}$ is the root-mean-square of the emf of the generator, R is the resistance, C is the capacitance and L is the inductance. (In Part (*a*), $C = L = 0$, in Part (*b*), $R = L = 0$ and in Part (*c*), $R = C = 0$.

Picture the Problem The impedance of an ac circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. We can evaluate the given expression for P_{av} first for $X_L = X_C = 0$ and then for $R = 0$.

(*a*) For $X = 0$, $Z = R$ and:

$$P_{\text{av}} = \frac{R\mathcal{E}_{\text{rms}}^2}{Z^2} = \frac{R\mathcal{E}_{\text{rms}}^2}{R^2} = \boxed{\frac{\mathcal{E}_{\text{rms}}^2}{R}}$$

(*b*) and (*c*) If $R = 0$, then:

$$P_{av} = \frac{R\mathcal{E}_{rms}^2}{Z^2} = \frac{(0)\mathcal{E}_{rms}^2}{(X_L - X_C)^2} = \boxed{0}$$

Remarks: Recall that there is no energy dissipation in an ideal inductor or capacitor.

64 •• A series *RLC* circuit that has an inductance of 10 mH, a capacitance of 2.0 μF, and a resistance of 5.0 Ω is driven by an ideal ac voltage source that has a peak emf of 100 V. Find (*a*) the resonant frequency and (*b*) the root-mean-square current at resonance. When the frequency is 8000 rad/s, find (*c*) the capacitive and inductive reactances, (*d*) the impedance, (*e*) the root-mean-square current, and (*f*) the phase angle.

Picture the Problem We can use $\omega_0 = 1/\sqrt{LC}$ to find the resonant frequency of the circuit, $I_{rms} = \mathcal{E}_{rms}/R$ to find the rms current at resonance, the definitions of X_C and X_L to find these reactances at ω = 8000 rad/s, the definitions of Z and I_{rms} to find the impedance and rms current at ω = 8000 rad/s, and the definition of the phase angle to find δ.

(*a*) Express the resonant frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{1}{\sqrt{(10\,\text{mH})(2.0\,\mu\text{F})}} = \boxed{7.1\times10^3\ \text{rad/s}}$$

(*b*) Relate the rms current at resonance to $\mathcal{E}_{rms}$ and the impedance of the circuit at resonance:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{\mathcal{E}_{max}}{\sqrt{2}R} = \frac{100\,\text{V}}{\sqrt{2}(5.0\,\Omega)} = \boxed{14\,\text{A}}$$

(*c*) Express and evaluate X_C and X_L at ω = 8000 rad/s:

$$X_C = \frac{1}{\omega C} = \frac{1}{(8000\,\text{s}^{-1})(2.0\,\mu\text{F})} = 62.50\,\Omega = \boxed{63\,\Omega}$$

and

$$X_L = \omega L = (8000\,\text{s}^{-1})(10\,\text{mH}) = \boxed{80\,\Omega}$$

(*d*) Express the impedance in terms of the reactances, substitute the results from (*c*), and evaluate *Z*:

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$$
$$=\sqrt{(5.0\,\Omega)^2+(80\,\Omega-62.5\,\Omega)^2}$$
$$=18.2\,\Omega=\boxed{18\,\Omega}$$

(*e*) Relate the rms current at $\omega = 8000$ rad/s to $\mathcal{E}_{\text{rms}}$ and the impedance of the circuit at this frequency:

$$I_{\text{rms}}=\frac{\mathcal{E}_{\text{rms}}}{Z}=\frac{\mathcal{E}_{\max}}{\sqrt{2}Z}=\frac{100\,\text{V}}{\sqrt{2}(18.2\,\Omega)}$$
$$=\boxed{3.9\,\text{A}}$$

(*f*) δ is given by:

$$\delta=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

Substitute numerical values and evaluate δ:

$$\delta=\tan^{-1}\left(\frac{80\,\Omega-62.5\,\Omega}{5.0\,\Omega}\right)=\boxed{74^\circ}$$

65 •• [SSM] Find (*a*) the *Q* factor and (*b*) the resonance width (in hertz) for the circuit in Problem 64. (*c*) What is the power factor when $\omega = 8000$ rad/s?

Picture the Problem The *Q* factor of the circuit is given by $Q=\omega_0L/R$, the resonance width by $\Delta f = f_0/Q=\omega_0/2\pi Q$, and the power factor by $\cos\delta=R/Z$. Because *Z* is frequency dependent, we'll need to find X_C and X_L at $\omega = 8000$ rad/s in order to evaluate $\cos\delta$.

Using their definitions, express the *Q* factor and the resonance width of the circuit:

$$Q=\frac{\omega_0L}{R} \qquad (1)$$

and

$$\Delta f=\frac{f_0}{Q}=\frac{\omega_0}{2\pi Q} \qquad (2)$$

(*a*) Express the resonance frequency for the circuit:

$$\omega_0=\frac{1}{\sqrt{LC}}$$

Substituting for ω_0 in equation (1) yields:

$$Q=\frac{L}{\sqrt{LC}R}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

Substitute numerical values and evaluate *Q*:

$$Q=\frac{1}{5.0\,\Omega}\sqrt{\frac{10\,\text{mH}}{2.0\,\mu\text{F}}}=14.1=\boxed{14}$$

(*b*) Substitute numerical values in equation (2) and evaluate Δf:

$$\Delta f = \frac{7.07\times10^{3}\ \text{rad/s}}{2\pi(14.1)} = \boxed{80\,\text{Hz}}$$

(*c*) The power factor of the circuit is given by:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{5.0\,\Omega}{\sqrt{(5.0\,\Omega)^2 + \left((8000\,\text{s}^{-1})(10\,\text{mH}) - \dfrac{1}{(8000\,\text{s}^{-1})(2.0\,\mu\text{F})}\right)^2}} = \boxed{0.27}$$

66 •• FM radio stations typically operate at frequencies separated by 0.20 MHz. Thus, when your radio is tuned to a station operating at a frequency of 100.1 MHz, the resonance width of the receiver circuit should be much smaller than 0.20 MHz, so that you do not receive a signal from stations operating at adjacent frequencies. Assume your receiving circuit has a resonance width of 0.050 MHz. When tuned in to this particular station, what is the *Q* factor of your circuit?

Picture the Problem We can use its definition, $Q = f_0/\Delta f$ to find the *Q* factor for the circuit.

The *Q* factor for the circuit is given by:

$$Q = \frac{f_0}{\Delta f}$$

Substitute numerical values and evaluate *Q*:

$$Q = \frac{100.1\,\text{MHz}}{0.050\,\text{MHz}} \approx \boxed{2.0\times10^{3}}$$

67 •• A coil is connected to a 60-Hz ac generator with a peak emf equal to 100 V. At this frequency, the coil has an impedance of 10 Ω and a reactance of 8.0 Ω. (*a*) What is the peak current in the coil? (*b*) What is the phase angle between the current and the applied voltage? (*c*) A capacitor is put in series with the coil and the generator. What capacitance is required so that the current is in phase with the generator emf? (*d*) What is the peak voltage measured across this capacitor?

Picture the Problem We can use $I_{\text{peak}} = \mathcal{E}_{\text{peak}}/Z$ to find the current in the coil and the definition of the phase angle to evaluate δ. We can equate X_L and X_C to find the capacitance required so that the current and the voltage are in phase. Finally, we can find the voltage measured across the capacitor by using $V_C = IX_C$.

(*a*) Express the current in the coil in terms of the potential difference across it and its impedance:

$$I_{\text{peak}} = \frac{\mathcal{E}_{\text{peak}}}{Z}$$

Substitute numerical values and evaluate I_{peak}:

$$I_{\text{peak}} = \frac{100\,\text{V}}{10\,\Omega} = \boxed{10\,\text{A}}$$

(*b*) The phase angle δ is given by:

$$\delta = \cos^{-1}\left(\frac{R}{Z}\right) = \sin^{-1}\left(\frac{X_L}{Z}\right)$$

Substitute numerical values and evaluate δ:

$$\delta = \sin^{-1}\left(\frac{8.0\,\Omega}{10\,\Omega}\right) = \boxed{53^\circ}$$

(*c*) Express the condition on the reactances that must be satisfied if the current and voltage are to be in phase:

$$X_L = X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_L} = \frac{1}{2\pi f X_L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(8.0\,\Omega\right)} = 332\,\mu\text{F}$$
$$= \boxed{0.33\,\text{mF}}$$

(*d*) Express the potential difference across the capacitor:

$$V_C = I_{\text{peak}} X_C$$

Relate the peak current in the circuit to the impedance of the circuit when $X_L = X_C$:

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R}$$

Substitute for I to obtain:

$$V_C = \frac{V_{\text{peak}} X_C}{R} = \frac{V_{\text{peak}}}{2\pi f C R}$$

Relate the impedance of the circuit to the resistance of the coil:

$$Z = \sqrt{R^2 + X^2} \Rightarrow R = \sqrt{Z^2 - X^2}$$

Substituting for R yields:

$$V_C = \frac{V_{\text{peak}}}{2\pi fC\sqrt{Z^2 - X^2}}$$

Substitute numerical values and evaluate V_C:

$$V_C = \frac{100\text{ V}}{2\pi\left(60\text{ s}^{-1}\right)(332\,\mu\text{F})\sqrt{(10\,\Omega)^2 - (8.0\,\Omega)^2}} = \boxed{0.13\text{ kV}}$$

68 •• An ideal 0.25-H inductor and a capacitor are connected in series with an ideal 60-Hz generator. An digital voltmeter is used to measure the rms voltages across the inductor and capacitor independently. The voltmeter reading across the capacitor is 75 V and that across the inductor is 50 V. (*a*) Find the capacitance and the rms current in the circuit. (*b*) What is the rms voltage across the series combination of the capacitor and the inductor?

Picture the Problem We can find C using $V_C = I_{\text{rms}}X_C$ and I_{rms} from the potential difference across the inductor. In the absence of resistance in the circuit, the measured rms voltage across both the capacitor and inductor is $V = \left|V_L - V_C\right|$.

(*a*) Relate the capacitance C to the potential difference across the capacitor:

$$V_C = I_{\text{rms}}X_C = \frac{I_{\text{rms}}}{2\pi fC} \Rightarrow C = \frac{I_{\text{rms}}}{2\pi fV_C}$$

Use the potential difference across the inductor to express the rms current in the circuit:

$$I_{\text{rms}} = \frac{V_L}{X_L} = \frac{V_L}{2\pi fL}$$

Substitute for I_{rms} to obtain:

$$C = \frac{V_L}{(2\pi f)^2 LV_C}$$

Substitute numerical values and evaluate C:

$$C = \frac{50\text{ V}}{\left[2\pi\left(60\text{ s}^{-1}\right)\right]^2(0.25\text{ H})(75\text{ V})} = \boxed{19\,\mu\text{F}}$$

(*b*) Express the measured rms voltage V across both the capacitor and the inductor when $R = 0$:

$$V = \left|V_L - V_C\right|$$

Substitute numerical values and evaluate V: $V=\left|50\,\text{V}-75\,\text{V}\right|=\boxed{25\,\text{V}}$

69 •• **[SSM]** In the circuit shown in Figure 29-42 the ideal generator produces an rms voltage of 115 V when operated at 60 Hz. What is the rms voltage between points (*a*) *A* and *B*, (*b*) *B* and *C*, (*c*) *C* and *D*, (*d*) *A* and *C*, and (*e*) *B* and *D*?

Picture the Problem We can find the rms current in the circuit and then use it to find the potential differences across each of the circuit elements. We can use phasor diagrams and our knowledge of the phase shifts between the voltages across the three circuit elements to find the voltage differences across their combinations.

(*a*) Express the potential difference between points *A* and *B* in terms of I_{rms} and X_L:

$$V_{AB}=I_{\text{rms}}X_L \qquad (1)$$

Express I_{rms} in terms of $\mathcal{E}$ and Z:

$$I_{\text{rms}}=\frac{\mathcal{E}}{Z}=\frac{\mathcal{E}}{\sqrt{R^2+\left(X_L-X_C\right)^2}}$$

Evaluate X_L and X_C to obtain:

$$X_L=2\pi fL=2\pi\left(60\,\text{s}^{-1}\right)\left(137\,\text{mH}\right)=51.648\,\Omega$$

and

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(25\,\mu\text{F}\right)}=106.10\,\Omega$$

Substitute numerical values and evaluate I_{rms}:

$$I_{\text{rms}}=\frac{115\,\text{V}}{\sqrt{\left(50\,\Omega\right)^2+\left(51.648\,\Omega-106.10\,\Omega\right)^2}}=1.5556\,\text{A}$$

Substitute numerical values in equation (1) and evaluate V_{AB}:

$$V_{AB}=\left(1.5556\,\text{A}\right)\left(51.648\,\Omega\right)=80.344\,\text{V}=\boxed{80\,\text{V}}$$

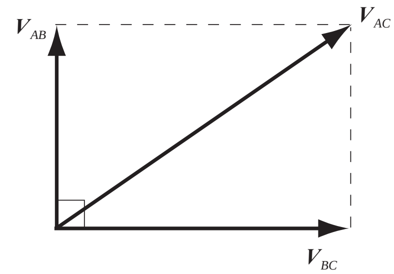
(*b*) Express the potential difference between points *B* and *C* in terms of I_{rms} and R:

$$V_{BC}=I_{\text{rms}}R=\left(1.5556\,\text{A}\right)\left(50\,\Omega\right)=77.780\,\text{V}=\boxed{78\,\text{V}}$$

(*c*) Express the potential difference between points *C* and *D* in terms of I_{rms} and X_C:

$$V_{CD} = I_{\text{rms}} X_C = (1.5556\,\text{A})(106.10\,\Omega)$$
$$= 165.05\,\text{V} = \boxed{0.17\,\text{kV}}$$

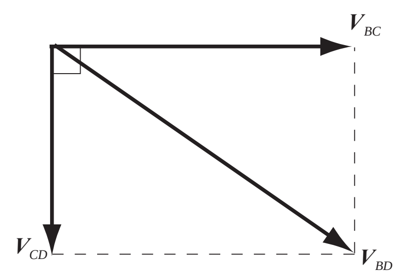
(*d*) The voltage across the inductor leads the voltage across the resistor as shown in the phasor diagram to the right:



Use the Pythagorean theorem to find V_{AC}:

$$V_{AC} = \sqrt{V_{AB}^2 + V_{BC}^2}$$
$$= \sqrt{(80.0\,\text{V})^2 + (77.780\,\text{V})^2}$$
$$= 111.58\,\text{V} = \boxed{0.11\,\text{kV}}$$

(*e*) The voltage across the capacitor lags the voltage across the resistor as shown in the phasor diagram to the right:



Use the Pythagorean theorem to find V_{BD}:

$$V_{BD} = \sqrt{V_{CD}^2 + V_{BC}^2}$$
$$= \sqrt{(165.05\,\text{V})^2 + (77.780\,\text{V})^2}$$
$$= 182.46\,\text{V} = \boxed{0.18\,\text{kV}}$$

70 •• When an *RLC* series circuit is connected to a 120-V rms, 60-Hz line, the rms current in the circuit is 11 A and this current leads the line voltage by 45º. (*a*) Find the average power supplied to the circuit. (*b*) What is the resistance in the circuit? (*c*) If the inductance in the circuit is 50 mH, find the capacitance in the circuit. (*d*) Without changing the inductance, by how much should you change the capacitance to make the power factor equal to 1? (*e*) Without changing the capacitance, by how much should you change the inductance to make the power factor equal to 1?

Picture the Problem We can use $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta$ to find the power supplied to the circuit and $P_{\text{av}} = I_{\text{rms}}^2 R$ to find the resistance. In Part (*c*) we can relate the capacitive reactance to the impedance, inductive reactance, and resistance of the circuit and solve for the capacitance *C*. We can use the condition on X_L and X_C at resonance to find the capacitance or inductance you would need to add to the circuit to make the power factor equal to 1.

(*a*) Express the power supplied to the circuit in terms of $\mathcal{E}_{rms}$, I_{rms}, and the power factor $\cos\delta$:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = (120\,\text{V})(11\,\text{A})\cos 45° = 933\,\text{W} = \boxed{0.93\,\text{kW}}$$

(*b*) Relate the power dissipated in the circuit to the resistance of the resistor:

$$P_{av} = I_{rms}^2 R \Rightarrow R = \frac{P_{av}}{I_{rms}^2}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{933\,\text{W}}{(11\,\text{A})^2} = 7.71\,\Omega = \boxed{7.7\,\Omega}$$

(*c*) Express the capacitance of the capacitor in terms of its reactance:

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} \quad (1)$$

Relate the capacitive reactance to the impedance, inductive reactance, and resistance of the circuit:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Express the impedance of the circuit in terms of the rms emf $\mathcal{E}$ and the rms current I_{rms}:

$$Z^2 = \frac{\mathcal{E}^2}{I_{rms}^2}$$

Equating these two expressions yields:

$$\frac{\mathcal{E}^2}{I_{rms}^2} = R^2 + (X_L - X_C)^2$$

Solve for $|X_L - X_C|$:

$$|X_L - X_C| = \sqrt{\frac{\mathcal{E}^2}{I_{rms}^2} - R^2}$$

Note that because *I* leads $\mathcal{E}$, the circuit is capacitive and $X_C > X_L$. Hence:

$$|X_L - X_C| = -(X_L - X_C)$$

and

$$X_C = X_L + \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2}$$
$$= 2\pi fL + \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2}$$

Substitute numerical values and evaluate X_C:

$$X_C = 2\pi\left(60\,\text{s}^{-1}\right)\left(50\,\text{mH}\right) + \sqrt{\frac{\left(120\,\text{V}\right)^2}{\left(11\,\text{A}\right)^2} - \left(7.71\,\Omega\right)^2}$$
$$= 18.8\,\Omega + 7.7\,\Omega = 26.6\,\Omega$$

Substitute in equation (1) and evaluate C:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(26.6\,\Omega\right)} = 99.9\,\mu\text{F}$$
$$= \boxed{0.10\,\text{mF}}$$

(*d*) Let $C_{\text{pf}=1}$ represent the capacitance required to make $\cos\delta = 1$. The necessary change in capacitance is given by:

$$\Delta C = C_{\text{pf}=1} - C = C_{\text{pf}=1} - 99.9\,\mu\text{F}$$

Relate $C_{\text{pf}=1}$ to X_L:

$$X_L = X_C = \frac{1}{2\pi fC_{\text{pf}=1}}$$

Solving for $C_{\text{pf}=1}$ yields:

$$C_{\text{pf}=1} = \frac{1}{2\pi fX_L}$$

Substitute for $C_{\text{pf}=1}$ in the expression for ΔC to obtain:

$$\Delta C = \frac{1}{2\pi fX_L} - 99.9\,\mu\text{F}$$

Substitute numerical values and evaluate ΔC:

$$\Delta C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(18.8\,\Omega\right)} - 99.9\,\mu\text{F}$$
$$= \boxed{41\,\mu\text{F}}$$

(*e*) Let $L_{\text{pf}=1}$ represent the inductance required to make $\cos\delta = 1$. The necessary change in inductance is given by:

$$\Delta L = L_{\text{pf}=1} - L = L_{\text{pf}=1} - 50\,\text{mH}$$

Relate $L_{\mathrm{pf}=1}$ to X_C: $$X_L = X_C = 2\pi f L_{\mathrm{pf}=1}$$

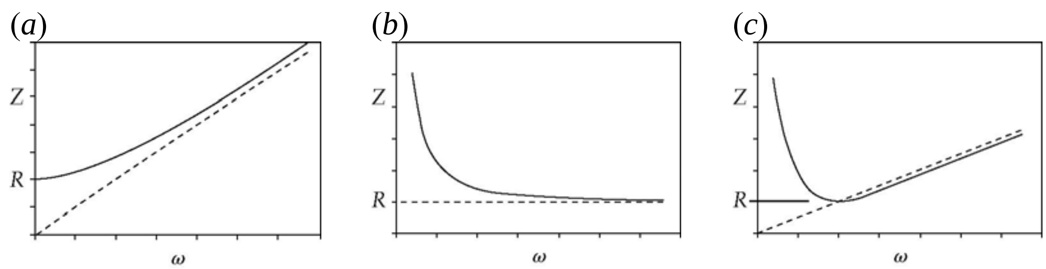
Solving for $L_{\mathrm{pf}=1}$ yields: $$L_{\mathrm{pf}=1} = \frac{X_C}{2\pi f}$$

Substitute for $L_{\mathrm{pf}=1}$ in the expression for ΔL to obtain: $$\Delta L = \frac{X_C}{2\pi f} - 50\ \mathrm{mH}$$

Substitute numerical values and evaluate ΔL: $$\Delta L = \frac{26.6\ \Omega}{2\pi\left(60\ \mathrm{s}^{-1}\right)} - 50\ \mathrm{mH} = \boxed{20\ \mathrm{mH}}$$

71 •• Plot the circuit impedance versus the angular frequency for each of the following circuits. (*a*) A driven series *LR* circuit, (*b*) a driven series *RC* circuit, and (*c*) a driven series *RLC* circuit.

Picture the Problem The impedance for the three circuits as functions of the angular frequency is shown in the three figures below. Also shown in each figure (dashed line) is the asymptotic approach for large angular frequencies.



72 •• In a driven series *RLC* circuit, the ideal generator has a peak emf equal to 200 V, the resistance is 60.0 Ω and the capacitance is 8.00 μF. The inductance can be varied from 8.00 mH to 40.0 mH by the insertion of an iron core in the solenoid. The angular frequency of the generator is 2500 rad/s. If the capacitor voltage is not to exceed 150 V, find (*a*) the peak current and (*b*) the range of inductances that is safe to use.

Picture the Problem We can find the maximum current in the circuit from the maximum voltage across the capacitor and the reactance of the capacitor. To find the range of inductance that is safe to use we can express Z^2 for the circuit in terms of $\mathcal{E}^2_{\mathrm{peak}}$ and I^2_{peak} and solve the resulting quadratic equation for L.

(*a*) Express the peak current in terms of the maximum potential difference across the capacitor and its $$I_{\mathrm{peak}} = \frac{V_{C,\mathrm{peak}}}{X_C} = \omega C V_{C,\mathrm{peak}}$$

reactance:

Substitute numerical values and evaluate I_{peak}:

$$I_{\text{peak}} = (2500\,\text{rad/s})(8.00\,\mu\text{F})(150\,\text{V}) = \boxed{3.00\,\text{A}}$$

(*b*) Relate the maximum current in the circuit to the emf of the source and the impedance of the circuit:

$$I_{\text{peak}} = \frac{\mathcal{E}_{\text{peak}}}{Z} \Rightarrow Z^2 = \frac{\mathcal{E}_{\text{peak}}^2}{I_{\text{peak}}^2}$$

Express Z^2 in terms of R, X_L, and X_C:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Substitute to obtain:

$$\frac{\mathcal{E}_{\text{peak}}^2}{I_{\text{peak}}^2} = R^2 + (X_L - X_C)^2$$

Evaluating X_C yields:

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500\,\text{rad/s})(8.00\,\mu\text{F})} = 50.0\,\Omega$$

Substitute numerical values to obtain:

$$\frac{(200\,\text{V})^2}{(3.00\,\text{A})^2} = (60.0\,\Omega)^2 + ((2500\,\text{rad/s})L - 50.0\,\Omega)^2$$

Solving for L yields:

$$L = \frac{50.0\,\Omega \pm \sqrt{844\,\Omega^2}}{2500\,\text{s}^{-1}}$$

Denoting the solutions as L_+ and L_-, find the values for the inductance:

$$L_+ = 31.6\,\text{mH and } L_- = 8.38\,\text{mH}$$

The ranges for L are:

$$\boxed{8.00\,\text{mH} < L < 8.38\,\text{mH}}$$

and

$$\boxed{31.6\,\text{mH} < L < 40.0\,\text{mH}}$$

73 •• A certain electrical device draws an rms current of 10 A at an average power of 720 W when connected to a 120-V rms, 60-Hz power line.
(*a*) What is the impedance of the device? (*b*) What series combination of resistance and reactance would have the same impedance as this device? (*c*) If the current leads the emf, is the reactance inductive or capacitive?

Picture the Problem We can find the impedance of the circuit from the applied emf and the current drawn by the device. In Part (*b*) we can use $P_{av} = I_{rms}^2 R$ to find R and the definition of the impedance of a series *RLC* circuit to find $X = X_L - X_C$.

(*a*) Express the impedance of the device in terms of the current it draws and the emf provided by the power line:

$$Z = \frac{\mathcal{E}_{rms}}{I_{rms}}$$

Substitute numerical values and evaluate Z:

$$Z = \frac{120\,\text{V}}{10\,\text{A}} = \boxed{12\,\Omega}$$

(*b*) Use the relationship between the average power supplied to the device and the rms current it draws to express R:

$$P_{av} = I_{rms}^2 R \Rightarrow R = \frac{P_{av}}{I_{rms}^2}$$

Substitute numerical values and evaluate R:

$$R = \frac{720\,\text{W}}{(10\,\text{A})^2} = 7.20\,\Omega = \boxed{7.2\,\Omega}$$

The impedance of a series *RLC* circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

or

$$Z^2 = R^2 + (X_L - X_C)^2$$

Solving for $X_L - X_C$ yields:

$$X = X_L - X_C = \sqrt{Z^2 - R^2}$$

Substitute numerical values and evaluate X:

$$X = \sqrt{(12\,\Omega)^2 - (7.20\,\Omega)^2} = \boxed{10\,\Omega}$$

(*c*) If the current leads the emf, the reactance is capacitive.

74 •• A method for measuring inductance is to connect the inductor in series with a known capacitance, a known resistance, an ac ammeter, and a variable-frequency signal generator. The frequency of the signal generator is varied and the emf is kept constant until the current is maximum. (*a*) If the capacitance is 10 μF, the peak emf is 10 V, the resistance is 100 Ω, and the rms current in the circuit is maximum when the driving frequency is 5000 rad/s, what is the value of the inductance? (*b*) What is the maximum rms current?

Picture the Problem (*a*) We can use the fact that when the current is a maximum,

$X_L = X_C$, to find the inductance of the circuit. In Part (*b*), we can find $I_{\text{rms, max}}$ from $\mathcal{E}_{\text{peak}}$ and the impedance of the circuit at resonance.

(*a*) Relate X_L and X_C at resonance:

$$X_L = X_C \text{ or } \omega_0 L = \frac{1}{\omega_0 C}$$

Solve for L to obtain:

$$L = \frac{1}{\omega_0^2 C}$$

Substitute numerical values and evaluate L:

$$L = \frac{1}{\left(5000\,\text{s}^{-1}\right)^2 (10\,\mu\text{F})} = \boxed{4.0\,\text{mH}}$$

(*b*) Noting that, at resonance, $X = 0$, express $I_{\text{rms, max}}$ in terms of the applied emf and the impedance of the circuit at resonance:

$$I_{\text{rms, max}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\mathcal{E}_{\text{max}}}{\sqrt{2}Z}$$

$$= \frac{10\,\text{V}}{\sqrt{2}(100\,\Omega)} = \boxed{71\,\text{mA}}$$

75 •• A resistor and a capacitor are connected in parallel across an ac generator (Figure 29-43) that has an emf given by $\mathcal{E} = \mathcal{E}_{\text{peak}} \cos \omega t$. (*a*) Show that the current in the resistor is given by $I_R = (\mathcal{E}_{\text{peak}}/R) \cos \omega t$. (*b*) Show that the current in the capacitor branch is given by $I_C = (\mathcal{E}_{\text{peak}}/X_C) \cos(\omega t + 90^\circ)$. (*c*) Show that the total current is given by $I = I_{\text{peak}} \cos(\omega t + \delta)$, where $\tan \delta = R/X_C$ and $I_{\text{peak}} = \mathcal{E}_{\text{peak}}/Z$.

Picture the Problem Because the resistor and the capacitor are connected in parallel, the voltage drops across them are equal. Also, the total current is the sum of the current through the resistor and the current through the capacitor. Because these two currents are not in phase, we use phasors to calculate their sum. The amplitudes of the applied voltage and the currents are equal to the magnitude of the phasors. That is $\left|\vec{\boldsymbol{\mathcal{E}}}\right| = \mathcal{E}_{\text{peak}}$, $\left|\vec{\boldsymbol{I}}\right| = I_{\text{peak}}$, $\left|\vec{\boldsymbol{I}}_R\right| = I_{R,\text{ peak}}$, and $\left|\vec{\boldsymbol{I}}_C\right| = I_{C,\text{ peak}}$.

(*a*) The ac source applies a voltage given by $\mathcal{E} = \mathcal{E}_{\text{peak}} \cos \omega t$. Thus, the voltage drop across both the load resistor and the capacitor is:

$$\mathcal{E}_{\text{peak}} \cos \omega t = I_R R$$

The current in the resistor is in phase with the applied voltage:

$$I_R = I_{R,\text{ peak}} \cos \omega t$$

Because $I_{R,\,\text{peak}} = \dfrac{\mathcal{E}_{\text{peak}}}{R}$:

$$I_R = \boxed{\frac{\mathcal{E}_{\text{peak}}}{R}\cos\omega t}$$

(*b*) The current in the capacitor leads the applied voltage by 90°:

$$I_{\text{C}} = I_{\text{C, peak}}\cos(\omega t + 90°)$$

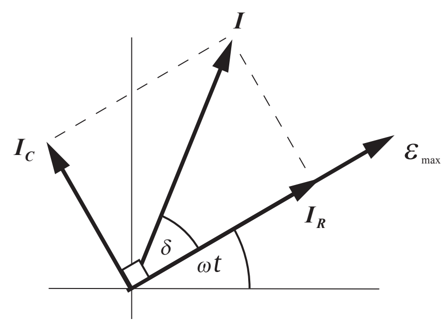
Because $I_{C,\text{peak}} = \dfrac{\mathcal{E}_{\text{peak}}}{X_{\text{C}}}$:

$$I_{\text{C}} = \boxed{\frac{\mathcal{E}_{\text{peak}}}{X_{\text{C}}}\cos(\omega t + 90°)}$$

(*c*) The net current *I* is the sum of the currents through the parallel branches:

$$I = I_R + I_C$$

Draw the phasor diagram for the circuit. The projections of the phasors onto the horizontal axis are the instantaneous values. The current in the resistor is in phase with the applied voltage, and the current in the capacitor leads the applied voltage by 90°. The net current phasor is the sum of the branch current phasors $\left(\vec{\boldsymbol{I}} = \vec{\boldsymbol{I}}_{\text{C}} + \vec{\boldsymbol{I}}_{\text{R}}\right)$.



The peak current through the parallel combination is equal to $\mathcal{E}_{\text{peak}}/Z$, where Z is the impedance of the combination:

$$I = I_{\text{peak}}\cos(\omega t - |\delta|),$$

where $I_{\text{peak}} = \dfrac{\mathcal{E}_{\text{peak}}}{Z}$

From the phasor diagram we have:

$$I_{\text{peak}}^2 = I_{R,\,\text{peak}}^2 + I_{C,\,\text{peak}}^2$$
$$= \left(\frac{\mathcal{E}_{\text{peak}}}{R}\right)^2 + \left(\frac{\mathcal{E}_{\text{peak}}}{X_C}\right)^2$$
$$= \mathcal{E}_{\text{peak}}^2\left(\frac{1}{R^2} + \frac{1}{X_C^2}\right) = \frac{\mathcal{E}_{\text{peak}}^2}{Z^2}$$

where $\dfrac{1}{Z^2} = \dfrac{1}{R^2} + \dfrac{1}{X_C^2}$

Solving for I_{peak} yields:

$$I_{\text{peak}} = \boxed{\frac{\mathcal{E}_{\text{peak}}}{Z}} \text{ where } Z^{-2} = R^{-2} + X_C^{-2}$$

From the phasor diagram:

$$I = \boxed{I_{\text{peak}}\cos(\omega t + \delta)}$$

where

$$\tan\delta = \frac{I_C}{I_R} = \frac{\dfrac{\mathcal{E}_{\text{peak}}}{X_C}}{\dfrac{\mathcal{E}_{\text{peak}}}{R}} = \boxed{\dfrac{R}{X_C}}$$

76 ••• Figure 29-44 shows a plot of average power P_{av} versus generator frequency ω for a series *RLC* circuit driven by an ac generator. The average power P_{av} is given by Equation 29-56. The full width at half-maximum, $\Delta\omega$, is the width of the resonance curve between the two points, where P_{av} is one-half its maximum value. Show that for a sharply peaked resonance, $\Delta\omega \approx R/L$ and that $Q \approx \omega_0/\Delta\omega$ in this case (Equation 29-58). *Hint: The half-power points occur when the denominator of Equation 29-56 is equal to twice the value it has at resonance; that is, when* $L^2\left(\omega^2-\omega_0^2\right)+\omega^2R^2 \approx 2\omega_0^2R^2$. *Let ω_1 and ω_2 be the solutions of this equation. Then, show that $\Delta\omega = \omega_2 - \omega_1 \approx R/L$.*

Picture the Problem We can use the condition determining the half-power points to obtain a quadratic equation that we can solve for the frequencies corresponding to the half-power points. Letting ω_1 be the half-power frequency that is less than ω_0 and ω_2 be the half-power frequency that is greater than ω_0 will lead us to the result that $\Delta\omega = \omega_2 - \omega_1 \approx R/L$. We can then use the definition of Q to complete the proof that $Q \approx \omega_0/\Delta\omega$.

Equation 29-56 is:

$$P_{\text{av}} = \frac{V_{\text{app, rms}}^2 R\omega^2}{L^2\left(\omega^2-\omega_0^2\right)^2+\omega^2R^2}$$

The half-power points occur when the denominator of Equation 29-56 is twice the value near resonance; that is, when:

$$L^2\left(\omega^2-\omega_0^2\right)^2+\omega^2R^2 \approx 2\omega_0^2R^2$$

or

$$L^2\left[\left(\omega-\omega_0\right)\left(\omega+\omega_0\right)\right]^2+\omega^2R^2 \approx 2\omega_0^2R^2$$

For a sharply peaked resonance, $\omega+\omega_0 \approx 2\omega_0$. Hence:

$$L^2\left[\left(\omega-\omega_0\right)\left(2\omega_0\right)\right]^2+\omega^2R^2 \approx 2\omega_0^2R^2$$

or

$$4\omega_0^2L^2\left(\omega-\omega_0\right)^2+\omega^2R^2 \approx 2\omega_0^2R^2$$

Let ω_1 be a solution to this equation. Noting that, for a sharply peaked resonance, $\omega_1 \approx \omega_0$, it follows that:

$$4\omega_0^2L^2\left(\omega_1-\omega_0\right)^2+\omega_0^2R^2 \approx 2\omega_0^2R^2$$

or, simplifying,

$$\left(\omega_1-\omega_0\right)^2 \approx \frac{R^2}{4L^2}$$

Solving for ω_1 yields:

$$\omega_1 \approx \omega_0 - \frac{R}{2L}$$

where we've used the minus sign because $\omega_1 < \omega_0$.

Similarly for ω_2:

$$\omega_2 \approx \omega_0 + \frac{R}{2L}$$

where we've used the plus sign because $\omega_2 > \omega_0$.

Evaluating $\Delta\omega = \omega_2 - \omega_1$ yields:

$$\Delta\omega \approx \omega_0 + \frac{R}{2L} - \left(\omega_0 - \frac{R}{2L}\right) = \boxed{\frac{R}{L}}$$

From the definition of Q:

$$\frac{R}{L} = \frac{\omega_0}{Q}$$

Substitute in the expression for $\Delta\omega$ to obtain:

$$\Delta\omega \approx \frac{\omega_0}{Q} \Rightarrow Q \approx \boxed{\frac{\omega_0}{\Delta\omega}}$$

77 ••• Show by direct substitution that $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$ (Equation 29-43*b*) is satisfied by $Q = Q_0 e^{-t/\tau}\cos\omega' t$, where $\tau = 2L/R$, $\omega' = \sqrt{1/(LC) - 1/\tau^2}$, and Q_0 is the charge on the capacitor at $t = 0$.

Picture the Problem We'll differentiate $Q = Q_0 e^{-t/\tau}\cos\omega' t$ twice and substitute this function and both its derivatives in the differential equation of the circuit. Rewriting the resulting equation in the form $A\cos\omega' t + B\sin\omega' t = 0$ will reveal that B vanishes. Requiring that $A\cos\omega' t = 0$ hold for all values of t will lead to the result that $\omega' = \sqrt{1/(LC) - 1/\tau^2}$.

Equation 29-43*b* is:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

Assume a solution of the form:

$$Q = Q_0 e^{-t/\tau}\cos\omega' t$$

Differentiate $Q(t)$ twice to obtain:

$$\begin{aligned}\frac{dQ}{dt} &= Q_0 \frac{d}{dt}\left[e^{-t/\tau}\cos\omega' t\right] \\ &= Q_0\left(e^{-t/\tau}\frac{d}{dt}\cos\omega' t + \cos\omega' t\frac{d}{dt}e^{-t/\tau}\right) \\ &= Q_0 e^{-t/\tau}\left(-\omega'\sin\omega' t - \frac{1}{\tau}\cos\omega' t\right)\end{aligned}$$

and

$$\frac{d^2Q}{dt^2} = Q_0 \frac{d}{dt}\left[e^{-t/\tau}\left(-\omega' \sin \omega' t - \frac{1}{\tau}\cos \omega' t\right)\right]$$

$$= Q_0 e^{-t/\tau}\left[\left(\frac{1}{\tau^2} - \omega'^2\right)\cos \omega' t + \frac{2\omega'}{\tau}\sin \omega' t\right]$$

Substitute these derivatives in the differential equation and simplify to obtain:

$$LQ_0 e^{-t/\tau}\left[\left(\frac{1}{\tau^2} - \omega'^2\right)\cos \omega' t + \frac{2\omega'}{\tau}\sin \omega' t\right] + RQ_0 e^{-t/\tau}\left(-\omega' \sin \omega' t - \frac{1}{\tau}\cos \omega' t\right)$$
$$+ \frac{1}{C} Q_0 e^{-t/\tau} \cos \omega' t = 0$$

Because Q_0 and $e^{-t/\tau}$ are never zero, divide them out of the equation and simplify to obtain:

$$L\left(\frac{1}{\tau^2} - \omega'^2\right)\cos \omega' t + \frac{2L\omega'}{\tau}\sin \omega' t - \omega' R \sin \omega' t - \frac{R}{\tau}\cos \omega' t + \frac{1}{C}\cos \omega' t = 0$$

Rewriting this equation in the form A$\cos\omega' t$ + B$\sin\omega' t$ = 0 yields:

$$\left(R\omega' - R\omega'\right)\sin \omega' t + \left[L\left(\frac{1}{\tau^2} - \omega'^2\right) + \frac{1}{C} - \frac{R}{\tau}\right]\cos \omega' t = 0$$

or

$$\left[L\left(\frac{1}{\tau^2} - \omega'^2\right) + \frac{1}{C} - \frac{R}{\tau}\right]\cos \omega' t = 0$$

If this equation is to hold for all values of *t*, its coefficient must vanish:

$$L\left(\frac{1}{\tau^2} - \omega'^2\right) + \frac{1}{C} - \frac{R}{\tau} = 0$$

Solving for ω' yields:

$$\omega' = \boxed{\sqrt{\frac{1}{LC} - \left(\frac{1}{2L}\right)^2}},$$

the condition that must be satisfied if $Q = Q_0 e^{-t/\tau}\cos \omega' t$ is the solution to Equation 29-43*b*.

78 ••• One method for measuring the magnetic susceptibility of a sample uses an *LC* circuit consisting of an air-core solenoid and a capacitor. The resonant frequency of the circuit without the sample is determined and then measured again with the sample inserted in the solenoid. Suppose you have a solenoid that is 4.00 cm long, 3.00 mm in diameter, and has 400 turns of fine wire. You have a sample that is inserted in the solenoid and completely fills the air space. Neglect end effects. (*a*) Calculate the inductance of your empty solenoid. (*b*) What value for the capacitance of the capacitor should you choose that the resonance frequency of the circuit without a sample is exactly 6.0000 MHz? (*c*) When a sample is inserted in the solenoid, you determine that the resonance frequency drops to 5.9989 MHz. Use your data to determine the sample's susceptibility.

Picture the Problem We can use $L=\mu_0 n^2 A\ell$ to determine the inductance of the empty solenoid and the resonance condition to find the capacitance of the sample-free circuit when the resonance frequency of the circuit is 6.0000 MHz. By expressing L as a function of f_0 and then evaluating df_0/dL and approximating the derivative with $\Delta f_0/\Delta L$, we can evaluate χ from its definition.

(*a*) Express the inductance of an air-core solenoid:

$$L=\mu_0 n^2 A\ell$$

Substitute numerical values and evaluate L:

$$L=\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{4.00\,\text{cm}}\right)^2\frac{\pi}{4}(3.00\,\text{cm})^2(4.00\,\text{cm})=3.553\,\text{mH}=\boxed{3.55\,\text{mH}}$$

(*b*) Express the condition for resonance in the LC circuit:

$$X_L=X_C\Rightarrow 2\pi f_0 L=\frac{1}{2\pi f_0 C}\qquad(1)$$

Solving for C yields:

$$C=\frac{1}{4\pi^2 f_0^2 L}$$

Substitute numerical values and evaluate C:

$$C=\frac{1}{4\pi^2(6.0000\,\text{MHz})^2(3.553\,\text{mH})}=1.9803\times10^{-13}\ \text{F}=\boxed{0.198\,\text{pF}}$$

(*c*) Express the sample's susceptibility in terms of L and ΔL:

$$\chi=\frac{\Delta L}{L}\qquad(2)$$

Solve equation (1) for f_0:

$$f_0=\frac{1}{2\pi\sqrt{LC}}$$

Differentiate f_0 with respect to L:

$$\frac{df_0}{dL} = \frac{1}{2\pi\sqrt{C}}\frac{d}{dL}L^{-1/2} = -\frac{1}{4\pi\sqrt{C}}L^{-3/2}$$
$$= -\frac{1}{4\pi L\sqrt{LC}} = -\frac{f_0}{2L}$$

Approximate df_0/dL by $\Delta f_0/\Delta L$:

$$\frac{\Delta f_0}{\Delta L} = -\frac{f_0}{2L} \text{ or } \frac{\Delta f_0}{f_0} = -\frac{\Delta L}{2L}$$

Substitute in equation (2) to obtain:

$$\chi = -2\frac{\Delta f_0}{f_0}$$

Substitute numerical values and evaluate χ:

$$\chi = -2\left(\frac{5.9989\,\text{MHz} - 6.0000\,\text{MHz}}{6.0000\,\text{MHz}_0}\right)$$
$$= \boxed{3.7\times10^{-4}}$$

The Transformer

79 • **[SSM]** A rms voltage of 24 V is required for a device whose impedance is 12 Ω. (*a*) What should the turns ratio of a transformer be, so that the device can be operated from a 120-V line? (*b*) Suppose the transformer is accidentally connected in reverse with the secondary winding across the 120-V-rms line and the 12-Ω load across the primary. How much rms current will then be in the primary winding?

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can use $V_2N_1 = V_1N_2$ and $N_1I_1 = N_2I_2$ to find the turns ratio and the primary current when the transformer connections are reversed.

(*a*) Relate the number of primary and secondary turns to the primary and secondary voltages:

$$V_{2,\,\text{rms}}N_1 = V_{1,\,\text{rms}}N_2 \qquad (1)$$

Solve for and evaluate the ratio N_2/N_1:

$$\frac{N_2}{N_1} = \frac{V_{2,\,\text{rms}}}{V_{1,\,\text{rms}}} = \frac{24\,\text{V}}{120\,\text{V}} = \boxed{\frac{1}{5}}$$

(*b*) Relate the current in the primary to the current in the secondary and to the turns ratio:

$$I_{1,\,\text{rms}} = \frac{N_2}{N_1}I_{2,\,\text{rms}}$$

Express the current in the primary winding in terms of the voltage across it and its impedance:

$$I_{2,\,\mathrm{rms}} = \frac{V_{2,\,\mathrm{rms}}}{Z_2}$$

Substitute for $I_{2,\,\mathrm{rms}}$ to obtain:

$$I_{1,\,\mathrm{rms}} = \frac{N_2}{N_1}\frac{V_{2,\,\mathrm{rms}}}{Z_2}$$

Substitute numerical values and evaluate $I_{1,\,\mathrm{rms}}$:

$$I_1 = \left(\frac{5}{1}\right)\left(\frac{120\,\mathrm{V}}{12\,\Omega}\right) = \boxed{50\,\mathrm{A}}$$

80 • A transformer has 400 turns in the primary and 8 turns in the secondary. (*a*) Is this a step-up or a step-down transformer? (*b*) If the primary is connected to a 120 V rms voltage source, what is the open-circuit rms voltage across the secondary? (*c*) If the primary rms current is 0.100 A, what is the secondary rms current, assuming negligible magnetization current and no power loss?

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can decide whether the transformer is a step-up or step-down transformer by examining the ratio of the number of turns in the secondary to the number of terms in the primary. We can relate the open-circuit rms voltage in the secondary to the primary rms voltage and the turns ratio.

(*a*) Because there are fewer turns in the secondary than in the primary it is a step-down transformer.

(*b*) Relate the open-circuit rms voltage $V_{2,\,\mathrm{rms}}$ in the secondary to the rms voltage $V_{1,\,\mathrm{rms}}$ in the primary:

$$V_{2,\,\mathrm{rms}} = \frac{N_2}{N_1}V_{1,\,\mathrm{rms}}$$

Substitute numerical values and evaluate $V_{2,\,\mathrm{rms}}$:

$$V_{2,\,\mathrm{rms}} = \frac{8}{400}(120\,\mathrm{V}) = \boxed{2.40\,\mathrm{V}}$$

(*c*) Because there are no power losses:

$$V_{1,\,\mathrm{rms}}I_{1,\,\mathrm{rms}} = V_{2,\,\mathrm{rms}}I_{2,\,\mathrm{rms}}$$

and

$$I_{2,\,\mathrm{rms}} = \frac{V_{1,\,\mathrm{rms}}}{V_{2,\,\mathrm{rms}}}I_{1,\,\mathrm{rms}}$$

Substitute numerical values and evaluate $I_{2,\,\mathrm{rms}}$:

$$I_{2,\,\mathrm{rms}} = \frac{120\,\mathrm{V}}{2.40\,\mathrm{V}}(0.100\,\mathrm{A}) = \boxed{5.00\,\mathrm{A}}$$

81 • The primary of a step-down transformer has 250 turns and is connected to a 120-V rms line. The secondary is to supply 20 A rms at 9.0 V rms. Find (*a*) the rms current in the primary and (*b*) the number of turns in the secondary, assuming 100 percent efficiency.

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can use $I_{1,\,\text{rms}}V_{1,\,\text{rms}} = I_{2,\,\text{rms}}V_{2,\,\text{rms}}$ to find the current in the primary and $V_{2,\,\text{rms}}N_1 = V_{1,\,\text{rms}}N_2$ to find the number of turns in the secondary.

(*a*) Because we have 100 percent efficiency:

$$I_{1,\,\text{rms}}V_{1,\,\text{rms}} = I_{2,\,\text{rms}}V_{2,\,\text{rms}}$$

and

$$I_{1,\,\text{rms}} = I_{2,\,\text{rms}}\frac{V_{2,\,\text{rms}}}{V_{1,\,\text{rms}}}$$

Substitute numerical values and evaluate $I_{1,\,\text{rms}}$:

$$I_{1,\,\text{rms}} = (20\text{ A})\frac{9.0\text{ V}}{120\text{ V}} = \boxed{1.5\text{ A}}$$

(*b*) Relate the number of primary and secondary turns to the primary and secondary voltages:

$$V_{2,\,\text{rms}}N_1 = V_{1,\,\text{rms}}N_2 \Rightarrow N_2 = \frac{V_{2,\,\text{rms}}}{V_{1,\,\text{rms}}}N_1$$

Substitute numerical values and evaluate N_2/N_1:

$$N_2 = \frac{9.0\text{ V}}{120\text{ V}}(250) \approx \boxed{19}$$

82 •• An audio oscillator (ac source) that has an internal resistance of 2000 Ω and an open-circuit rms output voltage of 12.0 V is to be used to drive a loudspeaker coil that has a resistance of 8.00 Ω. (*a*) What should be the ratio of primary to secondary turns of a transformer, so that maximum average power is transferred to the speaker? (*b*) Suppose a second identical speaker is connected in parallel with the first speaker. How much average power is then supplied to the two speakers combined?

Picture the Problem Note: In a simple circuit maximum power transfer from source to load requires that the load resistance equals the internal resistance of the source. We can use Ohm's law and the relationship between the primary and secondary currents and the primary and secondary voltages and the turns ratio of the transformer to derive an expression for the turns ratio as a function of the effective resistance of the circuit and the resistance of the speaker(s).

(*a*) Express the effective loudspeaker resistance at the primary of the transformer:

$$R_{\text{eff}} = \frac{V_{1,\,\text{rms}}}{I_{1,\,\text{rms}}}$$

Relate $V_{1,\,\text{rms}}$ to $V_{2,\,\text{rms}}$, N_1, and N_2:

$$V_{1,\,\text{rms}} = V_{2,\,\text{rms}}\frac{N_1}{N_2}$$

Express $I_{1,\,\text{rms}}$ in terms of $I_{2,\,\text{rms}}$, N_1, and N_2:

$$I_{1,\,\text{rms}} = I_{2,\,\text{rms}}\frac{N_2}{N_1}$$

Substitute for $V_{1,\,\text{rms}}$ and $I_{1,\,\text{rms}}$ and simplify to obtain:

$$R_{\text{eff}} = \frac{V_{2,\,\text{rms}}\dfrac{N_1}{N_2}}{I_{2,\,\text{rms}}\dfrac{N_2}{N_1}} = \left(\frac{V_{2,\,\text{rms}}}{I_{2,\,\text{rms}}}\right)\left(\frac{N_1}{N_2}\right)^2$$

Solve for N_1/N_2:

$$\frac{N_1}{N_2} = \sqrt{\frac{I_{2,\,\text{rms}}R_{\text{eff}}}{V_{2,\,\text{rms}}}} = \sqrt{\frac{R_{\text{eff}}}{R_2}} \qquad (1)$$

Evaluate N_1/N_2 for $R_{\text{eff}} = R_{\text{coil}}$:

$$\frac{N_1}{N_2} = \sqrt{\frac{2000\,\Omega}{8.00\,\Omega}} = 15.811 = \boxed{15.8}$$

(*b*) Express the power delivered to the two speakers connected in parallel:

$$P_{\text{sp}} = I_{1,\,\text{rms}}^2 R_{\text{eff}} \qquad (2)$$

Find the equivalent resistance R_{sp} of the two 8.00-Ω speakers in parallel:

$$\frac{1}{\boldsymbol{R}_{\text{sp}}} = \frac{1}{8.00\,\Omega} + \frac{1}{8.00\,\Omega} \Rightarrow R_{\text{sp}} = 4.00\,\Omega$$

Solve equation (1) for R_{eff} to obtain:

$$R_{\text{eff}} = R_2\left(\frac{N_1}{N_2}\right)^2$$

Substitute numerical values and evaluate R_{eff}:

$$R_{\text{eff}} = (4.00\,\Omega)(15.811)^2 = 1000\,\Omega$$

Find the current supplied by the source:

$$I_{1,\,\text{rms}} = \frac{V_{\text{rms}}}{R_{\text{tot}}} = \frac{12.0\,\text{V}}{2000\,\Omega + 1000\,\Omega} = 4.00\,\text{mA}$$

Substitute numerical values in equation (2) and evaluate the power delivered to the parallel speakers:

$$P_{\text{sp}} = (4.00\,\text{mA})^2(1000\,\Omega) = \boxed{16.0\,\text{mW}}$$

General Problems

83 • The distribution circuit of a residential power line is operated at 2000 V rms. This voltage must be reduced to 240 V rms for use within residences. If the secondary side of the transformer has 400 turns, how many turns are in the primary?

Picture the Problem We can relate the input and output voltages to the number of turns in the primary and secondary using $V_{2,\,\text{rms}}N_1 = V_{1,\,\text{rms}}N_2$.

Relate the output voltages $V_{2,\,\text{rms}}$ to the input voltage $V_{1,\,\text{rms}}$ and the number of turns in the primary N_1 and secondary N_2:

$$V_{2,\,\text{rms}} = \frac{N_2}{N_1}V_{1,\,\text{rms}} \Rightarrow N_1 = N_2\frac{V_{1,\,\text{rms}}}{V_{2,\,\text{rms}}}$$

Substitute numerical values and evaluate N_1:

$$N_1 = (400)\left(\frac{2000\,\text{V}}{240\,\text{V}}\right) = \boxed{3.33\times10^3}$$

84 •• A resistor that has a resistance R carries a current given by (5.0 A) sin $2\pi ft$ + (7.0 A) sin $4\pi ft$, where f = 60 Hz. (*a*) What is the rms current in the resistor? (*b*) If R =12 Ω, what is the average power delivered to the resistor? (*c*) What is the rms voltage across the resistor?

Picture the Problem We can use its definition, $I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$ to relate the rms current to the current carried by the resistor and find $\left(I^2\right)_{\text{av}}$ by integrating I^2.

(*a*) Express the rms current in terms of the $\left(I^2\right)_{\text{av}}$:

$$I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$$

Evaluate I^2:

$$\begin{aligned}I^2 &= \left[(5.0\,\text{A})\sin 2\pi ft + (7.0\,\text{A})\sin 4\pi ft\right]^2\\ &= \left(25\,\text{A}^2\right)\sin^2 2\pi ft + \left(70\,\text{A}^2\right)\sin 2\pi ft\sin 4\pi ft + \left(49\,\text{A}^2\right)\sin^2 4\pi ft\end{aligned}$$

Find $\left(I^2\right)_{\text{av}}$ by integrating I^2 from $t = 0$ to $t = T = 2\pi/\omega$ and dividing by T:

$$\begin{aligned}\left(I^2\right)_{\text{av}} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}&\left\{\left(25\,\text{A}^2\right)\sin^2 2\pi ft + \left(70\,\text{A}^2\right)\sin 2\pi ft\sin 4\pi ft\right.\\ &\left. + \left(49\,\text{A}^2\right)\sin^2 4\pi ft\right\}dt\end{aligned}$$

Use the trigonometric identity $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ to simplify and evaluate the 1st and 3rd integrals and recognize that the middle term is of the form sin*x*sin2*x* to obtain:

$$\left(I^2\right)_{av} = 12.5\,\text{A}^2 + 0 + 24.5\,\text{A}^2 = 37.0\,\text{A}^2$$

Substitute for $\left(I^2\right)_{av}$ and evaluate I_{rms}:

$$I_{rms} = \sqrt{37.0\,\text{A}^2} = \boxed{6.1\,\text{A}}$$

(*b*) Relate the power dissipated in the resistor to its resistance and the rms current in it:

$$P = I_{rms}^2 R$$

Substitute numerical values and evaluate *P*:

$$P = (6.08\,\text{A})^2(12\,\Omega) = \boxed{0.44\,\text{kW}}$$

(*c*) Express the rms voltage across the resistor in terms of *R* and I_{rms}:

$$V_{rms} = I_{rms}R$$

Substitute numerical values and evaluate V_{rms}:

$$V_{rms} = (6.08\,\text{A})(12\,\Omega) = \boxed{73\,\text{V}}$$

85 •• **[SSM]** Figure 29-45 shows the voltage versus time for a *square-wave* voltage source. If $V_0 = 12$ V, (*a*) what is the rms voltage of this source? (*b*) If this alternating waveform is rectified by eliminating the negative voltages, so that only the positive voltages remain, what is the new rms voltage?

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average of the voltage squared, $\left(V^2\right)_{av}$ and then use the definition of the rms voltage.

(*a*) From the definition of V_{rms} we have:

$$V_{rms} = \sqrt{\left(V_0^2\right)_{av}}$$

Noting that $-V_0^2 = V_0^2$, evaluate V_{rms}:

$$V_{rms} = \sqrt{V_0^2} = V_0 = \boxed{12\,\text{V}}$$

(*b*) Noting that the voltage during the second half of each cycle is now zero, express the voltage during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$V = V_0$$

Express the square of the voltage during this half cycle:

$$V^2 = V_0^2$$

Calculate $\left(V^2\right)_{\text{av}}$ by integrating V^2 from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by ΔT:

$$\left(V^2\right)_{\text{av}} = \frac{V_0^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{V_0^2}{\Delta T}\left[t\right]_0^{\frac{1}{2}\Delta T} = \tfrac{1}{2}V_0^2$$

Substitute to obtain:

$$V_{\text{rms}} = \sqrt{\tfrac{1}{2}V_0^2} = \frac{V_0}{\sqrt{2}} = \frac{12\ \text{V}}{\sqrt{2}} = \boxed{8.5\ \text{V}}$$

86 •• What are the average values and rms values of current for the two current waveforms shown in Figure 29-46?

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average current I_{av}, and the average of the current squared, $\left(I^2\right)_{\text{av}}$

From the definition of I_{av} and I_{rms} we have:

$$I_{\text{av}} = \frac{1}{\Delta T}\int_0^{\Delta T} I dt \text{ and } I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$$

<u>**Waveform (*a*)**</u> Express the current during the first half cycle of time interval ΔT:

$$I_a = \frac{4\ \text{A}}{\Delta T}t$$

where I is in A when t and T are in seconds.

Evaluate $I_{\text{av},\,a}$:

$$I_{\text{av},\,a} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{4.0\ \text{A}}{\Delta T}tdt = \frac{4.0\ \text{A}}{\left(\Delta T\right)^2}\int_0^{\Delta T} tdt$$

$$= \frac{4.0\ \text{A}}{\left(\Delta T\right)^2}\left[\frac{t^2}{2}\right]_0^{\Delta T} = \boxed{2.0\ \text{A}}$$

Express the square of the current during this half cycle:

$$I_a^2 = \frac{(4.0\ \text{A})^2}{(\Delta T)^2} t^2$$

Noting that the average value of the squared current is the same for each time interval ΔT, calculate $\left(I_a^2\right)_{\text{av}}$ by integrating I_a^2 from $t = 0$ to $t = \Delta T$ and dividing by ΔT:

$$\left(I_a^2\right)_{\text{av}} = \frac{1}{\Delta T}\int_0^{\Delta T} \frac{(4.0\ \text{A})^2}{(\Delta T)^2} t^2\,dt$$
$$= \frac{(4.0\ \text{A})^2}{(\Delta T)^3}\left[\frac{t^3}{3}\right]_0^{\Delta T} = \frac{16}{3}\ \text{A}^2$$

Substitute in the expression for $I_{\text{rms},\,a}$ to obtain:

$$I_{\text{rms},\,a} = \sqrt{\frac{16}{3}\ \text{A}^2} = \boxed{2.3\ \text{A}}$$

Waveform (*b*) Noting that the current during the second half of each cycle is zero, express the current during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$I_b = 4.0\ \text{A}$$

Evaluate $I_{\text{av},\,b}$:

$$I_{\text{av},\,b} = \frac{4.0\ \text{A}}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{4.0\ \text{A}}{\Delta T}[t]_0^{\frac{1}{2}\Delta T}$$
$$= \boxed{2.0\ \text{A}}$$

Express the square of the current during this half cycle:

$$I_b^2 = (4.0\ \text{A})^2$$

Calculate $\left(I_b^2\right)_{\text{av}}$ by integrating I_b^2 from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by ΔT:

$$\left(I_b^2\right)_{\text{av}} = \frac{(4.0\ \text{A})^2}{\Delta T}\int_0^{\frac{1}{2}\Delta} dt$$
$$= \frac{(4.0\ \text{A})^2}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = 8.0\ \text{A}^2$$

Substitute in the expression for $I_{\text{rms},\,b}$ to obtain:

$$I_{\text{rms},\,b} = \sqrt{8.0\ \text{A}^2} = \boxed{2.8\ \text{A}}$$

87 •• In the circuit shown in Figure 29-47, $\mathcal{E}_1 = (20\ \text{V})\cos 2\pi ft$, where $f = 180$ Hz; $\mathcal{E}_2 = 18$ V, and $R = 36\ \Omega$. Find the maximum, minimum, average, and rms values of the current in the resistor.

Picture the Problem We can apply Kirchhoff's loop rule to express the current in the circuit in terms of the emfs of the sources and the resistance of the resistor.

We can then find $I_{\max}$ and $I_{\min}$ by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. Finally, we can use $I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$ to derive an expression for I_{rms} that we can use to determine its value.

Apply Kirchhoff's loop rule to obtain: $\mathcal{E}_{1,\,\text{peak}} \cos\omega t + \mathcal{E}_2 - IR = 0$

Solving for I yields:

$$I = \frac{\mathcal{E}_{1,\,\text{peak}}}{R}\cos\omega t + \frac{\mathcal{E}_2}{R}$$

or

$$I = A_1 \cos\omega t + A_2$$

where $A_1 = \dfrac{\mathcal{E}_{1,\,\text{peak}}}{R}$ and $A_2 = \dfrac{\mathcal{E}_2}{R}$

Substitute numerical values to obtain:

$$\begin{aligned} I &= \left(\frac{20\,\text{V}}{36\,\Omega}\right)\cos\!\left(2\pi\left(180\,\text{s}^{-1}\right)t\right) + \frac{18\,\text{V}}{36\,\Omega} \\ &= \left(0.556\,\text{A}\right)\cos\!\left(1131\,\text{s}^{-1}\right)t + 0.50\,\text{A} \end{aligned}$$

The current is a maximum when $\cos\!\left(1131\,\text{s}^{-1}\right)t = 1$. Hence:

$$I_{\max} = 0.50\,\text{A} + 0.556\,\text{A} = \boxed{1.06\,\text{A}}$$

Evaluate $I_{\min}$:

$$I_{\min} = 0.50\,\text{A} - 0.556\,\text{A} = \boxed{-0.06\,\text{A}}$$

Because the average value of $\cos\omega t = 0$:

$$I_{\text{av}} = \boxed{0.50\,\text{A}}$$

The rms current is the square root of the average of the squared current:

$$I_{\text{rms}} = \sqrt{\left[I^2\right]_{\text{av}}} \qquad (1)$$

$\left[I^2\right]_{\text{av}}$ is given by:

$$\begin{aligned} \left[I^2\right]_{\text{av}} &= \left[\left(A_1 \cos\omega t + A_2\right)^2\right]_{\text{av}} \\ &= \left[A_1^2 \cos^2\omega t + 2A_1A_2\cos\omega t + A_2^2\right]_{\text{av}} \\ &= \left[A_1^2\cos^2\omega t\right]_{\text{av}} + \left[2A_1A_2\cos\omega t\right]_{\text{av}} + \left[A_2^2\right]_{\text{av}} \\ &= A_1^2\left[\cos^2\omega t\right]_{\text{av}} + 2A_1A_2\left[\cos\omega t\right]_{\text{av}} + A_2^2 \end{aligned}$$

Because $\left[\cos^2\omega t\right]_{\text{av}} = \frac{1}{2}$ and $\left[\cos\omega t\right]_{\text{av}} = 0$:

$$\left[I^2\right]_{\text{av}} = \tfrac{1}{2}A_1^2 + A_2^2$$

Substituting in equation (1) yields:

$$I_{\text{rms}} = \sqrt{\tfrac{1}{2}A_1^2 + A_2^2}$$

Substitute for A_1 and A_2 to obtain:

$$I_{\text{rms}} = \sqrt{\tfrac{1}{2}\left(\frac{\mathcal{E}_1}{R}\right)^2 + \left(\frac{\mathcal{E}_2}{R}\right)^2}$$

Substitute numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \sqrt{\tfrac{1}{2}\left(\frac{20\text{ V}}{36\,\Omega}\right)^2 + \left(\frac{18\text{ V}}{36\,\Omega}\right)^2} = \boxed{0.64\text{ A}}$$

88 •• Repeat Problem 87 if the resistor is replaced by a 2.0-μF capacitor.

Picture the Problem We can apply Kirchhoff's loop rule to obtain an expression for charge on the capacitor as a function of time. Differentiating this expression with respect to time will give us the current in the circuit. We can then find $I_{\max}$ and $I_{\min}$ by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. Finally, we can use $I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$ to derive an expression for I_{rms} that we can use to determine its value.

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_{1\,\text{peak}}\cos\omega t + \mathcal{E}_2 - \frac{q(t)}{C} = 0$$

Solving for $q(t)$ yields:

$$q(t) = C\left(\mathcal{E}_{1\,\text{peak}}\cos\omega t + \mathcal{E}_2\right) = A_1\cos\omega t + A_2$$

where

$$A_1 = C\mathcal{E}_{1\,\text{peak}} \text{ and } A_2 = C\mathcal{E}_2$$

Differentiate this expression with respect to t to obtain the current as a function of time:

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(A_1\cos\omega t + A_2\right) = -\omega A_1\sin\omega t$$

Substituting numerical values yields:

$$I = -2\pi(180\text{ Hz})(2.0\,\mu\text{F})\sin\left(2\pi(180\text{ Hz})t\right) = (-2.26\text{ mA})\sin\left(1131\text{ s}^{-1}\right)t$$

The current is a minimum when $\sin\left(1131\text{s}^{-1}\right)t = 1$. Hence:

$$I_{\min} = \boxed{-2.3\text{mA}}$$

The current is a maximum when $\sin\left(1131\text{s}^{-1}\right)t = -1$. Hence:

$$I_{\max} = \boxed{2.3\text{mA}}$$

Because the dc source sees the capacitor as an open circuit and the average value of the sine function over a period is zero:

$$I_{\text{av}} = \boxed{0}$$

The rms current is the square root of the average of the squared current:

$$I_{\text{rms}} = \sqrt{\left[I^2\right]_{\text{av}}} \qquad (1)$$

$\left[I^2\right]_{\text{av}}$ is given by:

$$\begin{aligned}
\left[I^2\right]_{\text{av}} &= \left[(A_1 \cos\omega t + A_2)^2\right]_{\text{av}} \\
&= \left[A_1^2 \cos^2\omega t + 2A_1A_2\cos\omega t + A_2^2\right]_{\text{av}} \\
&= \left[A_1^2 \cos^2\omega t\right]_{\text{av}} + \left[2A_1A_2\cos\omega t\right]_{\text{av}} + \left[A_2^2\right]_{\text{av}} \\
&= A_1^2\left[\cos^2\omega t\right]_{\text{av}} + 2A_1A_2\left[\cos\omega t\right]_{\text{av}} + A_2^2
\end{aligned}$$

Because $\left[\cos^2\omega t\right]_{\text{av}} = \frac{1}{2}$ and $\left[\cos\omega t\right]_{\text{av}} = 0$:

$$\left[I^2\right]_{\text{av}} = \tfrac{1}{2}A_1^2 + A_2^2$$

Substituting in equation (1) yields:

$$I_{\text{rms}} = \sqrt{\tfrac{1}{2}A_1^2 + A_2^2}$$

Substitute for A_1 and A_2 to obtain:

$$\begin{aligned}
I_{\text{rms}} &= \sqrt{\tfrac{1}{2}(C\mathcal{E}_1)^2 + (C\mathcal{E}_2)^2} \\
&= C\sqrt{\tfrac{1}{2}(\mathcal{E}_1)^2 + (\mathcal{E}_2)^2}
\end{aligned}$$

Substitute numerical values and evaluate I_{rms}:

$$\begin{aligned}
I_{\text{rms}} &= (2.0\ \mu\text{F})\sqrt{\tfrac{1}{2}(20\ \text{V})^2 + (18\ \text{V})^2} \\
&= \boxed{46\ \mu\text{A}}
\end{aligned}$$

89 ••• [SSM] A circuit consists of an ac generator, a capacitor and an ideal inductor—all connected in series. The emf of the generator is given by $\mathcal{E}_{\text{peak}}\cos\omega t$. (*a*) Show that the charge on the capacitor obeys the equation $L\dfrac{d^2Q}{dt^2} + \dfrac{Q}{C} = \mathcal{E}_{\text{peak}}\cos\omega t$. (*b*) Show by direct substitution that this equation is satisfied by $Q = Q_{\text{peak}}\cos\omega t$ where $Q_{\text{peak}} = -\dfrac{\mathcal{E}_{\text{peak}}}{L\left(\omega^2 - \omega_0^2\right)}$. (*c*) Show that the current can be written as $I = I_{\text{peak}}\cos(\omega t - \delta)$, where $I_{\text{peak}} = \dfrac{\omega\mathcal{E}_{\text{peak}}}{L\left|\omega^2 - \omega_0^2\right|} = \dfrac{\mathcal{E}_{\text{peak}}}{\left|X_L - X_C\right|}$ and $\delta = -90^\circ$ for $\omega < \omega_0$ and $\delta = 90^\circ$ for $\omega > \omega_0$, where ω_0 is the resonance frequency.

Picture the Problem In Part (*a*) we can apply Kirchhoff's loop rule to obtain the 2^{nd} order differential equation relating the charge on the capacitor to the time. In Part (*b*) we'll assume a solution of the form $Q = Q_{\text{peak}}\cos\omega t$, differentiate it twice, and substitute for d^2Q/dt^2 and Q to show that the assumed solution satisfies the differential equation provided $Q_{\text{peak}} = -\dfrac{\mathcal{E}_{\text{peak}}}{L\left(\omega^2 - \omega_0^2\right)}$. In Part (*c*) we'll use our results from (*a*) and (*b*) to establish the result for I_{peak} given in the problem statement.

(*a*) Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E} - \frac{Q}{C} - L\frac{dI}{dt} = 0$$

Substitute for $\mathcal{E}$ and rearrange the differential equation to obtain:

$$L\frac{dI}{dt} + \frac{Q}{C} = \mathcal{E}_{\max}\cos\omega t$$

Because $I = dQ/dt$:

$$\boxed{L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \mathcal{E}_{\max}\cos\omega t}$$

(*b*) Assume that the solution is:

$$Q = Q_{\text{peak}}\cos\omega t$$

Differentiate the assumed solution twice to obtain:

$$\frac{dQ}{dt} = -\omega Q_{\text{peak}}\sin\omega t$$

and

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_{\text{peak}}\cos\omega t$$

Substitute for $\dfrac{dQ}{dt}$ and $\dfrac{d^2Q}{dt^2}$ in the differential equation to obtain:

$$-\omega^2 LQ_{\text{peak}}\cos\omega t + \frac{Q_{\text{peak}}}{C}\cos\omega t = \mathcal{E}_{\text{peak}}\cos\omega t$$

Factor $\cos\omega t$ from the left-hand side of the equation:

$$\left(-\omega^2 LQ_{\text{peak}} + \frac{Q_{\text{peak}}}{C}\right)\cos\omega t = \mathcal{E}_{\text{peak}}\cos\omega t$$

If this equation is to hold for all values of t it must be true that:

$$-\omega^2 LQ_{\text{peak}} + \frac{Q_{\text{peak}}}{C} = \mathcal{E}_{\text{peak}}$$

Solving for Q_{peak} yields:

$$Q_{\text{peak}} = \frac{\varepsilon_{\text{peak}}}{-\omega^2 L + \dfrac{1}{C}}$$

Factor L from the denominator and substitute for $1/LC$ to obtain:

$$Q_{\text{peak}} = \frac{\varepsilon_{\text{peak}}}{L\left(-\omega^2 + \dfrac{1}{LC}\right)}$$

$$= \boxed{-\frac{\varepsilon_{\text{peak}}}{L\left(\omega^2 - \omega_0^2\right)}}$$

(*c*) From (*a*) and (*b*) we have:

$$I = \frac{dQ}{dt} = -\omega Q_{\text{peak}} \sin \omega t$$

$$= \frac{\omega \varepsilon_{\text{peak}}}{L\left(\omega^2 - \omega_0^2\right)} \sin \omega t = I_{\text{peak}} \sin \omega t$$

$$= \boxed{I_{\text{peak}} \cos(\omega t - \delta)}$$

where

$$I_{\text{peak}} = \boxed{\frac{\omega \varepsilon_{\text{peak}}}{L\left|\omega^2 - \omega_0^2\right|}} = \frac{\varepsilon_{\text{peak}}}{\dfrac{L}{\omega}\left|\omega^2 - \omega_0^2\right|}$$

$$= \frac{\varepsilon_{\text{peak}}}{\left|\omega L - \dfrac{1}{\omega C}\right|} = \boxed{\frac{\varepsilon_{\text{peak}}}{\left|X_L - X_C\right|}}$$

If $\omega > \omega_0$, $X_L > X_C$ and the current lags the voltage by 90° ($\delta = 90°$).

If $\omega < \omega_0$, $X_L < X_C$ and the current leads the voltage by 90°($\delta = -90°$).