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| 1- Successive peaks are one-half period apart. Hence the elapsed time between the peaks is $\frac{1}{2}T = \frac{1}{2f} = \frac{1}{2(60^{-1})} = \fbox{8.33 ms} $. |
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| 2- First express the initial rms voltage in terms of the peak voltage:  $$V\_{rms} = \frac{V\_{peak}}{\sqrt{2}}$$  Then expres the doubled rms voltage in terms of the new peak voltage $V\_{peak}^\prime$:  $$2V\_{rms} = \frac{V\_{peak}^\prime}{\sqrt{2}}$$  After you have done that you have to divide the second of these equations by the first and simplify to obtain:  $$\frac{2V\_{rms}}{V\_{rms}} = \frac{\frac{V\_{peak}^\prime}{\sqrt{2}}}{\frac{V\_{peak}}{\sqrt{2}}} \Rightarrow 2 = \frac{V\_{peak}^\prime}{V\_{peak}}$$  Now you only have to solve for $V\_{peak}^\prime$:  $$V\_{peak}^\prime = 2 V\_{peak} \Rightarrow \fbox{(a)} \text{ is the correct answer.}$$ |
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| 3 - The inductance of an inductor is independent of the frequency of the circuit and is determined only by its construction. The reactance is dependent on the frequency, hence \fbox{(b)} is correct. |
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| 4- The inductive reactance of an inductor and the frequency are related by $X\_L = \omega L$. Therefore, when $\omega$ is doubled, $X\_L$ will double as well. \fbox{(a)} is correct. |
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| 5- The capacitive reactance of a capacitor varies with the frequency and are related by $X\_C = \frac{1}{\omega C}$. Therefore, if $\omega$ change to its double then $X\_C$ will halve. \fbox{(c)} is correct. |
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| 6 - The answer to both questions is yes. (a) During current magnitude increase in the inductor, the inductor absorbs power from the generator. (b) During current decrease in the inductor, the inductor supplies power to the generator. |
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| 7 - (a) The capacitor absorbs power from the generator while the magnitude of the charge is accumulating on either plate of the capacitor. (b) The capacitor supplies power to the generator whenever the magnitude of the charge on either plate of the capacitor is decreasing. |
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| 8 - (a) First substitute the SI units of inductance and capacitance then simplify to obtain:  $$\frac{V \cdot s}{A} \cdot \frac{C}{V} = \frac{s}{\frac{C}{S}} \cdot C = \fbox{$s^2$}$$  (b) First substitute the SI units of inductance divided by resistance then simplify:  $$\frac{\frac{V \cdot s}{A}}{\Omega} = \frac{\frac{V \cdot s}{A}}{\frac{V}{A}} = \fbox{s}$$ |
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| 9 - The rms current through the resistor $I\_{rms}$ is directly proportional to $\omega$ as shown here:  $$I\_{rms} = \frac{\mathcal{E}\_{rms}}{R} = \frac{\mathcal{E}\_{peak}}{\sqrt{2}} = \frac{NBA}{\sqrt{2}}\omega$$  therefore, \fbox{(a)} is correct. |
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| 10 - The natural frequency of an $LC$ circuit is given by  $$f\_0 = \frac{1}{2\pi\sqrt{LC}}$$  We now express the natural frequencies of the circuit before and after the inductance is tripled:  $$f\_0 = \frac{1}{2\pi\sqrt{LC}} \text{ and } f\_0^\prime = \frac{1}{2\pi\sqrt{L^\prime C^\prime}}$$  We divide the second equation by the first and simplify:  $$\frac{f\_0^\prime}{f\_0} = \frac{\frac{1}{2\pi\sqrt{L^\prime C^\prime}}}{\frac{1}{2\pi\sqrt{LC}}} = \sqrt{\frac{LC}{L^\prime C^\prime}}$$  We take into account that the natural frequency is unchanged:  $$1 = \sqrt{\frac{LC}{L^\prime C^\prime}} \Rightarrow \frac{LC}{L^\prime C^\prime} = 1 \Rightarrow C^\prime = \frac{L}{L^\prime}C$$  Therefore when the inductance is tripled:  $$C^\prime = \frac{L}{3L}C = \frac{1}{3}C$$  Hence \fbox{(b)} is the correct answer. |
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| 11 - The maximum energy stored in the electric field ofthe capacitor is given by  $$U\_e = \frac{Q^2}{2C}$$  and the maximum energy stored in the magnetic field of the inductor is given by  $$U\_m = \frac{LI^2}{2}.$$  That is because energy is conserved in a LC circuit and oscillates between the inductor and the capacitor,  $$U\_c = U\_m = U\_{total},$$  therefore answer \fbox{(a)} is correct. |
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| 12 - (a) True. The $Q$ factor and the width of the resonance curve at half power are related according to  $$Q = \frac{\omega\_0}{\Delta \omega}$$  this means that they are inversely proportional to each other.\\  (b) True. Circuit's natural frequency depends only on the inductance $L$ of the inductor and the capacitance $C$ of the capacitor and is given by  $$\omega = \frac{1}{\sqrt{LC}}.$$  (c) True. The impedance of and $RLC$ circuit is given by  $$Z = \sqrt{R^2 + (X\_L - X\_C)^2}.$$  At resonance  $$X\_L = X\_C \text{ and so } Z = R.$$  (d) True. The phase angle $\delta$ is related to $X\_L$ and $X\_C$ according to  $$\delta = \tan^{-1} \left( \frac{X\_L - X\_C}{R} \right)$$  At resonance $X\_L = X\_C$ and so $\delta = 0.$ |
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| 13 - (a) False. The power factor given by  $$\cos \delta = \frac{R}{\sqrt{(X\_L - X\_C)^2 +R^2}},$$  is close to 1.\\  (b) False. The power factor is given by  $$\cos \delta = \frac{R}{\sqrt{(X\_L - X\_C)^2 +R^2}}.$$  (c) True. The resonance frequency for a driven series $RLC$ circuit depends only on $L$ and $C$ is given by  $$\omega\_{res} = \frac{1}{\sqrt{LC}}$$  (d) True. At resonance $X\_L - X\_C = 0$ and so $Z = R$ and the peak current is given by $I\_{peak} = V\_{app,peak}/R.$\\  (e) True. The capacitive reactance varies inversely with the driving frequency nd the inductive reactance varies directly with the driving frequency, hance at frequencies well below the resonance frequency the capacitive reactance is larger than the inductive reactance.\\  (f) True. For frequencies below the resonant frequency, the circuit is more capacitive than inductive and the phase constant $phi$ is negative. This means that the current leads the applied voltage. |
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| 14 - The power curves received by the radio have widht, hence the two frequencies coming from the radio stations can overlap as a result and you can receive signals from both stations. |
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| 15 - (a) True. The power factor is given by  $$\cos \delta = \frac{R}{\sqrt{\left( \omega L - \frac{1}{\omega C}\right)^2 + R^2}},$$  therefore for values of $\omega$ that are much higher or much lower than the resonant frequency, the term in parentheses becomes very large and $\cos \delta$ approaches to zero.\\  (b) False. When the resonance curve is reasonably narrow, the $Q$ factor can be approximated by  $$Q = \omega\_0 / \Delta \omega.$$  Hence a large value for $Q$ corresponds to a narrow resonance curve.\\  (c) True. See figure 29-21 |
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| 16 - Subscript 1 and 2 correspond to primary and secondary respectively. We assume no loss of power in the transformer, we can equate the power in the primary circuit to the power in the secondary circuit and solve for the rms current in th primary windings.\\  Assuming no power loss in the transformer:  $$P\_1 = P\_2$$  Substituting for $P$ we obtain:  $$I\_{1,rms}V\_{1,rms} = I\_{2,rms}V\_{2,rms}$$  We solve for $I$ and simplify:  $$I\_{1,rms} = \frac{I\_{2,rms}V\_{2,rms}}{V\_{1,rms}} = \frac{P\_2}{V\_{1,rms}}$$  hence answer $\fbox{(a)}$ is correct. |
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| 17 - (a) False. A transformer is a device used to raise or lower the voltage in a circuit.\\  (b) True. A transformer is a device used to raise or lower the voltage in a circuit.\\  (c) True. If energy is to be conserved, the product of the current and voltage must be constant.\\  (d) True. Because the product of current and voltage in the primary and secondary circuits is the same, increasing the current in the secondary results in a lowering (or stepping down) of the voltage.\\  (e) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the voltage in order to make her hair dryer work properly.\\  (f) True. Because electrical energy is provided at a higher voltage in Europe, the visitor would want to step-up the current (and decrease the voltage) in order to make his razor work properly. |
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| 18 - (a) First we relate the resistance and inductive reactance of the plant's total load to Z and $\delta$:  $$R = Z \cos \delta \text{ and } X\_L = Z \sin \delta$$  We then express Z in terms of the rms current $I\_{rms}$ in the power lines and the rms voltage $\mathcal{E}\_{rms}$ at the plant:  $$Z = \frac{\mathcal{E}\_{rms}}{I\_{rms}}$$  after express teh power delivered to the plant in terms of $\mathcal{E}\_{rms}, I\_{rms} \text{ and } \delta$ and solve for $I\_{rms}:$  $$P\_{av} = \mathcal{E}\_{rms}I\_{rms}\cos \delta \text{ and } I\_{rms} = \frac{P\_{av}}{\mathcal{E}\_{rms}\cos \delta}$$  then substitute to obtain:  $$Z = \frac{\mathcal{E}\_{rms}^2 \cos\delta}{P\_{av}}$$  Afterwards substitute numerical values and evaluate Z:  $$Z = \frac{(40 kV)^2 \cos 25^{\circ}}{2.3MW} = 630 \Omega$$  Substitute numerical values and evaluate for R and $X\_L:$  $$R = (630 \Omega)\cos 25^{\circ} = 571\Omega = \fbox{$0.57k\Omega$} \text{ and } X\_L = (630\Omega)\sin 25^{\circ} = 266\Omega = \fbox{$0.27k\Omega$}$$  (b) Find the current in the power lines:  $$I\_{rms} = \frac{2.3MW}{(40kV)\cos 25^{\circ}} = 63.4 A = \fbox{63A}$$  Then apply Kirchhoff's loop rule to the circuit:  $$\mathcal{E}\_{sub} - I\_{rms}R\_{trans} - I\_{rms}Z\_{tot} = 0$$  Solve for $\mathcal{E}\_{sub}:$  $$\mathcal{E}\_{sub} = I\_{rms}(R\_{trans} + Z\_{tot})$$  Substitute numerical values and evaluate $\mathcal{E}\_{sub}:$  $$\mathcal{E}\_{sub} = (63.4A)(5.2\Omega + 630\Omega) = \fbox{40.3kV}$$  (c) The power lost in transmission is:  $$P\_{trans} = I\_{trans}^2 R\_{trans} = (63.4A)^2(5.2\Omega) = 20.9kW = \fbox{21kW}$$  (d) Express the cost savings $\Delta C$ in terms of the difference in energy consumption $P\_{25^{\circ}} - P\_{18^{\circ}}\Delta t$ and the per-unit cost $u$ of the energy:  $$\Delta C = (P\_{25^{\circ}} - P\_{18^{\circ}})\Delta tu$$  Express the power lost in transmission when $\delta = 18^{\circ}:$  $$P\_{18^{\circ}} = I\_{18^{\circ}}^2R\_{trans}$$  Then find the current in the transmission lines when $\delta = 18^{\circ}:$  $$I\_{18^{\circ}} = \frac{2.3MW}{(40kV)\cos 18^{\circ}} = 60.5A$$  Evaluate $P\_{18^{\circ}}:$  $$P\_{18^{\circ}} = (60.5A)^2(5.2\Omega) = 19 kW$$  Substitute numerical values and evaluate $\Delta C:$  $$\Delta C = (20.9kW - 19kW)(16 h/d)\left( 30 \frac{d}{\text{month}}\right) \left( \frac{\$0.14}{kW \cdot h}\right) = \fbox{\$128}$$  (e) The required capacitance is given by:  $$C = \frac{1}{2\pi fX\_C}$$  We then relate the new phase $\delta$ to the inductive reactance $X\_L,$ the reactance due to the added capacitance $X\_C,$ and the resistance of the load R:  $$\tan \delta = \frac{X\_L - X\_C}{R} \Rightarrow X\_C = X\_L - R \tan \delta$$  We substitute for $X\_C:$  $$C = \frac{1}{2\pi f(X\_L R \tan \delta)}$$  We then substitute numerical values and evaluate C:  $$C = \frac{1}{2\pi(60s^{-1})(266\Omega - (571\Omega)\tan 18^{\circ})} = \fbox{$33\mu F$}$$ |
|  |
| 19 - We use $P\_{av} = \mathcal{E}\_{rms}I\_{rms}$ to find $I\_{rms},$ $I\_{peak} = \sqrt{2}I\_{rms}$ to find $I\_{peak},$ and $P\_{peak} = I\_{peak} \mathcal{E}\_{peak}$ to find $P\_{peak}.$\\  (a) We find the relationship of average power delivered by the source to the rms  voltage across the bulb and the rms current through it:  $$P\_{av} = \mathcal{E}\_{rms}I\_{rms} \Rightarrow I\_{rms} = \frac{P\_{av}}{\mathcal{E}\_{rms}}$$  We then substitute numerical values and evaluate $I\_{rms}:$  $$I\_{rms} = \frac{100W}{120V} = 0.8333A = \fbox{0.833A}$$  (b) We do express $I\_{peak}$ in terms of $I\_{rms}:$  $$I\_{peak} = \sqrt{2}I\_{rms}$$  substitute for $I\_{rms}$ and evaluate $I\_{peak}:$  $$I\_{peak} = \sqrt{2}(0.8333A) = 1.1785A = \fbox{1.18A}$$  (c) Afterwards we need to express the maximum power in terms of the maximum voltage and maximum current, and substitute numerical values evaluating $P\_{peak}:$  $$P\_{peak} = I\_{peak} \mathcal{E}\_{peak}$$  $$P\_{peak} = (1.1785A)\sqrt{2}(120V) = \fbox{200W}$$ |
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| 20 - We can use $I\_{peak} = \sqrt{2}I\_{rms}$ to find the largest peak current that the breaker can carry and $P\_{av} = I\_{rms}V\_{rms}$ to find the average power supplied by the circuit.\\  (a) We do express $I\_{peak}$ in terms of $I\_{rms}$ and relate the average power to the rms current and voltage:  $$I\_{peak} = \sqrt{2}I\_{rms} = \sqrt{2}(15A) = \fbox{21A}$$  $$P\_{av} = I\_{rms}V\_{rms} = (15A)(120V) = \fbox{1.8kW}$$ |
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| 21 - Utilize $X\_L = \omega L$ to determine the reactance of the inductor at any frequency. We do so by expressing the inductive reactance as a function of $f:$  $$X\_L = \omega L = 2\pi fL$$  (a) At $f = 60Hz:$  $$X\_L = 2\pi (60s^{-1})(1mH) = \fbox{$0.38\Omega$}$$  (b) At $f = 600Hz:$  $$X\_L = 2\pi (600s^{-1})(1mH) = \fbox{$3.77\Omega$}$$  (c) At $f = 6kHz:$  $$X\_L = 2\pi (6ks^{-1})(1mH) = \fbox{$37.7\Omega$}$$ |
|  |
| 22 - We can use $X\_L = \omega L$ to find the inductance of the inductor at any frequency.\\  (a) We do relate the reactance or the inductor the its inductance and solve for $L$ doing its evaluation:  $$X\_L = \omega L = 2\pi fL \Rightarrow L = \frac{X\_L}{2\pi f}$$  $$L = \frac{100 \Omega}{2\pi (80s^{-1})} = 0.199H = \fbox{0.20 H}$$  (b) At 160 Hz:  $$X\_L = 2\pi (160s^{-1})(0.199H) = \fbox{$0.20k\Omega$}$$ |
|  |
| 23 - If we equate reactances of the capacitor and inductor we then can solve for the frequency.\\  We do express the reactance of the inductor, then we express the reactance of the capacitor equating these reactances:  $$X\_L = \omega L = 2\pi fL$$  $$X\_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$  $$2\pi fL = \frac{1}{2\pi fC} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$  We finally substitute numerical values and evaluate f:  $$f = \frac{1}{2\pi}\sqrt{\frac{1}{(10\mu F)(1mH)}} = \fbox{1.6kHz}$$ |
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| 24 - We can use $X\_C = 1 / \omega C$ to find the reactance of the capacitor at any frequency. By expressing the capacitive reactance as a function of $f:$  $$X\_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$  (a) At $f = 60 Hz$:  $$X\_C = \frac{1}{2\pi(60s^{-1})(1nF)} = \fbox{$2.65M\Omega$}$$  (b) At $f = 6.00 kHz$:  $$X\_C = \frac{1}{2\pi(6.00kHz)(1nF)} = \fbox{$26.5k\Omega$}$$  (c) At $f = 6.00 MHz$:  $$X\_C = \frac{1}{2\pi(6.00MHz)(1nF)} = \fbox{$26.5\Omega$}$$ |
|  |
| 25 - We can use $I\_{peak} = \mathcal{E}\_{peak} / X\_C$ and $X\_C = 1 / \omega C$ to express $I\_{peak}$ as a function of $\mathcal{E}\_{peak}, f$ and C. Once we evaluate $I\_{peak},$ we can use $I\_{rms} = I\_{peak} / \sqrt{2}$ to find $I\_{rms}$.\\  We begin by expressing $I\_{peak}$ in terms of $\mathcal{E}\_{peak}$ and $X\_C$$$I\_{peak} = \frac{\mathcal{E}\_{peak}}{X\_C}$$  We then express the capacitive reactance and subsitute for $X\_C$ and simplify:  $$I\_{peak} = 2\pi fC\mathcal{E}\_{peak}$$  (a) Substitute numerical values and evaluate $I\_{peak}:$  $$I\_{peak} = 2\pi(20s^{}-1)(20\mu F)(10V) = 25.1mA = \fbox{25mA}$$  (b) Express $I\_{rms} = \frac{I\_{peak}}{\sqrt{2}} = \frac{25.1mA}{\sqrt{2}} = 18mA$ |
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| 26 - We can use $X\_C = 1 / \omega C = 1 / 2\pi fC$ to relate the reactance of the capacitor to the frequency:  $$X\_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \Rightarrow f = \frac{1}{2\pi CX\_C}$$  (a) Find $f$ when $X\_C = 1.00 \Omega$:  $$f = \frac{1}{2\pi(10\mu F)(1.00\Omega)} = \fbox{16kHz}$$  (b) Find $f$ when $X\_C = 100 \Omega$:  $$f = \frac{1}{2\pi(10\mu F)(100\Omega)} = \fbox{0.16kHz}$$  (c) Find $f$ when $X\_C = 10.0m \Omega$:  $$f = \frac{1}{2\pi(10\mu F)(10.0m \Omega)} = \fbox{1.6MHz}$$ |
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| 27 - We can find the sum of the phasors $V\_1$ and $V\_2$ using this trigonometric identity  $$\cos \theta + \cos \phi = 2 \cos \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi)$$  then we use this sum to express $I$ as a function of time. In (b) we use a phasor diagram to obtain the same result and in (c) we use the phasor diagram appropriate to the given voltages to express the current as a function of time.\\  (a) Apply the Kirchhoff's loop rule to the circuit and solve for I:  $$V\_1 + V\_2 - IR = 0$$  $$I = \frac{V\_1 + V\_2}{R}$$  Then we use the trigonometric identity described above to find $V\_1 + V\_2:$  $$V\_1 + V\_2 = (5.0V)[\cos (\omega t - \alpha ) + \cos (\omega t + \alpha)] = (5V)[2\cos \frac{1}{2}(2\omega t)\cos \frac{1}{2} (-2\alpha)] =$$  $$(10V)\cos \frac{\pi}{6}\cos \omega t = (8.66V)\cos \omega t$$  We substitute for $V\_1 + V\_2$ and $R$ to obtain:  $$I = \frac{(8.66V)\cos \omega t}{25\Omega} = (0.346A)\cos \omega t = (0.35A)\cos \omega t$$  $$I\_{peak} = \fbox{0.35A}$$  (b) We express the magnitude of the current in $R:$  $$\lvert I \rvert = \frac{\lvert \vec{V} \rvert}{R}$$  The phasor diagram for the voltages is shown. We can find $\vec V$ using vector addition:  $$\lvert \vec V \rvert = 2 \lvert \vec V\_1 \rvert cos 30^{\circ} = 2(5.0V)\cos 30^{\circ} = 8.66V$$  We then substitute for $\lvert \vec V \rvert$ and $R$ to obtain: $\lvert I \rvert = \frac{8.66V}{25\Omega} = 0.346A$ and $I = (0.35A) \cos \omega t$ where $I\_{peak} = \fbox{0.35A}$  (c) The phasor diagram is shown. Note that the phase angle between $V\_1$ and $V\_2$ is now $90^{\circ}$. We use the Pythagorean theorem to find $\lvert \vec V \rvert$:  $$\lvert \vec V \rvert = \sqrt{\lvert \vec V\_1 \rvert ^2 + \lvert \vec V\_2 \rvert ^2} = \sqrt{(5.0V)^2 + (7.0V)^2} = 8.60V$$  Then we express $I$ as a function of $t$:  $$I = \frac{\lvert \vec V \rvert}{R}\cos (\omega t + \delta)$$  where $\delta = 45^{\circ} - (90^{\circ} - \alpha) = \alpha - 45^{\circ} = \tan ^{-1} \frac{7.0V}{5.0V} - 45^{\circ} = 9.462^{\circ} = 0.165rad$\\  Finally substitute numerical values and evaluate $I$:  $$I = \frac{8.60V}{25\Omega}\cos (\omega t + 0.165rad) = \fbox{$(0.34A)\cos (\omega t + 0.17rad)$}$$ |
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| 28 - We begin by substituting the units of the various physical quantities in $1 / \sqrt{LC}$ and $Q = \omega{\_0}L / R$ to establish their units.\\  (a) We do substitute the units for $\omega{\_0}, L$ and $C$ in the expression $1 / \sqrt{LC}$ and then simplify:  $$\frac{1}{\sqrt{H\cdot F}} = \frac{1}{\sqrt{(\Omega \cdot s)(\frac{s}{\Omega})}} = \frac{1}{\sqrt{s^2}} = \fbox{$s^{-1}$}$$  (b) We do substitute the units for $\omega{\_0}, L$ and $R$ in the expression $Q = \omega{\_0}L / R$ and simplify:  $$\frac{\frac{1}{s} \cdot \frac{V \cdot s}{A}}{\frac{V}{A}} = 1 \Rightarrow \fbox{unitless}$$ |
|  |
| 29 - $T = 2\pi / \omega$ and $\omega = 1 / \sqrt{LC}$ can be used to relate $T, f$ to $L$ and $C$.\\  (a) We express the period of oscillation:  $$T = \frac{2\pi}{\omega}$$  Taking into account that for a $LC$ circuit $\omega = 1 / \sqrt{LC}$, we substitute for $\omega$ obtaining:  $$T = 2\pi \sqrt{LC} = 2\pi \sqrt{(2.0mH)(20\mu F)} = \fbox{1.3ms}$$  From the equation for $\omega$ we solve for $L$:  $$L = \frac{T^2}{4\pi^2C} = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (60^{-1})(80\mu F)} = \fbox{88mH}$$ |
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| 30 - The expression $f\_0 = 1 / 2\pi \sqrt{LC}$ can be used for the resonance frequency of a $LC$ circuit showing that each circuit oscillates with the same frequency. For (b) we can use $I\_{peak} = \omega Q\_0,$ where $Q\_0$ is the charge of the capacitor at time zero, and the definition of capacitance $Q\_0 = CV$ to express $I\_{peak}$ in terms of $\omega, C, V$.\\  We do express the resonance frequency for a $LC$ circuit  $$f\_0 = \frac{1}{2\pi \sqrt{LC}}$$  (a) We separate the product of $L$ and $C$ for each circuit:  $$\text{Circuit 1:} L\_1C\_1 = L\_1C\_0,$$  $$\text{Circuit 2:} L\_2C\_2 = (2L\_1)(\frac{1}{2}C\_0) = L\_1C\_1,$$  $$\text{Circuit 3:} L\_3C\_3 = (\frac{1}{2}L\_1)(2C\_0) = L\_1C\_1,$$  $$L\_1C\_1 = L\_2C\_2 = L\_3C\_3$$  Therefore, the resonance frequencies of the three circuits are equal.\\  (b) We do express the I peak in terms of the charge stored in the capacitor and $Q\_0$ in terms of the capacitance of the capacitor and the potential difference across the capacitor, then we substitute:  $$I\_{peak} = \omega Q\_0, \quad Q\_0 = CV \Rightarrow I\_{peak} = \omega CV$$  We have I peak directly proportional to C when $\omega$ and V are held constant. Hence the circuit with a capacitance of $2C\_0$ has the greatest peak current. |
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| 31 - The energy stored in the electric field of the capacitor can be found using $U = \frac{1}{2}CV^2$, the frecuency can be found by these relations $\omega{\_0} = 2\pi f\_0 = 1 / \sqrt{LC}$, and I peak can be determined by using these relations $I\_{peak} = \omega Q\_0$ and $Q\_0 = CV$.\\  (a) We express the energy stored in the system as function of C and V, substituting its numerical values:  $$U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0\mu F)(30V)^2 = \fbox{2.3mJ}$$  (b) We express the resonance frequency in terms of L and C, and substitute numerical values:  $$\omega{\_0} = 2\pi f\_0 = \frac{1}{\sqrt{LC}} \Rightarrow f\_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi\sqrt{(10mH)(5.0\mu F)}} = \fbox{0.71kHz}$$  (c) I peak can be expressed in terms of the charge stored in the capacitor and $Q\_0$ can be expressed in terms of the capacitance of the capacitor and the potential difference accross the capacitor, using these two, we substitute for $Q\_0$:  $$I\_{peak} = \omega Q\_0, \quad Q\_0 = CV \Rightarrow I\_{peak} = \omega CV$$  $$I\_{peak} = 2\pi (712s^{-1})(5.0 \mu F)(30V) = \fbox{0.67A}$$ |
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| 32 - (a) To find the initial voltage across the coil we can apply the Kirchhoff's loop rule. (b) The total energy lost by the Joule heating is the total energy initially stored in the capacitor. (c) The natural frequency of the circuit is given by $f\_0 = 1 / 2\pi \sqrt{LC}$. (d) The quality factor can be found by its definition.\\  (a) Applying Kirchhoff's loop rule gives that initial voltage across the coil is \fbox{24.0V}\\  (b) All the energy initially stored in the capacitor will be dissipated as Joule heat in the resistor because the ideal inductor can not dissipate energy as heat:  $$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.00\mu F)(24.0V)^2 = \fbox{0.576mJ}$$  (c) We find the natural frequency of the circuit:  $$f\_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{(400mH)(2.00\mu F)}} = \fbox{178Hz}$$  (d) The quality factor definition is $Q = \omega{\_0} L / R$, we can substitute for $\omega{\_0}$ and simpify, finally we substitute numerical values:  $$Q = \frac{\frac{1}{\sqrt{LC}}L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1.00\Omega}\sqrt{\frac{400mH}{2.00\mu F}} = \fbox{447}$$ |
| 33 - We can obtain a differential equation for the circuit by applying Kirchhoff's loop rule, letting Q represent the instantaneous charge of the capacitor. Solving this equation we obtain an expression for the charge on the capacitor as a function of time and by differentiating this expression with respect to time, and expression for the current as a function of time.\\  We apply Kirchhoff's loop rule to a clockwise loop just after the switch is closed:  $$\frac{Q}{C} + L\frac{dI}{dt} = 0$$  Substitute $I = dQ/dt$:  $$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0, \text{ hence } \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$  The solution to this differential equation is: $Q(t) = Q\_0\cos (\omega t - \delta)$ where $\omega = \sqrt{1 / LC}$. Because $Q(0) = Q\_0,$ and $\delta = 0$ and $Q(t) = Q\_0\cos \omega t$. The current in the circuit is the derivative of Q with respect to t:  $$I = \frac{dQ}{dt} = \frac{d}{dt}[Q\_0 \cos \omega t] = -\omega Q\_0 \sin \omega t$$  (a) A spreadsheet program was used to plot the following graph showing both the charge on the capacitor and the current in the circuit as functions of time. L, C, and Q 0 were all arbitrarily set equal to one to obtain these graphs. Note that the current leads the charge by one-fourth of a cycle or 90 degrees.\\  (b) The equation for the current is: $I = -\omega Q\_0 \sin \omega t$, we can use the identity between sine and cosine functions: $-sin \theta = \cos \left( \theta + \frac{\pi}{2}\right)$. Using this identity we can rewrite the equation for the current:  $$I = -\omega Q\_0 \sin \omega t = \fbox{$\omega Q\_0 \cos \left( \omega t + \frac{\pi}{2}\right)$}$$  Therefore, the current leads the charge by 90 degrees. |
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| 34 - The ratio $V\_R$ to $V\_L$ can be used to find the resistance of the circuit. For (b) we can use the fact that in a LR circuit, $V\_L$ leads $V\_R$ by 90 degrees to find the ac input voltage.\\  (a) We do express the potential differences accross R and L in terms of the common current through these components and express R in thee terms.  $$V\_L = IX\_L = I\omega L, \quad V\_R = IR$$  $$\frac{V\_R}{V\_L} = \frac{IR}{I\omega L} = \frac{R}{\omega L} \Rightarrow R = \left( \frac{V\_R}{V\_L} \right)\omega L$$  We substitute numerical values and evaluate R:  $$R = \left( \frac{30V}{40V} \right) 2\pi (60s^{-1})(1.4H) = \fbox{$0.40k\Omega$}$$  (b) $V\_R$ leads $V\_L$ by 90 degrees in a LR circuit:  $$V\_{peak} = \sqrt{2}V\_{rms} = \sqrt{2}\sqrt{V\_R^2 + V\_L^2}$$  $$V\_{peak} = \sqrt{2} \sqrt{(30V)^2 + (40V)^2} = \fbox{71V}$$ |
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| 35 - The definition of $X\_L$ in a $LR$ circuit can be used to find $L$.\\  From the definition of impedance of a coil in terms of its resistance and inductive reactance we can solve for $X\_L$:  $$Z = \sqrt{R^2 + X\_L^2} \Rightarrow X\_L = \sqrt{Z^2 - R^2}$$  Remember that $X\_L$ can be expressed in terms of $L$, $X\_L = 2\pi fL$. If we equate these equations we can solve for L:  $$2\pi fL = \sqrt{Z^2 - R^2} \Rightarrow L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$  Now we only need to substitute numerical values:  $$L = \frac{\sqrt{(200\Omega)^2-(80.0\Omega)^2}}{2\pi (1.00kHz)} = \fbox{29.2mH}$$ |
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| 36 - The two output voltage signals can be expressed as the product of the current from each source and $R = 1.00 k\Omega$. The impedance definition and given voltage signals can help us for determine the currents due to each source.\\  (a) We do express the output voltage signals in terms of the potential difference across the resistor:  $$V\_{1,out} = I\_1R \quad V\_{2,out} = I\_2R$$  We need $I\_1$ and $I\_2$:  $$I\_1 = \frac{V\_1}{Z\_1} = \frac{(10.0V)\cos 100t}{\sqrt{(1.00k\Omega)^2+[(100s^{-1})(1.00H)]^2}} = (9.95mA)\cos 100t$$  $$I\_2 = \frac{V\_2}{Z\_2} = \frac{(10.0V)\cos 10^4t}{\sqrt{(1.00k\Omega)^2+[(10^4s^{-1})(1.00H)]^2}} = (0.995mA)\cos 10^4t$$  Substituting these values in voltage expressions:  $$V\_{1,out} = (1.00k\Omega)(9.95mA)\cos 100t = \fbox{$(9.95V)\cos 100t$}$$  $$V\_{2,out} = (1.00k\Omega)(0.995mA)\cos 10^4t = \fbox{$(0.995V)\cos 10^4t$}$$  where $\omega\_1 = 100 rad/s$ and $\omega\_2 = 10000 rad/s$ |
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| 37 - There is a relationship between the power factor and the average power supplied to the coil $P\_{av} = \mathcal{E}\_{rms}I\_{rms}\cos \delta$. In (b) $P\_{av} = I\_{rms}^2R$ this expression gives the needed relationship for R. The resistance of the coil can be found by $X\_L = \omega L = R \tan \delta$ this expression relates the resistance, phase angle and inductance. By noting if the circuit is inductive, we can decide if the current leads or lags the voltage.\\  (a) We express the average power supplied to the coil in terms of the power factor:  $$P\_{av} = \mathcal{E}\_{rms}I\_{rms} \cos \delta \Rightarrow \cos \delta = \frac{P\_{av}}{\mathcal{E}\_{rms}I\_{rms}}$$  Substituting numerical values:  $$\cos \delta = \frac{60W}{(120V)(1.5A)} = 0.333 = \fbox{0.33}$$  (b) We do express the power supplied by the source in terms of the resistance of the coil and substitute numerical values:  $$P\_{av} = I\_{rms}^2 \Rightarrow R = \frac{P\_{av}}{I\_{rms}^2} = \frac{60W}{(1.5A)^2} = 26.7\Omega = \fbox{$27\Omega $}$$  (c) We relate the inductive reactance to the resistance and phase angle, solving for L:  $$X\_L = \omega L = R \tan \delta \Rightarrow L = \frac{R \tan \delta}{\omega} = \frac{R \tan[\cos^{-1}(0.333)]}{2\pi f}$$  $$L = \frac{(26.7\Omega)\tan (70.5^{\circ})}{2\pi (60s^{-1})} = \fbox{0.20H}$$  (d) We just need to evaluate $X\_L$:  $$X\_L = (26.7\Omega)\tan (70.5^{\circ}) = 75.4\Omega$$  The circuite is inductive, hence the current lags the voltage.\\  (e) From part (a):  $$\delta = \cos^{-1}(0.333) = 72^{\circ}$$ |
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| 38 - (a) Use $I\_{peak} = \mathcal{E}\_{peak}/\sqrt{R^2 + (\omega L)^2}$ and $V\_{L,peak} = I\_{peak}X\_L = \omega LI\_{peak}$ to find the peak current in the circuit and the peak voltage across the inductor. (b) Once we've found $V\_{L,peak}$ we can find $V\_{L,rms}$ using $V\_{L,rms} = V\_{L,peak} / \sqrt{2}$. (c) We can use $P\_{av} = \frac{1}{2}I^2\_{rms}R$ to find the average power dissipation. (d) $U\_{L,peak} = \frac{1}{2}LI\_{peak}^2$ to find the peak and average magnetic energy stored in the inductor. The average energy stored in the magnetic field of the inductor can be found using $U\_{L,av} = \int P\_{av}dt$.\\  (a) Apply Kirchhoff's loop rule to the circuit:  $$\mathcal{E} - IZ = 0 \Rightarrow I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L)^2}}$$  Substituting numerical values to find for I:  $$I\_{peak} = \frac{(345V)\cos (150 \pi t)}{\sqrt{(40\Omega)^2} + [(150\pi s^{-1})(36mH)]^2} = (7.94A)\cos (150\pi t) = \fbox{7.9A}$$  (b) We can find $\mathcal{E}$ as follows:  $$\mathcal{E} = (345V) \cos (150 \pi t)$$  $$V\_{L,peak} = \fbox{345V}$$  $$V\_{L,rms} = \frac{V\_{L,peak}}{\sqrt{2}} = \frac{345V}{\sqrt{2}} = \fbox{244V}$$  (c) We can relate the average power dissipation to $I\_{peak}$ and R:  $$P\_{av} = I\_{rms}^2R = \left( \frac{I\_{peak}}{\sqrt{2}} \right)^2R = \frac{1}{2}I\_{peak}^2R$$  Substituting numerical values:  $$P\_{av} = \frac{1}{2}(7.94A)^2(40\Omega) = \fbox{1.3kW}$$  (d) The maximum energy stored in the magnetic field of the inductor can be found as follows:  $$U\_{L,peak} = \frac{1}{2}LI\_{peak}^2 = \frac{1}{2}(36mH)(7.94A)^2 = \fbox{1.1J}$$  From the definition $U\_{L,av} = \frac{1}{T}\int\_0^TU(t)dt$ and $U(t) = \frac{1}{2}L[I(t)]^2$. We do substitute $U(t)$ and obtain:  $$U\_{L,av} = \frac{L}{2T}\int\_0^T[I(t)]^2dt$$  We evaluate the integral:  $$U\_{L,av} = \frac{L}{2T}\left[ \frac{1}{2}I\_{peak}^2\right]T = \frac{1}{4}LI\_{peak}^2$$  Finally we do substitute numerical values and evaluate:  $$U\_{L,av} = \frac{1}{4}(36mH)(7.94A)^2 = \fbox{0.57J}$$ |
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| 39 - The relationship between $X\_L$ and $R$ define the power factor when the coil is driven at a frequency of 60 Hz and then from the definition of $X\_L$ we can relate the inductive reactance at 240 Hz to the inductive reactance at 60 Hz, finally we can use the power factor definition to determine it at 240 Hz.\\  We use the definition of power factor to relate $R$ and $X\_L$:  $$\cos \delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X\_L^2}} \Rightarrow \cos^2 \delta = \frac{R^2}{R^2 + X\_L^2}$$  We solve for 60 Hz and substitute for $\cos \delta$:  $$X\_L^2(60Hz) = R^2\left( \frac{1}{\cos^2 \delta} - 1 \right) = R^2\left( \frac{1}{(0.866)^2} - 1 \right) = \frac{1}{3}R^2$$  We use the definition of $X\_L$:  $$X\_L^2(f) = 4\pi f^2L^2 \text{ and } X\_L^2(f') = 4\pi f'^2L^2$$  Combining these two equations:  $$\frac{X\_L^2(f')}{X\_L^2(f)} = \frac{4\pi f'^2L^2}{4\pi f^2L^2} = \frac{f'^2}{f^2} \Rightarrow X\_L^2(f') = \left( \frac{f'}{f} \right)^2X\_L^2(f)$$  Finally we substitute numerical values:  $$X\_L^2(240Hz) = \left( \frac{240s^{-1}}{60s^{-1}} \right)^2 X\_L^2(60Hz) = 16\left( \frac{1}{3}R^2 \right) = \frac{16}{3}R^2$$  We do substitute in the first equation to obtain:  $$(\cos \delta)\_{240Hz} = \frac{R}{\sqrt{R^2 + \frac{16}{3}R^2}} = \sqrt{\frac{3}{19}} = \fbox{0.397}$$ |
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| 40 - The voltage drops across hte inductor and the resistor are equal because they are connected in parallel. The sum of the current through the resistor and through the inductor sum up the total current. The two currents are not in phase, phasors are needed to calculate their sum. The magnitude of the phasors are equal to the amplitudes of the applied voltage and the currents, i.e. $\lvert\mathcal{\vec E} \rvert = \mathcal{E}, \lvert \vec I \rvert = I\_{peak}, \lvert \vec I\_R\rvert = I\_{R,peak}, \lvert \vec I\_L \rvert = I\_{L,peak}$ \\  (a) The voltage given is supplied by the ac source $\mathcal{E} = \mathcal{E}\cos \omega t$. Therefore the voltage drop across the load resistor and the inductor is:  $$I\_R = I\_{R,peak}\cos \omega t$$  and as $I\_{R,peak} = \frac{\mathcal{E}\_{peak}}{R}$:  $$I\_R = \frac{\mathcal{E}\_{peak}}{R}\cos \omega t$$  (b) The current in the inductor lags the applied voltage by $90^{\circ}$:  $$I\_L = I\_{L,peak} \cos (\omega t - 90^{\circ})$$  and as $I\_{L,peak} = \frac{\mathcal{E}\_{peak}}{X\_L}$:  $$I\_L = \frac{\mathcal{E}\_{peak}}{X\_L}\cos (\omega t - 90^{\circ})$$  (c) The sum of the currents through the parallel branches sum up the net current $I$:  $$I = I\_R + I\_L$$  It helps if we draw the phasor diagram of the circuit. The projections of the phasors onto the horizontal axis are the instantaneous values. The current in the resistor is in phase with the applied voltage, and the current in the inductor lags the applied voltage by 90 degrees. The net current phasor is the sum of the branch current phasors $(\vec I = \vec I\_R + \vec I\_L)$\\  The peak current through the parallel combination is equal to $\mathcal{E}\_{peak} / Z,$ where $Z$ is the impedance of the combination:  $$I = I\_{peak} \cos (\omega t - \lvert \delta \rvert) \text{ where } I\_{peak} = \frac{\mathcal{E}}{Z}$$  Based on the phasor diagram we have:  $$I\_{peak}^2 = I\_{R,peak}^2 + I\_{L,peak}^2 = \left( \frac{\mathcal{E}\_{peak}}{R} \right)^2 + \left( \frac{\mathcal{E}\_{peak}}{X\_L} \right)^2$$  $$= \mathcal{E}\_{peak}^2\left( \frac{1}{R^2} + \frac{1}{X\_L^2} \right) = \frac{\mathcal{E}\_{peak}^2}{Z^2}$$  where $\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X\_L^2}$. We do solve for $I\_{peak}$ and yields:  $$I\_{peak} = \frac{\mathcal{E}\_{peak}}{Z} \text{ where } Z^{-2} = R^{-2} + X\_L^{-2}$$  And finally, from the phasor diagram:  $$I = I\_{peak} \cos (\omega t - \lvert \delta \rvert)$$  $$\text{where } \tan \lvert \delta \rvert = \frac{I\_{L,peak}}{I\_{R,peak}} = \frac{\frac{\mathcal{E}\_{peak}}{X\_L}}{\frac{\mathcal{E}\_{peak}}{R}} = \fbox{$\frac{R}{X\_L}$}$$ |
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| 41 - This equation reflects the voltage drops $\mathcal{E} = V\_1 + V\_2$, $V\_1$ is the voltage drop across $R$ and $V\_2$ is the voltage drop across the parallel combination of $L$ and $R\_L$, if we take the same vectorial equation we have the relation for the phasors. The current for the parallel combination $\vec I = \vec I\_{R\_L} + \vec I\_{L}.$ Also, $V\_1$ is in phase with $I$ and $V\_2$ is in phase with $I\_{R\_L}$. Draw the phasor diagram for the currents in the parallel combination, then add phasors for the voltages to the diagram.\\  We show the phasor diagram for the currents in the circuit and the modified diagram showing the voltage phasors.\\  The maximum current in the inductor $I\_{2,peak}$ is given by:  $$I\_{2,peak} = \frac{V\_{2,peak}}{Z\_2} \text{ where } Z\_2^{-2} = R\_L^{-2} + X\_L^{-2}$$  the $\tan \lvert \delta \rvert$ is given by:  $$\tan \lvert \delta \rvert = \frac{I\_{L,peak}}{I\_{R,peak}} = \frac{V\_{2,peak} / X\_L}{V\_{2,peak} / R\_L}$$  $$= \frac{R\_L}{X\_L} = \frac{R\_L}{\omega L} = \frac{R\_L}{2\pi fL}$$  Solving for $\lvert \delta \rvert$:  $$\lvert \delta \rvert = \tan^{-1} \left( \frac{R\_L}{2\pi fL} \right)$$  We apply the law of cosines to the triangle formed by the voltage phasors and obtain:  $$\mathcal{E}\_{peak}^2 = V\_{1,peak}^2 + V\_{2,peak}^2 + 2V\_{1,peak}V\_{2,peak}\cos \lvert \delta \rvert$$  $$I\_{peak}^2Z^2 = I\_{peak}^2R^2 + I\_{peak}^2Z\_2^2 + 2I\_{peak}RI\_{peak}Z\_2\cos \lvert \delta \rvert$$  Simplyfing for the current and Z:  $$Z^2 = R^2 + Z^2 + 2RZ\_2\cos \lvert \delta \rvert$$  $$Z = \sqrt{R^2 + Z\_2^2 + 2RZ\_2\cos \lvert \delta \rvert}$$  The maximum current $I\_{peak}$ in the circuit $I\_{peak} = \frac{\mathcal{E}\_{peak}}{Z}$\\  $I\_{rms}$ is related to $I\_{peak}$ according to: $I\_{rms} = \frac{1}{\sqrt{2}}I\_{peak}$\\  (a) We do substitute numerical values to get $\delta$:  $$\lvert \delta \rvert = \tan^{-1} \left( \frac{20.0\Omega}{2\pi (500Hz)(3.20mH)} \right) = \tan^{-1}\left( \frac{20.0\Omega}{10.053\Omega} \right) = 63.31^{\circ}$$  Solving for $Z\_2$ gives:  $$Z\_2^{-2} = R\_L^{-2} + X\_L^{-2} = \frac{1}{\sqrt{(20.0\Omega)^{-2}+(10.053\Omega)^{-2}}} = 8.982\Omega$$  For $Z$ we do substitute numerical values and evaluate:  $$Z = \sqrt{(4.00\Omega)^2 + (8.982\Omega)^2 + 2(4.00\Omega)(8.982\Omega)\cos 63.31^{\circ}} = 11.36\Omega$$  We do substitute values and evaluate for $I\_{peak}$:  $$I\_{peak} = \frac{100V}{11.36\Omega} = 8.806A$$  Once we have I peak, we do substitute for I rms:  $$I\_{rms} = \frac{1}{\sqrt{2}}(8.806A) = \fbox{6.23A}$$  The maximum and rms values of $V\_2$ are given by:  $$V\_{2,peak} = I\_{peak}Z\_2 = (8.806A)(8.982\Omega) = 79.095V$$  $$V\_{2,rms} = \frac{1}{\sqrt{2}}V\_{2,peak} = \frac{1}{\sqrt{2}}(79.095V) = 55.929V,$$  The rms values of $I\_{R\_L,rms} \text{ and } I\_{L,rms}$:  $$I\_{R\_L,rms} = \frac{V\_{2,rms}}{R\_L} = \frac{55.929V}{20.0\Omega} = \fbox{2.80A}$$  $$I\_{L,rms} = \frac{V\_{2,rms}}{X\_L} = \frac{55.929V}{10.053\Omega} = \fbox{5.53A}$$  (b) We do proceed as in (a) with $f = 2000Hz$ to obtain:  $$X\_L = 40.2\Omega, \lvert \delta \rvert = 26.4^{\circ}, Z\_2 = 17.9\Omega , Z = 21.6\Omega , I\_{peak} = 4.64A, \text{ and } I\_{rms} = \fbox{3.28A}$$  $$V\_{2,max} = 83.0V, V\_{2,rms} = 58.7V, I\_{R\_L,rms} = \fbox{2.94A}, \text{ and } I\_{L,rms} = \fbox{1.46A}$$  (c) The sum of the power dissipated in the two resistors equals the power delivered by the ac source. The fraction of the total power delivered by the source that is dissipated in load resistor is given by:  $$\frac{P\_{R\_L}}{P\_{R\_L} + P\_R} = \left( 1 + \frac{P\_R}{P\_{R\_L}} \right)^{-1} = \left( 1 + \frac{I\_{rms}^2 R}{I\_{R\_L,rms}^2 R\_L}\right)^{-1}$$  We substitute numerical values for $f = 500 Hz$:  $$\frac{P\_{R\_L}}{P\_{R\_L} + P\_R}\bigg|\_{f=500Hz} = \left( 1 + \frac{(6.23A)^2(4.00\Omega)}{(2.80A)^2(20.0\Omega)} \right)^{-1} = 0.502 = 50.2 \%$$  (d) Substitute numerical values for $f = 2000 Hz$ to obtain:  $$\frac{P\_{R\_L}}{P\_{R\_L} + P\_R}\bigg|\_{f=2000Hz} = \left( 1 + \frac{(3.28A)^2(4.00\Omega)}{(2.94A)^2(20.0\Omega)} \right)^{-1} = 0.800 = 80.0\%$$ |
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| 42 - Let's treat the ac and dc components separately. L acts like a short circuit for the component. Denote the peak value of the voltage supplied by $\mathcal{E}\_{1,peak}$, then use $P = \mathcal{E}\_2^2 / R$ to find the power dissipated in the resistors by the current from the ideal battery. We apply Kirchhoff's loop including to $L, R\_1, R\_2$ to derive an expression for the average power delivered to each resistor by the ac voltage source.\\  (a) The total power delivered to $R\_1$ and $R\_2$:  \begin{equation} \label{p1}  P\_1 = P\_{1,dc} + P\_{1,ac}  \end{equation}  \begin{equation} \label{p2}  P\_2 = P\_{2,dc} + P\_{2,ac}  \end{equation}  The dc power delivered to the resistors whose resistances are $R\_1$ and $R\_2$:  $$P\_{1,dc} = \frac{\mathcal{E}\_2^2}{R\_1} \text{ and } P\_{2,dc} = \frac{\mathcal{E}\_2^2}{R\_2}$$  We express the average ac power delivered to $R\_1$:  $$P\_{1,ac} = \frac{\mathcal{E}\_{1,rms}^2}{R\_1} = \frac{\mathcal{E}\_{1,peak}^2}{2R\_1}$$  Applying Kirchhoff's loop rule clockwise to the loop includes $R\_1, L, R\_2$:  $$R\_1I\_1 - Z\_2I\_2 = 0$$  We solve for $I\_2$:  $$I\_2 = \frac{R\_1}{Z\_2}I\_1 = \frac{R\_1 \mathcal{E}\_{1,peak}}{Z\_2R\_1} = \frac{\mathcal{E}\_{1,peak}}{Z\_2}$$  We express teh average ac power delivered to $R\_2$:  $$P\_{2,ac} = \frac{1}{2}I\_{2,rms}^2R\_2 = \frac{1}{2}\left( \frac{\mathcal{E}\_{1,peak}}{Z\_2} \right)^2 R\_2 = \frac{\mathcal{E}\_{1,peak}^2 R\_2}{2Z\_2^2}$$  Substituting in equations \ref{p1} and \ref{p2}:  $$P\_1 = \frac{\mathcal{E}\_2^2}{R\_1} + \frac{\mathcal{E}\_{1,peak}^2}{2R\_1}$$  $$P\_2 = \frac{\mathcal{E}\_2^2}{R\_2} + \frac{\mathcal{E}\_{1,peak}^2 R\_2}{2Z\_2^2}$$  Substituting numerical values and evaluate P:  $$P\_1 = \frac{(16V)^2}{10\Omega} + \frac{(20V)^2}{2(10\Omega)} = \fbox{46W}$$  $$P\_2 = \frac{(16V)^2}{8\Omega} + \frac{(20V)^2(8.0\Omega)}{2[(8\Omega)^2 + (2\pi \{100s^{-1}\}\{6.0mH\})^2]} = \fbox{52W}$$  (b) We proceed as in (a) to evaluate $P\_1$ and $P\_2$ with $f = 200 Hz$:  $$P\_1 = 25.6W + 20.0W = \fbox{46W}$$  $$P\_2 = 32.0W + 13.2W = \fbox{45W}$$  (c) We proceed as in (a) to evaluate $P\_1$ and $P\_2$ with $f = 800 Hz$:  $$P\_1 = 25.6W + 20.0W = \fbox{46W}$$  $$P\_2 = 32.0W + 1.64W = \fbox{34W}$$ |
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| 43 - The voltage across the resistor can be found using a phasor diagram. The phasor diagram is shown. Using the Pythagorean theorem we can express $V\_R$:  $$V\_R = \sqrt{\mathcal{E}\_{rms}^2} - V\_L^2 = \sqrt{(100V)^2 - (80V)^2} = \fbox{60V}$$ |
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| 44 - The phasor diagram for the RC high-pass filter is shown. $\vec V\_{app}$ and $\vec V\_R$ are the phasors for $V\_{in}$ and $V\_{out}$ respectively. Note that $\tan \delta = -X\_C / R.$ That $\delta$ is negative follows from the fact that $\vec V\_{app}$ lags $\vec V\_R$ by $\lvert \delta \rvert$. The projection of $\vec V\_{app}$ onto the horizontal axis is $V\_{app} = V\_{in},$ and the projection of $\vec V\_R$ onto the horizontal axis is $V\_R = V\_{out}$. We start by expressing $V\_{app}$:  $$V\_{app} = V\_{app,peak} \cos \omega t \text{ where } V\_{app,peak} = I\_{peak}Z$$  \begin{equation}\label{rsq}  \text{and } Z^2 = R^2 + X\_C^2  \end{equation}  And as $\delta < 0$:  $$\omega t + \lvert \delta \rvert = \omega t - \delta$$  $V\_R$ is given by:  $$V\_R = V\_{R,peak} \cos (\omega t - \delta) \text{ where } V\_{R,peak} = V\_H = I\_{peak}R$$  Then we solve the equation \ref{rsq} for Z and substitute for $X\_C$:  \begin{equation}\label{zsq}  Z = \sqrt{R^2 + \left( \frac{1}{\omega C} \right)^2}  \end{equation}  Now, because $V\_{out} = V\_R$ we can express:  $$V\_{out} = V\_{R,peak} = \cos (\omega t - delta) = I\_{in,peak}R\cos(\omega t - \delta) = \frac{V\_{in peak}}{Z}R \cos (\omega t - \delta)$$  We now use equation \ref{zsq} to substitute for Z:  $$V\_{out} = \frac{V\_{in peak}}{\sqrt{R^2 + \left( \frac{1}{\omega C} \right)^2}}R \cos (\omega t - \delta)$$  Simplifying this expression:  $$V\_{out} = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^{-2}}} \cos (\omega t - \delta) = \fbox{$V\_H \cos (\omega t - \delta)$}$$  $$\text{where } V\_H = \frac{V\_{in peak}}{\sqrt{1+(\omega RC)^{-2}}}$$  As $\omega \rightarrow \infty$:  $$V\_H \rightarrow \frac{V\_{in peak}}{\sqrt{1+(0)^2}} = V\_{in peak}$$  showing that the result is consistent with the highpass name for this circuit. |
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| 45 - The phasor diagram for the RC high-pass filter is shown. $\vec V\_{app}$ and $V\_R$ are the phasors for $V\_{in}$ and $V\_{out}$, respectively. The projection of $\vec V\_{app}$ onto the horizontal axis is $V\_{app} = V\_{in}$, and the projection of $\vec V\_R$ onto the horizontal axis is $V\_R = V\_{out}$.\\  (a) $\vec V\_{app}$ lags $V\_R$ by $\delta$:  $$\tan \delta = -\frac{V\_C}{V\_R} = -\frac{IX\_C}{IR} = -\frac{X\_C}{R}$$  Using the definition of $X\_C$ we obtain:  $$\tan \delta = -\frac{\frac{1}{\omega C}}{R} = -\frac{1}{\omega RC}$$  and solving for $\delta$:  $$\delta = \fbox{$\tan^{-1} \left[ -\frac{1}{\omega RC} \right]$}$$  (b) As $\omega \rightarrow 0$:  $$\delta \rightarrow \fbox{$-90^{\circ}$}$$  (c) As $\omega \rightarrow \infty$:  $$\delta \rightarrow \fbox{0}$$ |
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| 46 - The results found in problems 44 and 45 can be used to find $f\_{3dB}$ and to plot graphs of $log(V\_{out})$ versus $log(f)$ and $\delta$ versus $log(f)$.\\  (a) Using the result of problem 44 to express the ratio $V\_{out} / V\_{in peak}$:  $$\frac{V\_{out}}{V\_{in peak}} = \frac{\frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^{-2}}}}{V\_{in peak}} = \frac{1}{\sqrt{1 + (\omega RC)^{-2}}}$$  And now when $V\_{out} = V\_{in peak} / \sqrt{2}$:  $$\frac{1}{\frac{1}{\sqrt{1 + (\omega RC)^{-2}}}} = \frac{1}{\sqrt{2}}$$  Squaring both sides of the equation and solving for $\omega RC$ to obtain:  $$\omega RC = 1 \Rightarrow \omega = \frac{1}{RC} \Rightarrow f\_{3dB} = \frac{1}{2\pi RC}$$  We now substitute numerical values and evaluate:  $$f\_{3dB} = \frac{1}{2\pi (20k\Omega)(15nF)} = \fbox{0.53kHz}$$  (b) Using result from problem 44 we have:  $$V\_{out} = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^{-2}}}$$  And in problem 45 we showed that:  $$\delta = \tan^{-1} \left[ -\frac{1}{\omega RC} \right]$$  We rewrite these expressins in terms of $f\_{3dB}$:  $$V\_{out} = \frac{V\_{in peak}}{\sqrt{1 + \left( \frac{1}{2\pi fRC} \right)^2}} = \frac{V\_{peak}}{\sqrt{1 + \left( \frac{f\_{3dB}}{f}\right)^2}}$$  and  $$\delta = \tan^{-1} \left[ -\frac{1}{2\pi fRC} \right] = \tan^{-1} \left[ -\frac{f\_{3dB}}{f} \right]$$  A spreadsheet program to generate the data for a graph of $V\_{out}$ versus $f$ and $\delta$ versus $f$ is shown, including formulas used to calculate the quantities in columns:  \begin{tabular}{| l | c | c |}  \hline  Cel & Formula/Content & Algebraic Form \\  \hline  B1 & 2.00E+03 & R \\  \hline  B2 & 1.50E-08 & C \\  \hline  B3 & 1 & $V\_{in peak}$\\  \hline  B4 & 531 & $f\_{3dB}$\\  \hline  A8 & 53 & $0.1f\_{3db}$\\  \hline  C8 & \$B\$3/SQRT(1+(1(\$B\$4/A8))\^{} 2) & $\frac{V\_{in peak}}{\sqrt{1 + \left( \frac{f\_{3dB}}{f} \right)^2}}$\\  \hline  D8 & LOG(C8) & $log(V\_{out}$\\  \hline  E8 & ATAN(-\$B\$4/A8) & $\tan^{-1}\left[ -\frac{f\_{3dB}}{f} \right]$\\  \hline  F8 & E8\*180/PI() & $\delta$ in degress\\  \hline  \end{tabular}  The graph of $log(V\_{out}$ versus $log(f)$ is shown for $V\_{in peak} = 1V$.\\  A graph of $\delta$ in degrees as a function of $log(f)$ is shown.\\  As shown by the spreadsheet program, we can see that when $f = f\_{3dB}$, $\delta \approx \fbox{$-44.9^{\circ}$}$. This result agrees with its calculated value of $\fbox{$-45.0^{\circ}$}$ |
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| 47 - Using the Kirchhoff's loop rule we can obtain a differential equation relating the input, capacitor, and resistor voltages. Because the voltage drop across the ressistor is small compared to the voltage drop across the capacitor, we can express the voltage drop across the capacitor in terms of the input voltage.\\  First apply Kirchhoff's loop rule to the input side of the filter:  $$V(t) - V\_C -IR = 0$$  where $V\_C$ is the potential difference across the capacitor.\\  The we substitute for $V(t)$ and I to obtain:  $$V\_{in peak}\cos \omega t - V\_C - R\frac{dQ}{dt} = 0$$  And because $Q = CV\_C$:  $$\frac{dQ}{dt} = \frac{d}{dt}[CV\_C] = C\frac{dV\_C}{dt}$$  Substituting we obtain:  $$V\_{peak} \cos \omega t - V\_C - RC\frac{dVC}{dt} = 0$$  the differential equation describing the potential difference across the capacitor.\\  There is no significant change in the voltage signal during one time constant, so we can express:  $$\frac{dV\_C}{dt} = 0 \Rightarrow RC\frac{dV\_C}{dt} = 0$$  Substituting for $RC\frac{dV\_C}{dt}$ yields:  $$V\_{in peak} \cos \omega t - V\_C = 0$$  and  $$V\_C = V\_{in peak}\cos \omega t$$  As a consequence, the potential difference across the resistor is given by:  $$V\_R = RC\frac{dV\_C}{dt} = \fbox{$RC\frac{d}{dt}[V\_{in peak}\cos \omega t]$}$$ |
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| 48 - We use the expression found for $V\_H$ in problem 44 and the definition of $\beta$ given in the problem statement to show that every time the frequency is halved, the output drops by 6 dB.\\  This equation we have from problem 44:  $$V\_H = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^{-2}}}$$  or  $$\frac{V\_H}{V\_{in peak}} = \frac{1}{\sqrt{1 + (\omega RC)^{-2}}}$$  We need to express this ratio in terms of $f$ and $f\_{3dB}$, we simplify as well:  $$\frac{V\_H}{V\_{peak}} = \frac{1}{\sqrt{1 + \left( \frac{f\_{3dB}}{f} \right)^2}} = \frac{f}{\sqrt{f\_{3dB}^2\left( \frac{f^2}{f\_{3dB}^2} \right)}}$$  For $f << f\_{3dB}$:  $$\frac{V\_H}{V\_{peak}} \approx \frac{f}{\sqrt{f\_{3dB}^2 \left( 1 + \frac{f^2}{f\_{3dB}^2} \right)}} = \frac{f}{f\_{3dB}}$$  Using the $\beta$ definition we have:  $$\beta = 20log\_{10} \left( \frac{V\_H}{V\_{peak}} \right)$$  Substituting for $V\_H / V\_{peak}$ we obtain:  $$\beta = 20log\_{10} \left( \frac{f}{f\_{3dB}} \right)$$  If we double the frequency we obtain:  $$\beta' = 20log\_{10} \left( \frac{\frac{1}{2}f}{f\_{3dB}} \right)$$  The decibel level change is given by:  $$\Delta \beta = \beta' - \beta = 20 log\_{10}\left(\frac{\frac{1}{2}f}{f\_{3dB}}\right) - 20log\_{10}\left(\frac{f}{f\_{3dB}}\right) = 20log\_{10}(1/2) \approx \fbox{-6dB}$$ |
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| 49 - The instantaneous power dissipated in the resistor can be expressed, then using the fact that the average value of the square of the cosine function over one cycle is half to establish the given result.\\  The instantaneous power P(t) dissipated in the resistor is:  $$P(t) = \frac{V\_{out}^2}{R}$$  The output voltage is:  $$V\_{out} = V\_H\cos (\omega t - \delta)$$  Taking from problem 44:  $$V\_H = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^{-2}}}$$  Substitute in the expression for P(t):  $$P(t) = \frac{V\_H^2}{R} \cos (\omega t - \delta) = \frac{V\_{in peak}^2}{R[1 + (\omega RC)^{-2}]} \cos (\omega t - \delta)$$  And taking in consideration that the average value of the square of the cosine function over one cycle is one half:  $$P\_{av} = \fbox{$\frac{V\_{in peak}^2}{2R[1 + (\omega RC)^{-2}]}$}$$ |
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| 50 - We should solve the expression for $V\_H$ from problem 44 for the required capacitance of the capacitor.\\  Taking the expression from problem 44:  $$V\_H = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^{-2}}}$$  And we do require that:  $$\frac{V\_H}{V\_{in peak}} = \frac{1}{\sqrt{1 + (\omega RC)^{-2}}} = \frac{1}{10}$$  or  $$\sqrt{1 + (\omega RC)^{-2}} = 10$$  If we do solve for C yields:  $$C = \frac{1}{\sqrt{99}\omega R} = \frac{1}{2\pi \sqrt{99}Rf}$$  Now we finally substitute numerical values and evaluate:  $$C = \frac{1}{2\pi(20k\Omega)(60Hz)} = \fbox{13nF}$$ |
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| 51 - The phasor diagram for the RC low-pass filter shows that $\vec V\_{app}$ and $\vec V\_C$ are the phasors for $V\_{in}$ and $V\_{out}$ respectively. The projection of $\vec V\_{app}$ onto the horizontal axis is $V\_{app} = V\_{in}$, the projection of $\vec V\_C$ onth the horizontal axis is $V\_C = V\_{out}$, $V\_{peak} = \lvert \vec V\_{app} \rvert$, and $\phi$ is the angle between $\vec V\_C$ and the horizontal axis.\\  (a)  We express $V\_{app}$:  $$V\_{app} = V\_{in peak} \cos \omega t \text{ where } V\_{in peak} = I\_{peak}Z$$  \begin{equation}\label{Zand}  \text{ and } Z^2 = R^2 + X\_C^2  \end{equation}  $V\_{out} = V\_C$ is given by:  $$V\_{out} = V\_{C,peak} = \cos \phi = I\_{peak}X\_C \cos \phi$$  Now we define $\delta$ as shown in the phasor diagram:  $$V\_{out} = I\_{peak}X\_C \cos (\omega t - \delta) = \frac{V\_{in peak}}{Z}X\_C \cos (\omega t - \delta)$$  Solving equation \ref{Zand} for Z and substituting $X\_C$:  \begin{equation}\label{zsqrt}  Z = \sqrt{R^2 + \left( \frac{1}{\omega C} \right)^2}  \end{equation}  We can use equation \ref{zsqrt} to substitute for Z and substitute for $X\_C$:  $$V\_{out} = \frac{V\_{in peak}}{\sqrt{R^2 + \left( \frac{1}{\omega C} \right)^2}} \left( \frac{1}{\omega C} \right) \cos (\omega t - \delta)$$  If we simplify further we can obtain:  $$V\_{out} = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^2}} \cos (\omega t - \delta)$$  or  $$V\_{out} = \fbox{$V\_L \cos (\omega t - \delta)$} \text{ where } V\_L = \fbox{$\frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^2}}$}$$  (b) We can note that as $\omega \rightarrow 0, V\_L \rightarrow V\_{peak}$. This is physically feasible as for low frequencies $X\_C$ is large and hence a larger peak input voltage will appear across it than appears across it for high frequencies.\\  Yet another note is that as $\omega \rightarrow \infty, V\_L \rightarrow 0$. This is physically feasible since, for high frequencies, $X\_C$ is small and, hence, a smaller peak voltage will appear across it than appears it for low frequencies.\\  Remarks: In Figures 29-19 and 29-20, $\delta$ is defined as the phase of the  voltage drop across the combination relative to the voltage drop across the resistor. |
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| 52 - The phasor diagram for the RC low-pass filter is shown. $\vec V\_{app}$ and $\vec V\_C$ are the phasors for $V\_{in}$ and $V\_{out}$ respectively. The projection of $\vec V\_{app}$ onto the horizontal axis is $V\_{app} = V\_{in}$ and the projection of $\vec V\_C$ onto the horizontal axis is $V\_C = V\_{out}$. $V\_{peak} = \lvert \vec V\_{app} \rvert$.\\  (a) Analyzing the phasor diagram we have:  $$\tan \delta = \frac{V\_R}{V\_C} = \frac{I\_{peak}R}{I\_{peak}X\_C} = \frac{R}{X\_C}$$  and from the $X\_C$ definition we obtain:  $$\tan \delta = \frac{R}{\frac{1}{\omega C}} = \omega RC$$  Solving for $\delta$:  $$\delta = \fbox{$\tan^{-1}(\omega RC)$}$$  (b) As $\omega \rightarrow 0, \delta \rightarrow \fbox{$0^{\circ}$}$. This behaviour makes sense physically in that, at low frequencies, $X\_C$ is very large compared to R and, as a consequence, $V\_C$ is in phase with $V\_{in}$.\\  As $\omega \rightarrow \infty, \delta \rightarrow \fbox{$90^{\circ}$}$. This behaviour makes sense physically in that, at high frequencies, $X\_C$ is very small compared to R and, as a consequence, $V\_C$ is out of phase with $V\_{in}$\\  Remarks: See the spreadsheet solution in the following problem for additional evidence that our answer for Part (b) is correct. |
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| 53 - In problems 51 and 52 we derived expressions for $V\_L$ and $\delta$ to plot graphs of $V\_L$ versus $f$ and $\delta$ versus $f$ for the low-pass filter of problem 51. We express $V\_L$ and $\delta$ as functions of $f\_{3dB}$ to simplify the spreadsheet program.\\  From problems 51 and 52 we have:  $$V\_L = \frac{V\_{in peak}}{\sqrt{1 + (\omega RC)^2}}$$  and  $$\delta = \tan^{-1} (\omega RC)$$  We express these expressions in terms of $f$ and obtain:  $$V\_L = \frac{V\_{in peak}}{\sqrt{1 + (2\pi fRC)^2}}$$  and  $$\delta = \tan^{-1} (2\pi fRC)$$  In the table we show a spreadsheet program to generate the data for graphs of $V\_L$ versus $f$ and $\delta$ versus $f$ for the low-pass filter. $V\_{in peak}$ has been arbitrarily set equal to 1. The formulas used to calculate the quantities are shown in the columns:\\  \begin{tabular}{| l | c | c |}  \hline  Cel & Formula/Content & Algebraic Form \\  \hline  B1 & 2.00E+03 & R \\  \hline  B2 & 1.50E-09 & C \\  \hline  B3 & 1 & $V\_{in peak}$\\  \hline  B8 & \$B\$3/SQRT(1+((2\*PI()\*A8\*1000\*\$B\$1\*\$B\$2)\^{}2)) & $\frac{V\_{in peak}}{\sqrt{1 + (2\pi fRC)^2}}$\\  \hline  C8 & ATAN(2\*PI()\*A8\*1000\*\$B\$1\*\$B\$2) & $\tan^{-1} (2\pi fRC)$\\  \hline  D8 & C8\*180/PI() & $\delta$ in degrees\\  \hline  \end{tabular}  A graph of $V\_L$ as function of $f$ is shown:\\  A graph of $\delta$ as a function of $f$ is shown:\\ |
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| 54 - The Kirchhoff's loop rule can be used to find a differential equation relating the input, capacitor, and resistor voltages. Then we assume a solution for this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution of these simultaneous equations will yield the amplitude of the output voltage.\\  First we apply Kirchhoff's loop rule to the input side of the filter to obtain $V(t) - IR V\_C = 0$, where $V\_C$ is the potential difference across the capacitor.\\  We substitute for $V(t)$ and I:  $$V\_{in peak} \cos \omega t - R\frac{dQ}{dt} - V\_C = 0$$  And because $Q = CV\_C$:  $$\frac{dQ}{dt} = \frac{d}{dt}[CV\_C] = C\frac{dV\_C}{dt}$$  Sustituting for $dQ / dt$:  $$V\_{in peak} \cos \omega t - RC \frac{dV\_C}{dt} - V\_C = 0$$  the differential equation describing the potential difference across the capacitor.\\  $V\_C$ is given by $V\_C = IX\_C = \frac{1}{\omega C}$. $V(t)$ varies rapidly meaning that $\omega >> 1$, therefore, $V\_C \approx 0$, and:  $$V\_{peak} \cos \omega t -RC \frac{dV\_C}{dt} = 0$$  Separating variables in the differential equation and solve for $V\_C$:  $$V\_C = \fbox{$\frac{1}{RC}\int V\_{peak} \cos \omega t dt$}$$ |
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| 55 - The phasor diagram for the trap filter is shown. $\vec V\_{app}$ and $\vec V\_L + \vec V\_C$ are the phasors for $V\_{in}$ and $V\_{out}$ respectively. The projection of $\vec V\_{app}$ onto the horizontal axis is $V\_{app} = V\_{in}$, and the projection of $\vec V\_L + \vec V\_C$ onto the horizontal axis is $V\_L + V\_C = V\_{out}$. We assume the impedance of the trap to be zero, then the frequency at which the circuits rejects signals will be shown. If we define $\Delta \omega = \lvert \omega - \omega\_{trap} \rvert$ and do we require that $\lvert Z\_{trap} \rvert = R$ will yield and expression for the bandwidth and reveal its dependence on R.\\  Expressing $V\_{app}$:  $$V\_{app} = V\_{app, peak} \cos \omega t$$  where $V\_{app, peak} = V\_{peak} = I\_{peak}Z$ and  \begin{equation}\label{zpeaks}  Z^2 = R^2 + (X\_L - X\_C)^2  \end{equation}  On the other hand, $V\_{out}$ is given by:  $$V\_{out} = V\_{out, peak} \cos (\omega t - \delta)$$  where $V\_{out, peak} = I\_{peak}Z\_{trap}$ and $Z\_{trap} = X\_L - X\_C$.\\  Solving for Z in equation \ref{zpeaks} yields:  \begin{equation}\label{zsqrt}  Z^2 = \sqrt{R^2 + (X\_L - X\_C)^2}  \end{equation}  And because $V\_{out} = V\_L + V\_C$:  $$V\_{out} = V\_{out, peak} \cos (\omega t - \delta) = I\_{peak}Z\_{trap} \cos (\omega t - \delta)$$  $$= \frac{V\_{peak}}{Z}Z\_{trap} \cos (\omega t - \delta)$$  We can use equation \ref{zsqrt} to substitute for Z:  $$V\_{out} = \frac{V\_{peak}}{\sqrt{R^2 + Z\_{trap}^2}} Z\_{trap} \cos (\omega t - \delta)$$  Provided that $Z\_{trap} = 0$, we note that $V\_{out} = 0$, then set $Z\_{trap} = 0$ and obtain $Z\_{trap} = X\_L - X\_C = 0$. Substituting for $X\_L$ and $X\_C$ yields:  $$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \fbox{$\frac{1}{\sqrt{LC}}$}$$  (b) We have as bandwith:  \begin{equation}\label{bwt}  \Delta \omega = \lvert \omega - \omega\_{trap} \rvert  \end{equation}  We use bandwith definition involving the frequency at which $\lvert Z\_{trap} \rvert = R$. Then:  $$\omega L - \frac{1}{\omega C} = R \Rightarrow \omega^2 LC - 1 = \omega RC$$  And because $\omega\_{trap} = 1 / \sqrt{LC}$:  $$\left( \frac{\omega}{\omega\_{trap}} \right)^2 - 1 = \omega RC$$  We solve for $\omega^2 - \omega\_{trap}^2$:  $$\omega^2 - \omega\_{trap}^2 = (\omega - \omega\_{trap})(\omega + \omega\_{trap})$$  And because $\omega \approx \omega\_{trap}, \omega + \omega\_{trap} \approx 2\omega\_{trap}$:  $$\omega^2 - \omega\_{trap}^2 \approx 2\omega\_{trap}(\omega - \omega\_{trap})$$  Now substitute in equation \ref{bwt} to obtain:  $$\Delta \omega = \lvert \omega - \omega\_{trap} \rvert = \frac{RC\omega\_{trap}^2}{2} = \fbox{$\frac{R}{2L}$}$$ |
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| 56 - The output voltage will mirror input voltage minus a 0.60 V drop for voltages greater than 0.60 V. When the voltage is below 0.60 V, the output voltage will be zero. A spreadsheet program was utilized to plot the following graph. The peak voltage and angular frequency were both arbitrarily set equal to one.\\ |
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| 57 - The time constant for the RC circuit and the frequency of the input signal can be related through the use of the potential difference decay across the capacitor. An approximate value for C can be found by expanding the exponential factor in the expression for $V\_C$. The C value will limit the variation in the output voltage by less than 50 percent.\\  We find the voltage across the capacitor:  $$V\_C = V\_{in} e^{-t / RC}$$  If we expend the exponential factor:  $$e^{-t / RC} \approx 1 - \frac{1}{RC}t$$  And for a decay of less than 50 percent:  $$1 - \frac{1}{RC}t \le 0.5 \Rightarrow C \le \frac{2}{R}t$$  And finally, the voltage is positive at every cycle, $t = 1 / 60 s$:  $$C \le \frac{2}{1.00k\Omega}\left( \frac{1}{60}s \right) = \fbox{$33\mu F$}$$ |
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| 58 - The current lags the voltage across the inductor and leads the voltage across a capacitor. If we use $I\_{L, peak} = \mathcal{E}\_{peak} / X\_L$ and $I\_{C, peak} = \mathcal{E}\_{peak}X\_C$ to find the amplitudes for these currents. The current in the generator is zero under resonance conditions, that is, when $\lvert I\_L \rvert = \lvert I\_C \rvert$. For finding the currents in the inductor and capacitor at resonance, we can use the shared potential difference across them and their reactances together with our knowledge of the phase relationships just mentioned.\\  (a) We do express the currents amplitudes through the inductor and capacitor:  $$I\_{L\_ peak} = \frac{\mathcal{E}\_{peak}}{X\_L} = \frac{\mathcal{E}\_{peak}}{2\pi fL}$$  and  $$I\_{C\_ peak} = \frac{\mathcal{E}\_{peak}}{X\_C} = \frac{\mathcal{E}\_{peak}}{\frac{1}{2\pi fC}} = 2 \pi fC\mathcal{E}\_{peak}$$  Substituting numerical values:  $$I\_{L, peak} = \frac{100V}{(4.00H)2 \pi f} = \fbox{$\frac{25.0V/H}{2 \pi f}, \text{ lagging } \mathcal{E} \text{ by } 90^{\circ}$}$$  and  $$I\_{C, peak} = (25.0 \mu F)(100V) \omega = \fbox{$(2.5mV \cdot F)2 \pi f, \text{ leading } \mathcal{E} \text{ by } 90^{\circ}$}$$  (b) We do express the condition that $I = 0$:  $$\lvert I\_L \rvert = \lvert I\_C \rvert \text{ or } \frac{\mathcal{E}}{\omega L} = \frac{\mathcal{E}}{\frac{1}{\omega C}} = \omega C\mathcal{E} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$  Now we substitute numerical values and evaluate $\omega$:  $$\omega = \frac{1}{\sqrt{(4.00H)(25.0 \mu F)}} = \fbox{100 rad/s}$$  We use numerical values expressing the current in the inductor at $\omega = \omega\_0$:  $$I\_L = \left( \frac{25.0V/H}{100s^{-1}} \right) \cos \left( \omega t - \frac{\pi}{2} \right) = \fbox{$(250mA) \cos \left( \omega t - \frac{\pi}{2} \right)$}$$  where $\omega = 100 rad / s$  We also express the current in the capacitor at $\omega = \omega\_0$:  $$I\_C = (2.5mV \cdot F)(100s^{-1}) \cos \left( \omega t + \frac{\pi}{2} \right)$$  $$= \fbox{$-(250mA) \cos \left( \omega t + \frac{\pi}{2} \right)$}$$  where $\omega = 100 rad / s$.\\  (d) We show the phasor diagram for the case where the inductive reactance is larger than the capacitive reactance. |
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| 59 - If we differentiate with respect to time we can find I as a function of time. For (b), C can be found using $\omega = 1 / \sqrt{LC}$. The energy stored in the magnetic field of the inductor is given by $U\_m = \frac{1}{2}LI^2$ and the energy stored in the electric field of the capacitor is given by $U\_e = \frac{1}{2}\frac{Q^2}{C}$.\\  (a) We differentiate the charge with respect to time for obtaining the current:  $$I(t) = \frac{dQ}{dt} = \frac{d}{dt}\left[ (15 \mu C) \cos \left( \omega t + \frac{\pi}{4} \right) \right] = -(15 \mu C)(1250s^{-1}) \sin \left( \omega t + \frac{\pi}{4} \right)$$  $$-(18.75mA) \sin \left( \omega t + \frac{\pi}{4} \right) = \fbox{$-(19mA) \sin \left( \omega t + \frac{\pi}{4} \right)$}$$  where $\omega = 1250 rad / s$\\  (b) We have a relation for C, L and $\omega$, solving for C and evaluating at numerical values:  $$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(1250s^{-1})(28mH)} = 22.86 \mu F = \fbox{$23 \mu F$}$$  (c) For the magnetic energy at time t we have:  $$U\_m(t) = \frac{1}{2}LI^2 = \frac{1}{2}(28mH)(18.75mA)^2 \sin^2 \left( \omega t + \frac{\pi}{4} \right) = \fbox{$(4.9 \mu J) \sin^2 \left( \omega t + \frac{\pi}{4} \right)$}$$  where $\omega = 1250 rad / s$.\\  We now find the electrical energy stored in the capacitor $U\_e = \frac{1}{2} \frac{Q^2}{C}$ as a function of time:  $$U\_e(t) = \frac{1}{2} \frac{(15 \mu F)^2}{22.86 \mu F} \cos^2 \left( \omega t + \frac{\pi}{4} \right)$$  $$= (4.92 \mu J) \cos^2 \left( \omega t + \frac{\pi}{4} \right)$$  where $\omega = 1250 rad / s$.\\  The total energy sum stored in electric and magnetic field is the sum of $U\_e(t)$ and $U\_m(t)$:  $$U = (4.92 \mu J) \sin^2 \left( \omega t + \frac{\pi}{4} \right) + (4.92 \mu J) \cos^2 \left( \omega t + \frac{\pi}{4} \right) = \fbox{$4.9 \mu J$}$$ |
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| 60 - The capacitance of a dielectric field capacitor definition and the expression for the resonance frequency of an LC circuit can be used to derive an expression for the fractional change in the thicknes of the dielectric in terms of the resonance frequency and the frequency of the circuit when the dielectric is under copression. Afterwards, we can use this expression for $\Delta t / t$ to calculate the Young's modulus for the dielectric material.\\  We begin by using th definition for the Young's modulus of the dielectric material:  \begin{equation}\label{my}  Y = \frac{stress}{strain} = \frac{\Delta P}{\Delta t / t}  \end{equation}  We let t to be the initial thickness of the dielectric, expressing the initial capacitance of the capacitor:  $$C\_0 = \frac{\kappa \epsilon\_0 A}{t}$$  We express the capacitance of the capacitor when it is under compression:  $$C\_C = \frac{\kappa \epsilon\_0 A}{t - \Delta t}$$  Now we express the resonance frequency of the capacitor before the dielectric is compressed:  $$\omega\_0 = \frac{1}{\sqrt{C\_0L}} = \frac{1}{\sqrt{\frac{\kappa \epsilon\_0 AL}{t}}}$$  And now when the dielectric is compressed:  $$\omega\_C = \frac{1}{\sqrt{C\_CL}} = \frac{1}{\sqrt{\frac{\kappa \epsilon\_0 AL}{t - \Delta t}}}$$  Simplifying by expressing the ratio of $\omega\_C$ and $\omega\_0$:  $$\frac{\omega\_C}{\omega\_0} = \frac{\sqrt{\frac{\kappa \epsilon\_0 AL}{t}}}{\sqrt{\frac{\kappa \epsilon\_0 AL}{t - \Delta t}}} = \sqrt{1 - \frac{\Delta t}{t}}$$  We can expand the radical binomially to obtain:  $$\frac{\omega\_C}{\omega\_0} = \left( 1 - \frac{\Delta t}{t} \right)^{1/2} \approx 1 - \frac{\Delta t}{2t}$$  provided that $\Delta t << t$.\\  Solving for $\Delta t / t$:  $$\frac{\Delta t}{t} = 2 \left( 1 - \frac{\omega\_c}{\omega\_0} \right)$$  Subsituting in equation \ref{my}:  $$Y = \frac{\Delta P}{2 \left( 1 - \frac{\omega\_c}{\omega\_0} \right)}$$  And finally using the numerical values we evaluate Y:  $$Y = \frac{(800atm)(102.325kPa/atm)}{2 \left( 1 - \frac{116MHz}{120MHz} \right)} = \fbox{$1.22 \times 10^9 N / m^2$}$$ |
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| 61 - The capacitor can be modeled as the equivalent of two capacitors connected in parallel. Let $C\_1$ by the capcacitance of the dielectric-filled capacitor and $C\_2$ be the air-filled capacitor. We will derive expressions for each capacitance and then add them together to obtain $C(x)$. We can then use the given resonance frequency when $x = w / 2$ and the given value for L to evaluate $C\_0$. In part (b) we can use our result for $C(x)$ and the relationship between $f, L$ and $C(x)$ at resonance to express $f(x)$.\\  (a) Let's express the equivalent capacitance of the two capacitors in parallel:  \begin{equation}\label{cx}  C(x) = C\_1 + C\_2 = \frac{\kappa \epsilon\_0 A\_1}{d} + \frac{\epsilon\_0 A\_2}{d}  \end{equation}  We need to express $A\_2$ in terms of the total area of a capacitor plate A, w, and the distance x:  $$\frac{A\_2}{A} = \frac{x}{w} \Rightarrow A\_2 = A \frac{x}{w}$$  We do express $A\_1$ in terms of A and $A\_2$:  $$A\_1 = A - A\_2 = A \left( 1 - \frac{x}{w} \right)$$  Substituting in equation \ref{cx} and simplifying:  $$C(x) = \frac{\kappa \epsilon\_0 A}{d}\left( 1 - \frac{x}{w} \right) + \frac{\epsilon\_0 A x}{d w}$$  $$= \frac{\epsilon\_0 A}{d} \left[ \kappa \left( 1 - \frac{x}{w} \right) + \frac{x}{w} \right] = \kappa C\_0 \left[ 1 - \frac{\kappa - 1}{kw} x \right]$$  where $C\_0 = \frac{\epsilon\_0 A}{d}$  We find $C(w / 2)$:  $$C \left( \frac{w}{2} \right) = \kappa C\_0 \left[ 1- \frac{(\kappa - 1)w}{2kw} \right] = \kappa C\_0 \left[ 1 - \frac{\kappa - 1}{2\kappa} \right] = C\_0 \frac{\kappa + 1}{2}$$  Now for the resonance frequency of the circuit in terms of L and $C(x)$ we express:  \begin{equation}\label{fx}  f(x) = \frac{1}{2 \pi \sqrt{LC(x)}}  \end{equation}  We do evaluate $f(w / 2)$:  $$f \left( \frac{w}{2} \right) = \frac{1}{2 \pi \sqrt{LC\_0 \frac{\kappa + 1}{2}}} = \frac{1}{2 \pi} \sqrt{\frac{2}{(\kappa + 1)LC\_0}}$$  Solving for $C\_0$ we obtain:  $$C\_0 = \frac{1}{2 \pi^2 f^2 \left( \frac{w}{2} \right)L(\kappa + 1)}$$  Now we substitute numerical values and evaluate $C\_0$:  $$C\_0 = \frac{1}{2 \pi^2 (90MHz)^2 \left( \frac{20cm}{2} \right)(2.0mH)(4.8 + 1)} = \fbox{5.4fF}$$  (b) Substitute for $C(x)$ in equation \ref{fx}:  $$f(x) = \frac{1}{2 \pi \sqrt{L\kappa C\_0 \left[ 1 - \frac{\kappa - 1}{\kappa w} x \right]}}$$  And finally substitute numerical values and evaluate $f(x)$:  $$f(x) = \frac{1}{2 \pi \sqrt{(2.0mH)(4.8)(5.39 \times 10^{-16}F) \left[ 1 - \frac{4.8 - 1}{4.8(0.20m)} x \right]}} = \fbox{$\frac{70MHz}{\sqrt{1-(4.0m^{-1})x}}$}$$  \section{Driven RLC Circuits} |
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| 62 - In the diagram is shown the relationship between $\delta, X\_L, X\_C, R$. This reference triangle can be used to express the power factor for the given circuit. In (b) we can find the rms current from the rms potential difference and the impedance of the circuit. The rms current and the resistance of the resistor can be used to find the average power delivered by the source.\\  (a) The power factor is defined as:  $$\cos \delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X\_L - X\_C)^2}}$$  We have no inductance in the circuit, therefore $X\_L = 0$ and  $$\cos \delta = \frac{R}{\sqrt{R^2 + X\_C^2}} = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$  Substituting numerical values to evaluate $\cos \delta$:  $$\cos \delta = \frac{80\Omega}{\sqrt{(80\Omega)^2 + \frac{1}{(400s^{-1})^2(20 \mu F)^2}}} = \fbox{0.54}$$  (b) For this exercise, express the rms current in the circuit:  $$I\_{rms} = \frac{\mathcal{E}\_{rms}}{Z} = \frac{\frac{\mathcal{E}\_{max}}{\sqrt{2}}}{\sqrt{R^2 + X\_C}}$$  $$\frac{\mathcal{E}\_{max}}{\sqrt{2} \sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$  Substituting numerical values and evaluate $I\_{rms}$:  $$I\_{rms} = \frac{20V}{\sqrt{2}\sqrt{(80\Omega)^2 + \frac{1}{(400s^{-1})^2(20 \mu F)^2}}} = 95.3mA = \fbox{95mA}$$  (c) The average power that the generator delivers is $P\_{av} = I\_{rms}^2R$, substituting numerical values and evaluating $P\_{av}$:  $$P\_{av} = (95.3mA)^2(80\Omega) = \fbox{0.73W}$$ |
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| 63 - $Z = \sqrt{R^2 + (X\_L - X\_C)^2}$ gives the immpedance of an ac circuit. If we make $X\_L = X\_C = 0$ and then $R = 0$, we can evaluate the impedance expression for $P\_{av}$.\\  (a) For $X = 0, Z = R$:  $$P\_{av} = \frac{R\mathcal{E}\_{rms}^2}{Z^2} = \frac{R\mathcal{E}\_{rms}^2}{R^2} = \fbox{$\frac{R\mathcal{E}\_{rms}^2}{R}$}$$  (b) and (c). If $R = 0$, then:  $$P\_{av} = \frac{R\mathcal{E}\_{rms}^2}{Z^2} = \frac{(0)\mathcal{E}\_{rms}^2}{(X\_L - X\_C)^2} = \fbox{0}$$  Remarks: in an ideal inductor or capacitor there is no energy dissipation. |
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| 64 - The resonant frequency of the circuit can be found using $\omega\_0 = 1 / \sqrt{LC}$, the rms current at resonance can be found using $I\_{rms} = \mathcal{E}\_{rms} / R$. We can find the reactances at $\omega = 8000 rad / s$, by using the definitions of $X\_C, X\_L$. The definitions of $Z, I\_{rms}$ to find the impedance and rms current at $\omega = 8000 rad / s$, and the definition of the phase angle to find $\delta$.\\  (a) We begin by expressing the resonant frequency $\omega\_0$ in terms of L and C and substitute numerical values, evaluating:  $$\omega\_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10mH)(2.0 \mu F)}} = \fbox{$7.1 \times 10^3 rad / s$}$$  (b) We need to find a relationship between the rms current at resonance and $\mathcal{E}\_{rms}$ and the impedance of the circuit at resonance as well:  $$I\_{rms} = \frac{\mathcal{E}\_{rms}}{R} = \frac{\mathcal{E}\_{max}}{\sqrt{2}R} = \frac{100V}{\sqrt{2}()5.0\Omega} = \fbox{14A}$$  (c) We do express and evaluate $X\_C$ and $X\_L$ at $\omega = 8000 rad / s$:  $$X\_C = \frac{1}{\omega C} = \frac{1}{(8000s^{-1})(2.0 \mu F)} = 62.50 \Omega = \fbox{$63 \Omega$}$$  and  $$X\_L = \omega L = (8000 s^{-1})(10 mH) = \fbox{$80\Omega$}$$  (d) The impedance needs to be expressed in terms of reactances, and substitute results from (c) and evaluate Z:  $$Z^2 = \sqrt{R^2 + (X\_L - X\_C)^2} = \sqrt{(5.0 \Omega)^2 + (80 \Omega - 62.5 \Omega)^2} = 18.2 \Omega = \fbox{$18 \Omega$}$$  (e) We need to relate the rms current at $\omega = 8000 rad / s$ to $\mathcal{E}\_{rms}$ and the impedance of the circuit at this same frequency:  $$I\_{rms} = \frac{\mathcal{E}\_{rms}}{Z} = \frac{\mathcal{E}\_{max}}{\sqrt{2}Z} = \frac{100V}{\sqrt{2}(18.2 \Omega)} = \fbox{3.9 A}$$  (f) We find $\delta$ as follows:  $$\delta = \tan^{-1} \left( \frac{X\_L - X\_C}{R} \right) = \tan^{-1} \left( \frac{80 \Omega - 62.5 \Omega}{5.0 \Omega} \right) = \fbox{$74^{\circ}$}$$ |
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| 65 - $Q = \omega L / R$ gives us the Q factor of the circuit, the resonance width by $\Delta f = f\_0 / Q = \omega\_0 / 2 \pi Q$, and the power factor by $\cos \delta = R / Z$. Z is frequency dependent, hence we need to find $X\_C$ and $X\_L$ at $\omega = 8000 rad / s$ to be able to evaluate $\cos \delta$.\\  Let's use above definitions to express Q factor and resonance width of the circuit:  \begin{equation}\label{cu}  Q = \frac{\omega\_0 L}{R}  \end{equation}  and  \begin{equation}\label{delf}  \Delta f = \frac{f\_0}{Q} = \frac{\omega\_0}{2 \pi Q}  \end{equation}  (a) We begin by expressin the resonance frequency for the circuit:  $$\omega\_0 = \frac{1}{\sqrt{LC}}$$  Substituting in equation \ref{cu}:  $$Q = \frac{1}{\sqrt{LC} R} = \frac{1}{r}\sqrt{\frac{L}{C}}$$  Substituting numerical values and evaluating:  $$Q = \frac{1}{5.0 \Omega} \sqrt{\frac{10 mH}{2.0 \mu F}} = 14.1$$  Substitute numerical values in equation \ref{delf} and evaluate:  $$\Delta f = \frac{7.07 \times 10^3 rad / s}{2 \pi (14.1)} = \fbox{80 Hz}$$  (c) The power factor of the circuit is given by:  $$\cos \delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X\_L - X\_C)^2}} = \frac{R}{\sqrt{R^2 + \left( \omega L - \frac{1}{\omega C}\right)^2}}$$  Substituting numerical values an evaluating for $\cos \delta$:  $$\cos \delta = \frac{5.0 \Omega}{\sqrt{(5.0 \Omega)^2 + \left( (8000s^{-1})(10 mH) - \frac{1}{(8000s^{-1})(2.0 \mu F)} \right)^2}} = \fbox{0.27}$$ |
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| 66 - The Q factor for the circuit can be found from its definition $Q = f\_0 / \Delta f$. Only substitute the numerical values.  $$Q = \frac{100.1 MHz}{0.050 MHz} \approx \fbox{$2.0 \times 10^3$}$$ |
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| 67 - From the current definition $I\_{peak} = \mathcal{E}\_{peak} / Z$ to find the current in the coil and the definition of the phase angle to evaluate $\delta$. We can equate $X\_L$ and $X\_C$ to find the capacitance required so that the current and the voltage are in phase. $V\_C =IX\_C$ can be used to find the measured voltage across the capacitor.\\  (a) We express the current in the coil in terms of the potential difference across it and its impedance:  $$I\_{peak} = \frac{\mathcal{E}\_{peak}}{Z} = \frac{100 V}{10 \Omega} = \fbox{10 A}$$  (b) The phase angle $\delta$ is given by the next definition, and substitute numerical values to evaluate:  $$\delta = \cos^{-1} \left( \frac{R}{Z} \right) = \sin^{-1} \left( \frac{X\_L}{Z} \right) = \sin^{-1} \left( \frac{8.0 \Omega}{10 \Omega} \right) = \fbox{$53^{\circ}$}$$  (c) Expressing the condition on the on the reactances that must be satisfied if the current and voltage are to be in phase:  $$X\_L = X\_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X\_L} = \frac{1}{2 \pi fX\_L}$$  Substituting numerical values:  $$C = \frac{1}{2 \pi (60s^{-1})(8.0 \Omega)} = 332 \mu F = \fbox{0.33 mF}$$  (d) Expressing the potential difference across the capacitor:  $$V\_C = I\_{peak}X\_C$$  And relate the peak current in the circuit to the impedance of the circuit when $X\_L = X\_C$:  $$I\_{peak} = \frac{V\_{peak}}{R}$$  Substituting for the current:  $$V\_C = \frac{V\_{peak}X\_C}{R} = \frac{V\_{peak}}{2 \pi fCR}$$  Now we need to relate the impedance of the circuit to the resistance of the coil:  $$Z = \sqrt{R^2 + X^2} \Rightarrow R = \sqrt{Z^2 - X^2}$$  We substitute for R:  $$V\_C = \frac{V\_{peak}}{2 \pi fC \sqrt{Z^2 - X^2}}$$  Finally substitute numerical values and evaluate:  $$V\_C = \frac{100 V}{2 \pi (60s^{-1})(332 \mu F)\sqrt{(10 \Omega)^2 - (8.0 \Omega)^2}} = \fbox{0.13 kV}$$ |
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| 68 - $V\_C = I\_{rms}X\_C$ and $I\_{rms}$ from the potential difference across the inductor can serve to find C. If the resistance in the circuit vanishes, the measured rms voltage across both the capacitor and inductor is $V = \lvert V\_L - V\_C \rvert$.\\  (a) We find a relationship between the capacitance C and the potential difference across the capacitor:  $$V\_C = I\_{rms}X\_C = \frac{I\_{rms}}{2 \pi fC} \Rightarrow C = \frac{I\_{rms}}{2 \pi fV\_C}$$  Using the potential difference across the inductor to express the rms current in the circuit:  $$I\_{rms} = \frac{V\_L}{X\_L} = \frac{V\_L}{2 \pi fL}$$  Substituting for the $I\_{rms}$ and using numerical values:  $$C = \frac{V\_L}{(2 \pi f)^2LV\_C} = \frac{50 V}{[2 \pi (60s^{-1})]^2(0.25 H)(75 V)} = \fbox{$19 \mu F$}$$  (b) We need to express the measured rms voltage V across both the capacitor and the inductor when $R = 0$:  $$V = \lvert V\_L - V\_C \rvert$$  Finally we substitute numerical values and evaluate:  $$V = \lvert 50 V - 75 V \rvert = \fbox{25 V}$$ |
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| 69 - The potential differences across each of the circuit elements can be found by the rms current in the circuit. Phasor diagrams and our knowledge of the phase shifts between the voltages across the three circuit elements to find the voltage diferrences across their combinations.\\  (a) We do express the potential diffenrence between points A and B in terms of $I\_{rms}$ and $X\_L$:  \begin{equation}\label{voltab}  V\_{AB} = I\_{rms}X\_L  \end{equation}  Expressing $I\_{rms}$ in terms of $\mathcal{E}$ and Z:  $$I\_{rms} = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X\_L - X\_C)^2}}$$  We evaluate $X\_L$ and $X\_C$ to obtain:  $$X\_L = 2 \pi fL = 2 \pi (60s^{-1})(137 mH) = 51.648 \Omega$$  and  $$X\_C = \frac{1}{2 \pi fC} = \frac{1}{2 \pi (60s^{-1})(25 \mu F)} = 106.10 \Omega$$  We evaluate for $I\_{rms}$:  $$I\_{rms} = \frac{115 V}{\sqrt{(50 \Omega)^2 + (51.648 \Omega - 106.10 \Omega)^2}} = 1.5556 A$$  Substitute numerical values in equation \ref{voltab} and evaluate:  $$V\_{AB} = (1.5556 A)(51.648 \Omega) = 80.344 V = \fbox{80V}$$  (b) We express the potential difference between points B and C in terms of $I\_{rms}$ and R:  $$V\_{BC} = I\_{rms}R = (1.5556 A)(50 \Omega) = 77.780 V = \fbox{78 V}$$  (c) Expressing the potential difference between points C and D in terms of $I\_{rms}$ and $X\_C$:  $$V\_{CD} = I\_{rms}X\_C = (1.5556 A)(106.10 \Omega) = 165.05 V = \fbox{0.17 kV}$$  (d) The voltage across the inductor leads the voltage across the resistor as shown in the phasor diagram. We can use the Pythagorean theorem to find $V\_{AC}$  $$V\_{AC} = \sqrt{V\_{AB}^2 + V\_{BC}^2} = \sqrt{(80.0 V)^2 + (77.780 V)^2} = 111.58 V = \fbox{0.11 kV}$$  (e) The voltage across the capacitor lags the voltage across the resistor as shown in the phasor diagram and use the Pythagorean to find $V\_{BD}$:  $$V\_{BD} = \sqrt{V\_{CD}^2 + V\_{BC}^2}$$  $$= \sqrt{(165.05 V)^2 + (77.780 V)^2} = \fbox{182.46 V}$$ |
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| 70 - We can find the power supplied to the circuit by $P\_{av} = \mathcal{E}\_{rms}I\_{rms}\cos \delta$ and the resistance can be found by $P\_{av} = I\_{rms}^2R$. In (c) the impedance, inductive reactance, and resistance can be related to the capacitive reactance and solve for the capacitance C. We can use the condition on $X\_L$ and $X\_C$ at resonance to find the capacitance or inductance you would need to add to the circuit to make the power factor equal to 1.\\  (a) We begin by expressing the power supplied to the circuit in terms of $\mathcal{E}\_{rms}, I\_{rms}$ and the power factor $\cos \delta$ and substitute numerical values:  $$P\_{av} = \mathcal{E}\_{rms}I\_{rms} \cos \delta = (120 V)(11 A) \cos 45^{\circ} = 933 W$$  (b) We relate the resistance to the power dissipated in the circuit:  $$P\_{av} = I\_{rms}^2 \Rightarrow R = \frac{P\_{av}}{I\_{rms}^2}$$  Substituting numerical values and evaluating:  $$R = \frac{933 W}{(11 A)^2} = 7.71 \Omega$$  (c) Expressing the capacitance of the capacitor in terms of its reactance:  \begin{equation}\label{cfr}  C = \frac{1}{\omega X\_C} = \frac{1}{2 \pi f X\_C}  \end{equation}  We need to relate the capacitive reactance to the impedance, inductive reactance and resistance of the circuit:  $$Z^2 = R^2 + (X\_L - X\_C)^2$$  Expressing the impedance of the circuit in terms of the rms emf $\mathcal{E}$ and the rms current:  $$Z^2 = \frac{\mathcal{E}^2}{I\_{rms}^2}$$  And equating these expressions yields:  $$\frac{\mathcal{E}^2}{I\_{rms}^2} = R^2 + (X\_L -X\_C)^2$$  We need to solve for $\lvert X\_L - X\_C \rvert$:  $$\lvert X\_L - X\_C \rvert = \sqrt{\frac{\mathcal{E}^2}{I\_{rms}^2} - R^2}$$  I leads $\mathcal{E}$, then the circuit is capacitive and $X\_C > X\_L$.  $$\lvert X\_L - X\_C \rvert = -(X\_L - X\_C)$$  and  $$X\_C = X\_L + \sqrt{\frac{\mathcal{E}^2}{I\_{rms}^2} - R^2} = 2 \pi fL + \sqrt{\frac{\mathcal{E}^2}{I\_{rms}^2} - R^2}$$  Substituting numerical values and evaluating:  $$X\_C = 2 \pi (60s^{-1})(50 mH) + \sqrt{\frac{(120 V)^2}{(11 A)^2} - (7.71 \Omega)^2} = 26.6 \Omega$$  We substitute in equation \ref{cfr} and evaluate C:  $$C = \frac{1}{2 \pi (60s^{-1})(26.6 \Omega)} = 99.9 \mu F = 0.10 mF$$  (d) We'll represent the capacitance required to make $\cos \delta = 1$ as $C\_{pf = 1}$. The necessary change in capacitance is given by:  $$\Delta C = C\_{pf = 1} - C = C\_{pf = 1} - 99.9 \mu F$$  We find a relationship between $C\_{pf = 1}$ and $X\_L$:  $$X\_L = \frac{1}{2 \pi fC\_{pf = 1}}$$  Solving for $C\_{pf = 1}$  $$C\_{pf = 1} = \frac{1}{2 \pi fX\_L}$$  Substituting for $C\_{pf = 1}$ in the expression for $\Delta C$:  $$\Delta C = \frac{1}{2 \pi fX\_L} - 99.9 \mu F$$  Substituting for numefical values and evaluating:  $$\Delta C = \frac{1}{2 \pi (60s^{-1})(18.8 \Omega)} - 99.9 \mu F = 41 \mu F$$  (e) We represent the inductance required to make $\cos \delta = 1$ as $L\_{pf = 1}$. The necessary change in inductance is given by:  $$\Delta L = L\_{pf = 1} - L = L\_{pf = 1} - 50 mH$$  Relating $L\_{pf = 1}$ to $X\_C$:  $$X\_L = X\_C = 2 \pi fL\_{pf = 1}$$  Solving it:  $$L\_{pf = 1} = \frac{X\_C}{2 \pi f}$$  Substitute for $L\_{pf = 1}$ in the expression for $\Delta L$:  $$\Delta L = \frac{X\_C}{2 \pi f} - 50 mH$$  Substituting numerical values and evaluating:  $$\Delta L = \frac{26.6 \Omega}{2 \pi (60s^{-1})} - 50 mH = 20 mH$$ |
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| 71 - The figures for the impedance of the three circuits are shown.\\  Also shown in each figure is the asymptotic approach for large angular frequencies. |
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| 72 - The maximum current in the circuit can be found from the maximum voltage across the capacitor and the reactance of the capacitor. To find the range of inductance that is safe to use we can express $Z^2$ for the circuit in terms of $\mathcal{E}\_{peak}^2$ and $I\_{peak}^2$ and solve the resulting quadratic equation for L.\\  (a) We express the peak current in terms of the maximum potential difference across the capacitor and its resistance:  $$I\_{peak} = \frac{V\_{C, peak}}{X\_C} = \omega CV\_{C, peak}$$  Substituting numerical values:  $$I\_{peak} = (2500 rad / s)(8.00 \mu F)(150 V) = 3.00 A$$  (b) We do relate the maximum current in the circuit to the emf of the source and the impedance of the circuit:  $$I\_{peak} = \frac{\mathcal{E}\_{peak}}{Z} \Rightarrow Z^2 = \frac{\mathcal{E}\_{peak}^2}{I\_{peak}^2}$$  Expressing $Z^2$ in terms of $R, X\_L, X\_C$  $$Z^2 = R^2 + (X\_L - X\_C)^2$$  Substituting:  $$\frac{\mathcal{E}\_{peak}^2}{I\_{peak}^2} = R^2 + (X\_L - X\_C)^2$$  Evaluating $X\_C$:  $$X\_C = \frac{1}{\omega C} = \frac{1}{(2500 rad / s)(8.00 \mu F)} = 50.0 \Omega$$  We now substitute numerical values:  $$\frac{(200 V)^2}{(3.00 A)^2} = (60.0 \Omega)^2 + ((2500 rad / s)L - 50.0 \Omega)^2$$  Solving for L:  $$L = \frac{50.0 \Omega \pm \sqrt{844 \Omega^2}}{2500 rad / s}$$  We can denote solutions as $L\_{+}, L\_{-}$, and find the values for the inductance:  $$L\_{+} = 31.6 mH \text{ and } L\_{-} = 8.38 mH$$  The ranges for L:  $$\fbox{$8.00 mH < L < 8.38 mH$}$$  and  $$\fbox{$31.6 mH < L < 40.0 mH$}$$ |
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| 73 - The impedance of the circuit can be found from the applied emf and the current drawn by the device. For (b) we can use $P\_{av} = I\_{rms}^2R$ to find R and the definition of impedance of a series RLC circuit to find $X = X\_L - X\_C$.\\  (a) We express the impedance in terms of the emf provided by the power line and the current, we substitute numerical values:  $$Z = \frac{\mathcal{E}\_{rms}}{I\_{rms}} = \frac{120 V}{10 A} = \fbox{$12 \Omega$}$$  (b) We express R using the relationship between the average power supplied to the device and the rms current it draws:  $$P\_{av} = I\_{rms}^2R \Rightarrow R = \frac{P\_{av}}{I\_{rms}^2}$$  Substituting numerical values:  $$R = \frac{720 W}{(10 A)^2} = \fbox{$7.20 \Omega$}$$  The impedance of a series RLC circuit is given by:  $$Z = \sqrt{R^2 + (X\_L - X\_C)^2}$$  or  $$Z^2 = R^2 + (X\_L - X\_C)^2$$  Solving for $X\_L - X\_C$ yields:  $$X = X\_L - X\_C = \sqrt{Z^2 - R^2}$$  Substituting numerical values and evaluating:  $$X = \sqrt{(12 \Omega)^2 - (7.20 \Omega)^2} = \fbox{$10 \Omega$}$$  (c) The reactance is capacitive if the current leads the emf. |
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| 74 - (a) The fact that when the current is maximum, $X\_L = X\_C$, can be used to find the inductance of the circuit. (b) $\mathcal{E}\_{peak}$ and the impedance of the circuit at resonance can give $I\_{rms, max}$.\\  (a) Relate $X\_L$ and $X\_C$ at resonance:  $$X\_L = X\_C \text{ or } \omega\_0 L = \frac{1}{\omega\_0 C}$$  Solve for L:  $$L = \frac{1}{\omega\_0^2C}$$  Substituting numerical values and evaluating:  $$L = \frac{1}{(5000s^{-1})^2(10 \mu F)} = \fbox{4.0 mH}$$  (b) At resonance, $X = 0$, then express $I\_{rms, max}$ in terms of the applied emf and the impedance of the circuit at resonance:  $$I\_{rms, max} = \frac{\mathcal{E}\_{rms}}{Z} = \frac{\mathcal{E}\_{max}}{\sqrt{2}Z} = \frac{10 V}{\sqrt{2}(100 \Omega)} = \fbox{71 mA}$$ |
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| 75 - The capacitor and resistor are connected in parallel, then the voltage drops across then are equal. The total current is the sum of the current through the capacitor and the current through the resistor. These currents are not in phase, we need to calculate phasors to calculate their sum. The amplitudes of the applied voltage and the currents are equal to the magnitude of the phasors, that is $\lvert \mathcal{\vec E} \rvert = \mathcal{E}\_{peak}$, $\lvert \vec I \rvert = I\_{peak}$, $\lvert \vec I\_R \rvert = I\_{R, peak}$, and $\lvert \vec I\_C \rvert = I\_{C, peak}$.\\  (a) $\mathcal{E} = \mathcal{E}\_{peak} \cos \omega t$ is the voltage applied by the source. Thus, the voltage drop across both the load resistor and the capacitor is:  $$\mathcal{E}\_{peak} \cos \omega t = I\_RR$$  The current in the resistor is in phase with the applied voltage:  $$I\_R = I\_{R, peak} \cos \omega t$$  and because $I\_{R, peak} = \frac{\mathcal{E}\_{peak}}{R}$:  $$I\_R = \frac{\mathcal{E}\_{peak}}{R} \cos \omega t$$  (b) The current in the capacitor leads the applied voltage by 90 degrees:  $$I\_C = I\_{C, peak} \cos (\omega t + 90^{\circ})$$  And because $I\_{C, peak} = \frac{\mathcal{E}\_{peak}}{X\_C}$:  $$I\_C = \fbox{$\frac{\mathcal{E}\_{peak}}{X\_C} \cos (\omega t + 90^{\circ})$}$$  (c) The net current I is the sum of the currents through the parallel branches:  $$I = I\_R + I\_C$$  We need to draw a phasor diagram for the circuit. The projections of the phasors onto the horizontal axis are the instantaneous values. The current in the resistor is in phase with the applied voltage, and hte current in the capacitor leads the applied voltage by 90 degrees. The net current phasor is the sum of the branch current phasors.\\  The peak current throught the parallel combination is $\mathcal{E}\_{peak} / Z$, where Z is the impedance of the combination:  $$I = I\_{peak} \cos (\omega t - \lvert \delta \rvert)$$,  where  $$I\_{peak} = \frac{\mathcal{E}\_{peak}}{Z}$$  Analyzing the phasor diagram we do have:  $$I\_{peak}^2 = I\_{R, peak}^2 + I\_{C, peak}^2 = \left( \frac{\mathcal{E}\_{peak}}{R} \right)^2 + \left( \frac{\mathcal{E}\_{peak}}{X\_C} \right)^2$$  $$= \mathcal{E}\_{peak}^2 \left( \frac{1}{R^2} + \frac{1}{X\_C^2} \right) = \frac{\mathcal{E}\_{peak}^2}{Z^2}$$  where  $$\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X\_C^2}$$  Solving for $I\_{peak}$ yields:  $$I\_{peak} = \fbox{$\frac{\mathcal{E}\_{peak}}{Z}$} \text{ where } Z^{-2} = R^{-2} + X\_C^{-2}$$  And from the phasor diagram:  $$I = \fbox{$I\_{peak} \cos (\omega t + \delta)$}$$  where  $$\tan \delta = \frac{I\_C}{I\_R} = \frac{\frac{\mathcal{E}\_{peak}}{X\_C}}{\frac{\mathcal{E}\_{peak}}{R}} = \fbox{$\frac{R}{X\_C}$}$$ |
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| 76 - Determining the half power points using the condition can be used to tobtain the quadratic equation that we can solve for the frequencies corresponding to the half power points. Let $\omega\_1$ be the half power frequency that is less than $\omega\_0$ and $\omega\_2$ be the half power frequency that is greater than $\omega\_0$ will lead us to the result that $\Delta \omega = \omega\_2 - \omega\_1 \approx R / L$. We then can use the Q definition to complete the proof that $Q \approx \omega\_0 / \Delta \omega$.\\  Equation 29-56 is:  $$P\_{av} = \frac{V\_{app,rms}^2R\omega^2}{L^2(\omega^2 - \omega\_0^2) + \omega^2R^2}$$  When the denominator of equation 29-56 is twice the value near resonance, the half-power points occur:  $$L^2(\omega^2 - \omega\_0^2)^2 + \omega^2R^2 \approx 2 \omega\_0^2 R^2$$  or  $$L^2[(\omega - \omega\_0)(\omega + \omega\_0)]^2 + \omega^2 R^2 \approx 2 \omega\_0^2 R^2$$  For a sharply peaked resonance, $\omega + \omega\_0 \approx 2 \omega\_0$. Hence:  $$L^2[(\omega - \omega\_0)(2 \omega\_0)]^2 + \omega^2 R^2 \approx 2 \omega\_0^2 R^2$$  or  $$4 \omega\_0^2L^2(\omega - \omega\_0)^2 + \omega^2 R^2 \approx 2 \omega\_0^2 R^2$$  We represent with $\omega\_1$ a solution to this equation. And we note that for a sharply peaked resonance, $\omega\_1 \approx \omega\_0$, it follows:  $$4 \omega\_0^2L^2(\omega - \omega\_0)^2 + \omega^2 R^2 \approx 2 \omega\_0^2 R^2$$  if we simplify it:  $$(\omega - \omega\_0)^2 \approx \frac{R^2}{4L^2}$$  and solving for $\omega\_1$:  $$\omega\_1 \approx \omega\_0 - \frac{R}{2L}$$  where we used the minus sign because $\omega\_1 < \omega\_0$.\\  Similarly for $\omega\_2$  $$\omega\_2 \approx \omega\_0 - \frac{R}{2L}$$  where we used the plus sign because $\omega\_2 > \omega\_0$.\\  Evaluating $\Delta \omega = \omega\_2 - \omega\_1$:  $$\Delta \omega \approx \omega\_0 + \frac{R}{2L} - \left( \omega\_0 - \frac{R}{2L} \right) = \fbox{$\frac{R}{L}$}$$  Using the Q definition:  $$\frac{R}{L} = \frac{\omega\_0}{Q}$$  Substituting in the expression for $\Delta \omega$:  $$\Delta \omega \approx \frac{\omega\_0}{Q} \Rightarrow Q \approx \fbox{$\frac{\omega\_0}{\Delta \omega}$}$$ |
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| 77 - From the equation $Q = Q\_0 e^{-t / \tau} \cos \omega' t$ we can start and differentiate twice and then substitute this function and both its derivatives in the differential equation of the circuit. Then we can re-write the resulting equation in the form $A \cos \omega' t + B \sin \omega' t = 0$ will reveal that B is equal to zero. If we require that $A \cos \omega' t = 0$ to hold for all values of t will lead to the fact that $\omega' = \sqrt{1 / (LC) - 1 / \tau^2}$.\\  Taking equation 29-43b:  $$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$  We do assume a solution of the form:  $$Q = Q\_0e^{-t / \tau} \cos \omega' t$$  Differentiating twice Q(t):  $$\frac{dQ}{dt} = Q\_0 \frac{d}{dt}[e^{-t / \tau} \cos \omega' t] = Q\_0 \left( e^{-t / \tau} \frac{d}{dt} \cos \omega' t + \cos \omega' t \frac{d}{dt} e^{-t / \tau} \right)$$  $$= Q\_0 e^{-t / \tau} \left( -\omega' \sin \omega' t - \frac{1}{\tau} \cos \omega' t \right)$$  and  $$\frac{d^2Q}{dt^2} = Q\_0 \frac{d}{dt}\left[ e^{-t / \tau} \left( -\omega' \sin \omega' t - \frac{1}{\tau} \cos \omega' t \right) \right]$$  $$= Q\_0 e^{-t / \tau} \left[ \left( \frac{1}{\tau^2} -\omega'^2 \right) \cos \omega' t +\frac{2 \omega'}{\tau} \sin \omega' t \right]$$  We substitute these derivatives in the differential equation and simplify:  $$LQ\_0 e^{-t / \tau} \left[ \left( \frac{1}{\tau^2} -\omega'^2 \right) \cos \omega' t +\frac{2 \omega'}{\tau} \sin \omega' t \right] + RQ\_0e^{-t / \tau} \left( -\omega' \sin \omega' t - \frac{1}{\tau} \cos \omega' t \right)$$  $$+ \frac{1}{C}Q\_0e^{-t / \tau} \cos \omega' t = 0$$  Now, $Q\_0$ and $e^{-t / \tau}$ are never zero, so we can divide them out of the equation and simplify:  $$L \left( \frac{1}{\tau^2} -\omega'^2 \right) \cos \omega' t +\frac{2 L \omega'}{\tau} \sin \omega' t - \omega' R \sin \omega' t - \frac{R}{\tau} \cos \omega' t +\frac{1}{C} \cos \omega' t = 0$$  We can re-write this equation in the form $A \cos \omega' t + B \sin \omega' t = 0$:  $$(R \omega' - R \omega') \sin \omega' t + \left[ L \left( \frac{1}{\tau^2} - \omega'^2 \right) + \frac{1}{C} - \frac{R}{\tau} \right] \cos \omega' t = 0$$  or  $$\left[ L \left( \frac{1}{\tau^2} - \omega'^2 \right) + \frac{1}{C} - \frac{R}{\tau} \right] \cos \omega' t = 0$$  If this equation holds for all values of t, then its coefficient must be zero:  $$L \left( \frac{1}{\tau^2} - \omega'^2 \right) + \frac{1}{C} - \frac{R}{\tau} = 0$$  We now have to solve for $\omega'$  $$\omega' = \fbox{$\sqrt{\frac{1}{LC} - \left( \frac{1}{2L} \right)^2}$}$$  the condition that must be satisfied if $Q = Q\_0e^{-t / \tau} \cos \omega' t$ is the solution to equation 29-43b |
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| 78 - The inductance of the empty solenoid can be determined by using $L = \mu\_0 n^2 A\ell$ and the resonance condition to find the capacitance of the sample-free circuit when the resonance frequency of the circuit is 6.00 MHz. We can evaluate $\chi$ from its definition, if we express L as a function of $f\_0$ and then evaluate $df\_0 / dL$ and approximating the derivative with $\Delta f\_0 / \Delta L$.\\  (a) We express the inductance of an air-core solenoid:  $$L = \mu\_0 n^2 A\ell$$  Substituting numerical values and evaluating:  $$L = (4 \pi \times 10^{-7}N / A^2)\left( \frac{400}{4.00 cm} \right)^2 \frac{\pi}{4} (3.00 cm)^2(4.00 cm) = \fbox{3.553 mH}$$  (b) Expressing the condition for resonance in the LC circuit:  \begin{equation}\label{xlxc}  X\_L = X\_C \Rightarrow 2 \pi f\_0 L = \frac{1}{2 \pi f\_0 C}  \end{equation}  If we solve for C, it yields:  $$C = \frac{1}{4 \pi^2 f\_0^2 L}$$  Substituting numerical values and evaluating:  $$C = \frac{1}{4 \pi^2 (6.00 MHz)^2(3.553 mH)} = 1.9803 \times 10^{-13}F = \fbox{0.198 pF}$$  (c) We express the sample's susceptibility in terms of L and $\Delta L$:  \begin{equation}\label{chi}  \chi = \frac{\Delta L}{L}  \end{equation}  Now solve equation \ref{xlxc} for $f\_0$:  $$f\_0 = \frac{1}{2 \pi \sqrt{LC}}$$  Differentiating $f\_0$ with respect to L:  $$\frac{df\_0}{dL} = \frac{1}{2 \pi \sqrt{C}} \frac{d}{dL}L^{-1/2} = -\frac{1}{4 \pi \sqrt{C}}L^{-3/2}$$  $$= -\frac{1}{4 \pi L \sqrt{LC}} = -\frac{f\_0}{2L}$$  We make an approximation of $df\_0 / dL$ by $\Delta f\_0 / \Delta L$:  $$\frac{\Delta f\_0}{\Delta L} = -\frac{f\_0}{2L} \text{ or } \frac{\Delta f\_0}{f\_0} = -\frac{\Delta L}{2L}$$  Substituting in equation \ref{chi}:  $$\chi = -2\frac{\Delta f\_0}{f\_0}$$  Finally we substitute numerical values and evaluate:  $$\chi = -2 \left( \frac{5.9989 MHz - 6.0000 MHz}{6.0000 MHz} \right) = \fbox{$3.7 \times 10^{-4}$}$$ |
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| 79 - We'll use the subscript 1 to denote the primary and 2 the secondary. These relations can be used to find the turns ratio and the primary current when the transformer connections are reversed $V\_2N\_1 = V\_1N\_2$ and $N\_1I\_1 = N\_2I\_2$.\\  (a) Find the relationship between the number of primary and secondary turns to the primary and secondary voltages:  $$V\_{2,rms}N\_1 = V\_{1,rms}N\_2$$  Solve and evaluate for the ratio $N\_2 / N\_1$:  $$\frac{N\_2}{N\_1} = \frac{V\_{2,rms}}{V\_{1,rms}} = \frac{24 V}{120 V} = \fbox{$\frac{1}{5}$}$$  (b) We then relate the current in the primary to the current in the secondary and to the turns ratio:  $$I\_{1,rms} = \frac{N\_2}{N\_1}I\_{2,rms}$$  Then we express the current in the primary winding in terms of the voltage across it and its impedance:  $$I\_{2,rms} = \frac{V\_{2,rms}}{Z\_2}$$  Substituting for $I\_{2,rms}$ we obtain:  $$I\_{1,rms} = \frac{N\_2}{N\_1}\frac{V\_{2,rms}}{Z\_2}$$  Finally we substitute numerical values and evaluate:  $$I\_1 = \left( \frac{1}{5} \right) \left( \frac{120 V}{12 \Omega} \right) = \fbox{2 A}$$ |
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| 80 - We use subscript 1 for the primary and 2 for the secondary. The transformer is step-up or step-down depending on the examination of the ratio of the number of turns in the secondary to the number of turns in the primary. The open-circuit rms voltage in the secondary can be related to the primary rms voltage and the turns ratio.\\  (a) There are fewer turns in the secondary than in the primary, hence it is a step-down transformer.\\  (b) We need to relate open-circuit rms voltages, in the primary and secondary:  $$V\_{2,rms} = \frac{N\_1}{N\_1}V\_{1,rms}$$  Substituting numerical values and evaluating:  $$V\_{2,rms} = \frac{8}{400}(120 V) = \fbox{2.40 V}$$  There are no power losses, so we can write:  $$V\_{1,rms}I\_{1,rms} = V\_{2,rms}I\_{2,rms}$$  and  $$I\_{2,rms} = \frac{V\_{1,rms}}{V\_{2,rms}}I\_{1,rms}$$  Substituting numerical values and evaluating:  $$I\_{2,rms} = \frac{120 V}{2.40 V}(0.100 A) = \fbox{5.00 A}$$ |
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| 81 - We use subscript 1 for the primary and 2 for the secondary. $V\_{1,rms}I\_{1,rms} = V\_{2,rms}I\_{2,rms}$ can be used to find the current in the primary and $V\_{2,rms}N\_1 = V\_{1,rms}N\_2$ to find the number of turns in the secondary.\\  (a) We have 100 percent efficiency, therefore:  $$V\_{1,rms}I\_{1,rms} = V\_{2,rms}I\_{2,rms}$$  and  $$I\_{1,rms} = I\_{2,rms}\frac{V\_{2,rms}}{V\_{1,rms}}$$  Substituting numerical values and evaluating:  $$I\_{1,rms} = (20 A)\frac{9.0 V}{120 V} = \fbox{1.5 A}$$  (b) We need a relationship between the number of primary and secondary turns to the primary and secondary voltages:  $$V\_{2,rms}N\_1 = V\_{1,rms}N\_2 \Rightarrow N\_2 = \frac{V\_{2,rms}}{V\_{1,rms}}N\_1$$  Substituting numerical values and evaluating:  $$N\_2 = \frac{9.0 V}{120 V}(250) \approx \fbox{19}$$ |
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| 82 - For a simple circuit, a maximum power transfer from the source requires that the load resistance equal the internal resistance of the source. By Ohm's law and the relationship between the primary and secondary currents and the primary and secondary voltages and the turns ration of the transformer we can derive an expression for the turns ratio as a function of the effective resistance of the circuit and the resistance of the speakers.\\  (a) We begin by expressing hte effective loudspeaker resistance at the primary of the transformer:  $$R\_{eff} = \frac{V\_{1,rms}}{I\_{1,rms}}$$  Then we relate the $V\_{1,rms}$ to $V\_{2,rms}, N\_1$ and $N\_2$:  $$V\_{1,rms} = V\_{2,rms} \frac{N\_1}{N\_2}$$  Expressing the $I\_{1,rms}$ in terms of $I\_{2,rms}, N\_1, N\_2$:  $$I\_{1,rms} = I\_{2,rms} \frac{N\_2}{N\_1}$$  Substituting for $V\_{1,rms}, I\_{1,rms}$ and simplifying:  $$R\_{eff} = \frac{V\_{2,rms} \frac{N\_1}{N\_2}}{I\_{2,rms} \frac{N\_2}{N\_1}} = \left( \frac{V\_{2,rms}}{I\_{2,rms}} \right) \left( \frac{N\_1}{N\_2} \right)^2$$  Solving for $N\_1 / N\_2$:  \begin{equation}\label{n1n2}  \frac{N\_1}{N\_2} = \sqrt{\frac{I\_{2,rms}R\_{eff}}{V\_{2,rms}}} = \sqrt{\frac{R\_{eff}}{R\_2}}  \end{equation}  Evaluating for $R\_{eff} = R\_{coil}$:  $$\frac{N\_1}{N\_2} = \sqrt{\frac{2000 \Omega}{8.00 \Omega}} = \fbox{15.811}$$  (b) We then express the power delivered to the two speakers connected in parallel:  \begin{equation}\label{psp}  P\_{sp} = I\_{1,rms}^2R\_{eff}  \end{equation}  Finding the equivalent resistance $R\_{sp}$ of the two $8.00 \Omega$ speakers in parallel:  $$\frac{1}{R\_{sp}} = \frac{1}{8.00 \omega} + \frac{1}{8.00 \omega} \Rightarrow R\_{sp} = 4.00 \Omega$$  We now solve equation \ref{n1n2}:  $$R\_{eff} = R\_2 \left( \frac{N\_1}{N\_2} \right)^2$$  Substituting numerical values and evaluating:  $$R\_{eff} = (4.00 \Omega)(15.811)^2 = 1 k\Omega$$  We need to find the current supplied by the source:  $$I\_{1,rms} = \frac{V\_{rms}}{R\_{tot}} = \frac{12.0 V}{2000 V + 1000 V} = 4.00 mA$$  Finally we substitute numerical values in equation \ref{psp} and evaluate the power delivered to the parallel speakers:  $$P\_{sp} = (4.00 mA)^2(1000 \Omega) = \fbox{16.0 mW}$$ |
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| 83 - The relationship of number of turns to input and output voltages is what can be used to solve this problem: $V\_{2,rms}N\_1 = V\_{1,rms}N\_2$.\\  We start by relating the output voltage $V\_{2,rms}$ to the input voltage $V\_{1,rms}$ and the number of turns of the primary and secondary $N\_1, N\_2$:  $$V\_{2,rms} = \frac{N\_2}{N\_1}V\_{1,rms} \Rightarrow N\_1 = N\_2 \frac{V\_{1,rms}}{V\_{2,rms}}$$  Finally we substitute numerical values and evaluate:  $$N\_1 = (400)\left( \frac{2000 V}{240 V} \right) = \fbox{$3.33 \times 10 ^3$}$$ |
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| 84 - The definition $I\_{rms} = \sqrt{(I^2)\_{av}}$ can be used to relate the rms current to the current carried by the resistor and find $(I^2)\_{av}$ by integrating $I^2$.\\  (a) We do express the rms current in terms of the $(I^2)\_{av}$:  $$I\_{rms} = \sqrt{(I^2)\_{av}}$$  We do evaluate $I^2$:  $$I^2 = [(5.0 A) \sin 2 \pi ft + (7.0 A) \sin 4 \pi ft]^2$$  $$= (25 A^2) \sin^2 2 \pi ft + (70 A^2) \sin 2 \pi ft + (49 A^2) \sin^2 4 \pi ft$$  Now we find $(I^2)\_{av}$ by integrating $I^2$ from $t =0$ to $t = T = 2 \pi / \omega$ and dividing by T:  $$(I^2)\_{av} = \frac{\omega}{2 \pi}\int\_0^{2 \pi / \omega}[(25 A^2) \sin^2 2 \pi ft + (70 A^2) \sin 2 \pi ft \sin 4 \pi ft + (49 A^2) \sin^2 4 \pi ft]dt$$  Using the trigonometric identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ to simplify and evaluate the first and third integrals and recognize that the middle term is of the form $\sin x \sin 2x$ we obtain:  $$(I^2)\_{av} = 12.5 A^2 + 0 + 24.5 A^2 = 37.0 A^2$$  We substitute for $(I^2)\_{av}$ and evaluate $I\_{rms}$:  $$I\_{rms} = \sqrt{37.0 A^2} = \fbox{6.1 A}$$  (b) The power dissipated in the resistor can be related to its resistance and the rms current in it:  $$P = I\_{rms}^2 R$$  Substituting numerical values and evaluating:  $$P = (6.08 A)^2(12 \Omega) = \fbox{0.44 kW}$$  (c) In this exercise we do express the rms voltage across the resistor in terms of R and $I\_{rms}$:  $$V\_{rms} = I\_{rms}R = (6.08 A)(12 \Omega) = \fbox{73 V}$$  \paragraph{}  85 - The integral of a quantity over an interval divided by $\Delta T$ is the average of that quantity over a time interval $\Delta T$. This definition help us to find both the average of the voltage squared $(V^2)\_{av}$ and then use the definition of the rms voltage to finish the analysis.\\  (a) The $V\_{rms}$ definition:  $$V\_{rms} = \sqrt{(V\_0^2)\_{av}}$$  Keep in mind that $-V\_0^2 = V\_0^2$, and evaluate $V\_{rms}$:  $$V\_{rms} = \sqrt{(V^2)\_{av}} = V\_0 = \fbox{12 V}$$  (b) The voltage during the second half of each cycle is zero, we express the voltage during the first half cycle of the time interval $\frac{1}{2}\Delta T$:  $$V = V\_0$$  If we express the square of the voltage during this half cycle:  $$V^2 = V\_0^2$$  We need to calculate $(V^2)\_{av}$ by integrating $V^2$ from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by $\Delta T$:  $$(V^2)\_{av} = \frac{V\_0^2}{\Delta T} \int\_0^{\frac{1}{2}\Delta T}dt = \frac{V\_0^2}{\Delta T}[t]\_0^{\frac{1}{2}\Delta T} = \frac{1}{2}V\_0^2$$  Substituting numerical values:  $$V\_{rms} = \sqrt{\frac{1}{2}V\_0^2} = \frac{V\_0}{\sqrt{2}} = \frac{12 V}{\sqrt{2}} = \fbox{8.5 V}$$ |
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| 86 - The integral of a quantity over an interval divided by $\Delta T$ is the average of that quantity over a time interval $\Delta T$. This definition can help us to find the average current $I\_{av}$ and the average of the current squared, $(I^2)\_{av}$.\\  We begin using the definition of $I\_{av}$ and $I\_{rms}$:  $$I\_{av} = \frac{1}{\Delta T} \int\_0^{\Delta T} Idt \text{ and } I\_{rms} = \sqrt{(I^2)\_{av}}$$  (a) Waveform. We express the current during the first cycle of time interval $\Delta T$:  $$I\_a = \frac{4 A}{\Delta T}t$$  where I is in A when t and T are in seconds.\\  Then we evaluate $I\_{av,a}$:  $$I\_{av,a} = \frac{1}{\Delta T} \int\_0^{\Delta T} \frac{4.0 A}{\Delta T}tdt = \frac{4.0 A}{(\Delta T)^2} \int\_0^{\Delta T}tdt$$  $$= \frac{4.0 A}{(\Delta T)^2} \left[ \frac{t^2}{2} \right]\_0^{\Delta T} = \fbox{2.0 A}$$  Now express the square of the current during this half cycle:  $$I\_a^2 = \frac{(4.0 A)^2}{(\Delta T)^2}t^2$$  The average value of the current is the same for each time interval $\Delta T$, we calcultate $(I\_a^2)\_{av}$ by integrating $I\_a^2$ from $t = 0$ to $t = \Delta T$ and divide by $\Delta T$:  $$(I\_a^2)\_{av} = \frac{1}{\Delta T} \int\_0^{\Delta T} \frac{(4.0 A)^2}{(\Delta T)^3} \left[ \frac{t^3}{3} \right]\_0^{\Delta T} = \frac{16}{3}A^2$$  Substitute in the expression for $I\_{rms, a}$:  $$I\_{rms, a} = \sqrt{\frac{16}{3}A^2} = \fbox{2.3 A}$$  (b) Waveform. The current during the second half of each cycle is zero, therefore we can express the current during the first half cycle of the time interval $\frac{1}{2}\Delta T$:  $$I\_b = 4.0 A$$  Evaluate $I\_{av,b}$:  $$I\_{av,b} = \frac{4.0A}{\Delta T} \int\_0^{\frac{1}{2}\Delta T}dt = \frac{4.0 A}{\Delta T}[t]\_0^{\frac{1}{2}\Delta T} = \fbox{2.0 A}$$  Expressing the square of the current during this half cycle:  $$I\_b^2 = (4.0 A)^2$$  Now calculate $(I\_b^2)\_{av}$ by integrating $(I\_b^2)$ from $t = 0$ to $t = \frac{1}{2}\Delta T$ and divide by $\Delta T$:  $$(I\_b^2)\_{av} = \frac{(4.0 A)^2}{\Delta T} \int\_0^{\frac{1}{2}\Delta T}dt$$  $$= \frac{(4.0 A)^2}{\Delta T} [t]\_0^{\frac{1}{2}\Delta T} = 8.0 A^2$$  Finally we substitute in the expression for $I\_{rms, b}$:  $$I\_{rms, b} = \sqrt{8.0 A^2} = \fbox{2.8 A}$$ |
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| 87 - The current in the circuit in terms of the emf of the sources and the resistance of the resistor can be expressed if we utilize Kirchhoff's loop rule. Afterwards, we can find the minimum and maximum currents by considering the conditions under which the time-dependent factor in I will be maximum or minimum. Finally, we can use $I\_{rms} = \sqrt{(I^2)\_{av}}$ to derive an expression for $I\_{rms}$ that we can use to determine its value.\\  First we begin by applying Kirchhoff's loop rule:  $$\mathcal{E}\_{1,peak} \cos \omega t + \mathcal{E}\_2 - IR = 0$$  We do solve for I:  $$I = \frac{\mathcal{E}\_{1,peak}}{R} \cos \omega t + \frac{\mathcal{E}\_2}{R}$$  or  $$I = A\_1 \cos \omega t + A\_2$$  where  $$A\_1 = \frac{\mathcal{E}\_{1,peak}}{R} \text{ and } A\_2 = \frac{\mathcal{E}\_2}{R}$$  Substituting numerical values:  $$I = \left( \frac{20 V}{36 \Omega} \right) \cos (2 \pi (180s^{-1})t) + \frac{18 V}{36 \Omega}$$  $$= (0.556 A) \cos (1131s^{-1})t + 0.50 A$$  The current is a maximum when $\cos (1131s^{-1})t = 1$. Hence:  $$I\_{max} = 0.50 A + 0.556 A = \fbox{1.06 A}$$  We now evaluate $I\_{min}$:  $$I\_{min} = 0.50 A - 0-556 A = \fbox{-0.06 A}$$  And because the average value of $\cos \omega t = 0$:  $$I\_{av} = \fbox{0.50 A}$$  The rms current is the square root of the average of the squared current:  \begin{equation}\label{irms}  I\_{rms} = \sqrt{[I^2]\_{av}}  \end{equation}  where $[I^2]\_{av}$ is given by:  $$[I^2]\_{av} = [(A\_1 \cos \omega t + A\_2)^2]\_{av}$$  $$= [A\_1^2 \cos^2 \omega t + 2 A\_1 A\_2 \cos \omega t + A\_2^2]\_{av}$$  $$[A\_1^2 \cos^2 \omega t]\_{av} +[2A\_1A\_2 \cos \omega t]\_{av} + [A\_2^2]\_{av}$$  $$A\_1^2[\cos^2 \omega t]\_{av} +2A\_1A\_2[\cos \omega t]\_{av} + A\_2^2$$  And because $[\cos^2 \omega t]\_{av} = \frac{1}{2}$ and $[\cos \omega t]\_{av} = 0$:  $$[I^2]\_{av} = \frac{1}{2}A\_1^2 + A\_2^2$$  Substituting in equation \ref{irms}:  $$I\_{rms} = \sqrt{\frac{1}{2}A\_1^2 + A\_2^2}$$  $$I\_{rms} = \sqrt{\frac{1}{2} \left( \frac{\mathcal{E}\_1}{R} \right)^2 + \left( \frac{\mathcal{E}\_2}{R} \right)^2}$$  Finally we substitute numerical values and evaluate:  $$I\_{rms} = \sqrt{\frac{1}{2}\left( \frac{20 V}{36 \Omega} \right)^2 + \left( \frac{18 V}{36 \Omega} \right)^2} = \fbox{0.64 A}$$ |
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| 88 - We need to obtain an expression for charge on the capacitor as a function of time and this can be obtained using Kirchhoff's loop rule. If we differentiate this expression with respect to time will give us the current in the circuit. Then we can find $I\_{max}$ and $I\_{min}$ by considering the conditions under which the time dependent factor in I will be a maximum or a minimum. Finally, we can use $I\_{rms} = \sqrt{(I^2)\_{av}}$ to derive an expression for $I\_{rms}$ that we can use to determine its value.\\  We begin by applying the Kirchhoff's loop rule:  $$\mathcal{E}\_{1,peak} \cos \omega t + \mathcal{E}\_2 - \frac{q(t)}{C} = 0$$  Solving for q(t):  $$q(t) = C(\mathcal{E}\_{1,peak} \cos \omega t + \mathcal{E}\_2) = A\_1 \cos \omega t + A\_2$$  where  $$A\_1 = C\mathcal{E}\_{1, peak} \text{ and } A\_2 = C\mathcal{E}\_2$$  We differentiate this expression with respect to time to obtain the current as a function of time:  $$I = \frac{dq}{dt} = \frac{d}{dt}(A\_1 \cos \omega t + A\_2) = -\omega A\_1 \sin \omega t$$  We now substitute numerical values:  $$I = -2 \pi (180 Hz)(2.0 \mu F) \sin (2 \pi (180 Hz)t) = (-2.26 mA) \sin (1131s^{-1})t$$  Then we analyze that the current is at minimum when $\sin (1131s^{-1} = 1)$. Hence:  $$I\_{min} = \fbox{-2.3 mA}$$  And the current is at maximum when $\sin (1131s^{-1} = -1)$, then:  $$I\_{max} = \fbox{2.3 mA}$$  The dc source senses the capacitor as an open circuit and the average value fo the sine function over a period is zero, hence:  $$I\_{av} = \fbox{0}$$  The rms current is the square root f the average of the squared current:  \begin{equation}\label{irms2}  I\_{rms} = \sqrt{[I^2]\_{av}}  \end{equation}  where $[I^2]\_{av}$ is given by:  $$[I^2]\_{av} = [(A\_1 \cos \omega t + A\_2)^2]\_{av}$$  $$= [A\_1^2 \cos^2 \omega t + 2A\_1A\_2 \cos \omega t + A\_2^2]\_{av}$$  $$[A\_1^2 \cos^2 \omega t]\_{av} +[2A\_1A\_2 \cos \omega t]\_{av} + [A\_2^2]\_{av}$$  $$A\_1^2[\cos^2 \omega t]\_{av} +2A\_1A\_2[\cos \omega t]\_{av} + A\_2^2$$  And because $[\cos^2 \omega t]\_{av} = \frac{1}{2}$ and $[\cos \omega t]\_{av} = 0$:  $$[I^2]\_{av} = \frac{1}{2}A\_1^2 + A\_2^2$$  Substituting in equation \ref{irms}:  $$I\_{rms} = \sqrt{\frac{1}{2}A\_1^2 + A\_2^2}$$  Substituting for $A\_1, A\_2$:  $$I\_{rms} = \sqrt{\frac{1}{2}(C\mathcal{E}\_1)^2 + (C\mathcal{E}\_2)^2}$$  $$= C\sqrt{\frac{1}{2}(\mathcal{E}\_1)^2 + (\mathcal{E}\_2)^2}$$  Substituting numerical values and evaluating:  $$I\_{rms} = (2.0 \mu F)\sqrt{\frac{1}{2}(20 V)^2 + (18 V)^2} = \fbox{$46 \mu A$}$$ |
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| 89 - In part (a) we can obtain the second order differential equation relating the charge on the capacitor to the time by using the Kirchhoff's loop rule. In part (b) we take a solution of the form $Q = Q\_{peak} \cos \omega t$, differentiate it two times, and substitute for $d^2Q / dt^2$ and Q to show that the assumed solution satisfies the differential equation provided $Q\_{peak} = -\frac{\mathcal{E}\_{peak}}{L(\omega^2 -\omega\_0^2)}$. In part (c) we use the results from (a) and (b) to find a result for $I\_{peak}$ given the problem statement.\\  (a) We begin by applying the Kirchhoff's loop rule:  $$\mathcal{E} - \frac{Q}{C} - L\frac{dI}{dt} = 0$$  Substituting for $\mathcal{E}$ and rearranging the diffenrential equation:  $$L\frac{dI}{dt} + \frac{Q}{C} = \mathcal{E}\_{max} \cos \omega t$$  And taking into account that $I = dQ / dt$:  $$\fbox{$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \mathcal{E}\_{max} \cos \omega t$}$$  (b) We assume a solution of the form:  $$Q = Q\_{peak} \cos \omega t$$  Differentiating twice:  $$\frac{dQ}{dt} = -\omega Q\_{peak} \sin \omega t$$  and  $$\frac{d^2Q}{dt^2} = -\omega^2 Q\_{peak} \cos \omega t$$  Then substituting $\frac{dQ}{dt}$ and $\frac{d^2Q}{dt^2}$ in the differential equation:  $$-\omega^2LQ\_{peak} \cos \omega t + \frac{Q\_{peak}}{C} \cos \omega t = \mathcal{E}\_{peak} \cos \omega t$$  Taking $\cos \omega t$ as factor:  $$\left( -\omega^2 LQ\_{peak} + \frac{Q\_{peak}}{C} \right) \cos \omega t = \mathcal{E}\_{peak} \cos \omega t$$  For this equation to hold for all values of t, we need:  $$\left( -\omega^2 LQ\_{peak} + \frac{Q\_{peak}}{C} \right) = \mathcal{E}\_{peak}$$  Solving for $Q\_{peak}$:  $$Q\_{peak} = \frac{\mathcal{E}\_{peak}}{-\omega^2 L + \frac{1}{C}}$$  We now take L as factor and substitute for $1 / LC$:  $$Q\_{peak} = \frac{\mathcal{E}\_{peak}}{L\left( -\omega^2 + \frac{1}{LC} \right)} = \fbox{$-\frac{\mathcal{E}\_{peak}}{L(\omega^2 - \omega\_0^2)}$}$$  From (a) and (b) we have:  $$I = \frac{dQ}{dt} = -\omega Q\_{peak} \sin \omega t = \frac{\omega \mathcal{E}\_{peak}}{L(\omega^2 - \omega\_0^2)} \sin \omega t$$  $$= I\_{peak} \sin \omega t = \fbox{$I\_{peak \cos (\omega t - \delta)}$}$$  where  $$I\_{peak} = \fbox{$\frac{\omega \mathcal{E}\_{peak}}{L\lvert \omega^2 - \omega\_0^2 \rvert}$} = \frac{\mathcal{E}\_{peak}}{\frac{L}{\omega}\lvert \omega^2 - \omega\_0^2 \rvert}$$  $$= \frac{\mathcal{E}\_{peak}}{\lvert \omega L - \frac{1}{\omega C} \rvert} = \fbox{$\frac{\mathcal{E}\_{peak}}{\lvert X\_L - X\_C \rvert}$}$$  If $\omega > \omega\_0, X\_L > X\_C$ and the current lags the voltage by $90^{\circ} (\delta = 90^{\circ})$  If $\omega < \omega\_0, X\_L < X\_C$ and the current lags the voltage by $90^{\circ} (\delta = -90^{\circ})$ |
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