Quantum Mechanics

for Computing Apps

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Continental México

Contents

Etimology

Quantum

- The amount or quantity observably present, or available. [18th c.]
- (physics) The smallest possible, and therefore indivisible, unit of a given quantity or quantifiable phenomenon. [20th c.]
- (computing) The amount of time allocated for a thread to perform its work in a multithreaded environment.

On the shoulders of giants...

Copenhagen interpretation

Figure 1: Solvay Conference 1927 (17/29 Nobel prizes)

Framework

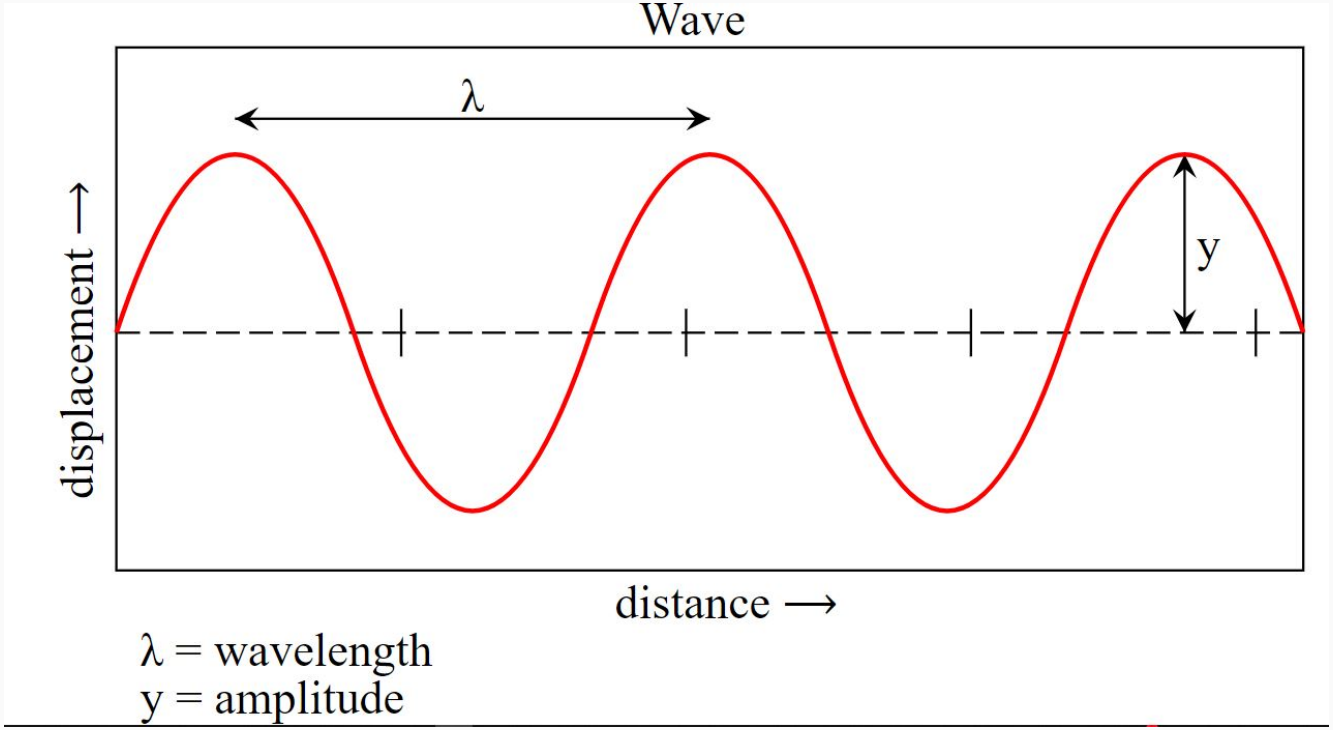
Concepts

Relativistic Energy: $E = mc^2$

Momentum: $p = mv$

Radiation: transmission of energy through space.

Waves



Frequency: $\nu = \frac{c}{\lambda}$

Electromagnetic Waves transport energy and momentum.

Waves: phase

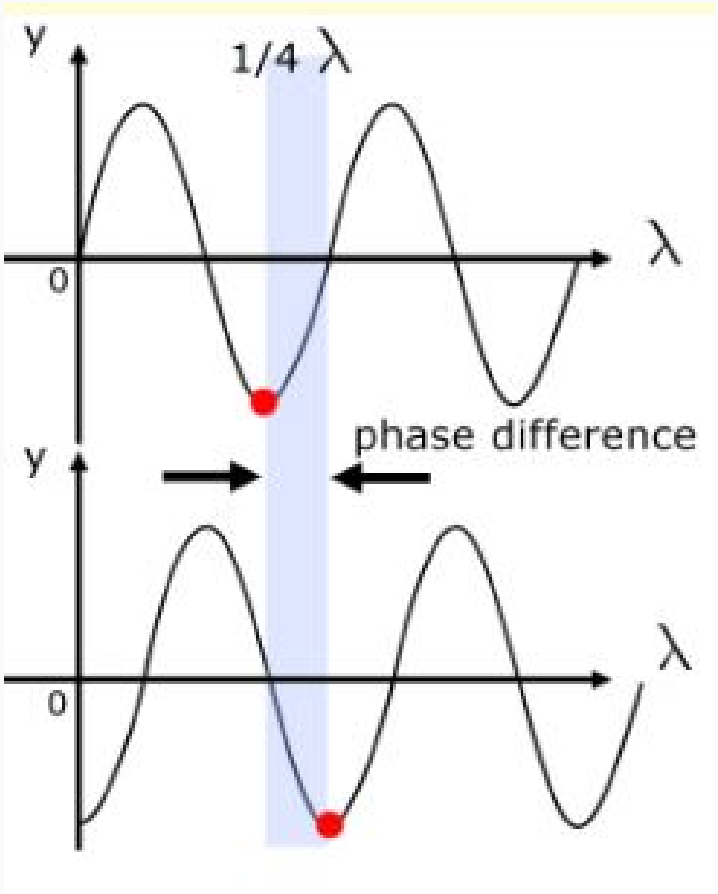


Figure 2: Wave phase

Waves: Interference

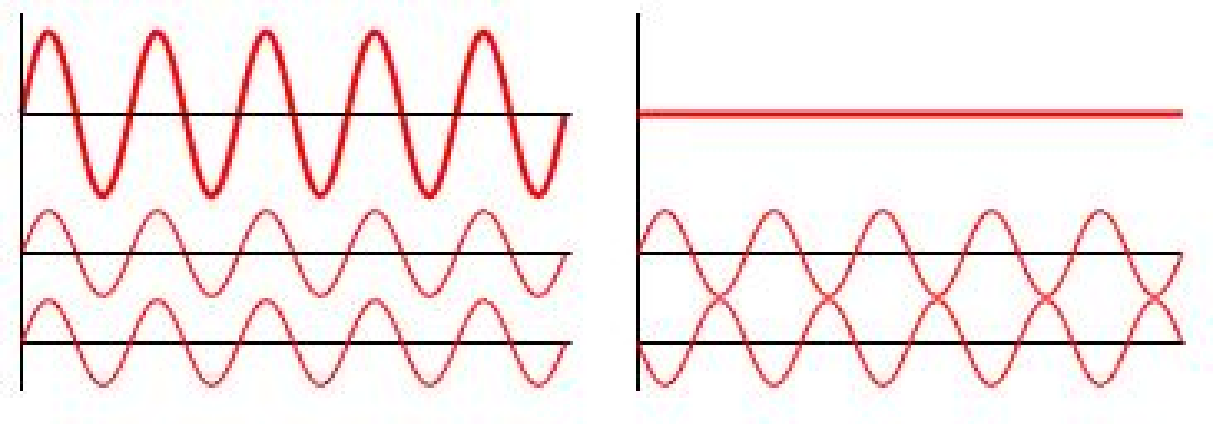


Figure 3: Waves interference

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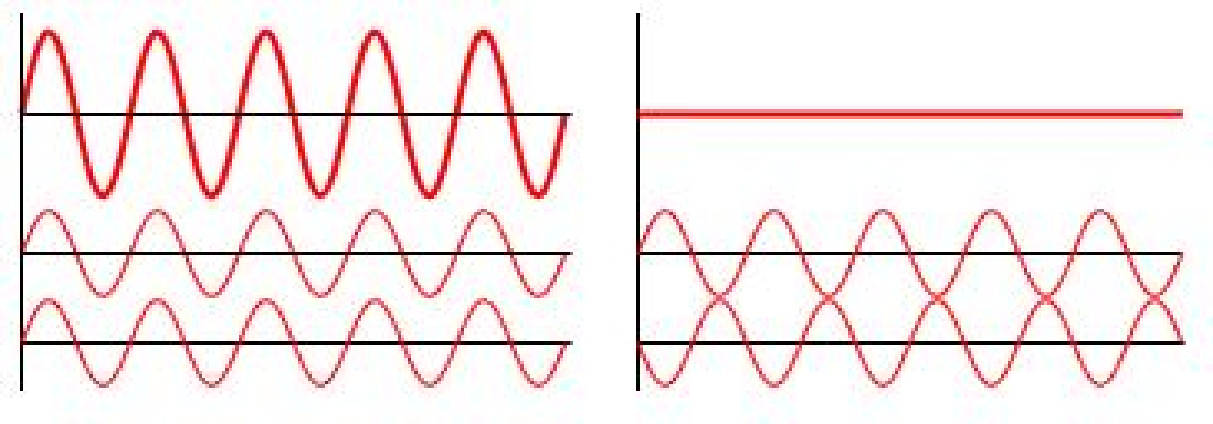
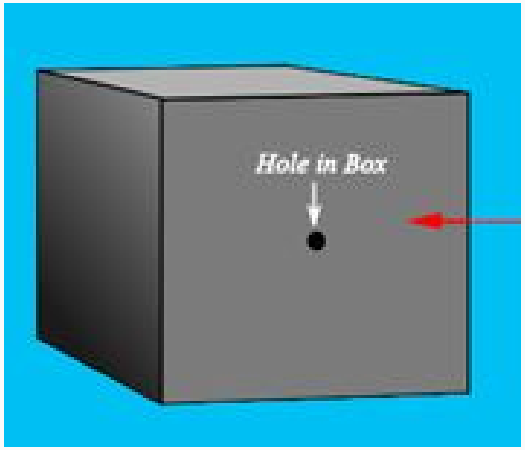


Figure 3: Waves interference

LASER: wave sources are perfectly coherent if they have a constant phase difference and the same frequency, and the same waveform.

Particle aspect of radiation

Blackbody radiation [1900]



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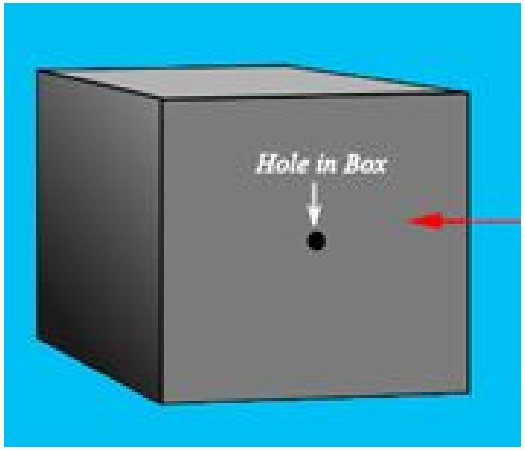

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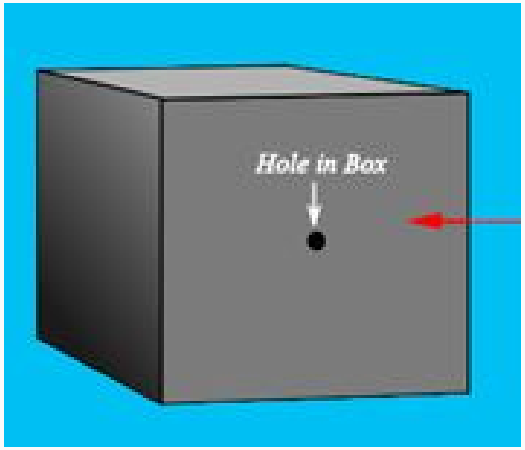

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$h\nu$ is the energy of a "quantum" of radiation, ν represents the frequency of an oscillating charge.

Particle behavior of Waves

- Planck's new idea, the *discrete* exchange of energy, solved the "ultraviolet catastrophe" as it matches experimental data.
- The spectrum of the blackbody radiation reveals the quantization of radiation, notably the particle behavior of electromagnetic waves.

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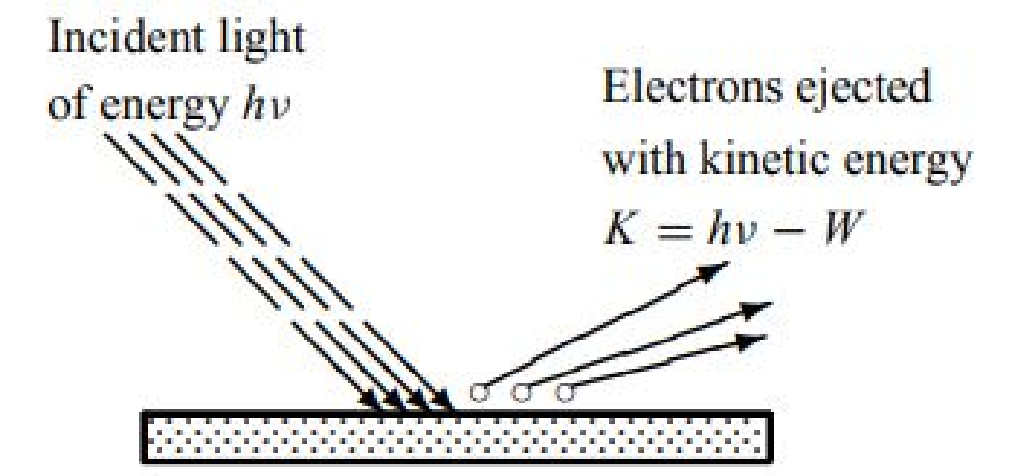
$h = 6.62x10^{-34} J \cdot s$
For a mass of 1 kg, length of 1 m, time 1 s, the Action $= 981x10^{2} J \cdot s$

Photoelectric Effect [1905]



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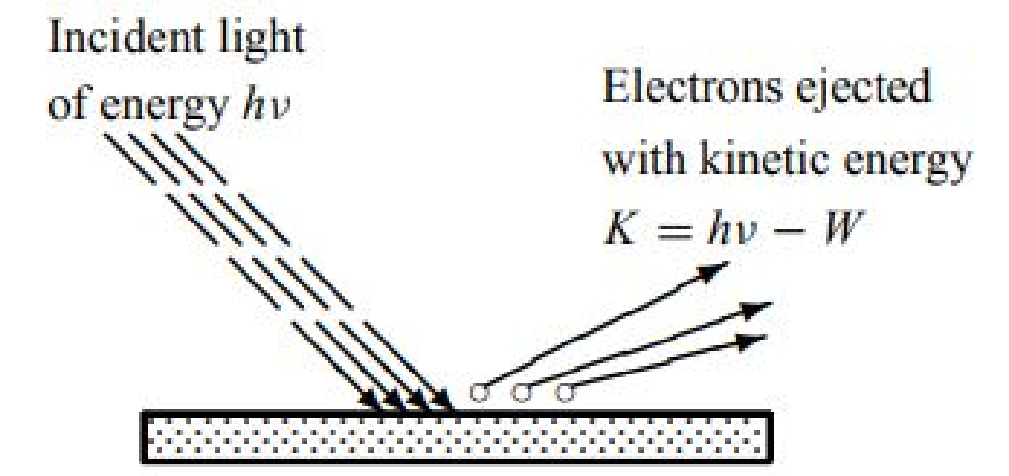

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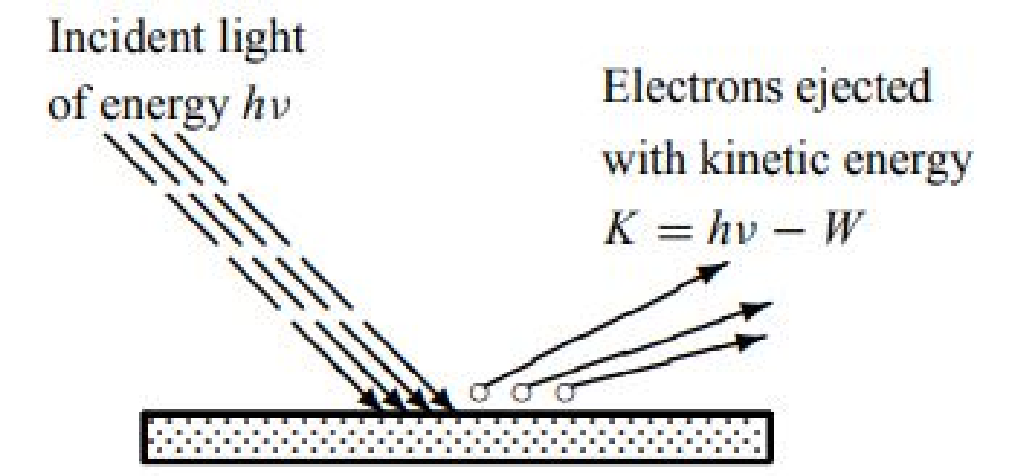

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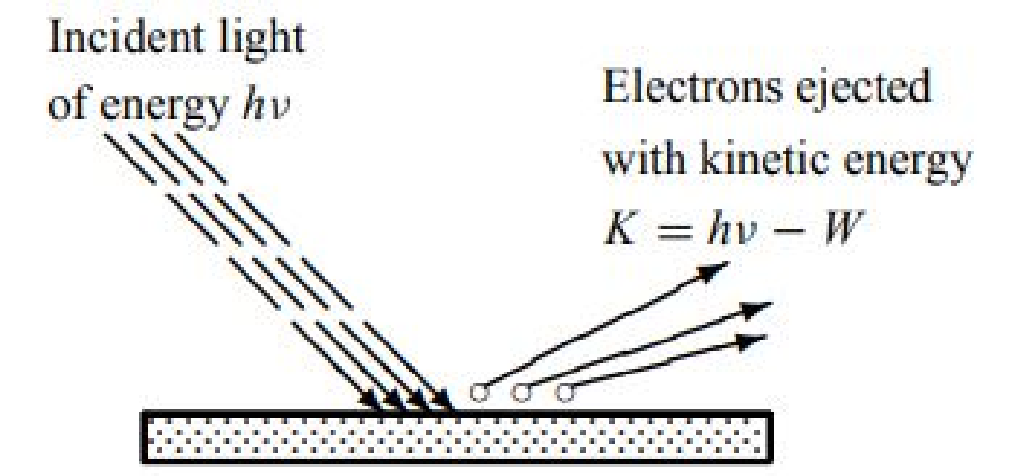

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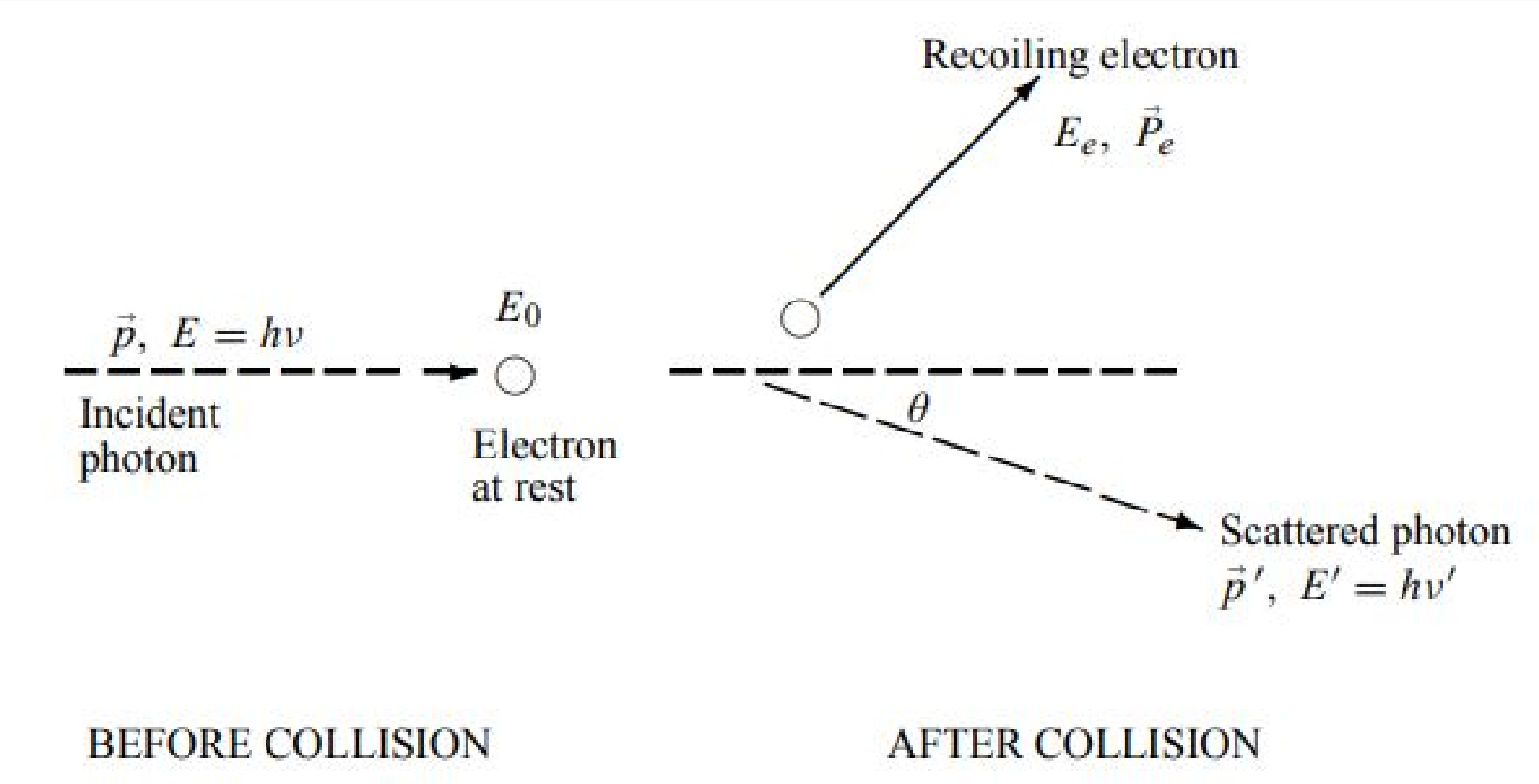
Photoelectric effect does provide compelling evidence for the corpuscular nature of the electromagnetic radiation.

Compton Effect [1923]



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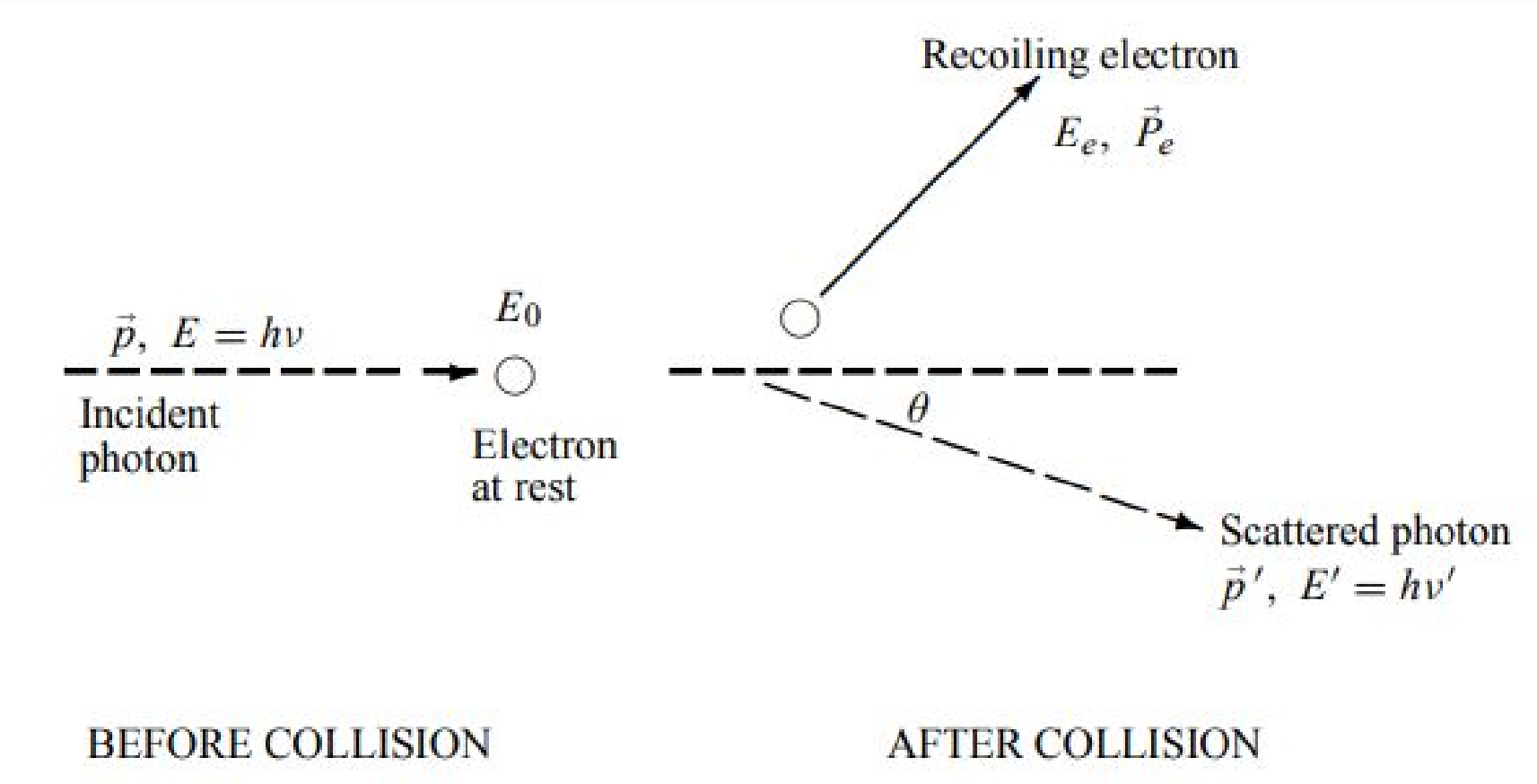
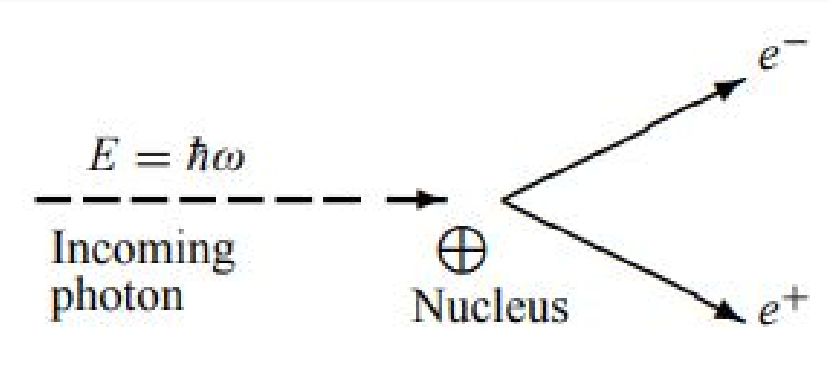


Compton effect confirms that photons behave like particles: they collide with electrons like material particles

Pair production, annihilation [1932]



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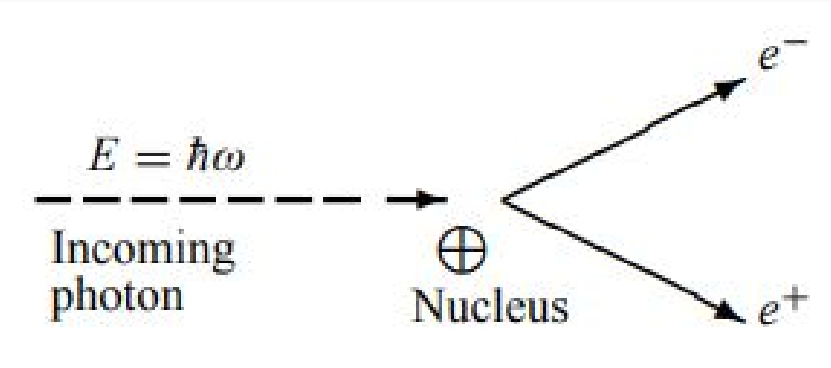

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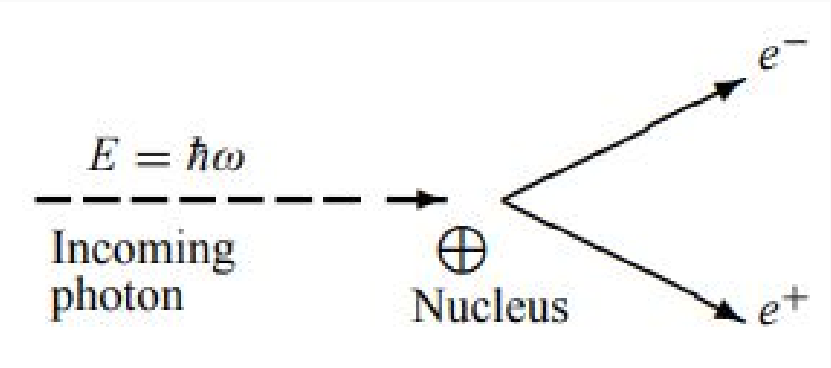

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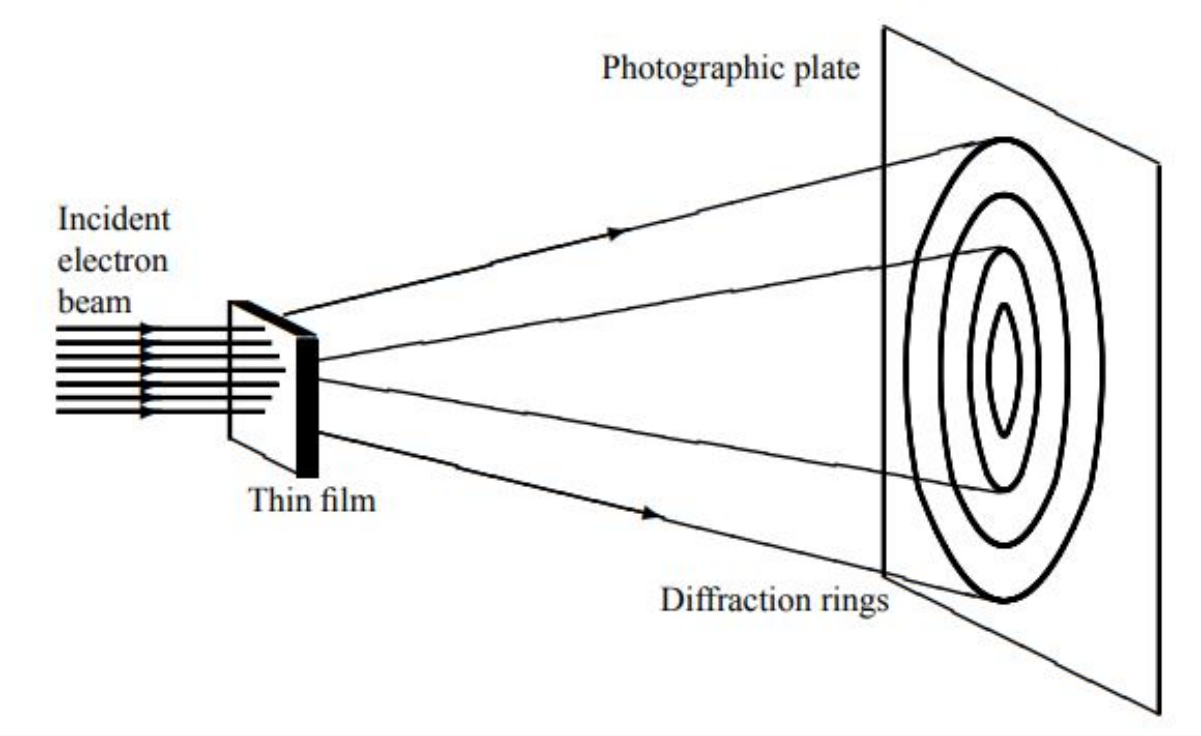
$$h\nu = E_{e^-} + E_{e^+} + E_N$$

$$= (m_e c^2 + k_{e^-}) + (m_e c^2 + k_{e^+}) + K_N$$

Wave Aspect of Particles

Matter Waves [1923]



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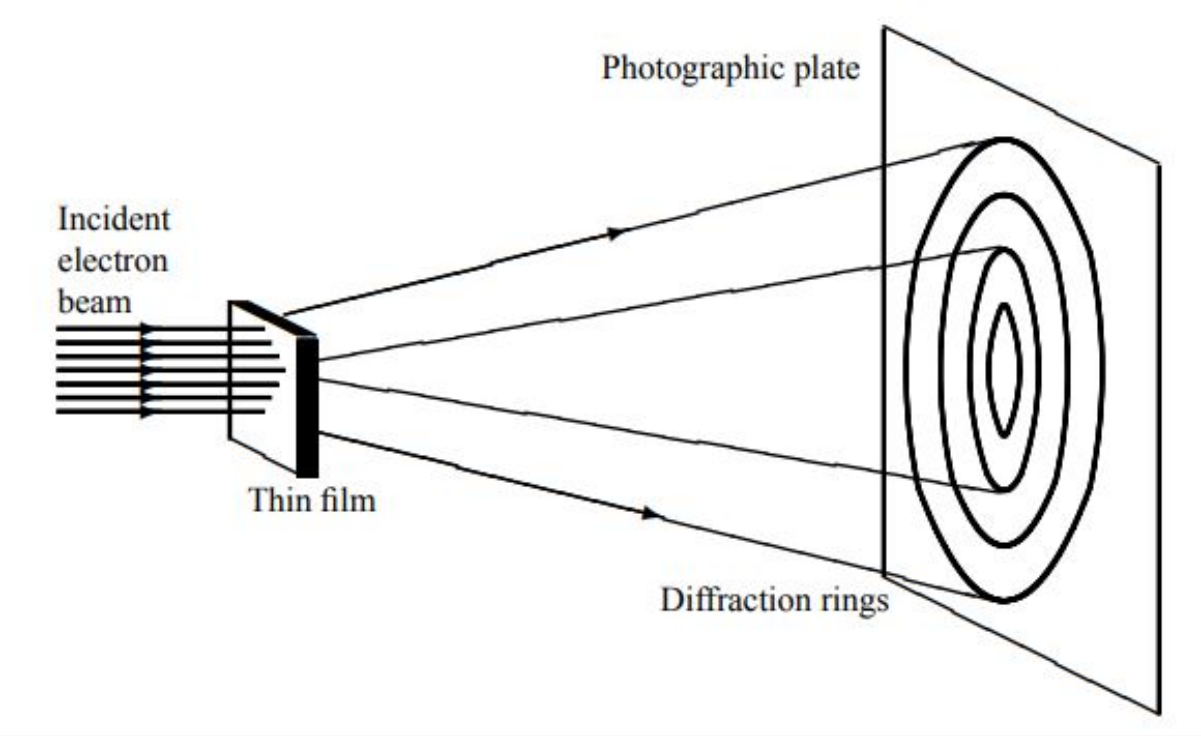

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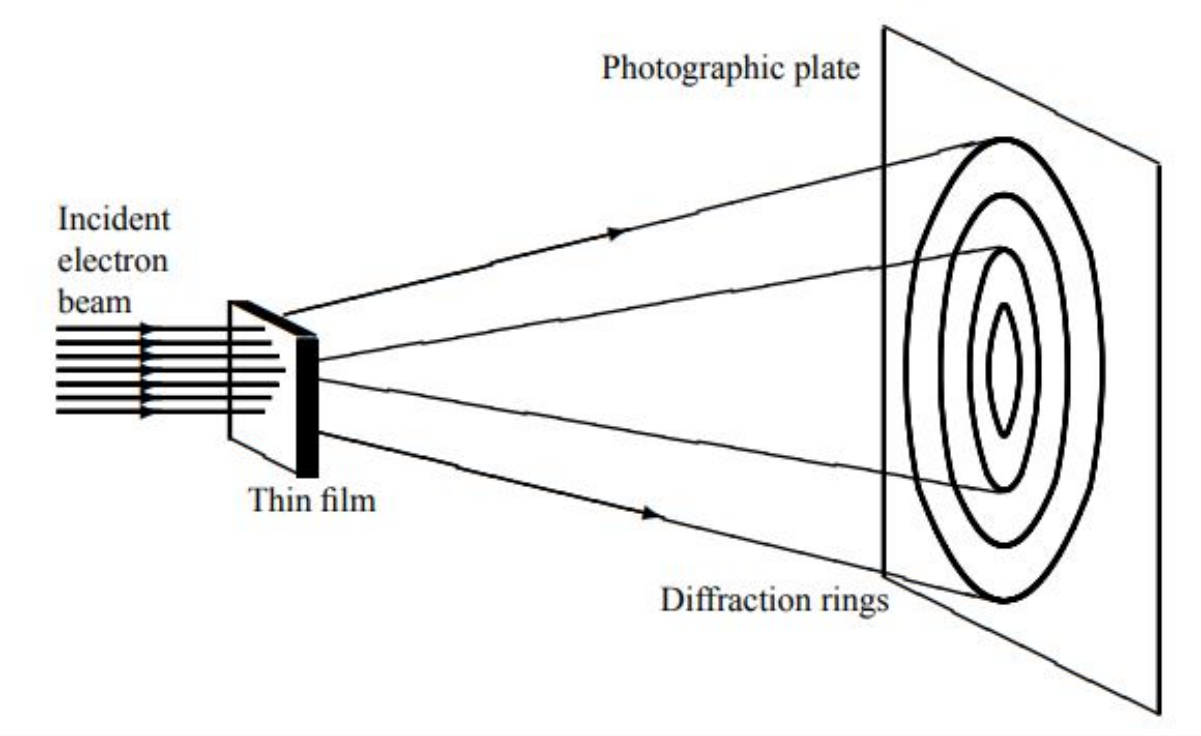

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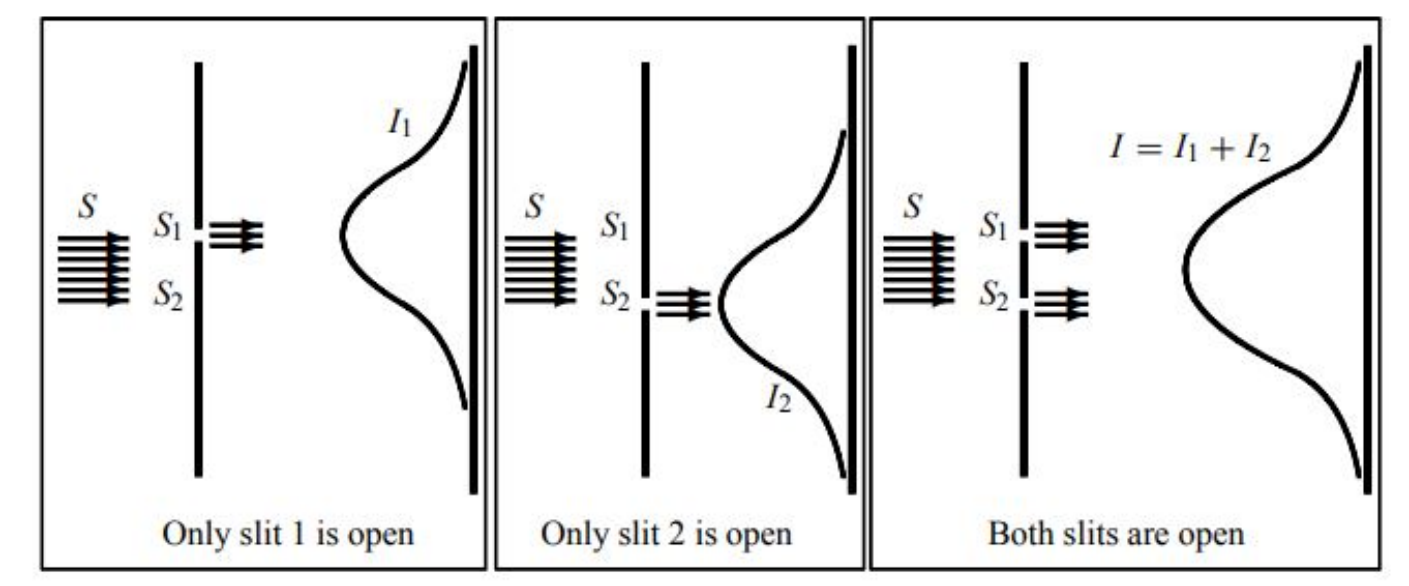
e.g. BuckminsterFullerene (C60) is the largest object observed to exhibit wave–particle duality

Particles vs Waves

Classical View of Particles and Waves



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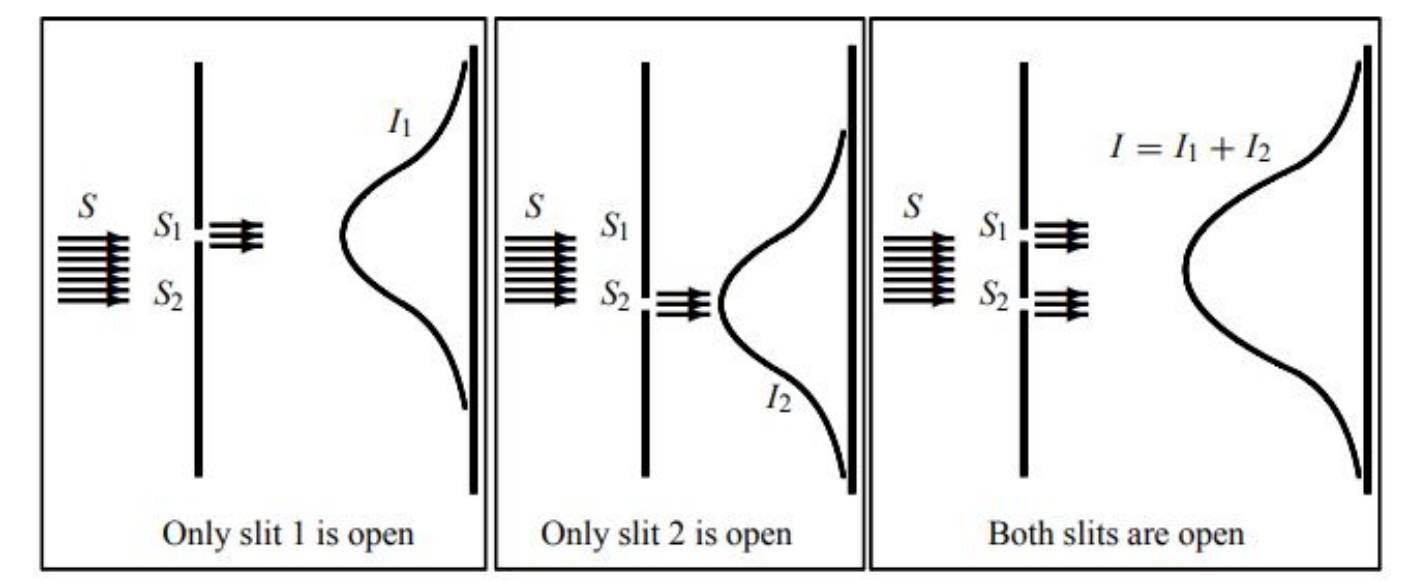

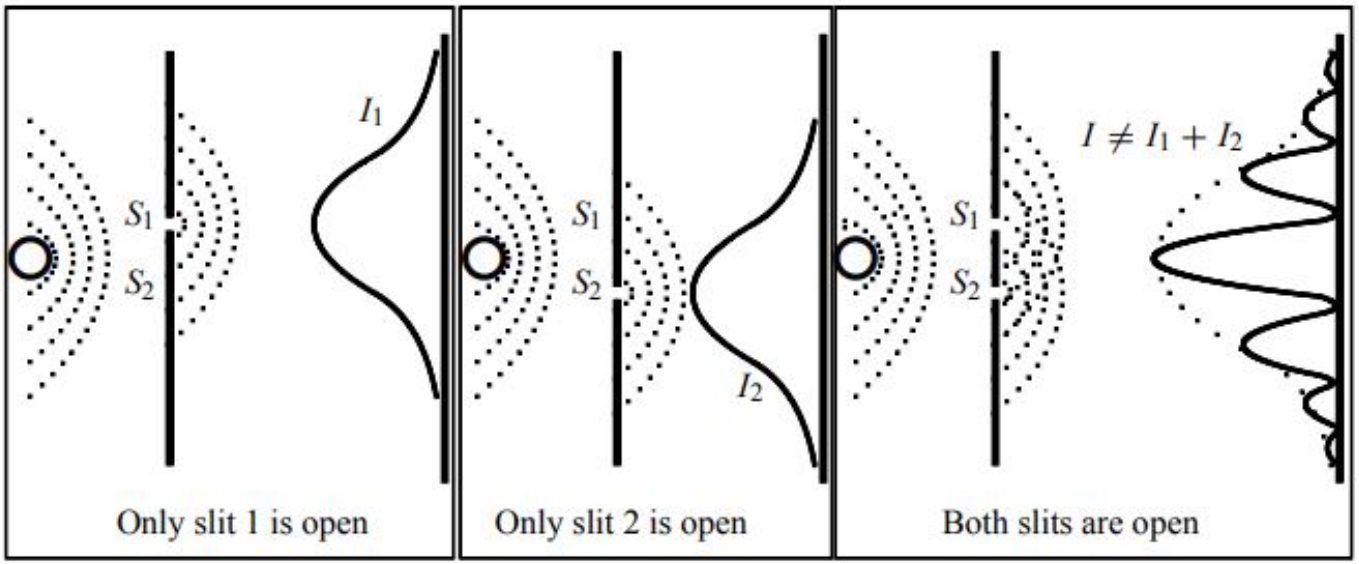

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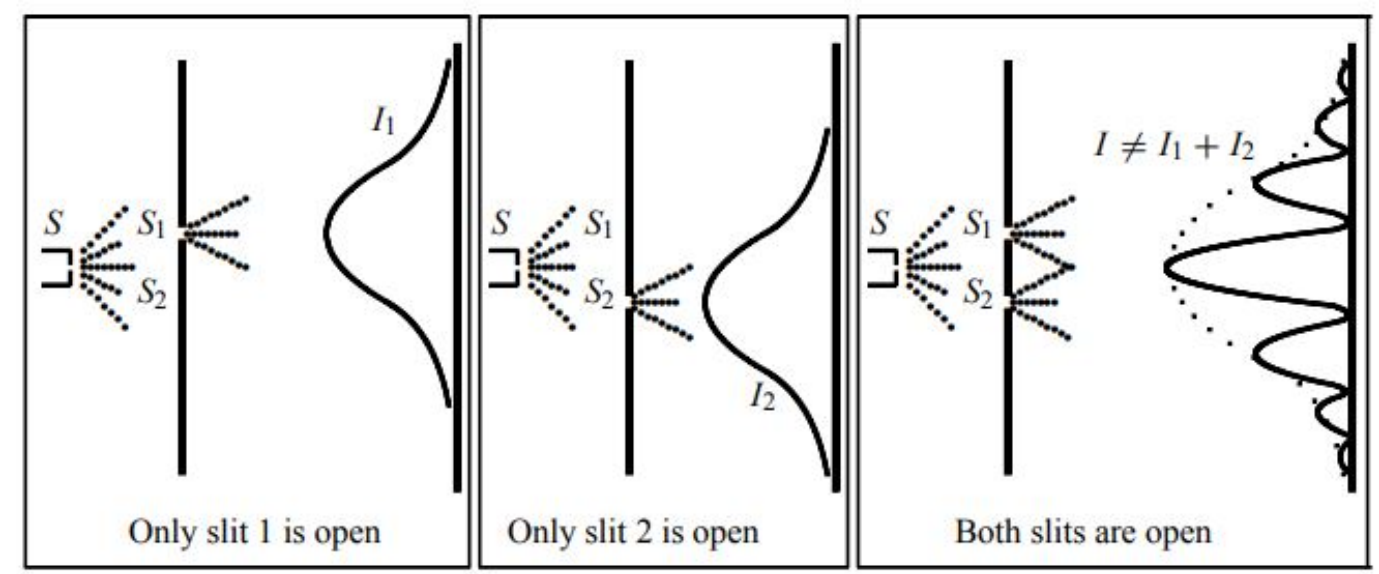

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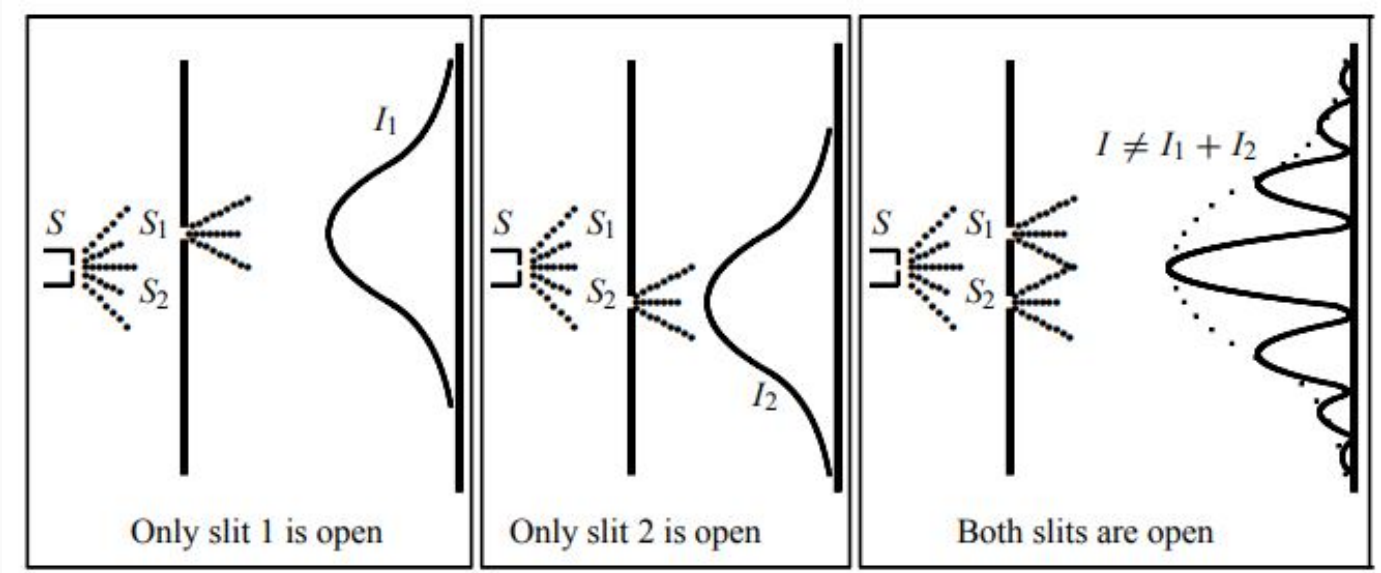

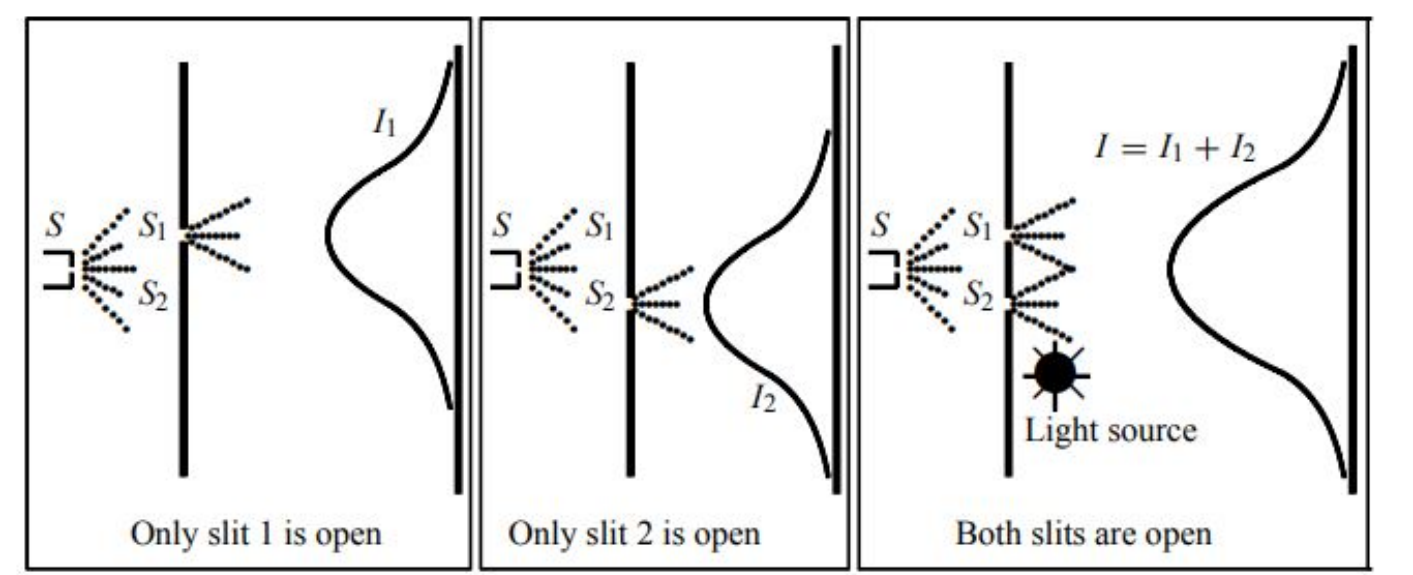

Indeterministic Nature of the Microphysical World

Double-slit experimet shows:

- microscopic material particles do give rise to interference patterns.
- it is impossible to trace the motion of individual electrons.
- electrons display both particle and wave properties.

Wave–Particle Duality: Complementarity

- In the realm of classical physics waves and particles are mutually exclusive.

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- An experiment designed to isolate the particle features of a quantum system gives no information about its wave features, and vice versa
- Particle and wave manifestations do not contradict or preclude one another, they are just complementary. Bohr.

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- If a particle is accurately localized (i.e., $\Delta x \to 0$), there will be total uncertainty about its momentum (i.e., $\Delta px \to \infty$)

Probabilistic Interpretation

In quantum mechanics the state (or one of the states) of a particle is described by a wave function $\psi(\vec{r}, t)$, corresponding to de Broglie wave of this particle. It describes the wave properties of a particle.

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Max Born interpreted as the probability of finding the particle somewhere in space:

$$\int_{allSpace} |\psi(\vec{r}, t)|^2 d^3 r = 1$$

where ψ is a solution of the Schrödinger equation.

Classic: Rutherford Atom [1911]

Fails to explain:

- atoms are stable (should lose energy)
- radiate energy over discrete frequency ranges (should emit over continuous range)

Quantum: Bohr Atom [1913]

Shown by experiment:

- atoms are stable
- radiate energy over discrete frequency ranges

- Only a discrete set of circular stable orbits are allowed.
- Emission or absorption of radiation can take place only when an electron jumps from one allowed orbit to another.

Postulates of Quantum Mechanics

- Spatial distribution of a particle is defined by a wave function.
- A state vector (wave function) $\psi(\vec{r}, t)$ contains all the information we need to know about the system and from which all needed physical quantities can be computed.
- Quantum postulates cannot be derived; they result from experiment.

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The square norm of wave function $|\psi(\vec{r}, t)|^2$ represents a position probability density, that is the probability of finding the particle at time t in a volume element.

Superposition Principle

- Digression on vectors in R^2
- Superposition of wave functions solutions of Schrödinger equation:

$$\psi(\vec{r}, t) = \alpha_1 \psi_1(\vec{r}, t) + \alpha_2 \psi_2(\vec{r}, t)$$

Measurement in Quantum Mechanics

- In QM the measurement process perturbs the system significantly.
- The act of measurement generally changes the state of the system

Computational Quantum Mechanics

Quantum Information

The minimum unit of quantum information is a quantum bit, qubit. Is a linear superposition of two orthogonal quantum states:

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

where α and β are arbitrary complex values satisfying $|\alpha|^2 + |\beta|^2 = 1$.