Robinson-Stanely Expectation Formulation

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The polynomial $A_n(x)$ from the theorem is defined as:

$$A_n(x) = \sum_{m=0}^{\binom{n}{x}} A_{n,m} x^m$$

We now carry out a small change of variables $x = \frac{p}{1-p}, p \in [0, 1]$:

$$A_n\left(\frac{p}{1-p}\right) = \sum_{m=0}^{\binom{n}{x}} A_{n,m}\left(\frac{p}{1-p}\right)^m$$

No we seek the probability of $G(V, E), |V| = n, |E| = m$ under the Erdos-Reni model $\mathcal{G}_{n,p}$. That is for every edge $e_i \in E$ we evaluate the probability under the model resulting in p^m times (conjunction) the probability of every edge $e_i \notin E$ we evaluate the probability under the Erdos-Reni model $(1-p)^{m-\binom{n}{2}}$ resulting in $p^n(1-p)^{\binom{n}{2}-m}$. It becomes immediately obvious that this probability can be factored from our substitution of x^m :

$$A_n\left(\frac{p}{1-p}\right) = \sum_{m=0}^{\binom{n}{x}} A_{n,m} p^m (1-p)^{\binom{n}{2}-m}(1-p)^{-\binom{n}{2}}$$

Simple factoring out.

$$A_n\left(\frac{p}{1-p}\right) = (1-p)^{-\binom{n}{2}} \sum_{m=0}^{\binom{n}{x}} A_{n,m} p^m (1-p)^{\binom{n}{2}-m}$$

Renaming probability:

$$A_n\left(\frac{p}{1-p}\right) = (1-p)^{-\binom{n}{2}} \sum_{m=0}^{\binom{n}{x}} A_{n,m} \mathbb{P}r_{n,p}[G]$$

We know from the definition of $A_{n,m}$ that it is the number of acyclic orientations in a graph $G(V, E), m, n$ Thus :

$$A_n\left(\frac{p}{1-p}\right) = (1-p)^{-\binom{n}{2}} \mathbb{E}_{n,p}[|AO(G)|] \quad \square$$