Introduction

The objective here is to give a short code implementation of GARCH for forecasting and volatility estimation.

The implementation is based on Kevin Sheppard's *arch* package for Python.

We start by importing the necessary packages and uploading a financial data (here using `yfinance` which obtains financial time series from Yahoo Finance).

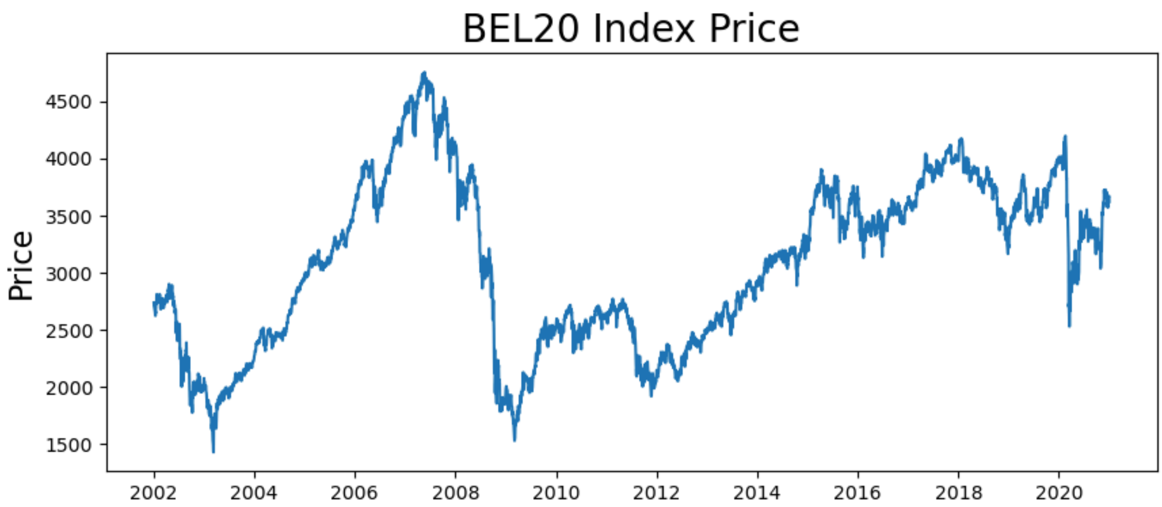
In [7]:
```
#In case not yet installed:
#%pip install arch
```

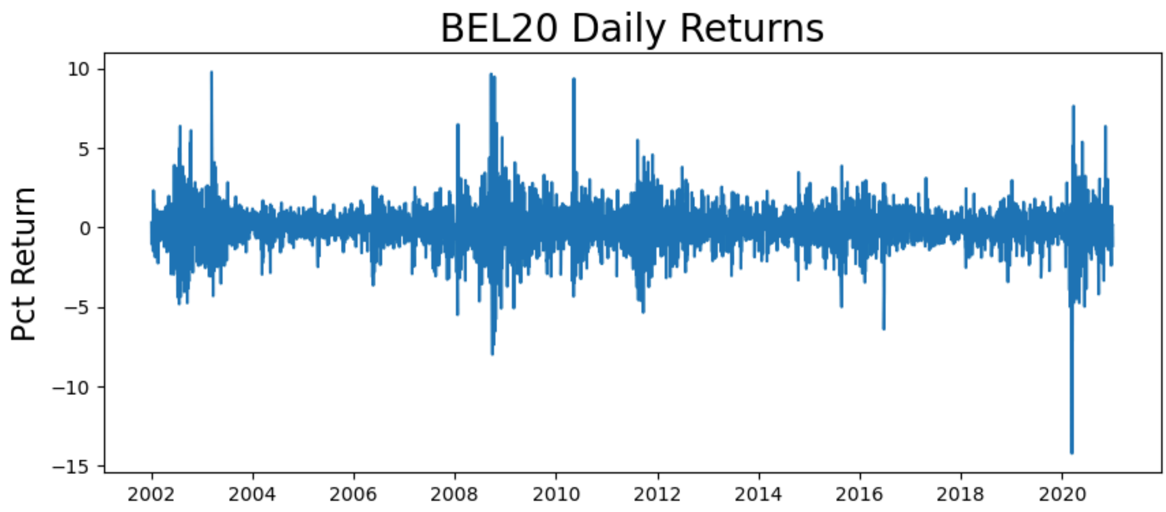
In [2]:
```
import pandas_datareader.data as web
from datetime import datetime, timedelta
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt
from arch import arch_model
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
import numpy as np
```

In [61]:
```
# Download needed data, here Belgium's BEL 20 (^BFX)
# Could be changed for other, like S&P 500 data (ticker ^GSPC)
ticker = "^BFX"
data = yf.download(ticker, start="2002-01-01", end="2021-01-01")

# Calculate daily returns
    # --> Using adjusted price (for dividends, splits...)
    #no need to account for other changes to estimate return
data['Daily Return'] = 100*data['Adj Close'].pct_change().dropna()
data=data.iloc[1:] #excluding first line since it is used to estimate returns

#Plot prices
plt.figure(figsize=(10,4))
plt.plot(data['Adj Close'])
plt.ylabel('Price', fontsize=16)
plt.title('BEL20 Index Price', fontsize=20)

#create returns variable
returns=data['Daily Return']

#Plot daily returns
plt.figure(figsize=(10,4))
plt.plot(returns)
plt.ylabel('Pct Return', fontsize=16)
plt.title('BEL20 Daily Returns', fontsize=20)
```

```
[*********************100%%**********************]  1 of 1 completed
```

Out[61]:
```
Text(0.5, 1.0, 'BEL20 Daily Returns')
```





PACF

Plotting the partial autocorrelation function (PACF) is a good first step to determine the parameters to be used in the GARCH (p,q) modelling.

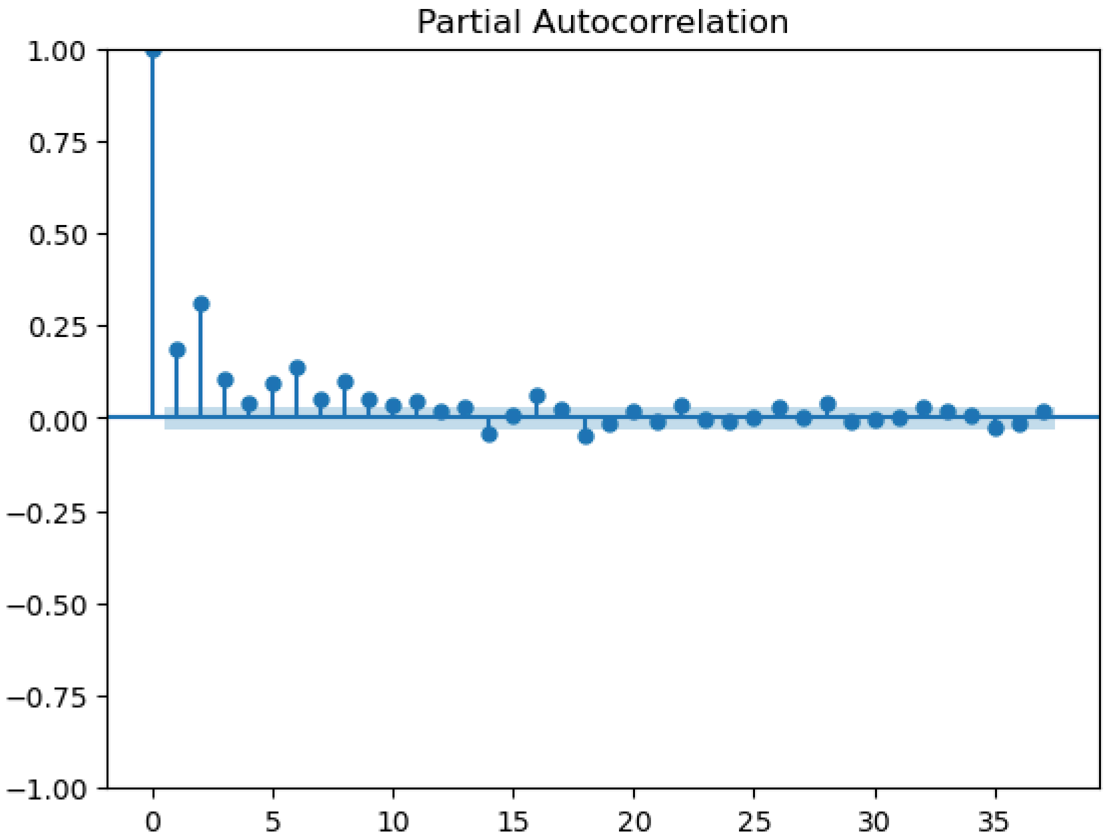
Since the error term follows a similar GARCH process (s.t. η is a white noise):

$$\epsilon_t = \eta_t \sqrt{\omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2}$$

Then ϵ_t^2 will be similar to an AR process, with its PACF's parameters showing if the 'direct effects' of the p autocorrelations are significant:

$$\epsilon_t^2 = \eta_t^2 (\omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2)$$

In [60]:
```
plot_pacf(returns**2)
plt.show()
```



GARCH (2,2) Estimation

After second autocorrelation it experiences faster decay, GARCH(2,2) can be a good first candidate

In [51]:
```
model = arch_model(returns, p=2, q=2)
model_fit = model.fit()
model_fit.summary()
```

```
Iteration:      1,   Func. Count:      8,   Neg. LLF: 95815.30235564295
Iteration:      2,   Func. Count:     21,   Neg. LLF: 27946.543291364876
Iteration:      3,   Func. Count:     32,   Neg. LLF: 1661009518.3369653
Iteration:      4,   Func. Count:     40,   Neg. LLF: 7160.017407202196
Iteration:      5,   Func. Count:     48,   Neg. LLF: 7252.431352953115
Iteration:      6,   Func. Count:     57,   Neg. LLF: 7128.015091515116
Iteration:      7,   Func. Count:     65,   Neg. LLF: 7141.818932261457
Iteration:      8,   Func. Count:     73,   Neg. LLF: 7030.957934604168
Iteration:      9,   Func. Count:     81,   Neg. LLF: 7034.699667872124
Iteration:     10,   Func. Count:     89,   Neg. LLF: 7029.768050827293
Iteration:     11,   Func. Count:     96,   Neg. LLF: 7029.7660791813005
Iteration:     12,   Func. Count:    103,   Neg. LLF: 7029.766002510643
Iteration:     13,   Func. Count:    110,   Neg. LLF: 7029.765998518107
Iteration:     14,   Func. Count:    116,   Neg. LLF: 7029.765998518431
Optimization terminated successfully    (Exit mode 0)
            Current function value: 7029.765998518107
            Iterations: 14
            Function evaluations: 116
            Gradient evaluations: 14
```

Out[51]:

Constant Mean - GARCH Model Results

Dep. Variable:	Daily Return	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-7029.77
Distribution:	Normal	**AIC:**	14071.5
Method:	Maximum Likelihood	**BIC:**	14110.5
		No. Observations:	4855
Date:	Tue, Jun 25 2024	**Df Residuals:**	4854
Time:	04:55:08	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0674	1.276e-02	5.281	1.288e-07	[4.236e-02,9.237e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0315	9.937e-03	3.167	1.541e-03	[1.199e-02,5.095e-02]
alpha[1]	0.1169	2.023e-02	5.777	7.619e-09	[7.722e-02, 0.157]
alpha[2]	0.0562	3.550e-02	1.584	0.113	[-1.335e-02, 0.126]
beta[1]	0.3234	0.217	1.492	0.136	[-0.101, 0.748]
beta[2]	0.4865	0.188	2.593	9.507e-03	[0.119, 0.854]

Covariance estimator: robust

- Since alpha 2 is not significant, while beta 2 still is, let's try GARCH(1,2)

In [62]:

```
model = arch_model(returns, p=1, q=2)
model_fit = model.fit()
model_fit.summary()
```

```
Iteration:      1,   Func. Count:      7,   Neg. LLF: 104862.33277570791
Iteration:      2,   Func. Count:     19,   Neg. LLF: 27963.747402753877
Iteration:      3,   Func. Count:     29,   Neg. LLF: 1334271792.7905927
Iteration:      4,   Func. Count:     36,   Neg. LLF: 7239.142234840261
Iteration:      5,   Func. Count:     43,   Neg. LLF: 2072082869.6169012
Iteration:      6,   Func. Count:     50,   Neg. LLF: 7035.034701609923
Iteration:      7,   Func. Count:     57,   Neg. LLF: 7037.115372067508
Iteration:      8,   Func. Count:     64,   Neg. LLF: 7031.925098457304
Iteration:      9,   Func. Count:     71,   Neg. LLF: 7041.127365428974
Iteration:     10,   Func. Count:     78,   Neg. LLF: 7031.518110551529
Iteration:     11,   Func. Count:     85,   Neg. LLF: 7034.864274995717
Iteration:     12,   Func. Count:     93,   Neg. LLF: 7031.4696062143785
Iteration:     13,   Func. Count:     99,   Neg. LLF: 7031.468943846021
Iteration:     14,   Func. Count:    105,   Neg. LLF: 7031.468943362935
Optimization terminated successfully    (Exit mode 0)
            Current function value: 7031.468943362935
            Iterations: 14
            Function evaluations: 105
            Gradient evaluations: 14
```

Out[62]:

Constant Mean - GARCH Model Results

Dep. Variable:	Daily Return	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-7031.47
Distribution:	Normal	**AIC:**	14072.9
Method:	Maximum Likelihood	**BIC:**	14105.4
		No. Observations:	4856
Date:	Tue, Jun 25 2024	**Df Residuals:**	4855
Time:	10:04:49	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0672	1.274e-02	5.279	1.302e-07	[4.226e-02,9.219e-02]

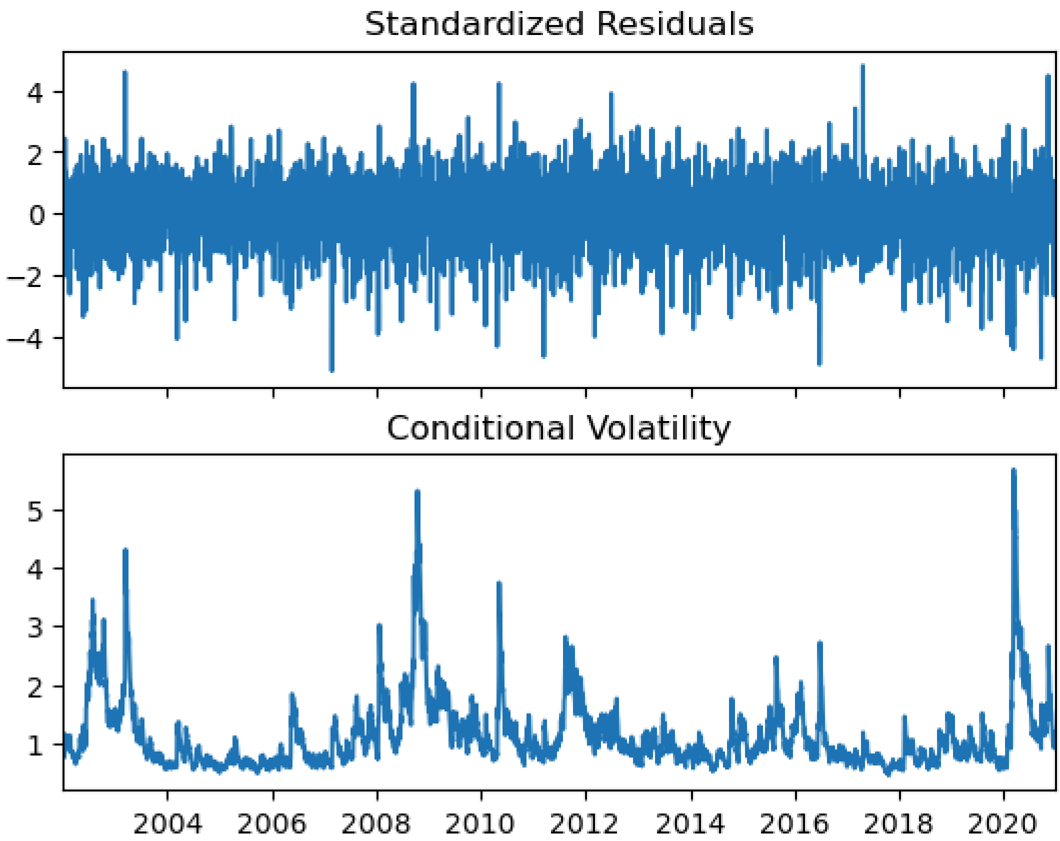
Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0210	5.425e-03	3.867	1.101e-04	[1.035e-02,3.161e-02]
alpha[1]	0.1157	1.920e-02	6.025	1.692e-09	[7.805e-02, 0.153]
beta[1]	0.8360	0.194	4.304	1.681e-05	[0.455, 1.217]
beta[2]	0.0370	0.180	0.205	0.838	[-0.317, 0.391]

Covariance estimator: robust

In [54]:

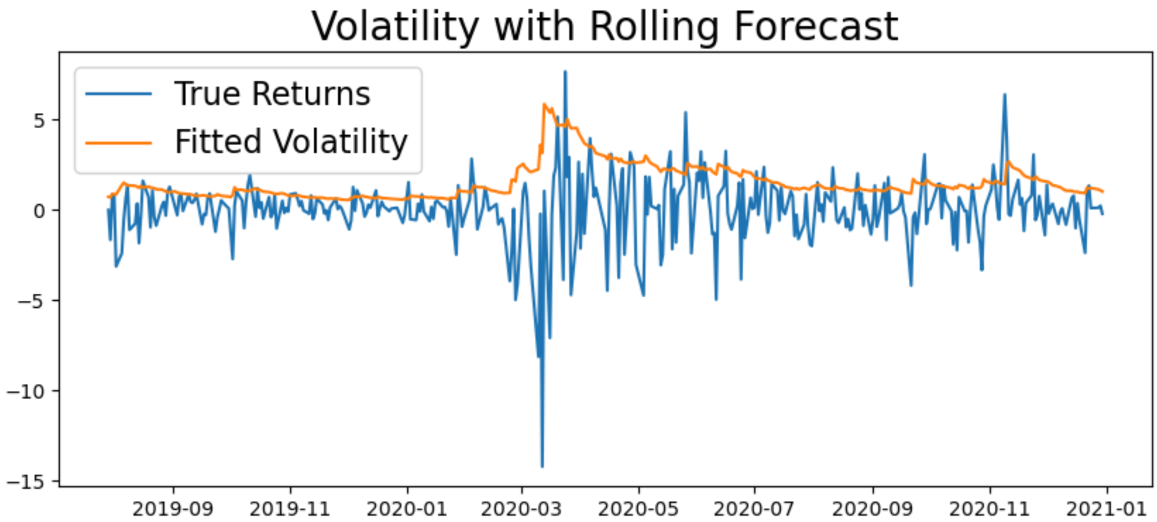
```
res = model.fit(update_freq=0, disp="off")
fig = res.plot()
```



- Now we move to the forecast of volatility

In [56]:
```
# Initialize an empty list to store rolling predictions
rolling_predictions = []

# Define the size of the test set (number of observations to forecast)
test_size = 365

# Rolling forecast process
for i in range(test_size):
    # Slice the training set to include all data up to the current point in the
    train = returns[:-(test_size-i)]

    # Initialize the GARCH(1,2) model on the training data
    model = arch_model(train, p=1, q=2)

    # Fit the model to the training data
    model_fit = model.fit(disp='off')

    # Forecast the next observation (horizon=1 means the next time step)
    pred = model_fit.forecast(horizon=1)

    # Append the predicted volatility (standard deviation) for the next time ste
    rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))

# Convert the rolling predictions list to a pandas Series, aligning it with the
rolling_predictions = pd.Series(rolling_predictions, index=returns.index[-test_s

# Plotting the results
plt.figure(figsize=(10,4))

# Plot the true returns for the test set period
```

```
true, = plt.plot(returns[-test_size:])

# Plot the rolling predictions of volatility
preds, = plt.plot(rolling_predictions)
plt.title('Volatility with Rolling Forecast', fontsize=20)
plt.legend(['True Returns', 'Fitted Volatility'], fontsize=16)

# Show the plot
plt.show()
```



Next steps

- Value at Risk estimation
- Use in different financial series
- Comparison of Monte Carlo and GARCH forecasts
- Backtesting performance (e.g. Mean absolute error)
- Model extensions

In [ ]: