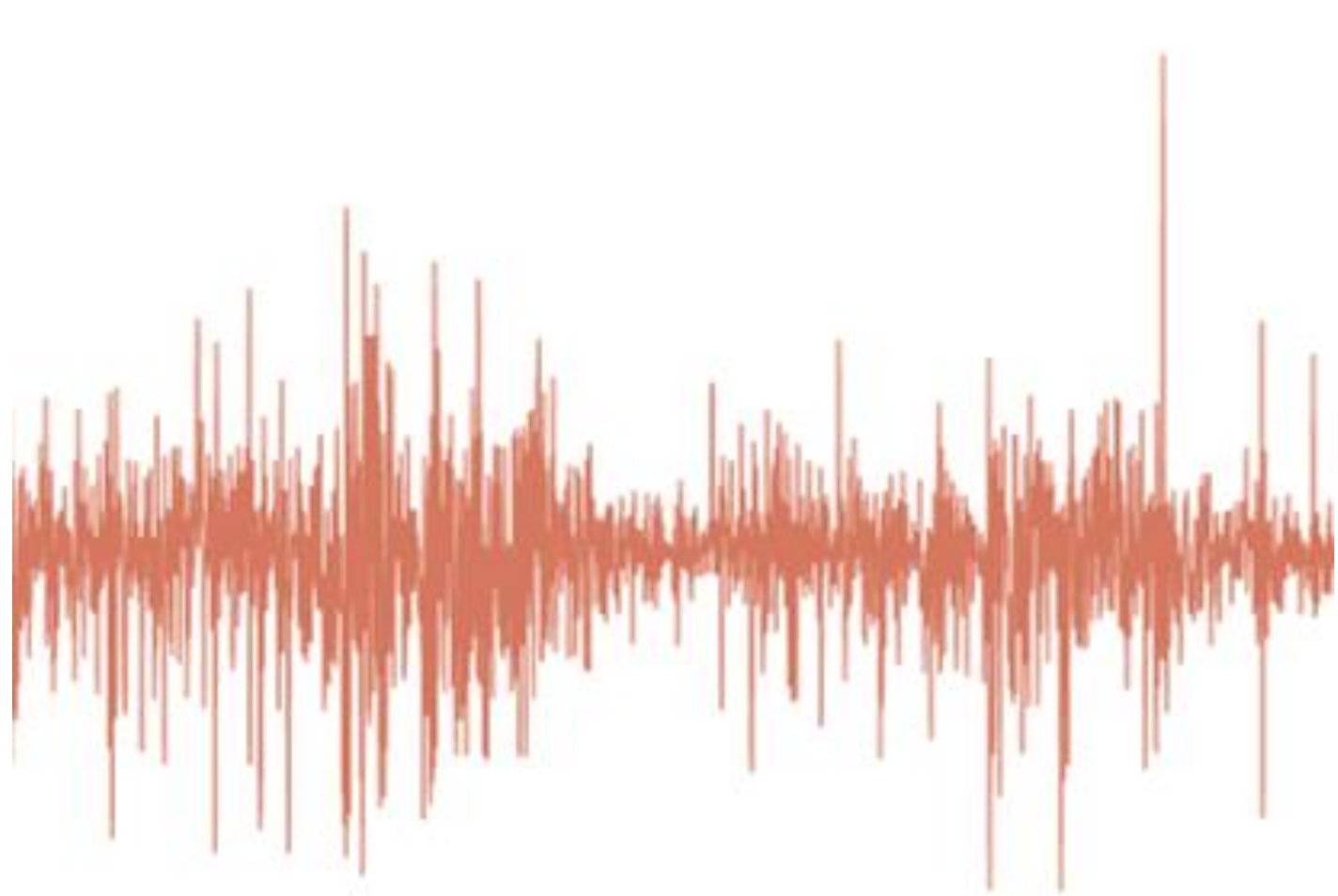


GARCH:

**How insights on conditional
variance can help our
predictions**

Francisco Pitthan



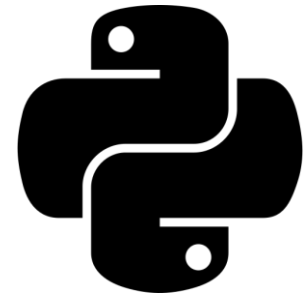
Objective



Why it
matters?



What
is it?



How to
implement?

Why it
matters?



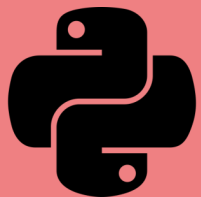
✓ Many series have time-dependent volatility

What
is it?



✓ Model with persistent & time-depend. volatility

How to
implement?



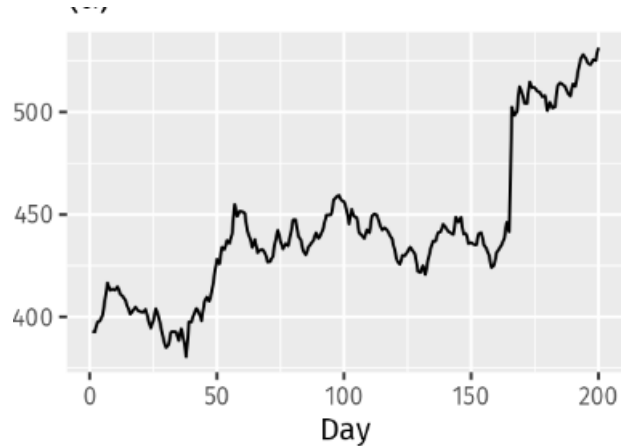
✓ Python implementation with *arch* package



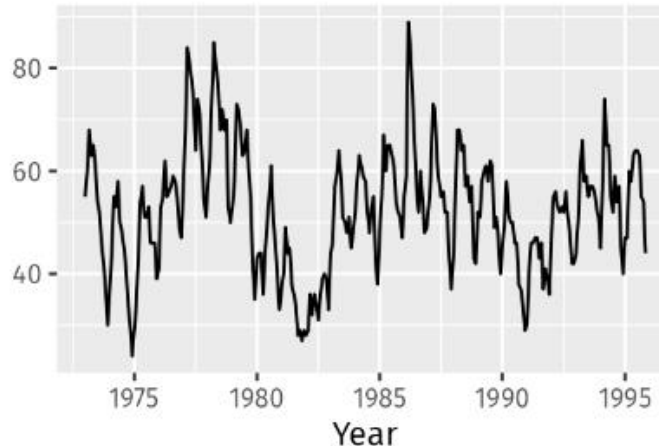
Why it
matters?

Forecasting models assume stationary time-series

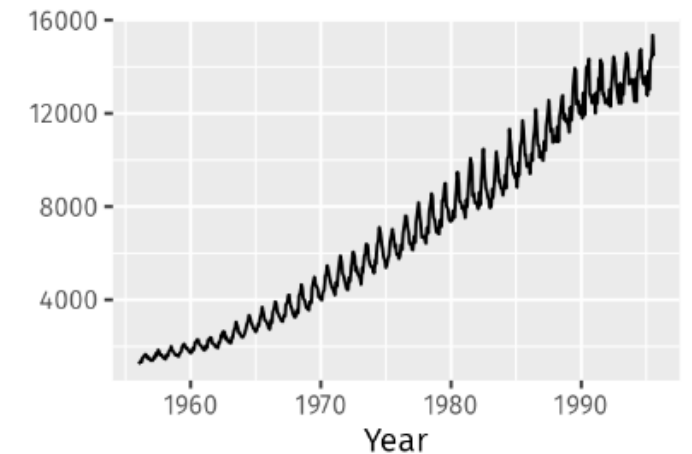
Which can be defined¹:



Constant mean
without trends



No seasonality



Constant variance
over time

1. Heckert et al. (2012)

Stationarity is important for modeling

E.g. AR(1)

$$y_t = \phi y_{t-1} + u_t$$

$$y_{t-1} = \phi y_{t-2} + u_{t-1}$$

$$y_t = \phi(\phi y_{t-2} + u_{t-1}) + u_t$$

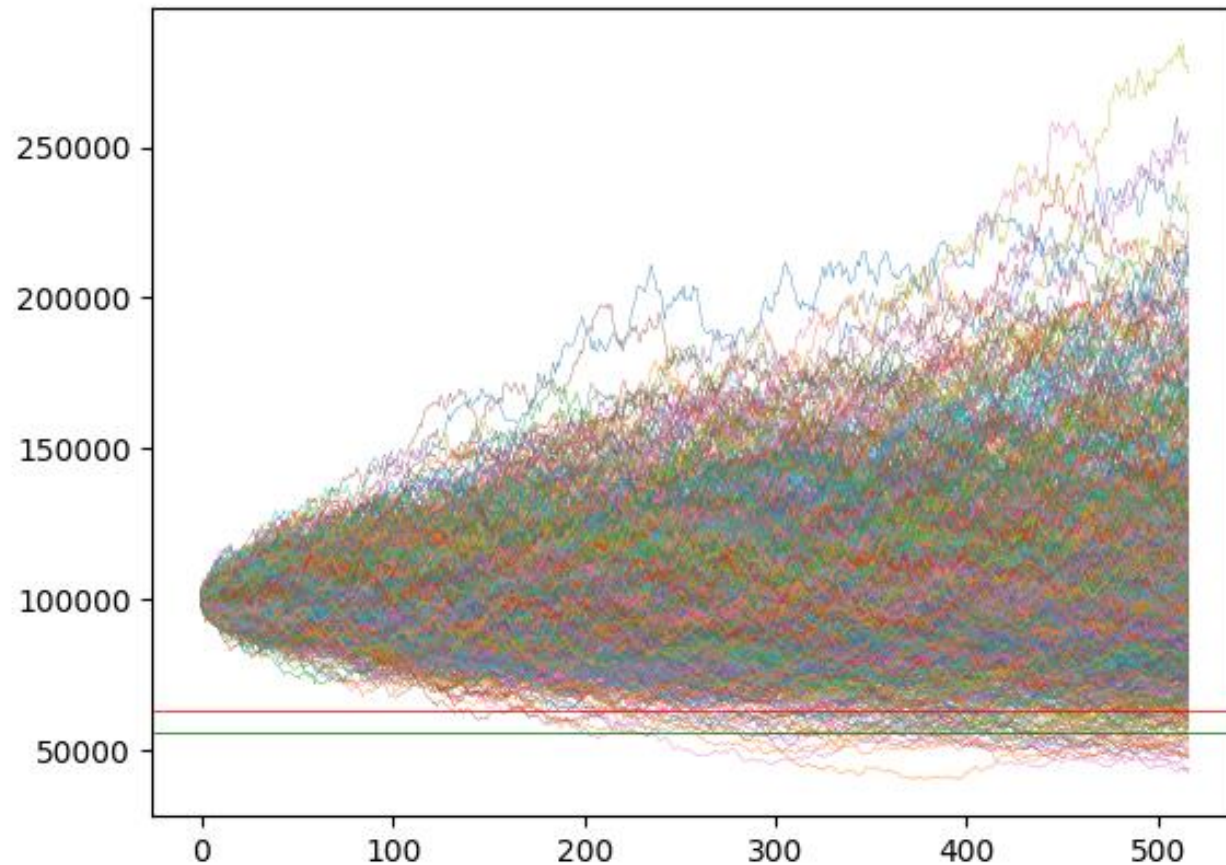
$$y_t = \phi^2 y_{t-2} + \phi u_{t-1} + u_t$$

$$y_t = \phi^m y_{t-m} + \sum_{j=0}^m \phi^j u_{t-j}, \quad |\phi| < 1$$

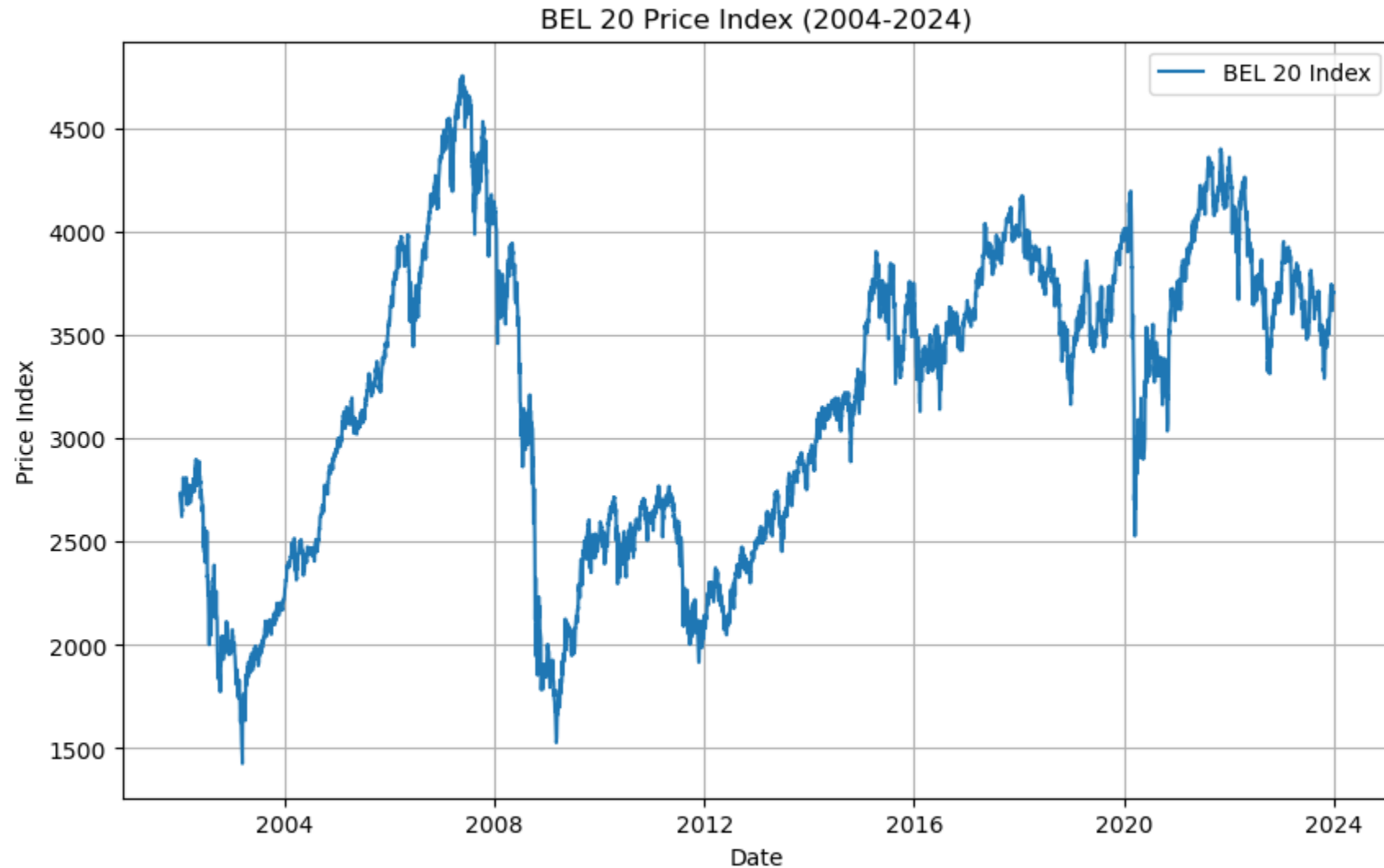
$$y_t = \sum_{j=0}^{\infty} \phi^j u_{t-j}$$

Can be an implicit assumption

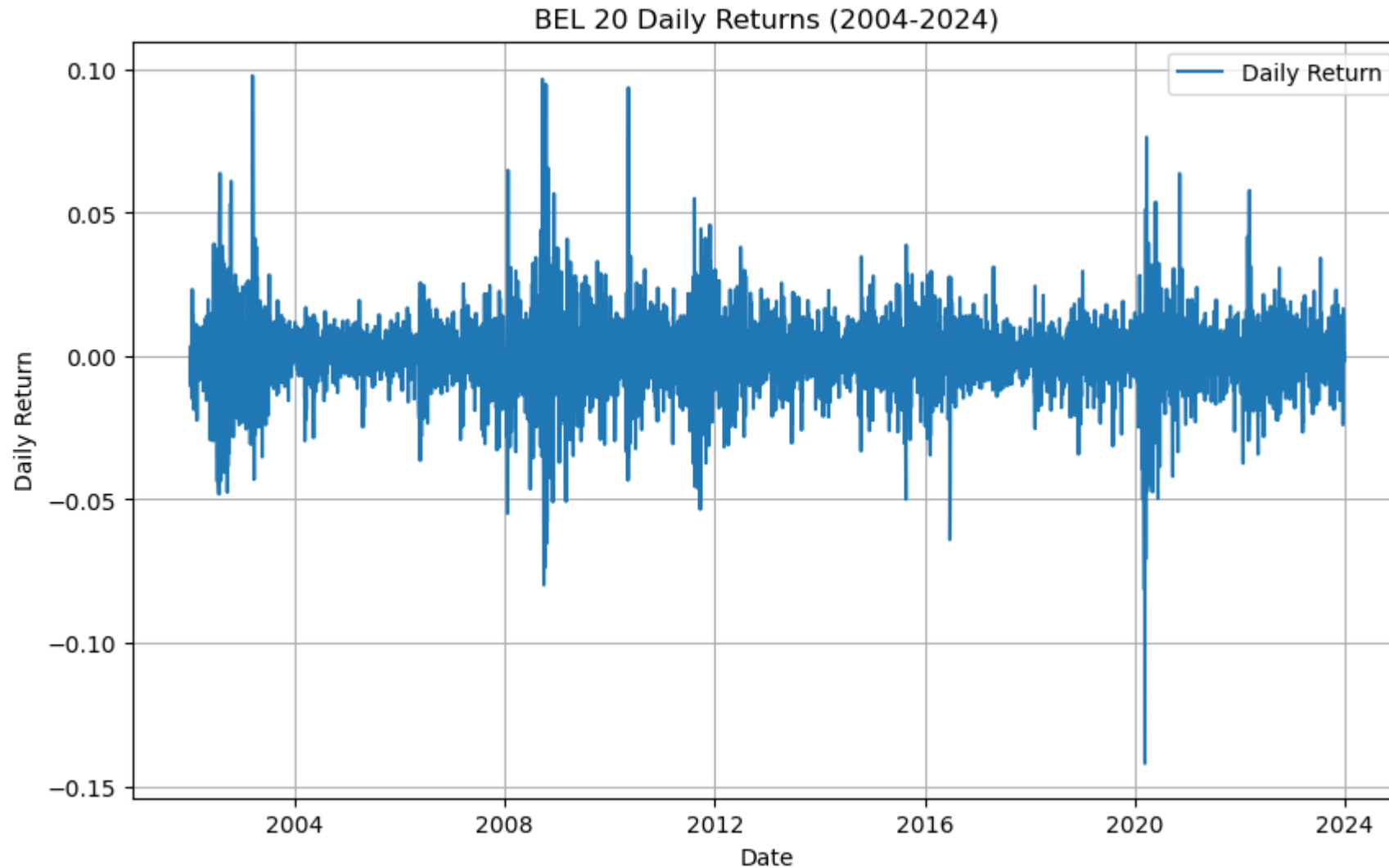
E.g. Montecarlo simulations:



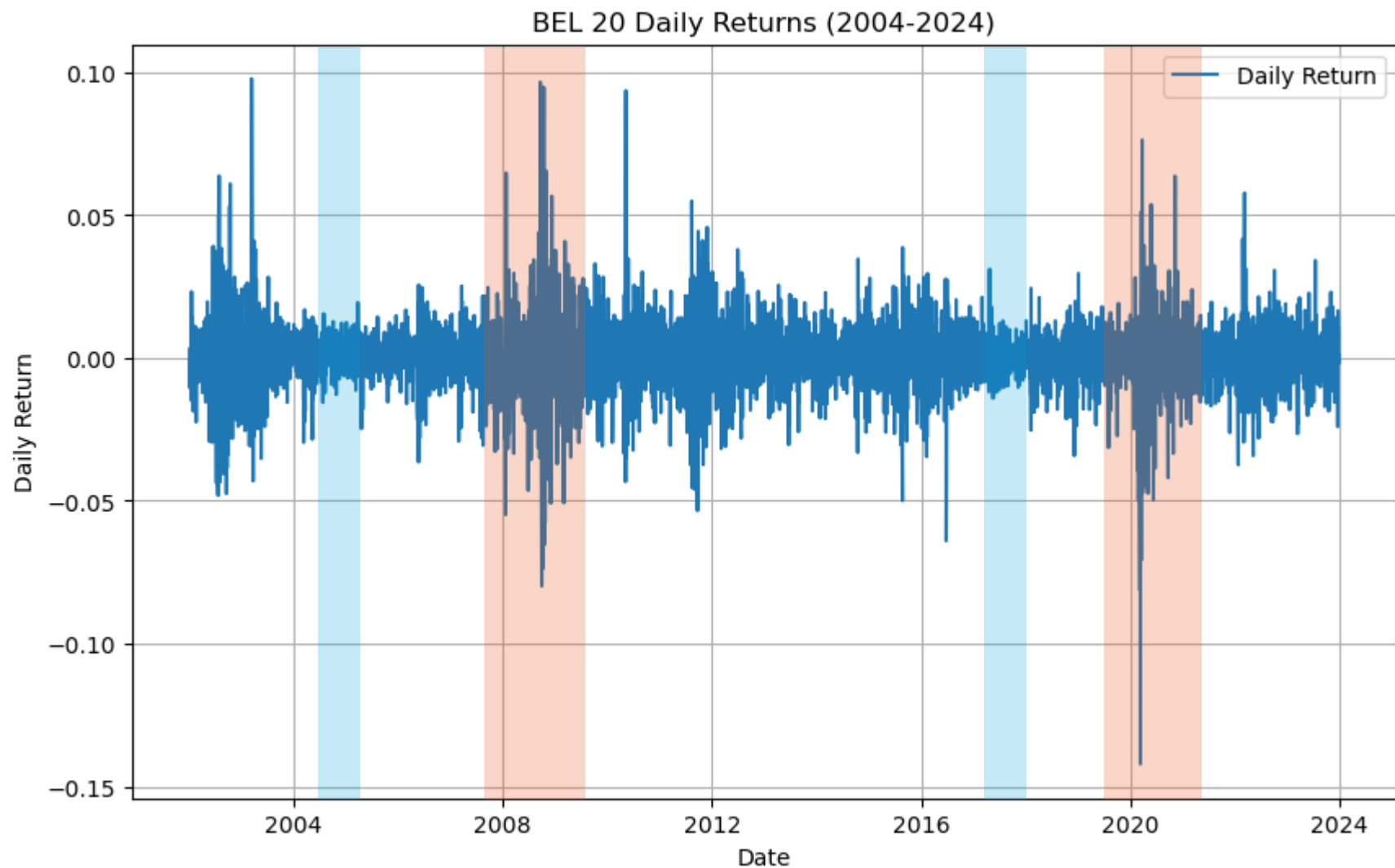
We realize many time series are not stationary



Can first difference solve it?



Variance can be clustered





What
is it?

ARCH & GARCH History



Robert Engle

Received Nobel in Economics (2003) for developing: “methods of analysing economic time series with time-varying volatility – *ARCH*”



Tim Bollerslev

He developed GARCH, the generalized extension of the *ARCH* model. He was a R. Engle’s student

ARCH & GARCH exploit vol's time-dependence

G eneralized

A uto

R egressive

C onditional

H eteroskedasticity

Heteroskedasticity

From Greek, 'different dispersion':

➤ In statistics, non-constant variance!

Conditional Heteroskedasticity

In cross-sections:

$$\text{Var}(\varepsilon_i | x_i) = \sigma_i^2$$

→ Covariates can
influence volatility

In time-series:

$$\text{Var}(\varepsilon_t | t) = \sigma_t^2$$

→ Time can
influence volatility

ARCH – Auto-Regressive Conditional Heteroskedasticity

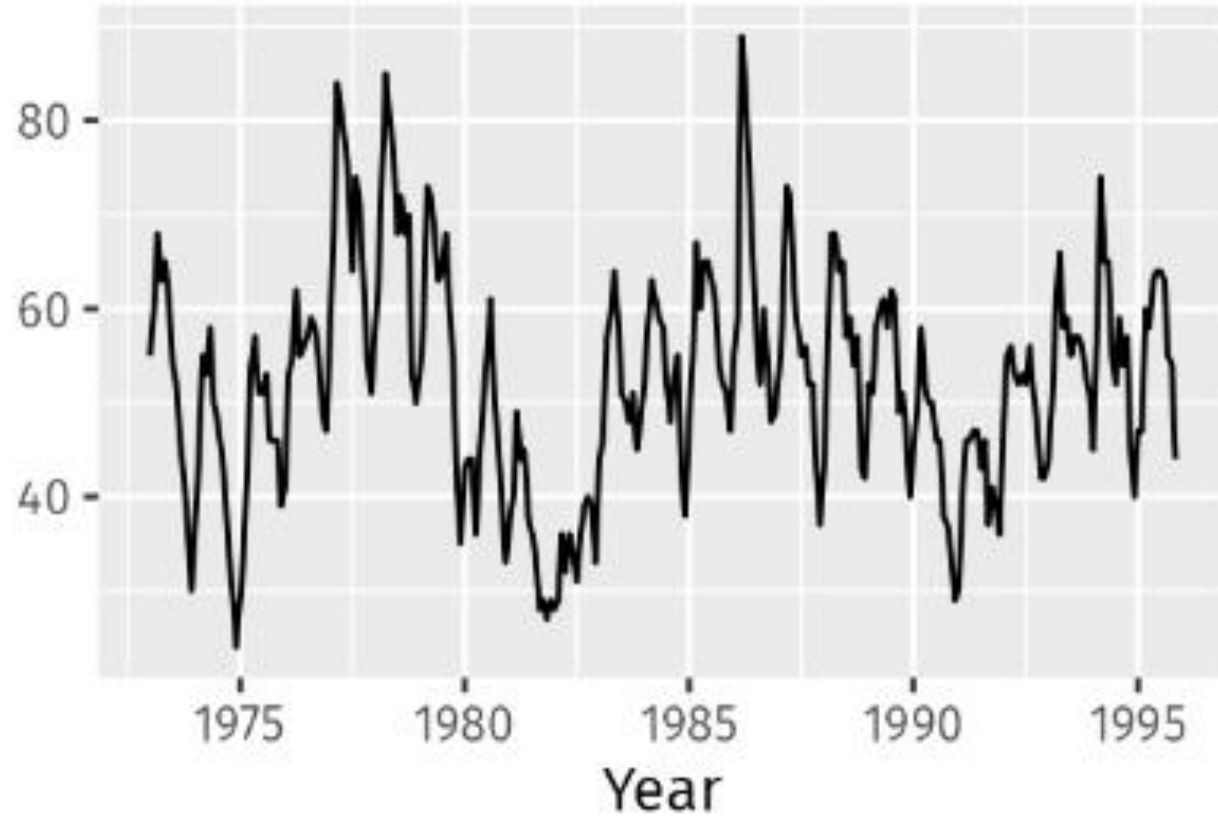
Assume the following model:

$$r_t = \mu_t + \varepsilon_t$$

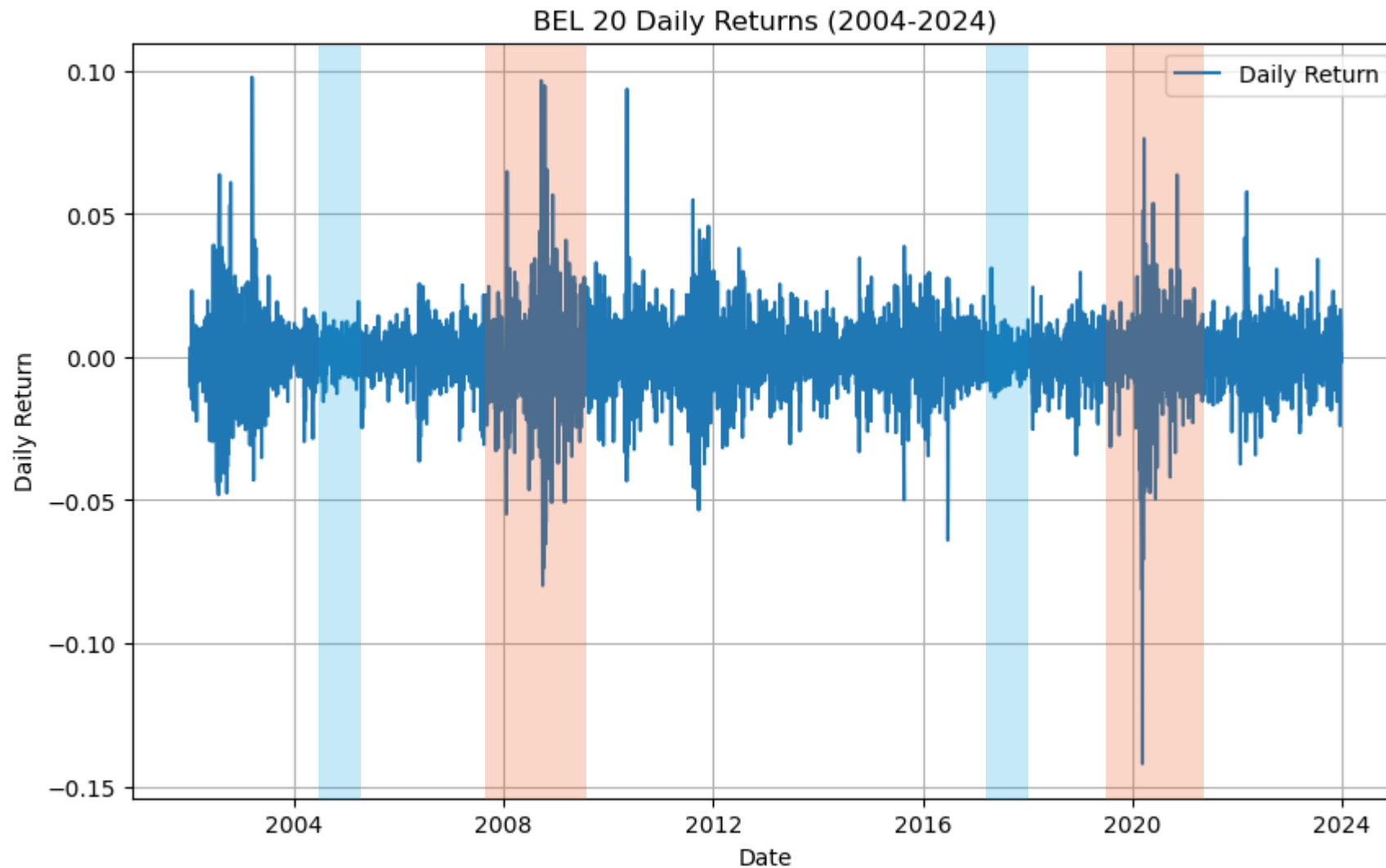
The residual ε_t follows an *ARCH* (q) process if:

$$\begin{cases} \varepsilon_t = \sigma_t \eta_t \text{ (s.t. } \eta_t \text{ is a White Noise)} \\ \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \end{cases}$$

ARCH can model peak variance....



....but not persistence of volatility



Generalized ARCH (GARCH)

The error ε_t follows a *GARCH* (p, q) process if:

$$\begin{cases} \varepsilon_t = \sigma_t \eta_t \text{ (s.t. } \eta_t \text{ is a White Noise)} \\ \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \end{cases}$$

GARCH (1,1) Assumptions

$$GARCH(1, 1) : \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

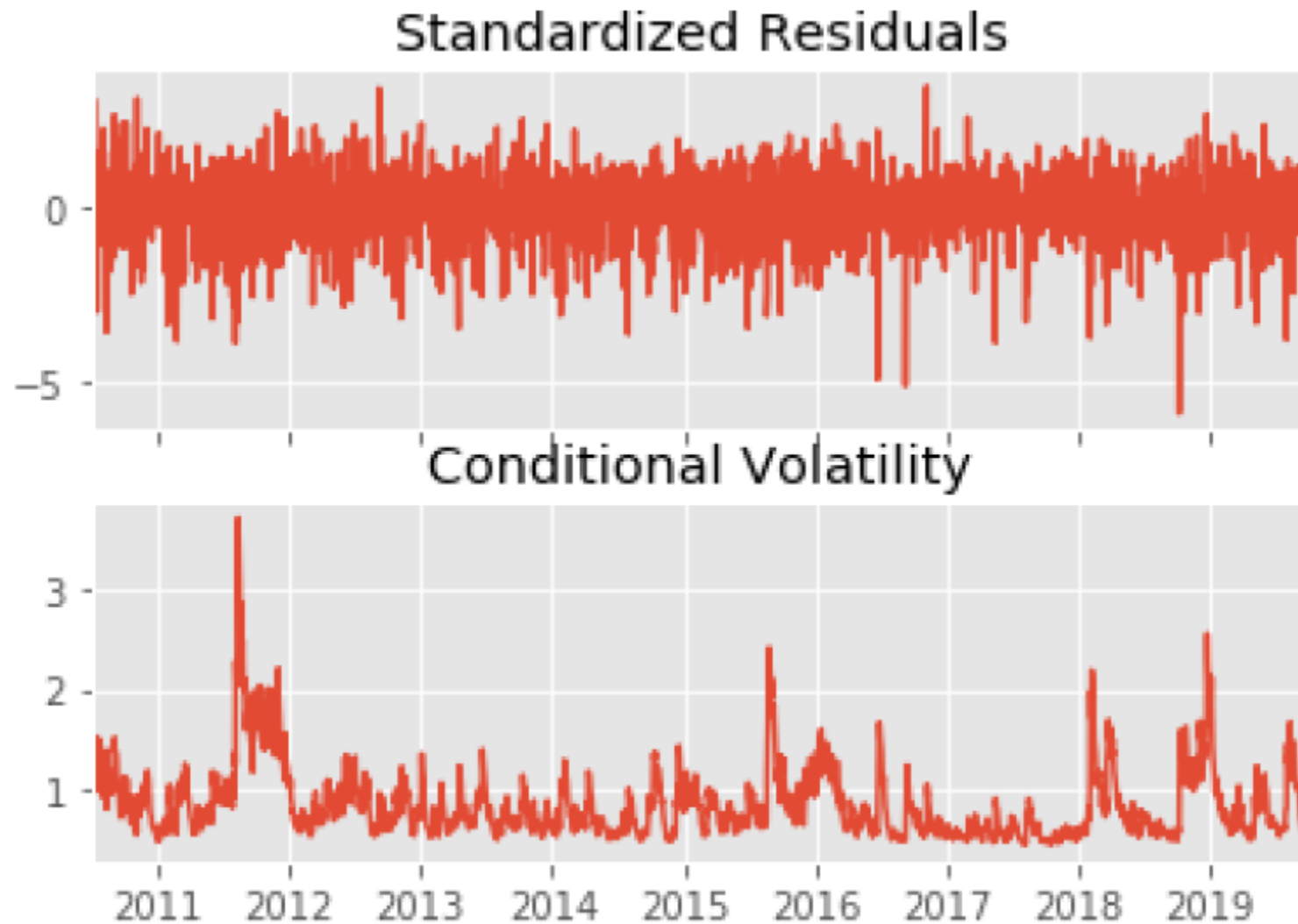
- $\omega, \alpha, \beta \geq 0 \rightarrow$ Positive variance
- $\alpha + \beta < 1 \rightarrow$ Mean-reverting variance

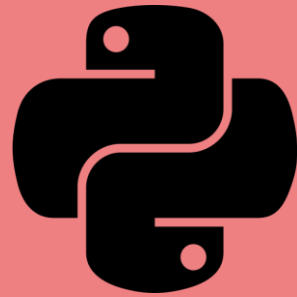
Long-run Variance in GARCH (1,1)

$$GARCH(1, 1) : \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

$$\frac{\omega}{(1 - \alpha - \beta)}$$

Residuals divided by standard deviation are stationary





How to
implement?

arch package in Python

```
from arch import arch_model
```

- Possibility to run both ARCH and GARCH models with different specifications.

```
basic_gm = arch_model(sp_data['Return'], p = 1, q = 1,  
                      mean = 'constant', vol = 'GARCH', dist = 'normal')
```


Let's see an example!

Next steps

- Use in different financial series
- Comparison of Monte Carlo and GARCH forecasts
- Backtesting performance (e.g. Mean absolute error)
- Model extensions

THANK YOU!!

GARCH:

How insights on conditional variance can help our predictions

Francisco Pitthan

