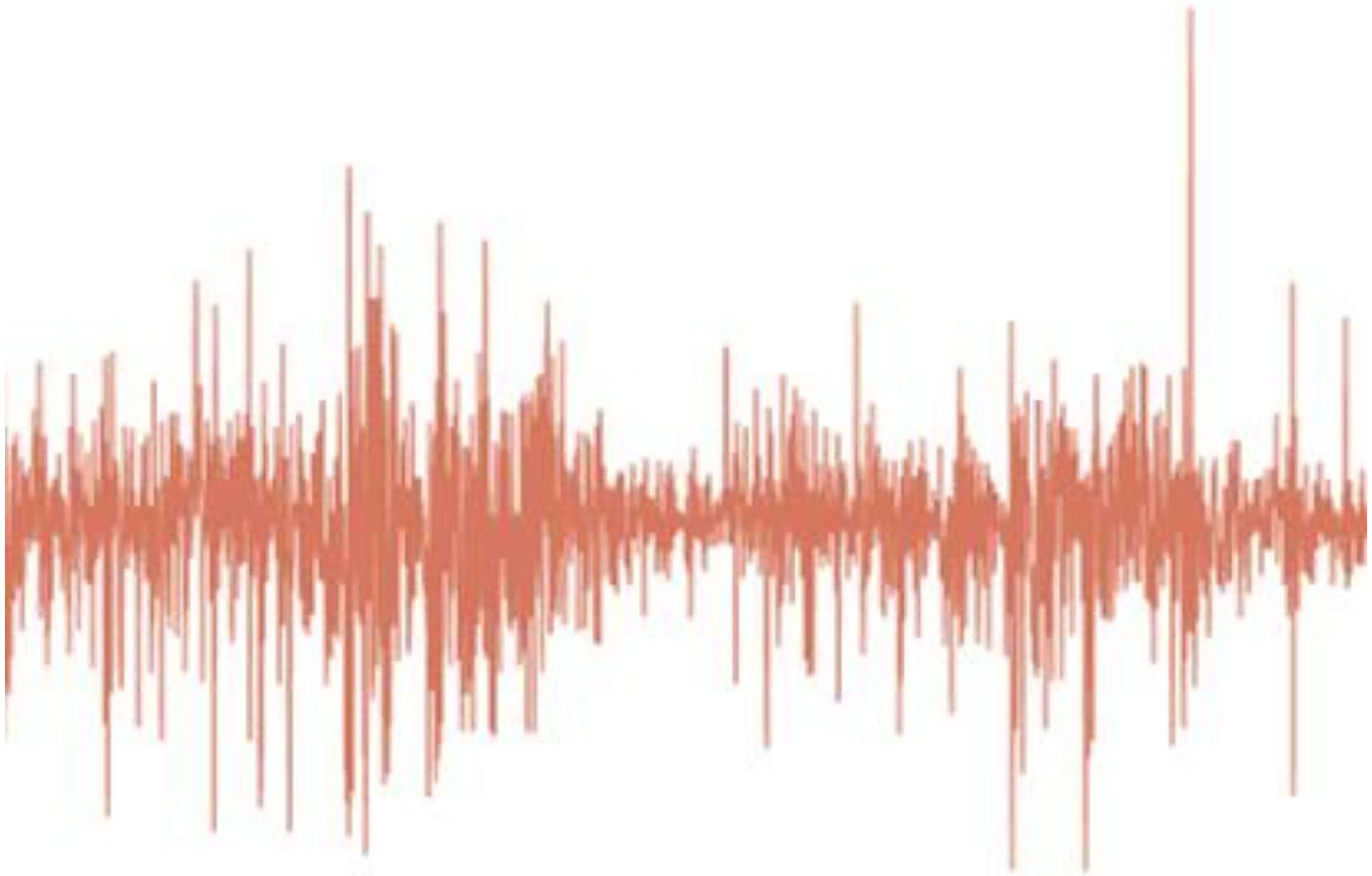
GARCH:

How insights on conditional variance can help our predictions



Francisco Pitthan

Objective

Why it matters?

What is it?

How to implement?

Why it matters?

✓ Many series have time-dependent volatility

What is it?

✓ Model with persistent & time-depend. volatility

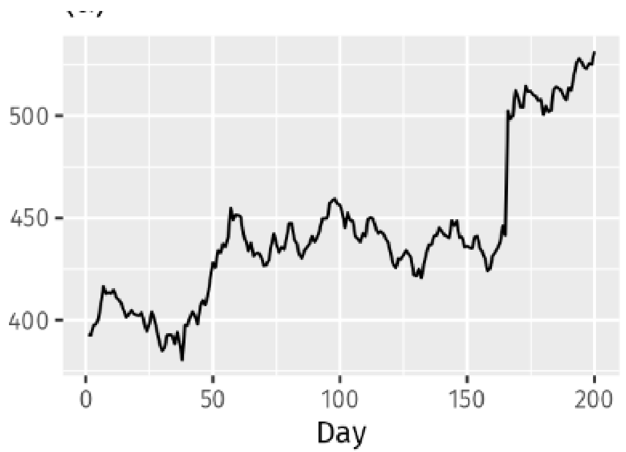
How to implement?

✓ Python implementation with *arch* package

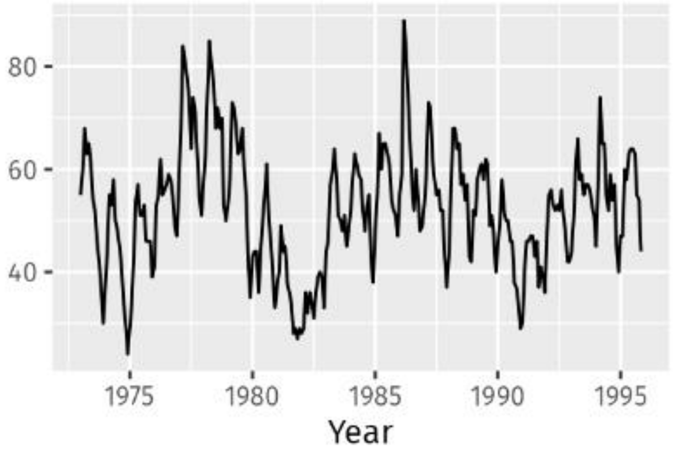
Why it matters?

Forecasting models assume stationary time-series

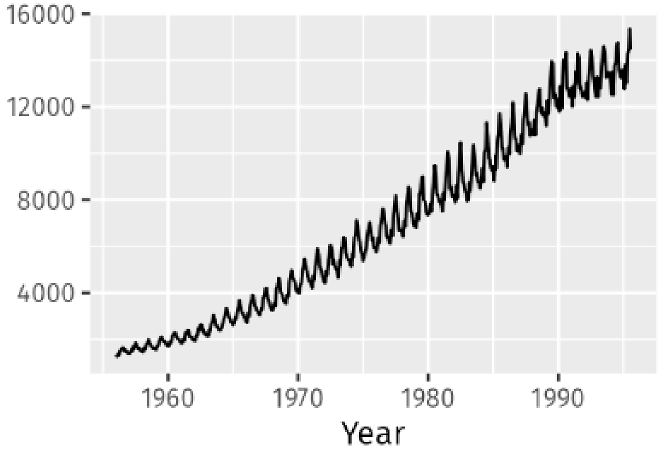
Which can be defined[1]:



Constant mean without trends



No seasonality



Constant variance over time

1. Heckert et al. (2012)

Stationarity is important for modeling

E.g. AR(1)

$$y_t = \phi y_{t-1} + u_t$$

$$y_{t-1} = \phi y_{t-2} + u_{t-1}$$

$$y_t = \phi(\phi y_{t-2} + u_{t-1}) + u_t$$

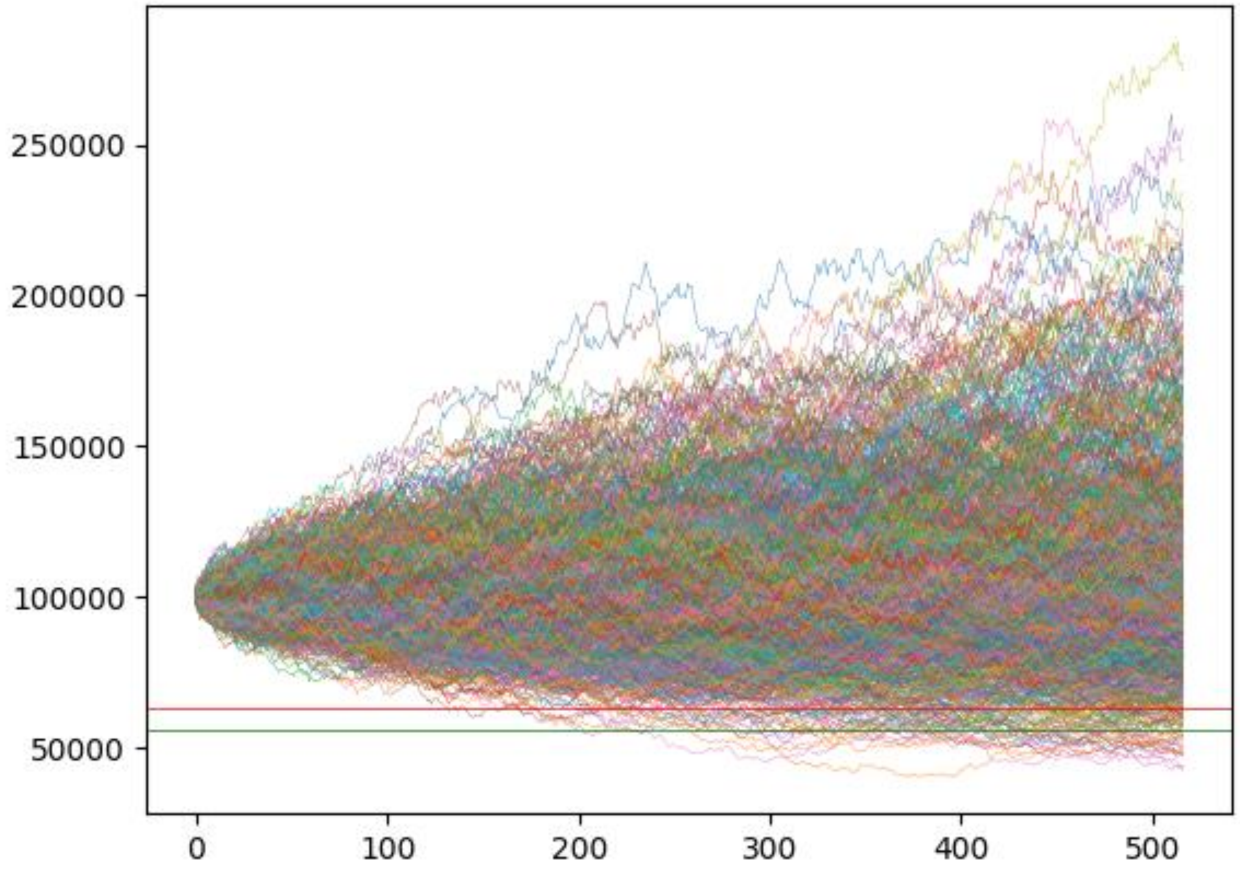
$$y_t = \phi^2 y_{t-2} + \phi u_{t-1} + u_t$$

$$y_t = \phi^m y_{t-m} + \sum_{j=0}^{m} \phi^j u_{t-j}, \quad |\phi| < 1$$

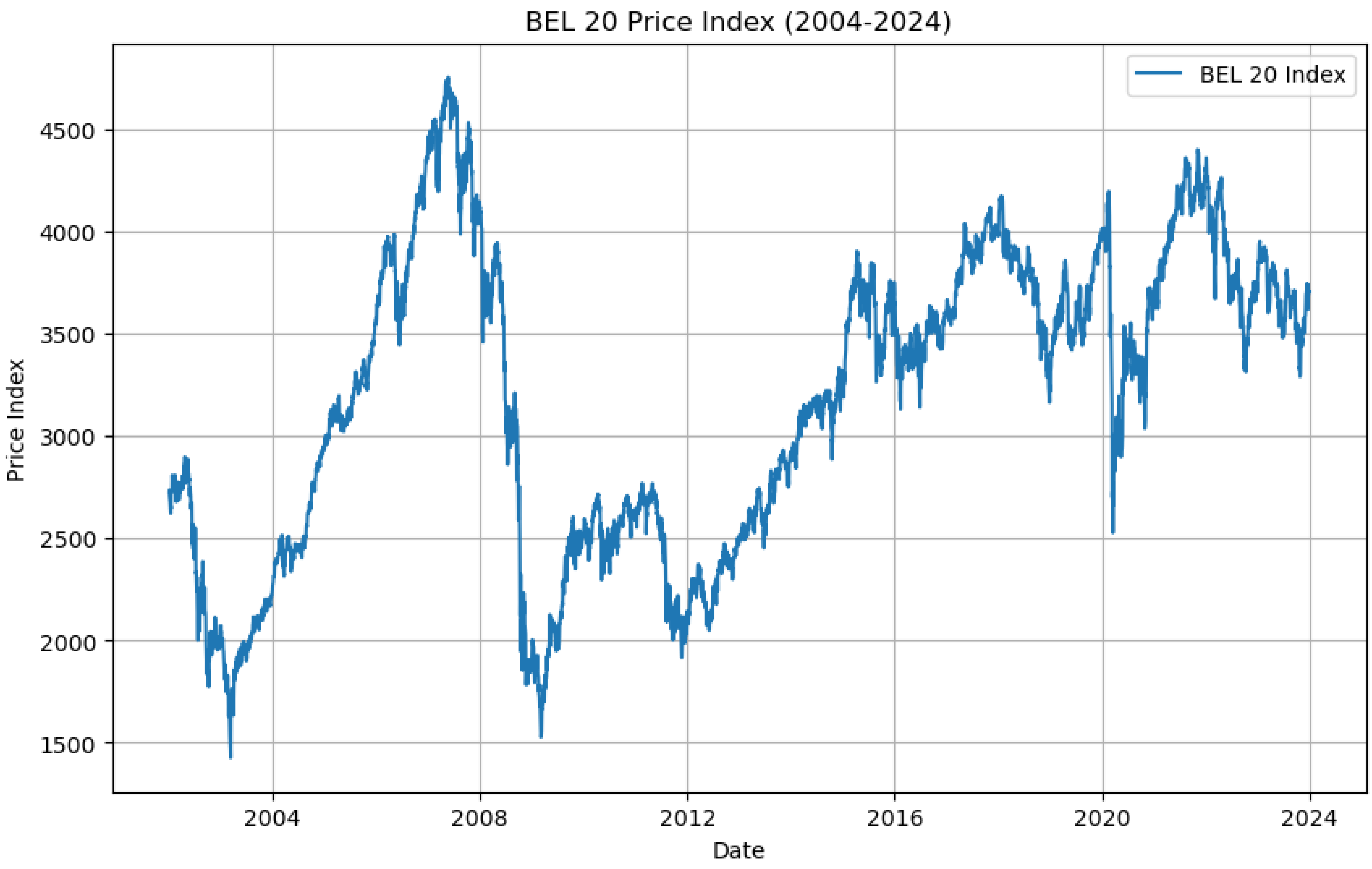
$$y_t = \sum_{j=0}^{\infty} \phi^j u_{t-j}$$

Can be an implicit assumption

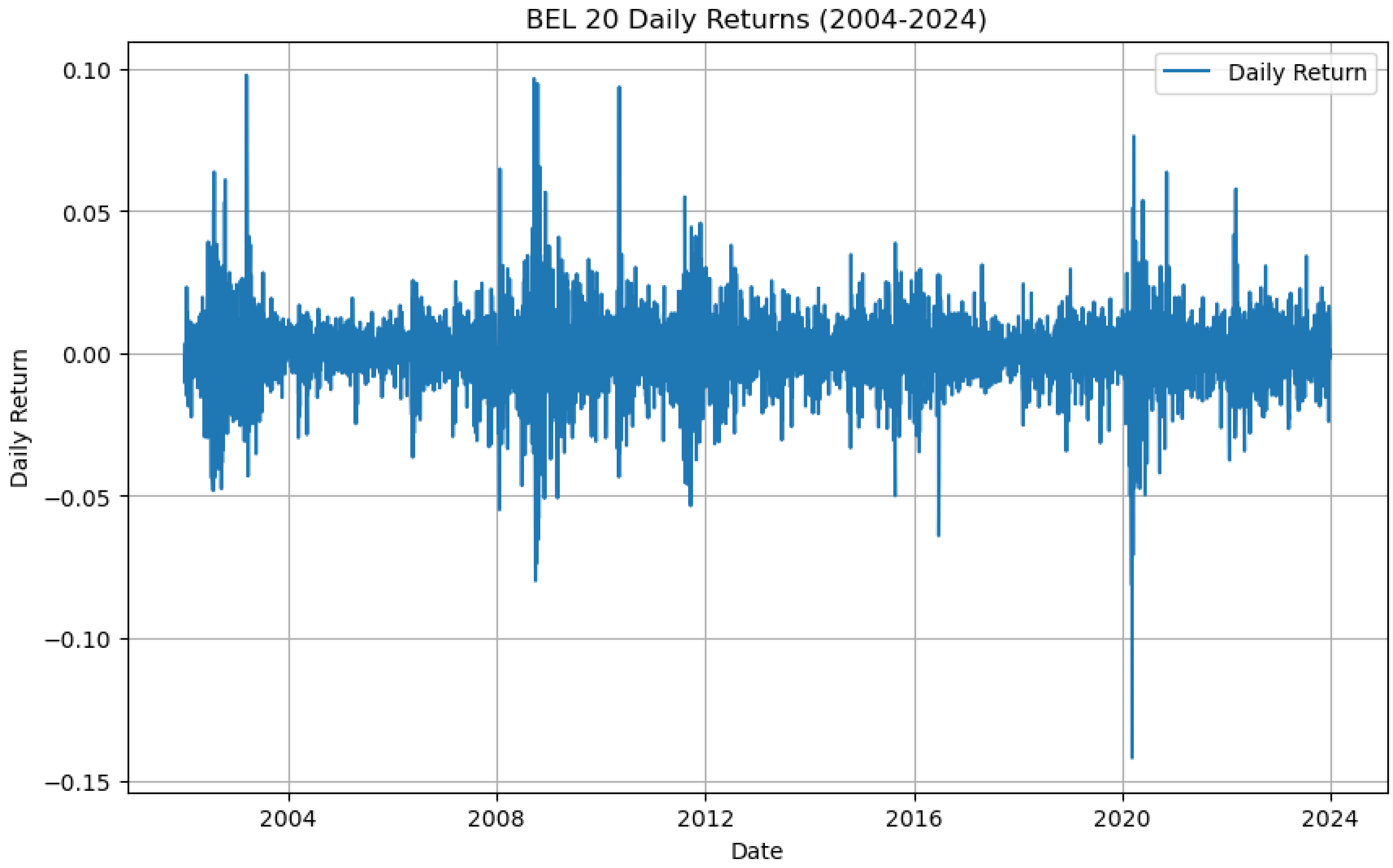
E.g. Montecarlo simulations:



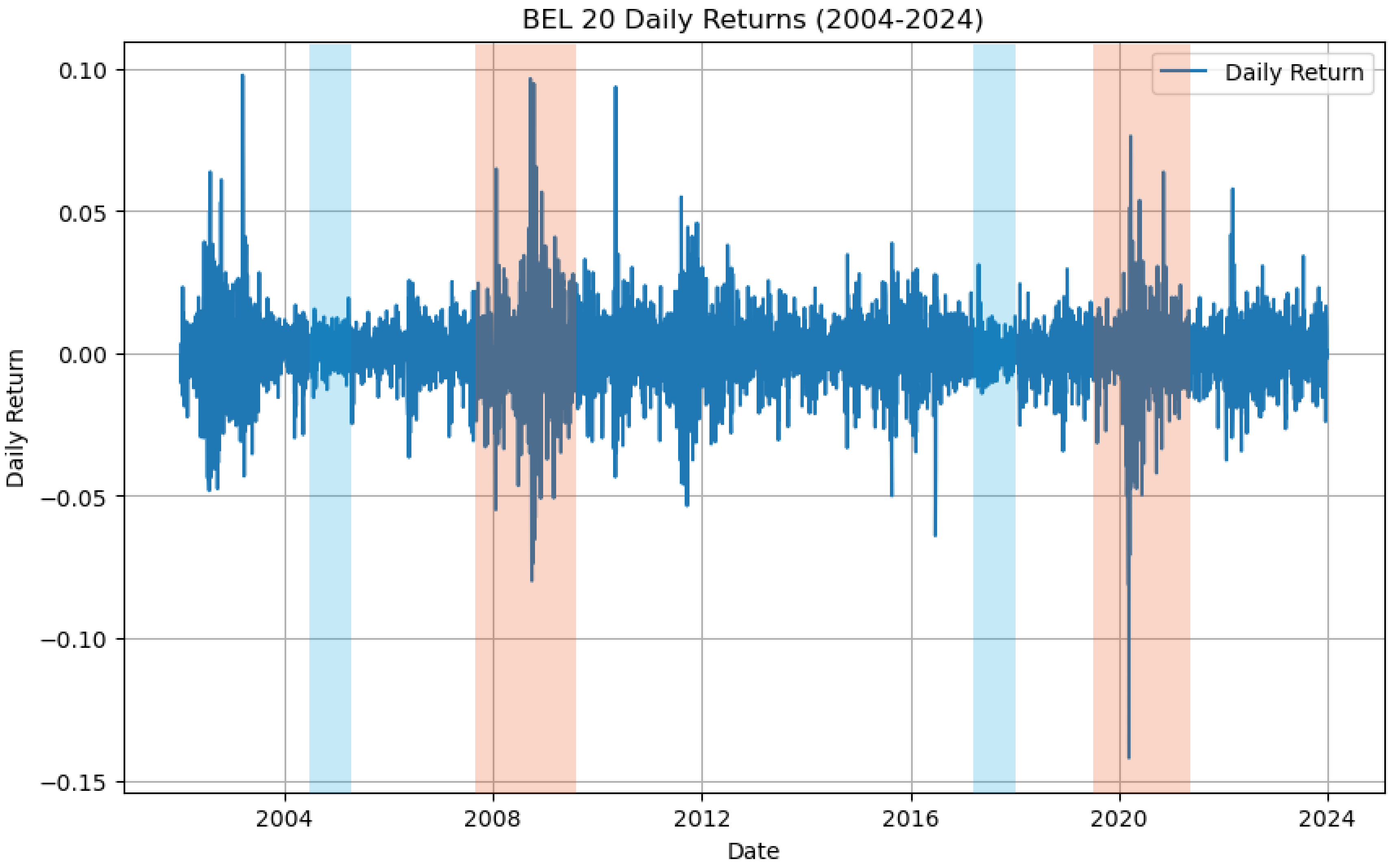
We realize many time series are not stationary



Can first difference solve it?



Variance can be clustered



What
is it?

ARCH & GARCH History

Robert Engle

Received Nobel in Economics (2003) for developing: “methods of analysing economic time series with time-varying volatility – *ARCH*”

Tim Bollerslev

He developed GARCH, the generalized extension of the *ARCH* model. He was a R. Engle’s student

ARCH & GARCH exploit vol's time-dependence

G eneralized

A uto

R egressive

C onditional

H eteroskedasticity

Heteroskedasticity

From Greek, ‘different dispersion’:

- In statistics, non-constant variance!

Conditional Heteroskedasticity

In cross-sections:

$$Var(\varepsilon_i | x_i) = {\sigma_i}^2$$

→Covariates can influence volatility

In time-series:

$$Var(\varepsilon_t | t) = {\sigma_t}^2$$

→Time can influence volatility

ARCH – Auto-Regressive Conditional Heteroskedasticity

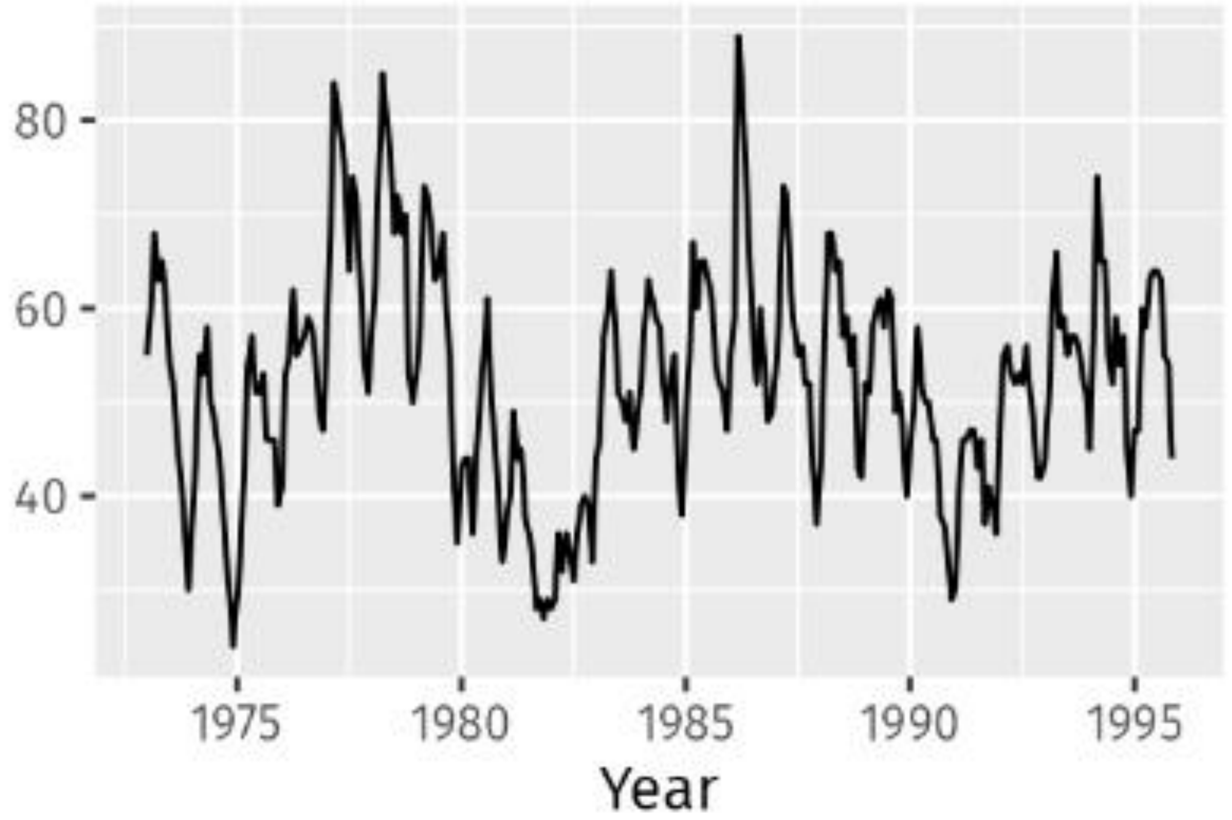
Assume the following model:

$$r_t = \mu_t + \varepsilon_t$$

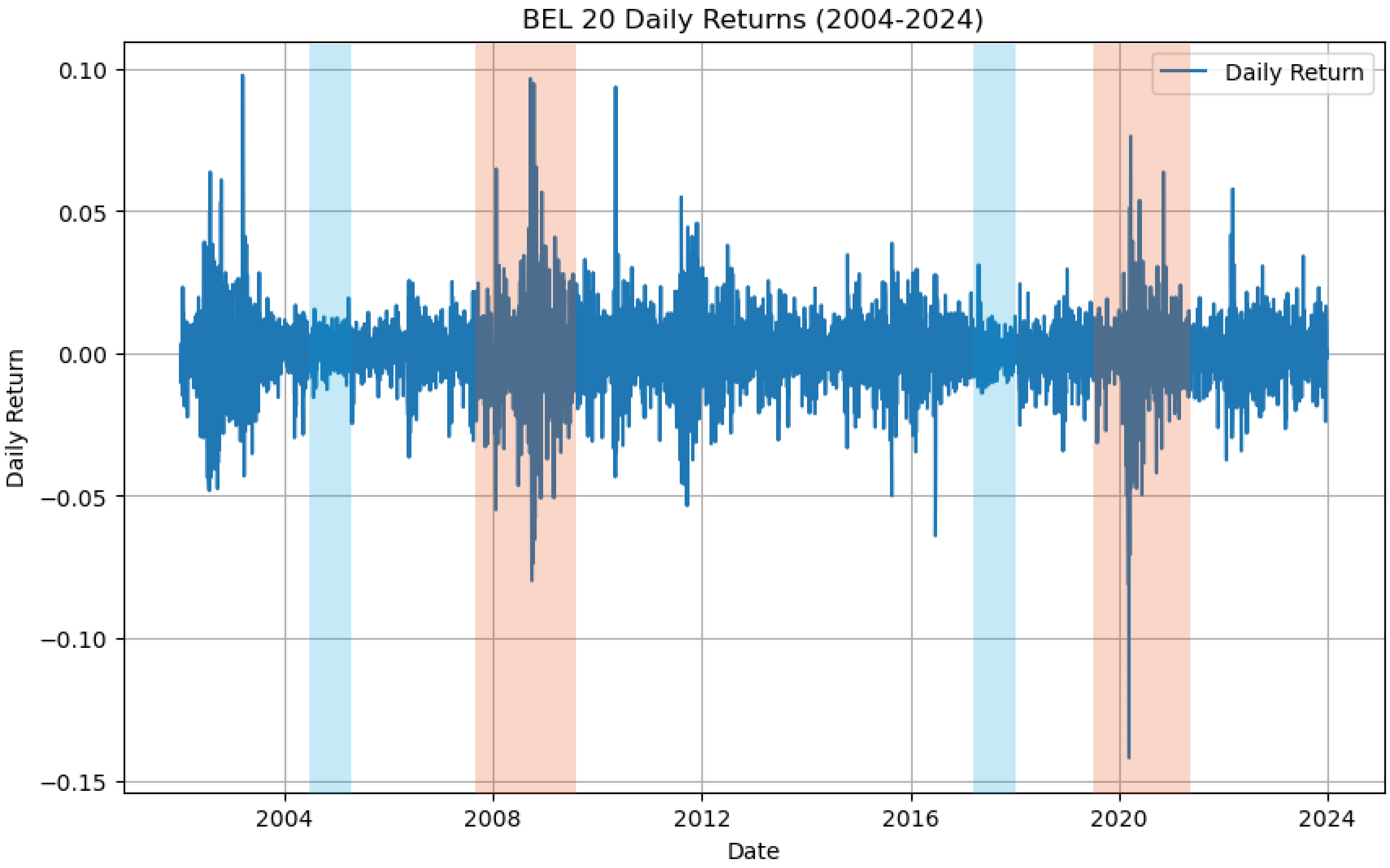
The error ε_t follows an *ARCH (q)* process if:

$$\begin{cases} \epsilon_t = \sigma_t \eta_t & \text{(s.t. } \eta_t \text{ is a White Noise)} \\ \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \end{cases}$$

ARCH can model peak variance….



….but not persistence of volatility



Generalized ARCH (GARCH)

The error ε_t follows a G*ARCH (p,q)* process if:

$$\begin{cases} \epsilon_t = \sigma_t \eta_t & \text{(s.t. } \eta_t \text{ is a White Noise)} \\ \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{cases}$$

GARCH (1,1) Assumptions

$$GARCH(1,1): \sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

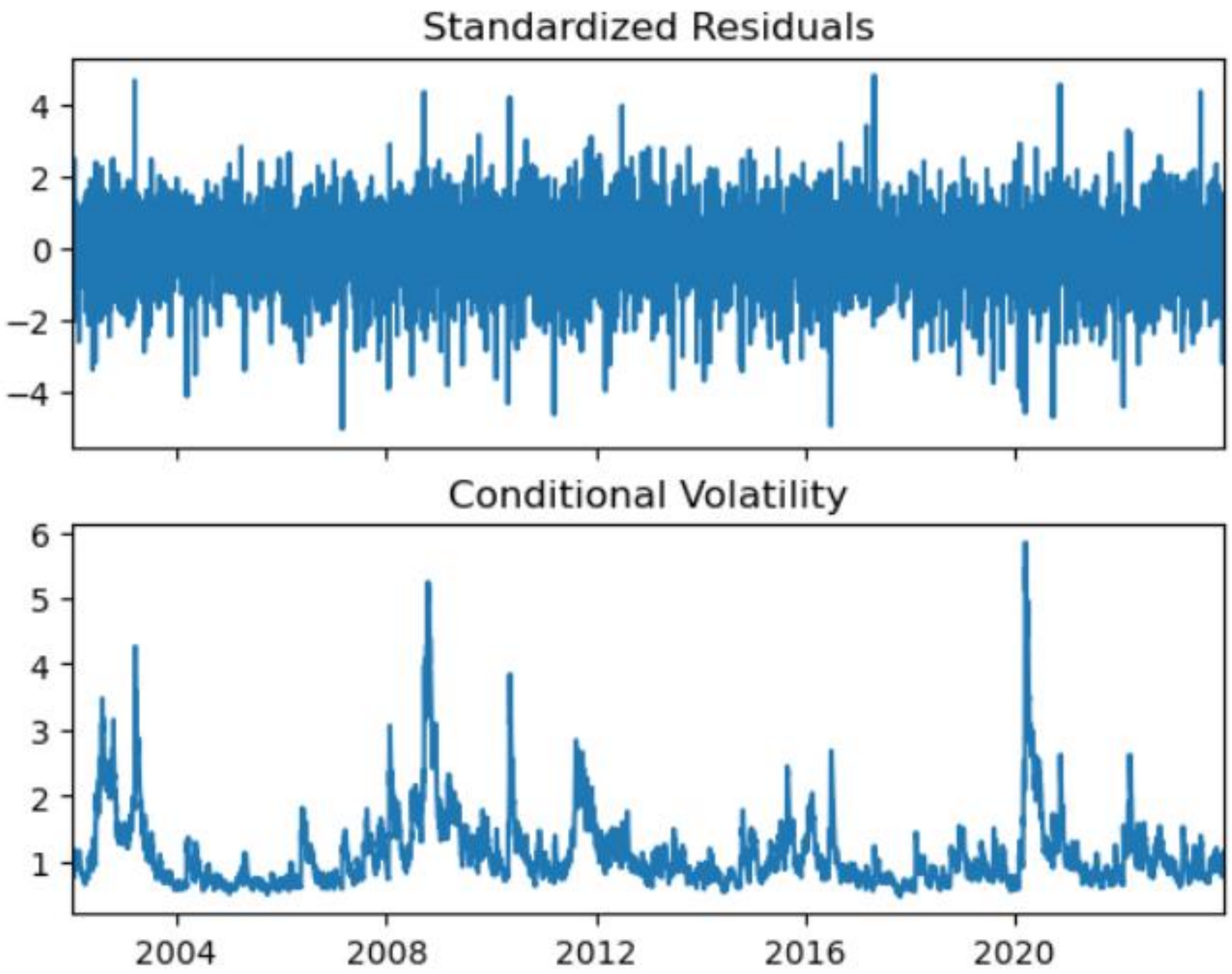
- $\omega, \alpha, \beta >= 0$ → Positive variance
- $\alpha + \beta < 1$ → Mean-reverting variance

Long-run Variance in GARCH (1,1)

$$\boxed{GARCH(1,1): \sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2}$$

$$\frac{\omega}{(1 - \alpha - \beta)}$$

Residuals divided by standard deviation are stationary



How to implement?

arch package in Python

```python
from arch import arch_model
```

- Possibility to run both ARCH and GARCH models with different specifications.

```python
basic_gm = arch_model(sp_data['Return'], p = 1, q = 1,
                      mean = 'constant', vol = 'GARCH', dist = 'normal')
```

Let’s see an example!

Next steps

- Value at Risk estimation
- Use in different financial series
- Comparison of Monte Carlo and GARCH forecasts
- Backtesting performance (e.g. Mean absolute error)
- Model extensions

THANK YOU!!

GARCH:

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