Set up situation.

Train drivers passing by each other. Really very fast trains. Actually had better be spaceships. With velocity v. Velocity could be measured in anything so we may need a conversion factor.

Each sees the other's measuring rods shrunk along the axis of movement by a factor of $\sqrt{1-\frac{v^2}{c^2}}$. This shrinkage factor is so important that it has its own name β (it is also much easier to write β in what follows. Here c stands for a conversion factor $299792458ms^{-1}$, which makes $\frac{v^2}{c^2}$ dimensionless. It did not matter what we used. What c means will make sense in a moment (forward ref).

(Fitzgerald contraction)

Note that because β depends on v^2 it is the same for positive and negativ velocities. In other words both scientists see the same shrinkage of the other. You very occasionally hear it said "rods contract if you move faster" but that sounds like it would be one-sided. How can both scientists think the other one's rod is contracting, who is right (neither?), what is going on. We aren't seeing the whole picture.

As velocity increases and gets closer to c then β tends to zero. (see graph). There is a singularity in β at v = c, more on this later.

