Chapter 1

Elementary trigonometry

We have met the ideas of angles and distances. The subject of trigonometry allows us to convert between the two. For example, if we know that the wheel of a steam engine has turned one eight of a complete turn ($\pi/4$ radians) then how much forward or backward movement will the piston of the engine have moved?

"Trigonometry" literally means (from Greek) the measurement of triangles ("trigons" though we don't call them that). In a way the name undersells its usefulness. In a sense you can use trigonometry even if there are no triangles around, although of course if there are at least three points in any problem you can always draw a triangle in to connect them.

Let us start with a simple example of the problem that we want to solve. Let us imagine that the circle depicted in figure 1 is vertical. O is the centre of the circle. The line passing horizontally through the circle we can take, arbitrarily, as our zero height line. If it helps to think of it as "ground level" then that is fine. The line OP is something that is rotating in that vertical plane. It might be an abstraction of (say) the arm of a crane, or a rocket launching vehicle.

Let us suppose that the arm has rotated through an angle θ. How high will the end of the arm (point P) be above the baseline.

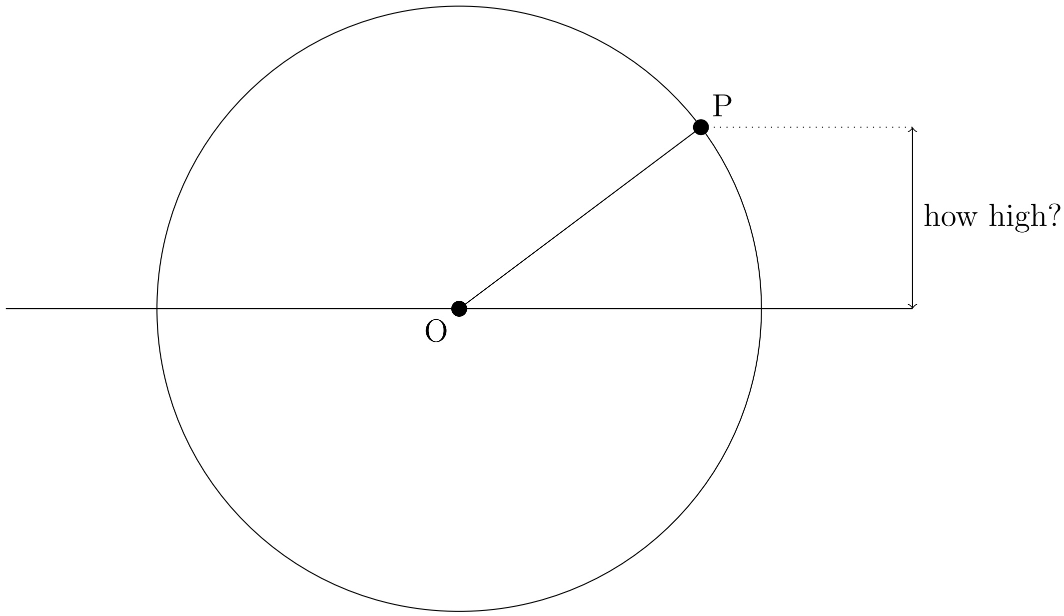

Figure 1.1: How high is P?

Well without more information this question is unanswerable because it depends on the size of the circle as well as the angle of OP from the "horizontal".

As it happens the size of the circle is not really critical, because if we scale up the size of the circle, we scale the height of P by the same amount. We might just as well work out the example for a circle with radius of 1. Once we have the answer, we can just multiply it by the radius if the circle we are interested in.

If you aren't happy with the idea that everything just scales up, I will prove it in the end notes of this chapter using similar triangles.

Even though we don't (yet) know exactly how the height of P behaves as it moves around the circle, there are a few things we do know. Let us draw a picture with four points (A, B, C, D) marked around it at the obvious four cardinal points of the circle and consider what happens to P's height as it travels around the circle. Mathematicians traditionally measure angles anticlockwise[1].

[1] why?

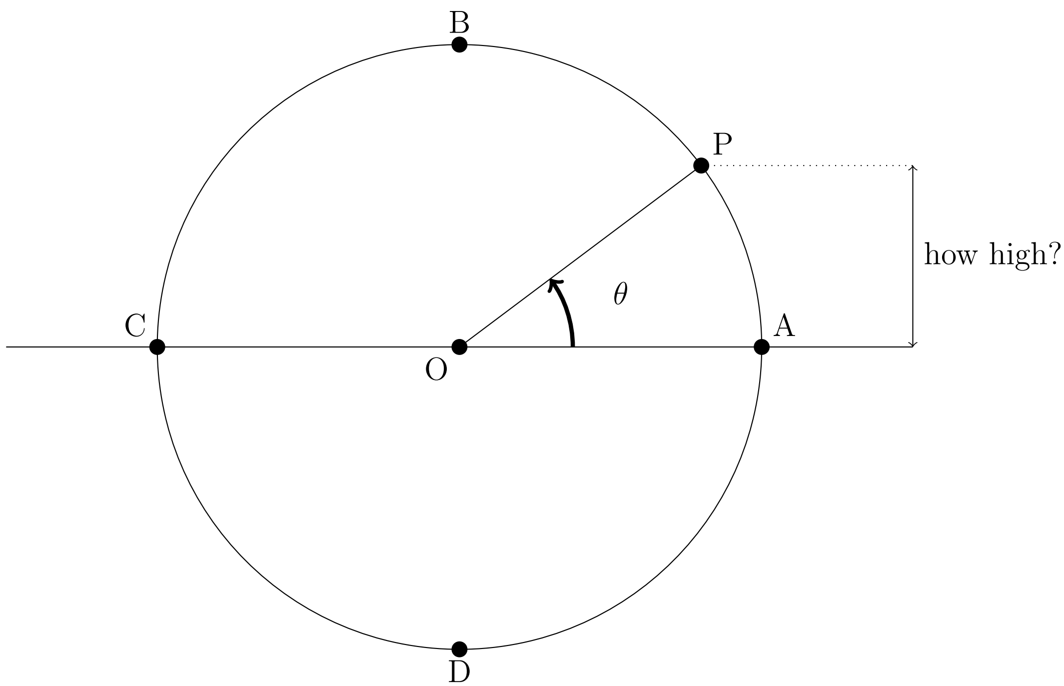

Figure 1.2: P's journey around the circle

When P starts out at A, it clearly has a height of zero. As it rotates anticlockwise, the height will increase until it reaches a maximum at B. At this point $\theta = \pi/2$radians. Since the circle has radius 1 (known in the trade as a "unit circle"), that maximum is 1.

Further round the circle, between $\pi/2$ and π radians, the height declines until at C (where $\theta = \pi$) the height is zero.

Further around the circle the arm is going to go below ground level. We can imagine that it is able to do so because someone has dug a ditch (not illustrated). Maybe it is the digging arm of a digger doing just that. The sensible thing to do here is treat the height of P as negative, because it is below our chosen zero.

Thus P then becomes negative from $\theta = \pi$ until $\theta = 3\pi/2$ at which point it has reached its minimum height -1. After this P begins to move back up until it arrives back at A. It should be clear that if θ grows beyond 2π the whole process will simply be repeated.

If we draw a graph of the height of P against the angle, this pattern becomes clearer.