

# AS3 - Option Pricing Project

## Black-Scholes and Binomial Lattice Methods

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## Overview

This project implements European option pricing using both the **Black-Scholes-Merton (BSM)** model and the **Binomial Lattice Method**. It is developed in **C++** following object-oriented principles. The application also computes the **Delta** of both Call and Put options.

## Structure

- **Option.h / Option.cpp**: Define the **Option** class, encapsulating parameters: strike price  $K$ , underlying price  $S$ , risk-free rate  $r$ , time to maturity  $T$ , and volatility  $\sigma$ .
- **Pricing\_Method.h**: Abstract class declaring two pure virtual functions: **BSM\_Pricer()** and **Binomial\_Pricer()**.
- **Option\_Price.h / Option\_Price.cpp**: Implements pricing methods for both models and computes the Greeks (Delta).
- **main.cpp**: User interface for inputting parameters and receiving pricing/delta results.
- **unit\_test.cpp**: Unit testing for both Call and Put options, validating price and Delta accuracy.

## Mathematical Models

### Black-Scholes Formula (European Call)

$$\begin{aligned}d_1 &= \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \\d_2 &= d_1 - \sigma\sqrt{T}, \\C &= S\Phi(d_1) - Ke^{-rT}\Phi(d_2), \\\Delta_{\text{call}} &= \Phi(d_1)\end{aligned}$$

### Put Option Price and Delta

$$\begin{aligned}P &= Ke^{-rT}\Phi(-d_2) - S\Phi(-d_1), \\\Delta_{\text{put}} &= \Phi(d_1) - 1\end{aligned}$$

where  $\Phi(\cdot)$  is the standard normal CDF.

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## Results

**Unit Testing:** Passed for both pricing models with acceptable numerical tolerance.

**Main Program:** Accepts repeated user input and returns price and Delta using both BSM and Binomial methods.

```
Test case 1 passed
Test case 2: BS and Binomial pricing and delta tests passed!
Test case 3 passed: Put option
Test case 4 passed: Put option
Test case 5: Consistency test passed!
All tests passed!
```

```
Enter Strike Price (K): 100
Enter Underlying Price (S): 100
Enter Risk-free Rate (r): 0.01
Enter Time to Maturity (T): 0.25
Enter Volatility (σ): 0.3
Enter Option Type (C for Call, P for Put): C
Black-Scholes Price: 6.09674
Black-Scholes Delta: 0.536519
Binomial Price: 6.09524
Binomial Delta: 0.536509
Enter 1 to continue, 0 to exit: 1
Enter Strike Price (K): 100
Enter Underlying Price (S): 100
Enter Risk-free Rate (r): 0.03
Enter Time to Maturity (T): 0.3
Enter Volatility (σ): 0.25
Enter Option Type (C for Call, P for Put): P
Black-Scholes Price: 4.99776
Black-Scholes Delta: -0.446625
Binomial Price: 4.9964
Binomial Delta: -0.446639
Enter 1 to continue, 0 to exit: 0
```