Population variance: $\sigma^2 = \mathbb{E}[(X - E(X))^2]$

Sample Variance: $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$

Note: The reason we divide by $n-1$ in s^2 & not n is because this makes s^2 an ~~b~~ unbiased estimator i.e. $\mathbb{E}[s^2] = \sigma^2$. This means that, on average, the sample variance is accurate.

Next, we unpack the formula for s^2 & try to arrive at the form that appears in exercise D:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i^2 - 2y_i\bar{y} + \bar{y}^2)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} y_i^2 - 2\bar{y} \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} \bar{y}^2 \right)$$

$$\boxed{\text{*Note: } \bar{y} = \sum_{i=1}^{n} y_i / n \Rightarrow n\bar{y} = \sum_{i=1}^{n} y_i}$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} y_i^2 - 2\bar{y}^2 n + n\bar{y}^2 \right)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2 \right)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} y_i^2 - n \left(\sum_{i=1}^{n} y_i / n \right)^2 \right)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} y_i^2 - \frac{n}{n^2} \left(\sum_{i=1}^{n} y_i \right)^2 \right)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} y_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} y_i \right)^2 \right)$$

Next, let $C_{T,j} = y_i$ $= \frac{1}{n-1} \left(\sum_{i=1}^{n} C_{T,j} - \frac{1}{n} \left(\sum_{i=1}^{n} C_{T,j} \right)^2 \right)$.

To find the sample standard deviation or $\sqrt{s^2} = s$, take square root.

$$s = \sqrt{\frac{\sum_{i=1}^{n} C_{T,j} - \frac{1}{n} \left(\sum_{i=1}^{n} C_{T,j} \right)^2}{n-1}}, \Rightarrow e^{-rT} \sqrt{\frac{\sum_{i=1}^{n} C_{T,j} - \frac{1}{n} \left(\sum_{i=1}^{n} C_{T,j} \right)^2}{n-1}}$$

Discounted = $e^{-rT} s$