Notes on Dynamics for cryptocurrency derivatives

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Abstract

To be filled.

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1. Ideas

1.1. Uncovered interest parity in the absence of bond

This section is mostly taken/copied from section 6.1 of Gudgeon *et al.* (2020) for idea generation. Uncovered interest parity (UIP) normally appear in the context of foreign exchange between two countries: domestic and foreign. An investor has the choice of whether to hold domestic or foreign assets. If the condition of UIP holds, a risk-neutral investor should be indifferent between holding the domestic or foreign assets because the exchange rate is expected to adjust such that returns are equivalent.

Example An investor starting with 1m GBP at $t = 0$ could either:

- receive an annual interest rate of $i_{\text{GBP}} = 3\%$, resulting in 1.03m GBP at $t = 1$
- or, immediately buy 1.23 USD at an exchange rate $S_{\text{GBP/USD}} = 0.813$, and receive an annual interest rate of $i_{\text{USD}} = 5\%$, resulting in 1.2915m USD at $t = 1$. Then, convert the USD with the exchange rate at $t = 1$, say $S_{\text{GBP/USD}} = 0.7974$, and get 1.03m GBP.

If UIP holds, despite the higher interest rate of the USD, the investor will be indifferent because the exchange rate between currencies offset the spread between interest rates. Mathematically, UIP is stated as

$$1 + R^{(i)} = (1 + R^{(j)}) \frac{\mathsf{E} S_{t+k}}{S_t},$$

where $R^{(i/j)}$ is the interest rate payable on asset i/j from time t to $t + k$, and S_t is the exchange rate ar time t.

Now the question is both $R^{(i/j)}$ are not known in advance due to the lack of a liquid bond market that investors can secure the future payoff by holding a cryptocurrency. However, the good news is that we have the observable historical short rate, $r_t^{(i)}$, and exchange rate $S_t^{(i/j)}$. The UIP condition in this case have to be adjusted to incorporate the fact that the investor consider also the uncertainty of the domestic and foreign short rate, i.e.[1]

$$\mathsf{E}\left(\exp\int_0^T r_t^{(i)}\mathrm{d}t\right) = \mathsf{E}\left(\exp\int_0^T r_t^{(j)}\mathrm{d}t\frac{S_{t+k}}{S_t}\right).$$

The above UIP condition open quite some questions

1. It seems necessary to model the joint dynamics of the foreign short rate and the exchange rate, such that the R.H.S. of the above equation can be evaluated.
2. Under which measure should we take the expectation of both sides? Any criteria of choosing the measure? No-arbitrage?
3. When will the above equation hold? When will not?
4. If the condition does not hold, are there any arbitrage opportunities?
5. What is the dynamics of $r^{(i/j)}$? For further development, we might want a parametrised stochastic model such that we can (i) perform measure change, (ii) price bonds, swaps, or any other derivatives easily, (iii) capture the interest rate dynamics nicely.

2. Literatures

2.1. Affine term structure models

1. The paper extends the affine class of term structure models to describe the joint dynamics of exchange rates and interest rates

[1]The AAVE and Compound interest rate are compounded every second, which is close enough to model the short rate payoff in a continuous compounding scheme.

References

Gudgeon, L., S. Werner, D. Perez, and W. J. Knottenbelt. Defi protocols for loanable funds: Interest rates, liquidity and market efficiency. In *Proceedings of the 2nd ACM Conference on Advances in Financial Technologies*, pages 92–112, 2020.