

# Low-Risk Alpha Without Low Beta

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September 2024

## Abstract

We propose a risk-managed approach to capturing the low-volatility anomaly. Leveraging multi-factor low-risk portfolios to a beta of 1.0 while controlling tracking error amplifies strategy returns and information ratios. Across developed and emerging markets, this levered low-risk strategy outperforms the market and traditional low-risk portfolios. Outperformance is driven by the strategy's low-risk tilt rather than leverage effects. Our results suggest that investors who are able to overcome leverage constraints are able to harvest the low-volatility anomaly more efficiently.

## Key Findings

- Leveraging multi-factor low-risk portfolios to a beta of 1.0 while controlling tracking error amplifies returns, outperforming both the market and traditional low-volatility portfolios.
- Strategy outperformance stems from the low-risk tilt, not leverage effects, supporting the role of leverage constraints in the anomaly's persistence.
- Investors able to use leverage can harvest the low-volatility anomaly more efficiently, enhancing returns while maintaining a market-like risk profile.

The low-volatility (or low-beta) effect may well be the most fundamental anomaly in financial markets. Whereas other anomalies, such as size, value, and momentum, imply that the Capital Asset Pricing Model (CAPM) is incomplete and needs to be augmented with additional factors, the low-beta anomaly challenges the core tenet that there exists a proportional relation between the market beta of a security and its expected return. Unfortunately, it is also one of the most difficult anomalies to exploit in practice, because low-beta portfolios exhibit a high active risk versus the market index (tracking error). Their typical return profile consists of substantial outperformance in bear markets, alternating with prolonged periods of underperformance in bull markets.

In this paper, we propose a novel leveraged low-risk strategy that is specifically designed to overcome this problem and deliver consistent outperformance regardless of the prevailing market environment. We first construct base-case low-risk portfolios which produce an ex-post beta of approximately 0.7 to the broad market, in line with the betas of low-volatility portfolios documented in the literature (Blitz and van Vliet, 2007). Inspired by the conservative investing formula of Blitz and van Vliet (2018) we incorporate value, momentum, and quality characteristics alongside low volatility.<sup>1</sup> We then apply a moderate amount of leverage (up to 40%) to these portfolios to increase their market beta to 1.0, bringing their absolute risk in line with the market while preserving their embedded tilt toward low-risk stocks. Importantly, we use a risk model to control the ex-ante tracking error of the leveraged low-risk portfolios relative to the market. As such, our approach is more efficient than simply scaling up a traditional low-risk portfolio proportionally or applying an index futures overlay to close the beta gap with the market index.

In a liquid developed markets universe over the period from 1986 to 2023, the information ratio (IR) of a leveraged risk-controlled low-risk strategy increases from 0.43 (no leverage, no risk control) to 0.92. This increase comes from both an increase in outperformance relative to the benchmark (3.32% to 5.92%) and a decrease in ex-post tracking error (7.75% to 6.43%). Although this increase in IR comes at the cost of a slightly lower Sharpe ratio (0.72 to 0.67), the maximum benchmark-relative drawdown drops from 22% to 12% for the leveraged risk-controlled low-risk strategy. Taken together, these results show how the combination of leverage and tracking error control provides access to the alpha of low-risk stocks without the low-beta tilt of traditional low-risk strategies. In other words, our approach transforms a defensive equity strategy that excels at capital preservation into a portfolio with market-like risk characteristics but a high expected outperformance.

Our approach is based on a root cause for the very existence of the low-volatility anomaly, namely leverage constraints. The theoretical basis for this was already provided by Brennan (1971) and Black (1972), who showed that the security market line flattens in the presence of leverage constraints, implying that low-beta stocks have higher returns than predicted by the CAPM. The general intuition is as follows. In the world of the CAPM, there is only one efficient portfolio, and investors simply lever or de-lever this portfolio based on their risk preference. However, when borrowing constraints prevent leverage, investors looking for a higher return have no other

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<sup>1</sup> We also include a Quality factor, given the known outperformance associated with these characteristics (Fama and French, 2015; Asness et al., 2019).

choice than to tilt their portfolios towards high-beta securities, in order to garner more of the equity risk premium. This increased (reduced) demand for high-beta (low-beta) stocks lowers (raises) their equilibrium expected returns. Although many more explanations have been proposed since (see Blitz, Falkenstein, and van Vliet (2014) for a comprehensive overview), there remains a consensus in the low-volatility literature that institutional leverage constraints play a vital role; see e.g., Ang et al. (2006, 2008), Asness et al. (2012), Baker et al. (2011, 2013), Baker and Haugen (2012), Blitz and van Vliet (2007), Blitz, van Vliet, and Baltussen (2019), Clarke et al. (2006), Frazzini and Pedersen (2014), Han and Lesmond (2011), Li et al. (2014), and Scherer (2011).

Black (1993) suggested that investors can leverage efficiently through their allocation. For instance, instead of investing 50% in regular stocks and 50% in bonds, one could choose to invest 70% in low-risk stocks and 30% in bonds. However, Swinkels et al. (2018) explain how this is generally discouraged by regulations such as Solvency II. Whereas the treatment of bonds is highly granular, with different capital charges depending on their credit rating and duration, the only distinction made for stocks is developed versus emerging markets, i.e., the risk of all stock portfolios within these markets is assumed to be the same. Thus, investors in the example above would face much higher capital charges. Leverage in the fixed income space is, in fact, quite accepted, e.g., the use of derivatives such as swaps to match the duration of assets and liabilities, but applying leverage to stock portfolios is typically only done by hedge funds.

Frazzini and Pedersen (2014) explicitly link the low-beta anomaly to leverage constraints by showing that when funding constraints tighten, betas tend to be compressed toward one and the risk-return relation becomes flatter. They construct a betting-against-beta (BAB) factor which goes long a leveraged low-beta portfolio and short a deleveraged high-beta portfolio, both targeting a beta of 1, resulting in a net beta of zero. In practice, however, investors aiming to exploit the low-volatility anomaly predominantly do so with unleveraged long-only portfolios. A well-known example is the MSCI Minimum Volatility Index series, which can be tracked by passive investors or serve as a benchmark for actively managed low-risk strategies. The popularity of unleveraged approaches illustrates that leverage aversion is real. However, we argue that sophisticated asset managers can help asset owners overcome leverage constraints by embedding leverage into a low-risk strategy, in such a way that the end product has the overall risk profile and general look and feel of a regular equity strategy.

The key contributions of our work are threefold. First, we propose an intuitive methodology to capture the low-volatility anomaly using leverage in a risk-controlled way. Second, we show that a leveraged low-risk strategy offers unique alpha above and beyond common asset pricing factors. Third, we provide comprehensive evidence on the effectiveness of this approach across geographies and market environments, demonstrating its robustness as an investment strategy. Our research expands the understanding of the low-volatility anomaly and presents a new way for investors to access the alpha associated with low-risk stocks without sacrificing market beta.

The main limitation of the proposed low-risk strategy is having sufficient access to leverage, and the treatment of leverage assumptions historically. In particular, access to leverage may decrease during a liquidity crunch, such as the Global Financial Crisis, which might be precisely when the expected benefits of leverage are the biggest. We believe that this is not a major concern because the amount of leverage we apply is modest (maximum 40%) and limited to the long side of the

portfolio, i.e., our approach does not involve short selling. Nevertheless, we demonstrate the sensitivity and robustness of a leveraged low-risk strategy to different financing cost levels. We find that the financing spread needs to be in excess of 600 basis points a year for the IR of the leveraged risk-controlled strategy to match the IR of the base non-leveraged strategy.

## DATA AND METHODOLOGY

### Data

Our main Developed Markets (DM) sample consists of MSCI World constituents at the end of every month from December 1985 to December 2023.<sup>2</sup> We additionally construct an Emerging Markets (EM) sample consisting of MSCI Emerging Markets constituents at the end of every month from December 1995 to December 2023.<sup>3</sup> We source stock returns from LSEG, US fundamental data from Compustat, and non-US fundamental data from Worldscope. We obtain the TED spread, 3-month treasury bill rate, and 90-day AA financial commercial paper interest rate from the FRED database.

We evaluate strategies in five different universes: DM, EM, North America (NA), Europe (EU), and Asia Pacific (APAC). North America consists of stocks listed in America and Canada, Europe consists of stocks listed in European countries, and Asia Pacific consists of stocks listed in Australia, Hong Kong, Japan, New Zealand, and Singapore that are in the parent DM universe. On average we have 2,008 stocks in the DM universe, 1,480 stocks in the EM universe, 652 stocks in the NA universe, 785 stocks in the EU universe, and 526 stocks in the APAC universe.

We source Fama-French factor portfolio returns from the Kenneth French data library. We follow Blitz et al. (2020) to construct large-cap-only factor returns and large-cap long-only factor returns. To compute the large-cap factors we take the underlying 2x3 Fama-French portfolios and use the large-cap portfolios to determine the large-cap factors (e.g., we compute the HML factor as BIG HiBM minus BIG LoBM). To compute the large-cap long-only factors, we take the large-cap top portfolio for each factor and then short a hedging portfolio. Following Blitz, Baltussen, and van Vliet (2020) we construct the hedging portfolio as the simple average of the nine large stock portfolios, derived from the 2x3 size-value, 2x3 size-profitability, and 2x3 size-investment portfolios.

### Base portfolio optimization

We use a portfolio optimization setting that mimics the construction of a real-world investment portfolio, applying realistic portfolio constraints and settings. We first present the settings for the unleveraged DM strategy that is the main focus of our study. For the EM and regional strategies, we typically increase stock-level limits by 50% to account for their reduced universe sizes. The base currency of all portfolios is the euro (EUR).

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<sup>2</sup> Prior to 2001 we use constituents of the FTSE Development Markets index as a proxy for MSCI World constituents.

<sup>3</sup> Prior to 2001 we use the 800 largest constituents of the S&P Global Broad Market Index as a proxy for MSCI EM constituents.

The portfolio exposure to countries, regions (defined as NA, EU, and APAC), GICS level one and level two sectors are restricted to deviate at most 10% from the capitalization-weighted benchmark exposure. The ex-ante beta (measured as the portfolio-weighted sum of historical beta calculated using the past 156 weekly returns) of the portfolio must be less than 80% of the benchmark’s market-capitalization weighted ex-ante beta. Portfolio weights must be non-negative (i.e., long-only). The maximum individual stock weight limit is the minimum of 1.5% or 20 times its benchmark weight.<sup>4</sup> The gross exposure of the portfolio is equal to 100%, i.e., fully invested.

Our portfolio optimization problem trades off expected returns, tracking error (TE), and turnover, subject to the previously described constraints. For a single time step the optimization objective is given by:

$$\max_w w'_{new}\mu - \gamma(w_{new} - w_{BM})'\Sigma(w_{new} - w_{BM}) - \kappa\|w_{new} - w_{old}\|_1, \quad (1)$$

where  $w_{new}$  are the optimized portfolio weights immediately after the rebalance,  $w_{old}$  are the portfolio weights immediately before the rebalance,  $w_{BM}$  are the benchmark weights,  $\mu$  are the expected alphas,  $\gamma$  is the weight of the TE penalty,  $\kappa$  is the weight of the turnover penalty, and  $\|\cdot\|_1$  denotes the L1 norm. We set  $\gamma$  to 1.0 and  $\kappa$  to 0.5. We rebalance the portfolios at the end of each month and round positions and trades smaller than 0.1 basis points to zero.

### Expected returns and risk

As inputs of expected returns  $\mu$ , we use a simple multi-factor score consisting of low risk (50%), value (16.67%), quality (16.67%), and momentum (16.67%). Combining low-risk factors with return factors is inspired by the conservative investing formula of Blitz and van Vliet (2018).<sup>5</sup> The low-risk factor is an equally weighted combination of past 260-day volatility, past 156-week volatility, past 260-day Dimson beta, and past 156-week beta, all calculated using returns in local currency. The value, quality, and momentum factors are represented by net payout yield, gross profitability to assets, and 12-1m price momentum, respectively. Each of the underlying signals is first converted to a robust z-score capped between  $-3$  and  $+3$ . The signals are then combined into a single multi-factor score by summing the z-scores and converting the summed z-scores into percentiles. Finally, these percentiles are converted to expected return estimates by rescaling them between  $-3\%$  and  $+3\%$ .

We use a standard variance-covariance (VCV) matrix ( $\Sigma$ ) that follows a latent factor model approach where we apply principal component analysis to the sample correlation matrix estimated using 5 years of daily returns data. We estimate separate VCV matrices for the DM, EM, NA, EU, and APAC universes.

### Leveraged portfolios

We construct long-only leveraged portfolios by optimizing a portfolio with the target gross exposure. We assume that any exposure greater than 100% is constructed using synthetic

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<sup>4</sup> This follows the approach used by the MSCI Minimum Volatility index series (MSCI Inc., 2018).

<sup>5</sup> We report robustness results for a low-volatility only strategy in the Appendix.

positions without the need for cash collateral<sup>6</sup> (e.g., for a 140% leveraged portfolio, 40% consists of synthetic positions) and that the synthetic portfolio involves financing costs equal to the risk-free rate. In our main analyses, we ignore all other costs to allow for a clean comparison of the different alternatives. In a separate section, we discuss the impact of transaction costs and the additional leverage costs that investors are likely to encounter in practice, such as a credit spread on top of the risk-free rate, not receiving full dividends and collateral costs.

In terms of portfolio construction, we set the amount of leverage to the level needed to achieve an ex-post beta between 0.95 and 1.05. For example, we use a leverage of 40% (gross market exposure of 140%) for the DM strategy to increase its ex-post beta from 0.7 to approximately 1.0. We increase all stock-level limits and portfolio constraints by the amount of leverage (e.g., a max weight of 1.5% becomes 2.1% for leverage of 40%, and portfolio constraints of 10% become 14%). We constrain the ex-ante beta of the leveraged portfolios to be within 10% bounds of the benchmark ex-ante beta.

### **Regional portfolios**

In addition to the core DM strategy, we also construct low-risk portfolios for EM and several regional investment universes. For these strategies we increase stock-level constraints by 50% (e.g., max weight from 1.5% to 2.25%) and reduce the turnover penalty by 20% (0.5 to 0.4), to account for the lower number of stocks in these investment universes. For EM we impose a portfolio constraint on China on-shore (A-Shares) and China off-shore (H-Shares) at 10%. Further, we constrain EM regions (Latin America, Asia, EMEA) at 10%. For non-EM regional strategies, we do not use regional constraints. Further, we estimate historical betas and VCV matrices in their respective universes, thereby producing regional variants. For example, for APAC, we calculate betas relative to the APAC value-weighted benchmark. We use these regional betas for both the portfolio constraints and the expected return inputs.

Exhibit 1 summarizes the key portfolio construction settings across the core and regional strategies and their respective leveraged strategies. For brevity, we do not include the extra EM constraints in this exhibit. For the leverage of each strategy, we use 40% leverage for DM, EU, and EM, 35% for APAC, and 30% for NA.

INSERT EXHIBIT 1 HERE

### **Portfolio statistics**

For each strategy we generate a series of statistics derived from the portfolio returns. We start with the return of the portfolio, which is the geometric average of the portfolio-weighted monthly returns. The excess return is the return in excess of the base currency risk-free return. Portfolio turnover (one way) is reported as an indicator for transaction costs. We compute volatility as the standard deviation of excess returns, and the Sharpe ratio (SR) is the ratio of the excess returns to volatility. The ex-post beta follows from a regression of the full-sample excess returns on the benchmark returns in excess of the risk-free return. We also report the annualized intercept of this regression as the alpha. Outperformance is the excess return above the benchmark excess

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<sup>6</sup> Note that the leverage of the portfolio can easily be adjusted to reach the desired gross exposure level. E.g., if 5% collateral is required the investor would hold 95% physical stocks and 45% synthetic positions.

return. Ex-post tracking error is the standard deviation of outperformance. Information ratio (IR) is the ratio of outperformance to ex-post tracking error. We additionally report absolute and relative drawdowns of the portfolio as the maximum peak-to-trough loss of the portfolio return or benchmark-relative outperformance, respectively.

We compare the excess return during different regimes, using the MSCI regional indices for the market, growth, value, large, and small stocks as regime indicators. For the market, we define an “up” regime as the time-periods when the market return is greater than or equal to zero, and “down” regimes as the time-periods when it is less than zero. We define HML up as the time-periods when the return of the Value index is greater than the Growth index, and HML down vice versa. SMB up is defined as the time-periods when the return of the Small cap index is greater than the Large cap index, and SMB down vice versa.

## **EMPIRICAL RESULTS**

### **Headline statistics**

Exhibit 2 reports the key portfolio statistics for three different Developed Markets strategies with different gross exposures (100%/140%) and tracking error control. The base case of fully invested with no leverage and no tracking error control (labeled “100”) corresponds to a standard low-risk portfolio which has an ex-post beta of 0.71 to the market, SR higher than the market (0.72 vs. 0.43), lower volatility than the market (12.7% vs. 15.5%), and thus significant CAPM alpha above the market (4.7% per year). This strategy also comes with a significant ex-post tracking error relative to the market (7.8%) which results in an IR of 0.43. By applying leverage of 40%, the beta increases (0.71 to 1.02), the IR increases (0.43 to 0.72), but the tracking error is roughly similar (7.8% to 8.5%).

INSERT EXHIBIT 2 HERE

When using both leverage and tracking error control we find a portfolio with an IR more than twice the base case (0.92 vs. 0.43), a lower tracking error (6.4% vs. 7.8%), a smaller maximum relative drawdown (12% vs. 22%), with only a modest reduction in SR (0.67 vs. 0.72). The introduction of leverage increases the beta of the portfolio closer to the market, effectively transforming the low-beta strategy into an approximately beta-1 strategy. The tracking error penalty is highly effective at reducing the ex-ante and ex-post tracking error but seems to overshoot slightly with regard to the ex-post beta. As the ex-post beta is a result of the process, and not something which can be explicitly controlled in the optimization, this increase could be a result of error maximization by introducing a risk model into the optimization process. Taken together, these results show how sacrificing a small amount of absolute risk-adjusted performance allows for significantly better benchmark relative risk-adjusted performance. Thus, this portfolio is more robust to the effects of general market movements.

Exhibit 3 plots the cumulative outperformance of the three strategies. The leveraged portfolio with no risk control (140) has the highest cumulative outperformance but this comes with a relatively higher tracking error. The tracking error-controlled leveraged portfolio (140TE) is able to closely match the 140 portfolio but with substantially lower tracking error, and thus delivers a

better overall relative risk-adjusted return profile. We observe two types of divergence occurring between the strategies. First, the outperformance of the leveraged strategies is particularly better during prolonged bull markets such as 1995 – 2000 and 2011 – 2020. During these periods the alpha of the lower volatility stocks is not enough to offset the performance drag stemming from lower market exposure. By correcting for the lower beta using leverage, we can better access the low-volatility alpha without sacrificing participation in market rallies. Second, the risk-controlled strategy (140TE) has a more stable profile on average, particularly during periods of market stress such as the Dot-Com Bubble and COVID-19 crisis. The profile across the full sample is that the 140TE strategy generally outperforms the unleveraged strategy but with less extreme outperformance during periods of market stress.

INSERT EXHIBIT 3 HERE

### **Detailed performance statistics**

Having discussed the broad statistics and outperformance of the leveraged risk-controlled strategies, we now dive deeper into the most salient statistics. Exhibit 4 presents the maximum absolute/relative drawdowns and SR/IR of the different strategies and the market. It is clear that all of the low-risk strategies outperform the market on a risk-adjusted basis and deliver significantly lower absolute drawdowns. When turning to market-relative performance, from a drawdown perspective the tracking error penalty has the biggest effect at reducing the maximum relative drawdown. The IR clearly shows a linear increase across the three strategies, demonstrating that there is significant value-add from jointly using leverage and a tracking error penalty.

INSERT EXHIBIT 4 HERE

Exhibit 5 shows several risk measures of the different strategies. The base-case, long-only low-volatility strategy has a sizeable tracking error. Merely using leverage to close the beta gap with the market index does not lower this tracking error. We only obtain a significant reduction of the tracking error when applying both leverage and a penalty on tracking error in the portfolio optimization step. Turning to the other risk metrics we observe that leveraging a low-volatility strategy to an ex-ante beta of 1 is highly effective at achieving an ex-post beta close to 1 and an ex-post volatility level that is roughly in line with market volatility. Thus, our approach successfully transforms a defensive equity strategy that excels at capital preservation into a more conventional active investment strategy that can keep up with the market in all environments.

INSERT EXHIBIT 5 HERE

The results so far have focused on full-sample statistics. Exhibit 6 shows the average outperformance during different market regimes. Specifically, we report results for market up/down, small-minus-big (SMB) up/down, and value-minus-growth (HML) up/down scenarios. First, the base strategy, due to its defensive nature, significantly outperforms during market-down environments but slightly lags when the market is up. We also observe this imbalance across SMB and HML regimes, where there are large differences between the up/down regimes. By contrast, the leveraged strategies (140 and 140TE) have positive expected



outperformance in all six scenarios. Thus, leverage is highly effective at providing added value regardless of the prevailing market, size, or value regime.

INSERT EXHIBIT 6 HERE

When constructing the low-risk portfolios we start with an investment universe in excess of 2,000 stocks (on average), which is then reduced to a portfolio of 100–200 stocks based on the optimization objective. Exhibit 7 reports the average portfolio exposure to the expected return forecast (i.e., the multi-factor score) quintiles relative to the benchmark. For example, “Q1” is the exposure of the portfolio to stocks in the top 20% of the expected return forecast relative to the benchmark exposure to these stocks. The primary effect of applying leverage is a massive increase in Q1 exposure. Thus, most of the leverage is used to buy more low-risk stocks. A secondary effect is that the underweights in Q2 and Q3 stocks are slightly reduced, especially when applying risk control. Although these stocks are slightly less attractive from an expected return perspective, some of them can be very effective at reducing tracking error. The least attractive stocks in Q4 and Q5 remain almost completely underweighted in all cases. Altogether, these portfolios are still obtaining strong exposure to the desired low-risk stocks, but a small amount of diversification can result in meaningful relative risk reduction.

INSERT EXHIBIT 7 HERE

### **Time-series risk reduction**

We now turn to a time-series risk perspective of the strategy. Exhibit 8 shows the ex-ante tracking error immediately after each monthly portfolio rebalance. From this plot, the benefit of using a tracking error penalty is immediately obvious. During “normal” periods, the tracking error difference between the different strategies is relatively low, showing that these portfolios likely have similar holdings and overall portfolio characteristics. However, during times of market stress, the ex-ante tracking error of the portfolios without risk control increases drastically. The strategy with tracking error control only has a modest increase in tracking error in these stress periods, resulting in a much more stable tracking error overall. This clearly shows the added benefit of active risk control, as the portfolios are able to significantly reduce relative risk whilst still delivering alpha associated with the low-volatility anomaly.

INSERT EXHIBIT 8 HERE

### **Spanning alpha**

To test whether the alpha associated with these strategies can be explained by Fama-French factors, we run spanning alpha regressions of the outperformance series on variations of Fama-French factors. It is important to align the methodology of the factors used to explain performance with the construction of the portfolio. The standard long-short Fama-French factors inflate the weight of illiquid small stocks and rely for 50% on short-selling stocks. As our portfolios are long-only and invested in large-capitalization universes, we conduct additional regressions using large-capitalization long-short factors and large-capitalization long-only factors.

Exhibit 9 presents the results of conducting these spanning alpha regressions. Across all specifications and strategies, we find positive alphas, albeit sometimes insignificant. As expected,

when regressing on the long-short factors, we find statistically insignificant alphas. As we progressively regress the outperformance on more “realistic” factors, the magnitude of the alphas increases, and the statistical significance becomes stronger, in line with the results of Blitz, van Vliet, and Baltussen (2019). We focus on the large-capitalization long-only factors. For the unleveraged (100) portfolio, we find a highly significant spanning alpha, driven by a highly significant negative loading on the market factor. As intended, the loading on the market becomes insignificant for the leveraged strategies. This also explains why the R-squared levels are much lower. The alphas increase to above 4%, despite higher loadings on some other priced factors.

INSERT EXHIBIT 9 HERE

## ROBUSTNESS TESTS

### Impact of costs

Financing the synthetic portfolio typically requires paying a credit spread over the risk-free rate to the lender. There is little transparency around what these spreads may be (often negotiated privately), and it is highly likely that these spreads will vary through time, particularly around times of market stress. As we use reasonably modest amounts of leverage (a maximum of 40%), the risk towards liquidity crunches will be limited. To test the sensitivity of our results to the financing spread level, the top-left panel of Exhibit 10 plots the IR of the strategies under different spread levels. We see that in order for the IR of the leveraged strategies to drop below the non-leveraged strategy the financing spread would have to be in excess of 600 basis points (bp).

INSERT EXHIBIT 10 HERE

To quantify the potential time-varying nature of the financing spread, we use the TED spread as a proxy for the financing cost a lender might charge.<sup>7</sup> We set a floor of 10 bp annual financing cost. This produces a time-series with an average of 46 bp cost per year. In addition, we consider trading costs, assumed to be a fixed 25 bp cost per trade, and receiving net instead of gross dividends on the synthetic positions, similar to not being able to reclaim dividend withholding taxes. Exhibit 11 shows that the combined impact of these effects is a drop in return of about 25 bp per annum for the base-case, unleveraged strategy, versus a return loss of around 100 bp for the leveraged strategies. Nevertheless, the leveraged strategies still massively outperform the base-case unleveraged strategy in terms of net outperformance and information ratio. Thus, accounting for costs does not alter any of our main conclusions.

INSERT EXHIBIT 11 HERE

To test the impact of trading cost assumptions on the strategies, the top-right panel of Exhibit 10 plots the IR of the different strategies when assuming different costs per trade. Up until a cost of at least 400 bp per trade, the IRs of the leveraged strategies are still above the IR of the base strategy. This shows that the strategies are extremely robust to trading costs, which is not

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<sup>7</sup> The TED spread was discontinued on January 21<sup>st</sup>, 2022, due to the end of the London Interbank Offer Rate (LIBOR). We proxy the TED spread by using the spread between 3-month AA commercial paper and 3-month Treasury bills between January 31<sup>st</sup>, 2022, and December 31<sup>st</sup>, 2023.

surprising given that the leveraged strategies only have about 20% additional turnover compared to the base-case strategy, as previously reported in Exhibit 2.

### **Leverage ratio sensitivity**

Throughout our analysis we used leverage levels calibrated on the desired ex-post portfolio characteristics. In order to test the impact of this touch of hindsight wisdom we examine how much the results change in case of different leverage levels. The bottom-left panel of Exhibit 10 presents the IRs when varying the assumed leverage for the DM strategy. We use the TED spread proxy for financing costs, and 30% dividend tax cost across these strategies. We see that for leverage between 10% and 60%, we are able to achieve consistently high IRs, far in excess of the unleveraged portfolios. This suggests that the choice of leverage level does not critically drive our results. In fact, 30% leverage yields a marginally higher IR than our baseline choice of 40%. The point of applying leverage is to move the ex-post beta closer to the benchmark, which can already be achieved with a modest amount of leverage.

### **TE penalty weight**

For simplicity we set the weight of the TE penalty  $\gamma$  equal to 1. This value does not necessarily lead to the highest IR. The bottom-right panel of Exhibit 10 shows the outperformance and tracking error for different values of  $\gamma$ . There is a clear trade-off between outperformance and tracking error, with both decreasing as we increase the value of  $\gamma$ . IR increases for  $\gamma$  up to 4 and starts to deteriorate thereafter. This shows that our results are robust to the choice of the risk aversion parameter  $\gamma$  and that the IR could be further improved by fine-tuning this setting.

### **Regional performance**

Finally, we evaluate the robustness of our leveraged risk-controlled strategy across various regions. Specifically, we test our strategy in EM, as well as sub-universes of the broad Developed Markets universe (NA, EU, and APAC). Exhibit 12 reports key portfolio statistics for the strategies and Exhibit 13 shows the ex-post beta, outperformance, maximum absolute/relative drawdowns and SR/IRs of the regional strategies and the market. Overall, we see that our results for DM generalize to the regional applications. Adding a tracking error penalty results in higher absolute but lower relative drawdowns. Further, we see in all regional strategies that the IRs increase relatively linearly as we introduce leverage and a tracking error penalty. Also similar to broad DM, the large increase in IR comes at the cost of only a small loss in SR.

INSERT EXHIBIT 12 HERE

INSERT EXHIBIT 13 HERE

## **CONCLUSION**

In this paper, we explore a novel approach to capturing the well-known low-volatility anomaly in equity markets. Our strategy consists of investing around 140% in low-risk stocks to lever the portfolio beta from 0.7 to 1.0, using a risk model to further control tracking error. This

methodology allows us to amplify the returns of the low-volatility effect while establishing a risk profile similar to the broad market.

Our empirical analysis across multiple geographic regions provides strong evidence for the effectiveness of this approach. The leveraged low-risk portfolios generate significant outperformance relative to both the market portfolio and traditional unleveraged low-risk portfolios. This outperformance is robust across time periods and markets and is not explained by exposure to common equity risk factors such as size and value. The results support the hypothesis that leverage constraints play a role in the persistence of the low-volatility anomaly, as proposed by Black (1972) and Frazzini and Pedersen (2014). By applying leverage to low-risk stocks, our strategy is able to more fully exploit the mispricing and extract the anomalous returns. At the same time, the use of a risk model to control tracking error helps to manage the risks associated with leverage and results in a more market-aware approach.

For investors, our results suggest that a risk-managed leveraged low-risk strategy can offer a powerful way to enhance portfolio returns. The strategy's significant and consistent outperformance, limited downside risk, and low correlations with other return sources make it an attractive diversifier and return enhancer. Compared to standard low-risk approaches, our methodology offers the potential for a more efficient capture of the premium. However, this comes with the requirement of having the ability to access and use leverage. Thus, the costs required to use leverage will be a determinant of the added benefits that can be enjoyed from following such a strategy. Nevertheless, if an investor is able to accept leverage in the portfolio, our results show that this allows access to the low-volatility anomaly without sacrificing participation in bull markets.

## **ACKNOWLEDGEMENTS**

We thank Machiel Zwanenburg, Matthias Hanauer, Harald Lohre, Pim van Vliet, and Dean Walsh for valuable comments. The views expressed in this paper are not necessarily shared by Robeco or its subsidiaries.

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## Exhibit 1: Portfolio optimization settings

This exhibit presents the core portfolio optimization settings used across the different low-risk strategies. The Base settings correspond to a Developed Markets strategy with no leverage. Leveraged settings correspond to the Base strategy with an assumed 40% leverage. Regional settings correspond to the Emerging Markets (EM), North American, Europe, and Asia Pacific strategies with no leverage. Regional Leveraged settings correspond to the regional settings with an assumed 40% leverage. For non-EM regional strategies, we do not apply a region constraint. Max. weight is the maximum individual absolute stock weight. Turnover penalty is the coefficient applied to the trades in the objective function. Risk penalty is the coefficient applied to the variance-covariance matrix. Sector, subsector, country, region, and ex-ante beta are the portfolio-level constraints set on each respective exposure.  $\beta_{BM}$  is the value-weighted ex-ante beta of the strategy's benchmark.

	Base	Leveraged	Regional	Regional Leveraged
<i>Stock limits</i>				
<b>Max. Weight</b>	MIN( $20w_{BM}$ , 1.5%)	MIN( $28w_{BM}$ , 2.1%)	MIN( $30w_{BM}$ , 2.25%)	MIN( $42w_{BM}$ , 3.15%)
<i>Objective function parameters</i>				
<b>Turnover Penalty <math>\kappa</math></b>	0.5	0.5	0.4	0.4
<b>Risk Penalty <math>\gamma</math></b>	0	0/1	0	0/1
<i>Portfolio constraints</i>				
<b>Sector</b>	$\pm 10\%$	$\pm 14\%$	$\pm 10\%$	$\pm 14\%$
<b>Subsector</b>	$\pm 10\%$	$\pm 14\%$	$\pm 10\%$	$\pm 14\%$
<b>Country</b>	$\pm 10\%$	$\pm 14\%$	$\pm 10\%$	$\pm 14\%$
<b>Region</b>	$\pm 10\%$	$\pm 14\%$	N/A	N/A
<b>Ex-ante Beta</b>	$< 0.8 \times \beta_{BM}$	$0.9 < \beta_{BM} < 1.1$	$< 0.8 \times \beta_{BM}$	$0.9 < \beta_{BM} < 1.1$

## Exhibit 2: Developed Markets strategy statistics

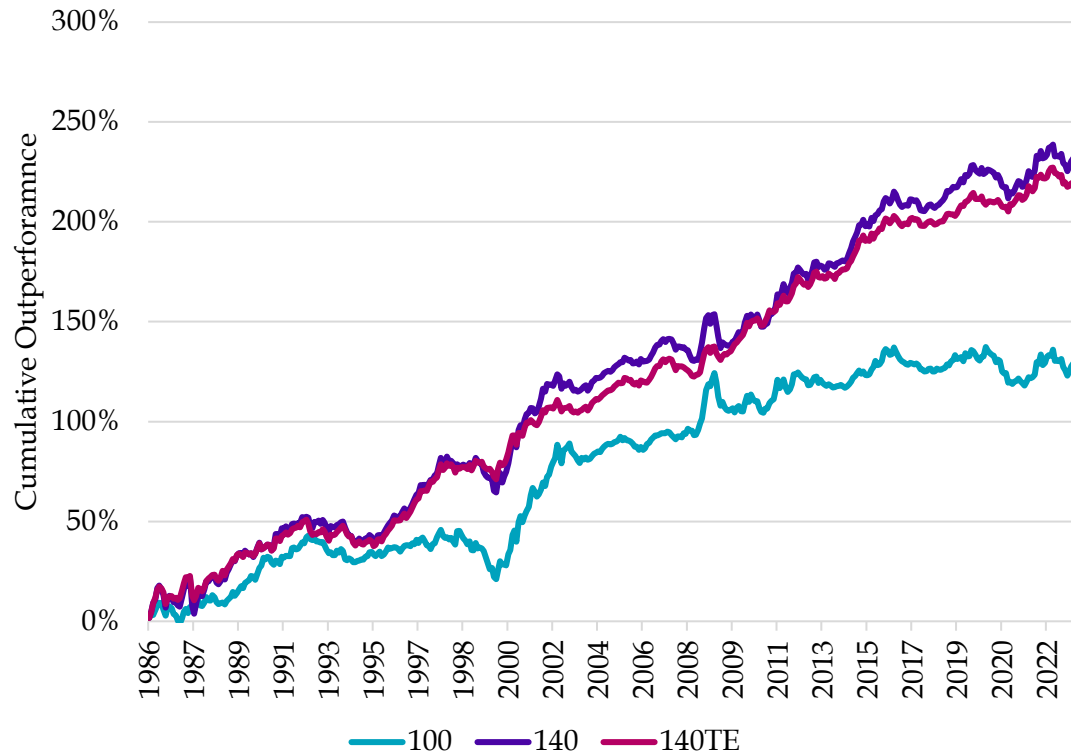
This exhibit presents key portfolio statistics for the Developed Markets strategies. The sample runs from December 1985 to December 2023.

	100	140	140TE
<b>Return</b>	12.41%	15.52%	15.24%
<b>Excess Return</b>	9.11%	12.13%	11.85%
<b>Volatility</b>	12.71%	17.93%	17.71%
<b>Sharpe Ratio</b>	0.72	0.68	0.67
<b>CAPM Alpha</b>	4.68%	6.10%	5.64%
<b>Relative Ex-ante Beta</b>	-0.35	-0.05	-0.02
<b>Ex-post Beta</b>	0.71	1.02	1.06
<b>Outperformance</b>	3.32%	6.18%	5.92%
<b>Ex-post Tracking Error</b>	7.75%	8.52%	6.43%
<b>Information Ratio</b>	0.43	0.72	0.92
<b>Max Absolute Drawdown</b>	-36%	-47%	-49%
<b>Max Relative Drawdown</b>	-22%	-17%	-12%
<b>Turnover</b>	49%	70%	71%
<b>Gross Exposure</b>	100%	140%	140%



### Exhibit 3: Cumulative outperformance of strategies

This exhibit plots the cumulative outperformance of three Developed Markets low-risk portfolios. “100” corresponds to the baseline long-only fully invested strategy with no risk control. “140” is a strategy optimized to 140% gross market exposure with no risk control. “140TE” is the “140” strategy with tracking error control applied.



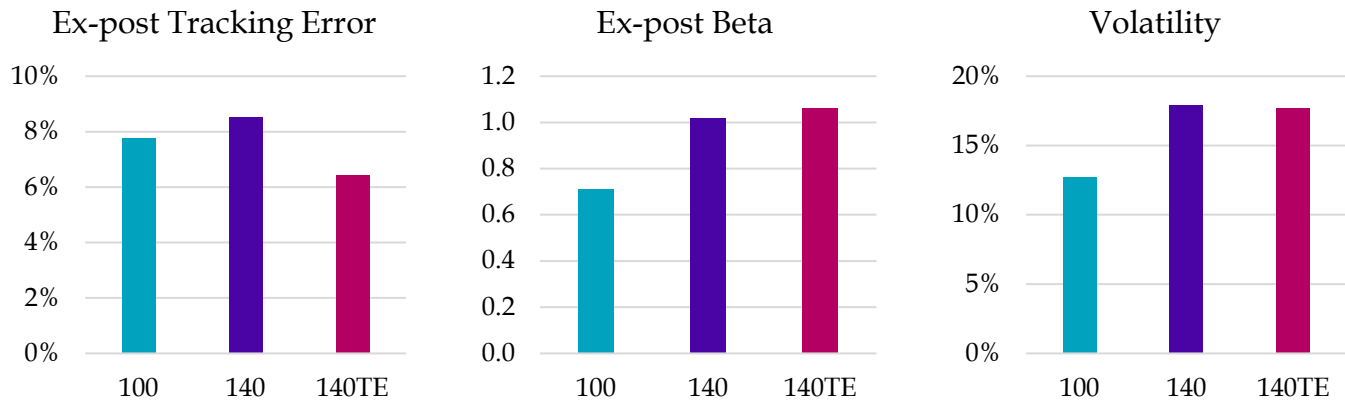
#### Exhibit 4: Drawdown and risk-adjusted performance

This exhibit plots the absolute and relative drawdowns and risk-adjusted performance ratios for the three different Developed Markets portfolios. The top row plots the maximum absolute peak-to-trough drawdown and the annualized Sharpe ratio, the statistics for the market are also included. The bottom row plots the benchmark relative maximum peak-to-trough drawdown and the annualized Information ratio (risk-adjusted benchmark relative performance).



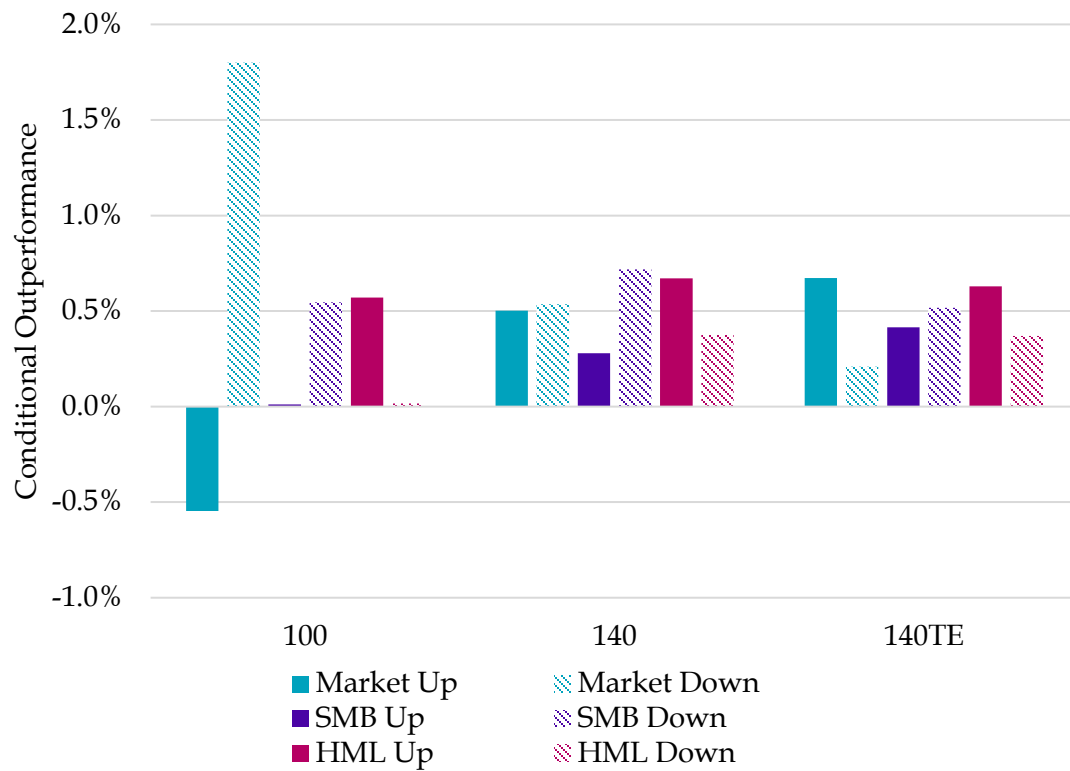
### Exhibit 5: Tracking error, beta, and volatility differences

This exhibit plots the realized risk measures of the three different portfolios. The first panel shows the ex-post tracking error (annualized standard deviation of active return). The second panel shows the ex-post beta relative to the benchmark (estimated using a regression). The third panel shows the annualized portfolio volatility.



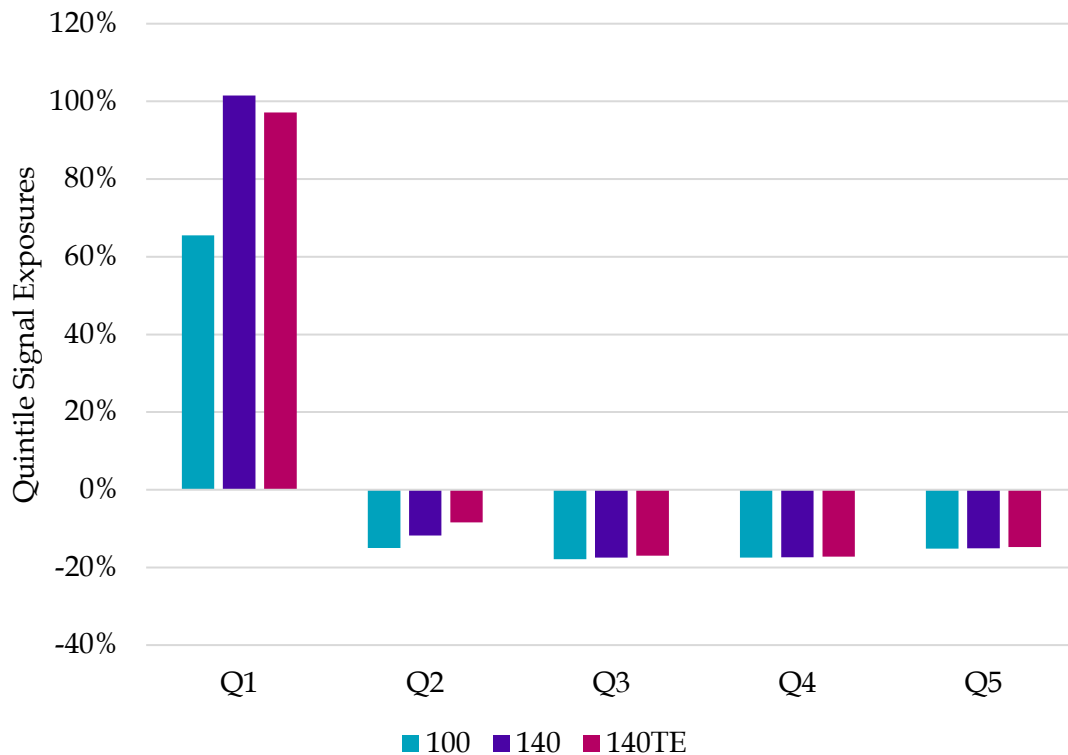
### Exhibit 6: Conditional outperformance

This exhibit plots the average monthly performance of the three portfolios in different market regimes. “Up” are the periods when the designated regime’s value is greater than or equal to zero, “Down” are the periods when the value is less than zero. Market corresponds to the benchmark return, SMB is the small-minus-big size factor, and HML is the high-minus-low value factor.



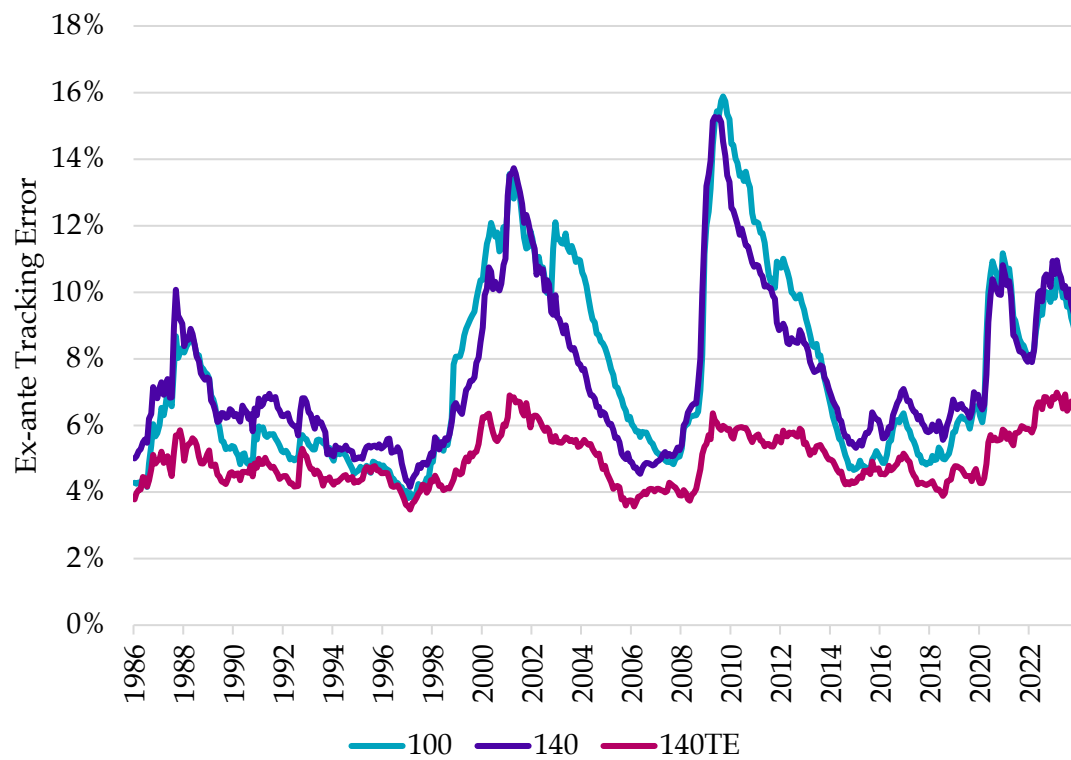
### Exhibit 7: Quintile exposures

This exhibit plots the benchmark relative exposure of the three different portfolios to the expected return signal. The expected return signal is split into quintiles and the active portfolio exposure to stocks in each quintile is summed. An exposure of zero corresponds to the same exposure as the benchmark, a positive value corresponds to higher than benchmark exposure, and a negative value corresponds to lower than benchmark exposure. “Q1” corresponds to stocks with the highest expected return, and “Q5” to stocks with the lowest expected return.



## Exhibit 8: Ex-ante tracking error through time

This exhibit plots the monthly ex-ante tracking error of the three different portfolios. The ex-ante tracking error is annualized and calculated using the variance-covariance matrix.



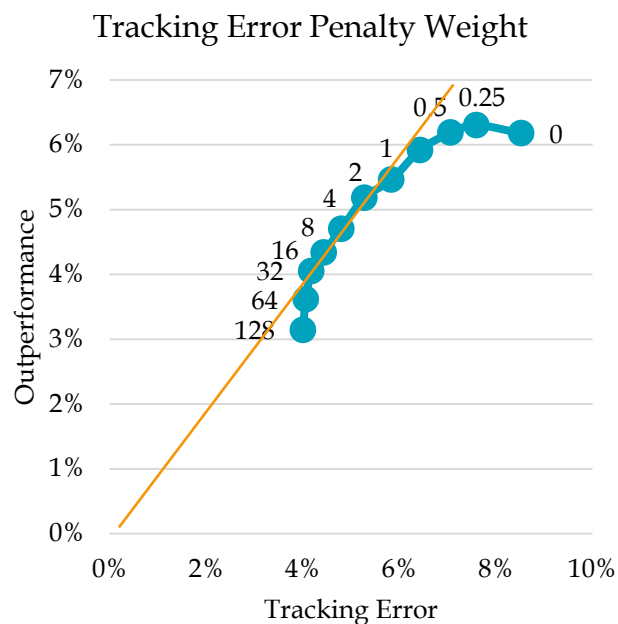
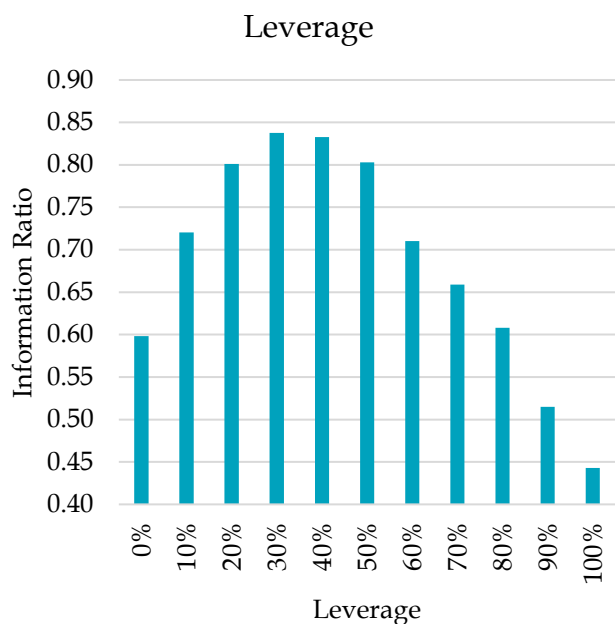
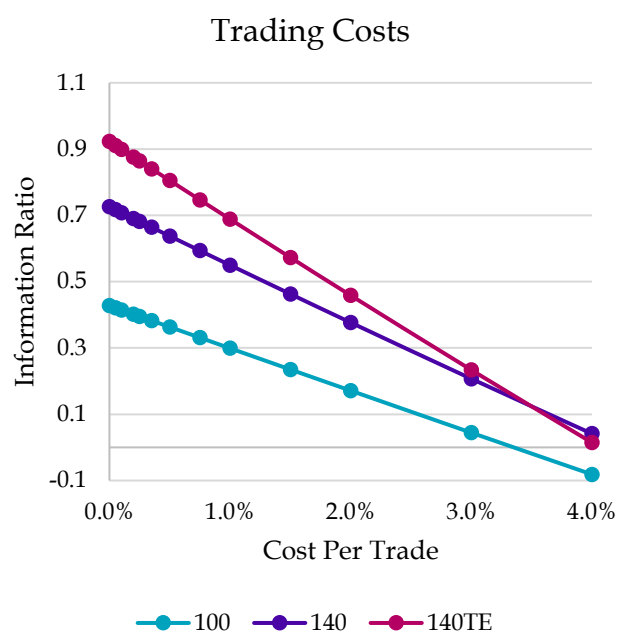
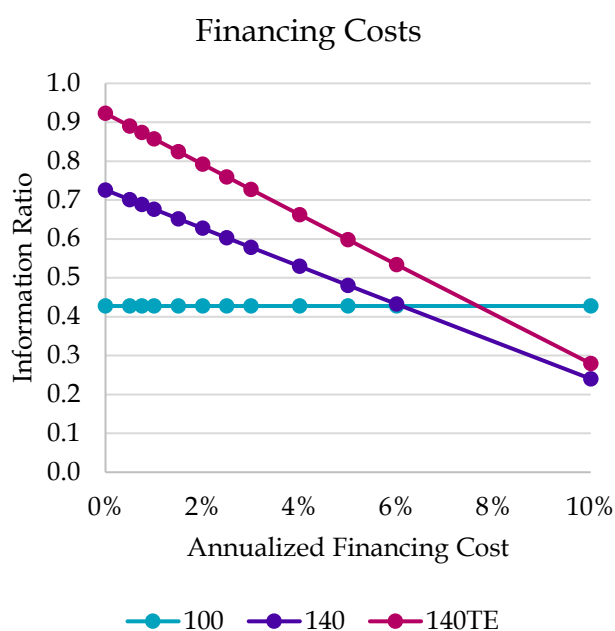
### Exhibit 9: Developed Markets spanning alphas

This exhibit presents the results of spanning alpha regressions for the Developed Markets strategies returns on variations of the Fama-French factors. Bold font indicates statistical significance at the 5% level.

	L/S Fama-French			Large-cap L/S Fama-French			Large-cap Long-only Fama-French		
	100	140	140TE	100	140	140TE	100	140	140TE
<b>Alpha</b>	1.21 (1.29)	2.02 (1.22)	2.22 (1.83)	<b>2.87</b> (3.30)	<b>3.59</b> (2.45)	<b>3.46</b> (3.04)	<b>3.72</b> (3.92)	<b>4.76</b> (3.00)	<b>4.35</b> (3.55)
<b>Market-RF</b>	<b>-0.218</b>	0.002	0.062	<b>-0.220</b>	0.013	0.068	<b>-0.265</b>	-0.044	0.025
<b>CMA</b>	<b>0.402</b>	<b>0.475</b>	<b>0.304</b>	<b>0.330</b>	<b>0.394</b>	<b>0.247</b>	<b>0.559</b>	<b>0.638</b>	<b>0.403</b>
<b>HML</b>	-0.059	-0.101	-0.022	-0.010	-0.030	0.038	0.116	0.142	<b>0.199</b>
<b>RMW</b>	<b>0.475</b>	<b>0.581</b>	<b>0.458</b>	<b>0.273</b>	<b>0.404</b>	<b>0.331</b>	<b>0.539</b>	<b>0.845</b>	<b>0.669</b>
<b>SMB</b>	0.106	0.039	0.068						
<b>WML</b>	<b>0.116</b>	<b>0.136</b>	<b>0.097</b>	<b>0.099</b>	<b>0.107</b>	<b>0.079</b>	0.102	<b>0.082</b>	0.075
<b>R-squared</b>	64%	28%	21%	62%	28%	22%	54%	18%	12%

## Exhibit 10: Financing costs, trading costs, leverage, and the tracking error penalty weight

The top row plots the sensitivity of the three strategies to the choice of financing costs and costs per trade. The reported strategies use 40% leverage, so a financing cost of 1% corresponds to 0.4% per year of cost. The bottom row plots the sensitivity to the chosen leverage ratio and the weight of the tracking error penalty for the Developed Markets portfolios. For the left figure only the level of leverage is changed; the other parameters remain unchanged. The right figure uses 40% leverage and assumes different values for the TE penalty weight  $\gamma$ . The straight line indicates the highest information ratio for  $\gamma = 4$ .





## Exhibit 11: Impact of costs

This exhibit presents selected portfolio statistics for the Developed Markets strategies after applying different costs. Trading costs are assumed as a fixed 0.25% cost per trade. Financing costs uses the TED spread until January 21<sup>st</sup>, 2022, and the spread between 3M commercial paper and 3M treasury bills thereafter, with a floor of 0.10% per year. We assume a 30% dividend tax cost.

<i>Panel A: Net Excess Return</i>			
	<b>100</b>	<b>140</b>	<b>140TE</b>
<b>No Cost</b>	9.11%	12.13%	11.85%
<b>–Trading Costs</b>	8.84%	11.73%	11.46%
<b>–Trading Costs–Financing</b>	8.84%	11.53%	11.25%
<b>–Trading Costs–Financing–Dividend Tax</b>	8.84%	11.12%	10.85%
<i>Panel B: Information Ratio</i>			
<b>No Cost</b>	0.43	0.73	0.92
<b>–Trading Costs</b>	0.40	0.68	0.86
<b>–Trading Costs–Financing</b>	0.40	0.66	0.83
<b>–Trading Costs–Financing–Dividend Tax</b>	0.40	0.61	0.77
<i>Panel C: Outperformance</i>			
<b>No Cost</b>	3.32%	6.18%	5.92%
<b>–Trading Costs</b>	3.07%	5.81%	5.55%
<b>–Trading Costs–Financing</b>	3.07%	5.62%	5.36%
<b>–Trading Costs–Financing–Dividend Tax</b>	3.07%	5.23%	4.97%

## Exhibit 12: Regional portfolio statistics

This exhibit presents key portfolio statistics of the three different strategies across regional variants. The regional variants are Emerging Markets (EM), North America (NA), Europe (EU), and Asia Pacific (APAC). Each regional variant uses a different set of portfolio construction settings, targeting the same characteristics as the core DM strategy.

	EM			NA			EU			APAC		
	100	140	140TE	100	130	130TE	100	140	140TE	100	135	135TE
<b>Gross Return</b>	11.75%	16.50%	14.94%	12.21%	14.39%	14.43%	12.30%	15.37%	14.56%	10.05%	12.60%	11.90%
<b>Gross Excess Return</b>	9.81%	14.47%	12.94%	8.91%	11.03%	11.07%	9.00%	11.98%	11.20%	6.82%	9.30%	8.62%
<b>Net Excess Return</b>	9.81%	13.68%	12.20%	8.91%	10.72%	10.76%	9.00%	11.47%	10.68%	6.82%	8.95%	8.26%
<b>Volatility</b>	14.37%	21.91%	21.50%	14.57%	19.04%	19.17%	12.68%	18.03%	17.87%	14.79%	20.63%	20.41%
<b>Sharpe Ratio</b>	0.68	0.62	0.57	0.61	0.56	0.56	0.71	0.64	0.60	0.46	0.43	0.40
<b>CAPM Alpha</b>	6.38%	8.96%	7.50%	3.30%	3.61%	3.44%	5.19%	6.33%	5.49%	5.05%	6.88%	6.20%
<b>Ex-post Beta</b>	0.60	0.94	0.96	0.76	1.01	1.05	0.70	1.01	1.04	0.66	0.96	1.01
<b>Outperformance</b>	5.11%	8.82%	7.41%	1.98%	3.67%	3.70%	4.02%	6.37%	5.62%	4.78%	6.87%	6.20%
<b>Ex-post Tracking Error</b>	11.23%	9.45%	7.45%	7.58%	7.57%	6.15%	7.65%	7.85%	6.49%	10.18%	10.17%	7.44%
<b>Information Ratio</b>	0.46	0.93	0.99	0.26	0.48	0.60	0.53	0.81	0.87	0.47	0.68	0.83
<b>Max Absolute Drawdown</b>	-42%	-60%	-58%	-34%	-43%	-49%	-44%	-57%	-57%	-54%	-67%	-69%
<b>Max Relative Drawdown</b>	-33%	-24%	-23%	-31%	-21%	-14%	-21%	-18%	-14%	-32%	-20%	-16%
<b>Turnover</b>	72%	110%	99%	62%	84%	81%	70%	102%	96%	69%	107%	98%
<b>Gross Exposure</b>	100%	140%	140%	100%	130%	130%	100%	140%	140%	100%	135%	135%

### Exhibit 13: Regional performance statistics

This exhibit plots the performance and risk statistics of the three different strategies across Developed Markets (DM) and the four regional variants. The regional variants are Emerging Markets (EM), North America (NA), Europe (EU), and Asia Pacific (APAC). Each regional variant uses a different set of portfolio construction settings, targeting the same characteristics as the core DM strategy.



## **APPENDIX: Low-volatility only strategy**

Following Blitz and van Vliet (2018) we have used a conservative investment strategy which selects stocks based on a multi-factor model consisting of low-volatility, low-beta, value, momentum, and quality factors. However, at the core of this strategy is the explicit beta target of approximately 0.7, which is achieved using a constraint on the ex-ante portfolio beta. Exhibit A1 and Exhibit A2 present the results using a stock-selection model consisting only of historical volatility (specifically past-260 day volatility and past-156 week volatility). Across both the portfolio statistics and spanning alphas, we find results consistent with our previous analysis. Naturally, the core results we present are stronger as they blend in additional performance drivers and thus the overall strategy benefits from superior diversification across factors. However, the leveraged low-risk strategy with risk control still significantly outperforms the base low-risk strategy. The cumulative outperformance of these low-volatility only strategies is shown in Exhibit A3, and we can clearly see that the added benefits of the application of leverage and risk-control still hold for this low-volatility stock selection strategy.

### Exhibit A1: Low-volatility only portfolio statistics

This exhibit presents key portfolio statistics of the three different strategies in the Developed Markets universe using a low-volatility only strategy. The strategy consists of a simple model which is 50% volatility estimated using the past 260 daily returns and 50% volatility estimated using the past 156 weekly returns.

	<b>100</b>	<b>140</b>	<b>140TE</b>
<b>Return</b>	10.05%	12.45%	12.49%
<b>Excess Return</b>	6.82%	9.15%	9.19%
<b>Volatility</b>	12.50%	17.62%	17.52%
<b>Sharpe Ratio</b>	0.55	0.52	0.52
<b>Ex-post Beta</b>	0.70	1.01	1.05
<b>Outperformance</b>	1.15%	3.36%	3.39%
<b>Tracking Error</b>	7.70%	7.92%	6.44%
<b>Information Ratio</b>	0.15	0.42	0.53

## Exhibit A2: Low-volatility only spanning alphas

This exhibit presents the spanning alphas of the three different strategies in the Developed Markets universe using a low-volatility only strategy. We present the spanning alphas over the L/S Fama-French factors, Large-cap L/S Fama-French factors, and Large-Cap Long-only Fama-French factors.

	L/S Fama-French			Large-cap L/S Fama-French			Large-cap Long-only Fama-French		
	100	140	140TE	100	140	140TE	100	140	140TE
<b>Alpha</b>	-0.26 (-0.29)	0.31 (0.21)	0.61 (0.50)	1.18 (1.41)	1.49 (1.12)	1.53 (1.37)	<b>2.15</b> (2.39)	2.73 (1.92)	<b>2.57</b> (2.17)
<b>Market-RF</b>	<b>-0.226</b>	0.005	0.052	<b>-0.229</b>	0.014	0.058	<b>-0.274</b>	-0.041	0.014
<b>CMA</b>	<b>0.353</b>	<b>0.392</b>	<b>0.307</b>	<b>0.315</b>	<b>0.372</b>	<b>0.278</b>	<b>0.548</b>	<b>0.617</b>	<b>0.473</b>
<b>HML</b>	0.030	0.048	0.038	0.050	0.064	0.061	0.147	0.207	0.161
<b>RMW</b>	<b>0.420</b>	<b>0.444</b>	<b>0.376</b>	<b>0.256</b>	<b>0.348</b>	<b>0.293</b>	<b>0.361</b>	<b>0.552</b>	<b>0.432</b>
<b>SMB</b>	0.103	0.023	0.032						
<b>WML</b>	<b>0.045</b>	<b>0.041</b>	<b>0.012</b>	<b>0.033</b>	<b>0.018</b>	<b>-0.002</b>	-0.014	<b>-0.078</b>	-0.089
<b>R-squared</b>	59%	21%	17%	58%	24%	19%	52%	15%	11%

### Exhibit A3: Low-volatility only cumulative outperformance

This exhibit presents the cumulative outperformance of the three different strategies in the Developed Markets universe using a low-volatility only strategy.

