

Copula-based hedging of cryptocurrencies

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joint work with

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Overview

Motivation

Copula-based hedging

Data

Results

Digital assets are here to stay

- ▶ Markets for cryptocurrencies are maturing:
 - Institutional investors are buying into it.
 - Regulators are working hard to make stablecoins “safe” (e.g. resolve issues of jurisdiction, financial stability).
 - Exchanges (e.g. CME) are issuing futures and options.
- ➡ We are in the middle of the digitalisation of financial markets...

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- ➡ We are in the middle of the digitalisation of financial markets...
(... and it's progressing rapidly.)

Digital assets are here to stay

The image shows a screenshot of the GARP (Global Association of Risk Professionals) website. At the top, there's a navigation bar with links for Home, COVID-19 Hub, Technology, Culture & Governance, Energy, Operational, Credit, Market, and More. A search icon is also present. Below the navigation is a main article thumbnail. The thumbnail features a dark background with a digital circuit board pattern and several candlestick charts representing price movements. The title of the article is "As Bitcoin Rises, Institutions Make Crypto Market Impact", with a subtitle "Barriers fall away but hedging remains a challenge; regulatory clarity will help". The date "Friday, February 26, 2021" and author "By John Hintze" are at the bottom of the thumbnail.

GARP Global Association of Risk Professionals

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- Investment Management -

As Bitcoin Rises, Institutions Make Crypto Market Impact

Barriers fall away but hedging remains a challenge; regulatory clarity will help

Friday, February 26, 2021

By John Hintze

<https://www.garp.org/#!/risk-intelligence/market/investment-management/a1Z1W000005kZDGUA2>

Bitcoin futures

- ▶ CME launched Bitcoin Futures in December 2017 and options on futures in January 2020
- ▶ Bitcoin Future:
 - Underlying: Bitcoin Reference Rate (BRR), based on relevant bitcoin transaction on certain exchanges
 - Maturities: nearest two Decembers and nearest six consecutive months
 - Settlement: cash
- ▶ <https://www.cmegroup.com/trading/equity-index/us-index/bitcoin.html>

Hedging cryptos

- ▶ Hedging Bitcoin exposure with Bitcoin futures:
 - Basis risk
 - BRR not traded
 - Ability of futures to hedge tail risks?

Hedging cryptos

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- ▶ Hedge other cryptos with Bitcoin?
 - High correlation
 - Tail risks, extreme events?

Index	1M Correlation Matrix							
	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	86.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.67%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	86.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.61%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	98.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	69.87%	59.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	26.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

Hedging cryptos

- ▶ Hedging Bitcoin exposure with Bitcoin futures:
 - Basis risk
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- ▶ Two directions:
 - Copulas
 - Risk measures

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Hedging spot with futures

- ▶ Hedge portfolio return: $R_t^h = R_t^S - h R_t^F$, where
 - R_t^S : spot return at time t
 - R_t^F : futures return at time t
 - h : hedge ratio
- ▶ Goal: Find optimal hedge ratio h^*
- ▶ Minimum-variance hedge ratio, e.g. Ederington (1979), assumes variance as risk measure and elliptical return distribution
- ▶ Extensions: risk measures, copulas, e.g. (Harris and Shen, 2006; Barbi and Romagnoli, 2014)

Copulas

Definition

A (bivariate) **copula** is a distribution function on $[0, 1]^2$ with standard uniform marginals.

- ▶ Copulas differ only through the dependence between the marginals.
- ▶ Sklar's Theorem (below) captures that copulas allow to separate
 - modelling of the marginals, and
 - modelling of the dependence structure.

Copulas

Theorem (Sklar's Theorem)

Let F be a joint distribution function with margins F_1, F_2 . Then, there exists a copula $C : [0, 1]^2 \rightarrow [0, 1]$ such that, for all $x, y \in \mathbb{R}$

$$F(x, y) = C(F_1(x), F_2(y)). \quad (1)$$

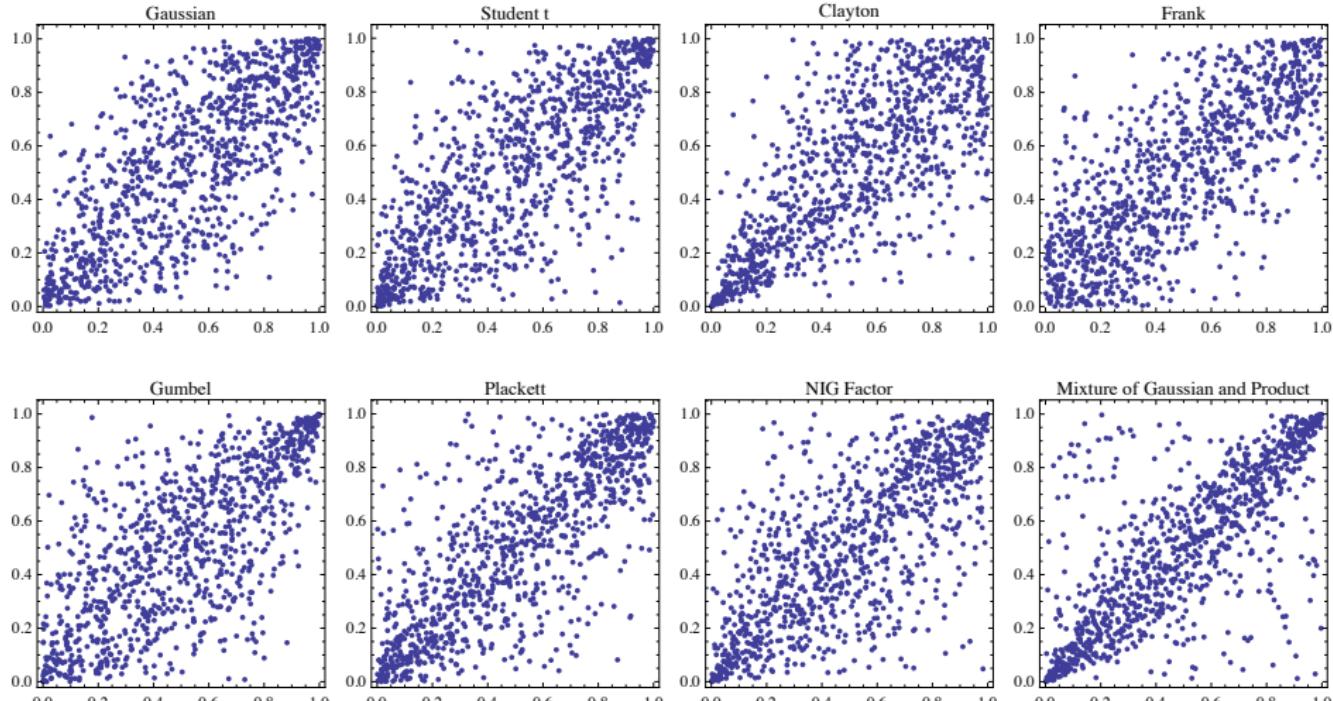
If the margins are continuous, then C is unique; otherwise C is unique on the range of the margins.

Conversely, if C is a copula and F_1, F_2 are univariate distribution functions, then the function F defined by (1) is a joint distribution function with margins F_1, F_2 .

- ▶ Representation of C in terms of F and its margins:

$$C(u, v) = F(F_1^{(-1)}(u), F_2^{(-1)}(v)).$$

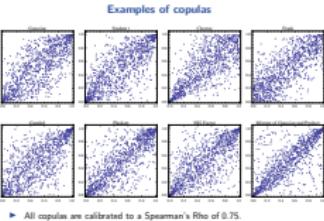
Examples of copulas



- ▶ All copulas are calibrated to a Spearman's Rho of 0.75.
- Copula-based hedging

Copula-based hedging of cryptocurrencies

└ Copula-based hedging



- Frank has no “tail”
- Mixture: df is linear combination of Gaussian and product copulas
- Factor NIG: Random variables are linear combination of joint factor and idiosyncratic component (again NIG under certain condition)
- Plackett: bivariate only; derived from the condition on a constant odds ratio for all values that bivariate continuous df can take.
- Which ones have tail dependence? Student-*t*, Clayton (one tail only), Gumbel (one tail only) (not: Gaussian, NIG factor, mixture, Plackett, Frank)

Copula-based hedging

Proposition (Barbi and Romagnoli (2014))

Let X and Y be two real-valued random variables with corresponding absolutely continuous copula C and continuous marginals F_X and F_Y . Then, the distribution of $Z = X - hY$ is given by

$$F_Z(x) = 1 - \int_0^1 D_1 C \left\{ u, F_Y \left\{ \frac{F_X^{(-1)}(u) - x}{h} \right\} \right\} du. \quad (2)$$

- Easy to show (e.g. McNeil *et al.* (2005)):

$$D_1 C(F_X(x), F_Y(y)) = \frac{\partial}{\partial u} C(u, v) = \mathbf{P}(Y \leq y | X = x).$$

Risk measures

- ▶ **Variance:** $\text{Var}(Z)$
- ▶ **Value-at-risk (VaR):** $\text{VaR}_\alpha = -F_Z^{(-1)}(1 - \alpha)$
- ▶ **Expected Shortfall (ES):** $\text{ES}_\alpha = -\frac{1}{1 - \alpha} \int_{-\infty}^{\alpha} F_Z^{(-1)}(p) dp.$

Risk measures

- ▶ **Spectral risk measures (SRM)** (Acerbi, 2002; Cotter and Dowd, 2006):

$$\rho_\phi = - \int_0^1 \phi(p) F_Z^{(-1)}(p) dp,$$

where q_p is the p -quantile of the return distribution and $\phi(s)$, $s \in [0, 1]$, is the so-called **risk aversion function**, a weighting function such that

$$(i) \quad \phi(p) \geq 0,$$

$$(ii) \quad \int_0^1 \phi(p) dp = 1,$$

$$(iii) \quad \phi'(p) \leq 0.$$

- ▶ SRM's are coherent risk measures.

Risk measures

- ▶ **Exponential spectral risk measure:** weighting function $\phi(p) = \lambda e^{-k(1-p)}$, where λ is an unknown positive constant, derived from exponential utility function:

$$\rho_\phi = \int_0^1 \phi(p) F_Z^{(-1)}(p) dp = \frac{k}{1 - e^{-k}} \int_0^1 e^{-k(1-p)} F_Z^{(-1)}(p) dp.$$

Optimal hedge ratio

- ▶ Hedge portfolio: $R_t^h = R_t^S - hR_t^F$, with h hedge ratio
- ▶ Optimal hedge ratio:

$$h^* = \operatorname{argmin}_h \rho(h),$$

where $\rho(h)$ is the risk of the hedge portfolio with hedge ratio h .

Overview

Motivation

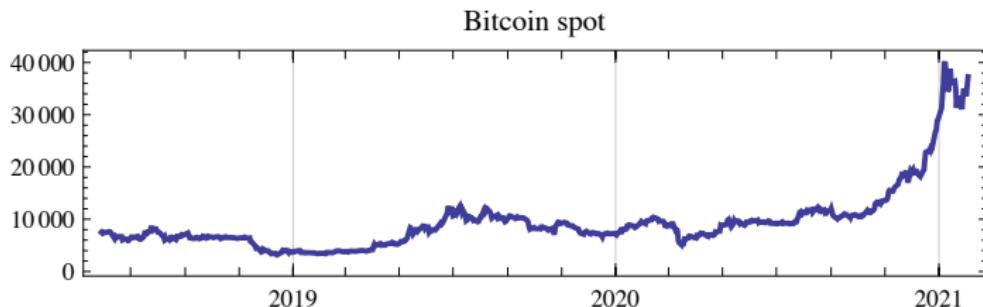
Copula-based hedging

Data

Results

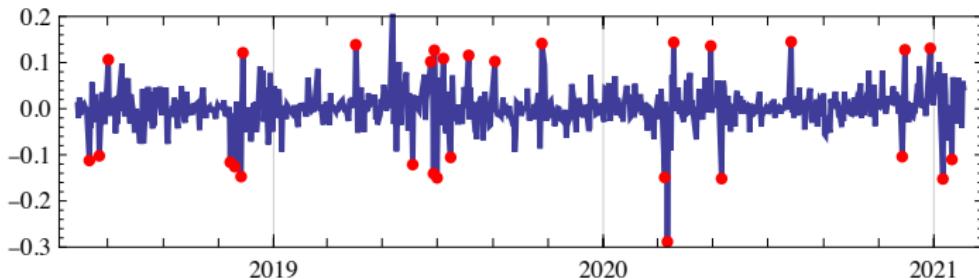
Data

- ▶ Daily log returns, 23pm CET
- ▶ 29 May 2018 through 3 Feb 2021
- ▶ Spot: Coingecko Bitcoin / USD
- ▶ Future: CME BTC Future
- ▶ Sources: Bloomberg, coingecko

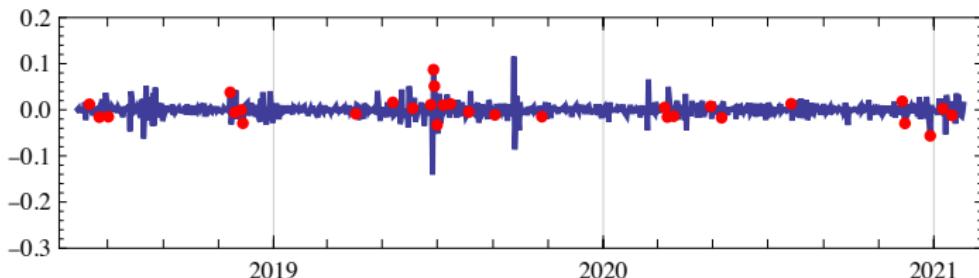


Time series

BTC spot returns with 30 most extreme observations

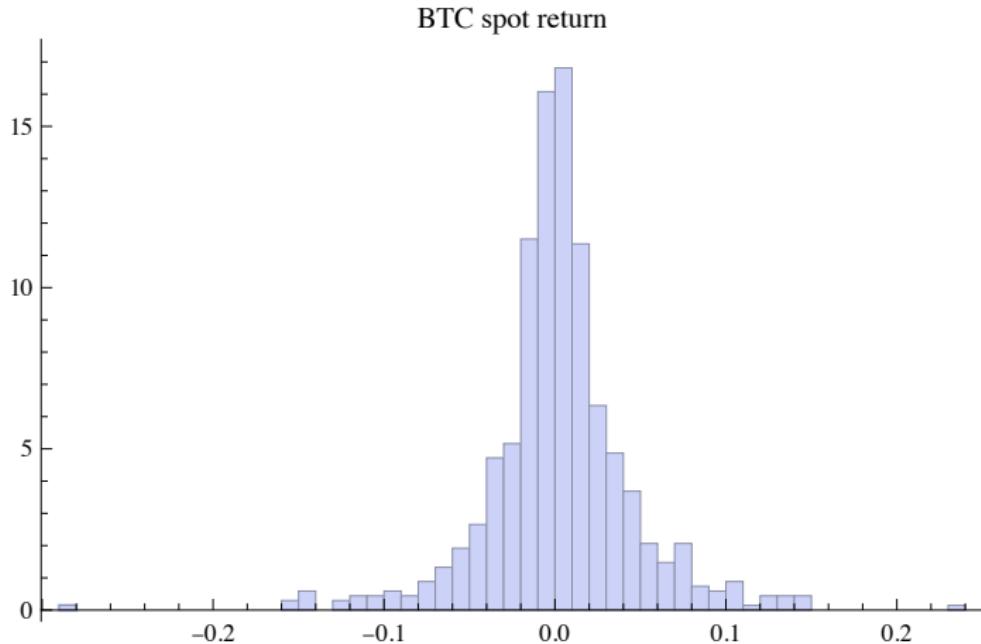


Return BTC spot minus future



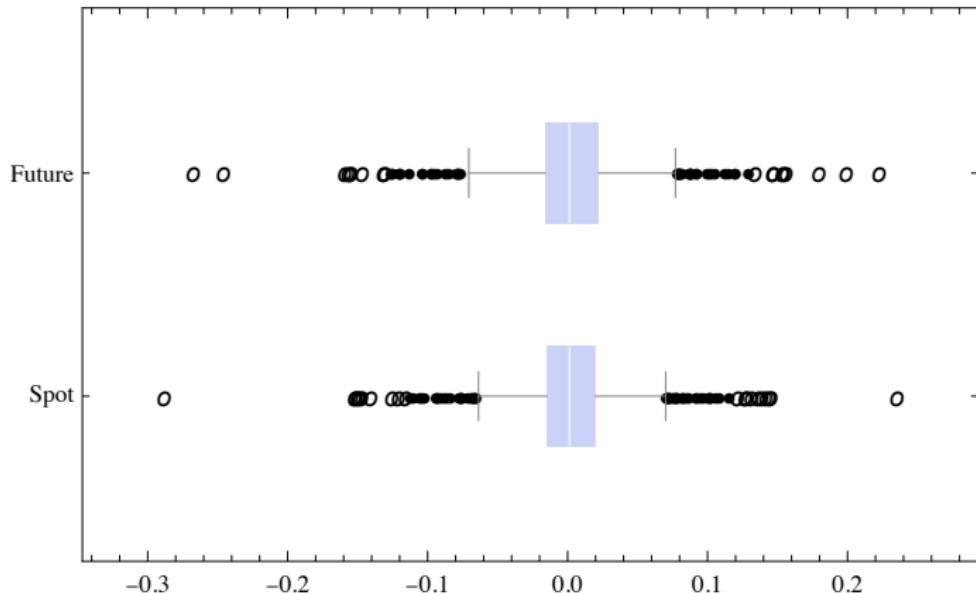
- i.e., hedge ratio of 1.

Distribution

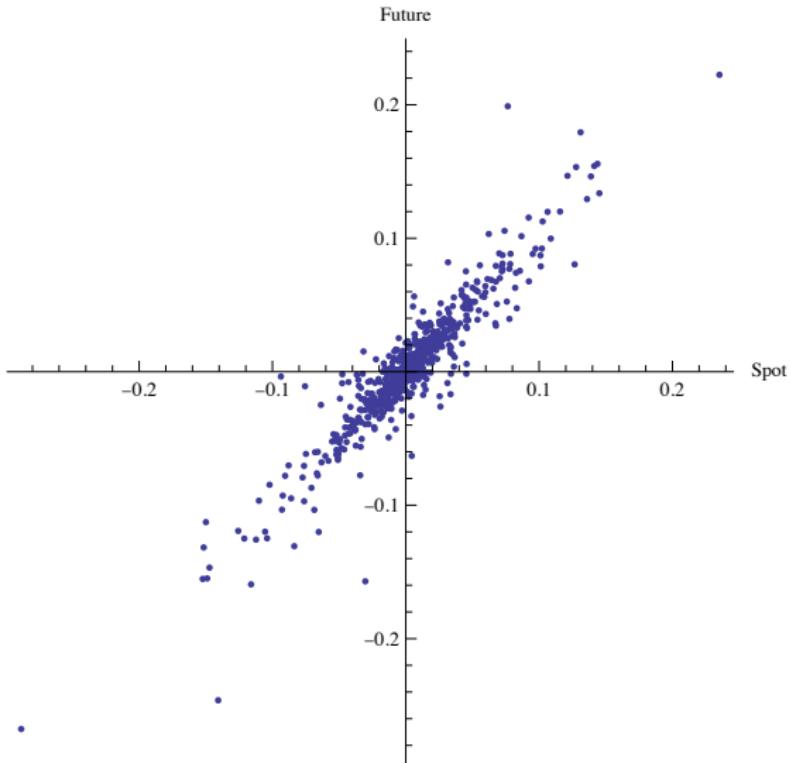


- ▶ Student t distribution: $\nu = 7.95$
- ▶ Generalised Pareto distribution (EVT): tail index $1/\xi = 4.92$

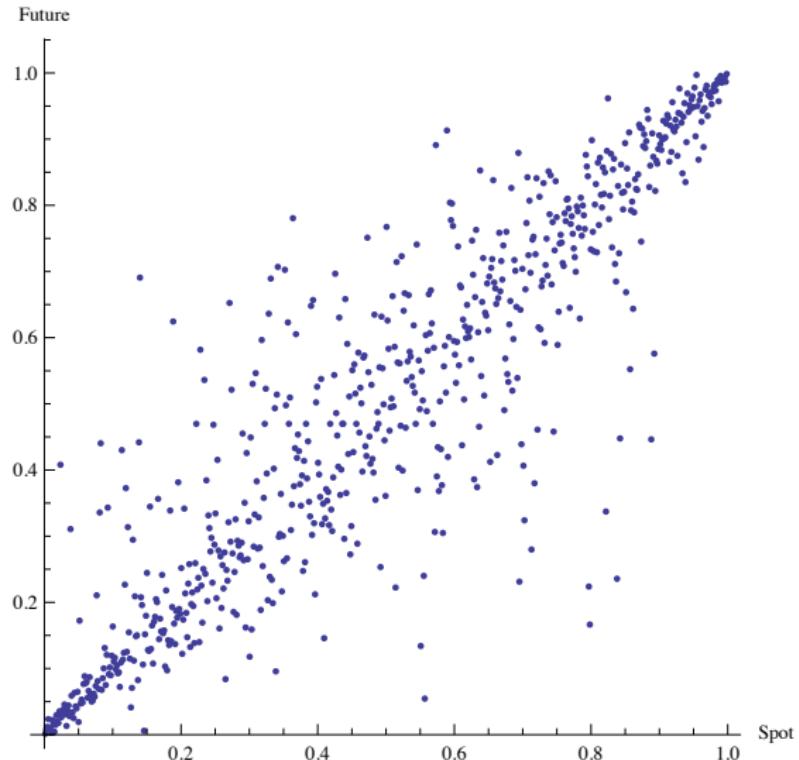
Spot and Future



Spot and Future



Spot and Future empirical copula



Overview

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Calibration

- ▶ Calibration of eight copula models (see slide [Copulas](#)) via **method of moments**, see e.g. (Genest and Rivest, 1993; Oh and Patton, 2013)
- ▶ “Moments” :
 - Spearman’s Rho
 - Quantile dependence at **0.05, 0.1, 0.9, 0.95** quantiles
- ▶ Margins follow empirical distribution
- ▶ Recalibration every 5 days with 300 data points

Copula-based hedging of cryptocurrencies

└ Results

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- Interesting to note that Student-*t* copula always calibrates to 2, resp. 4 (boundary solution)
- This is not the case if calibrating the data to a multivariate Student *t*.

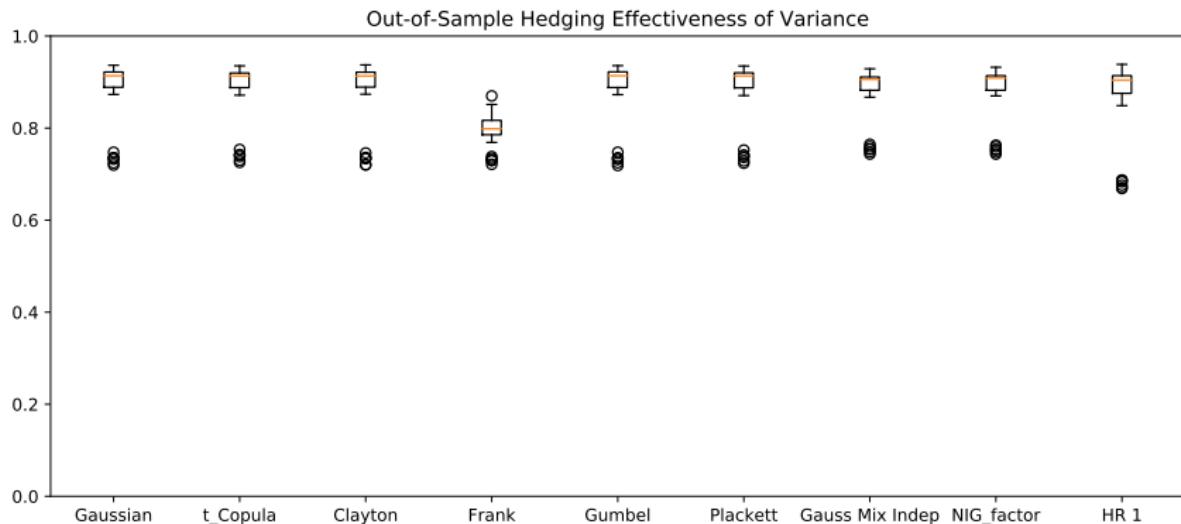
Hedge Effectiveness

- ▶ Hedge effectiveness (Ederington, 1979) captures percentage reduction in risk:

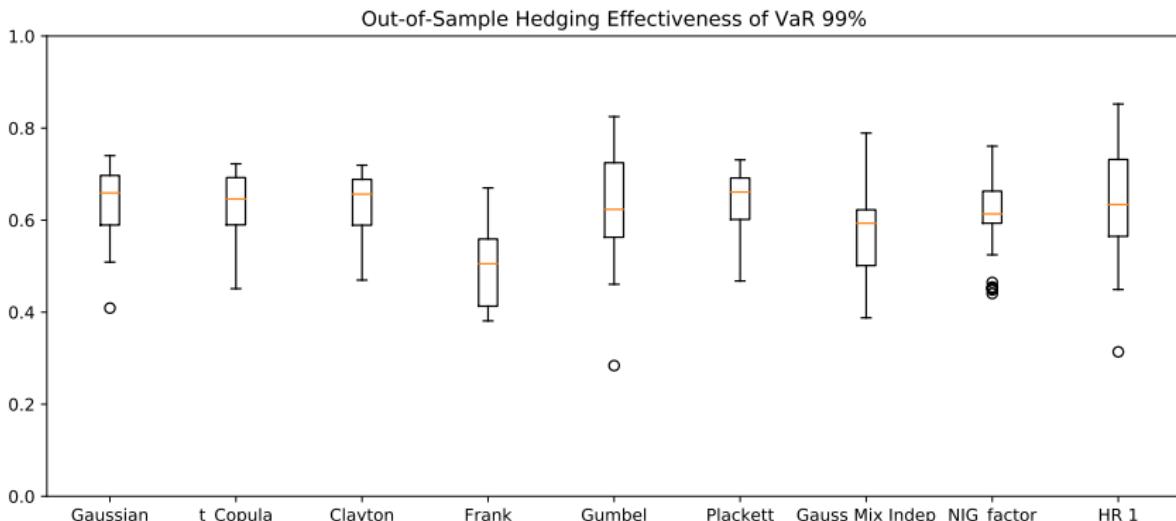
$$1 - \frac{\rho(R^h)}{\rho(R^S)}.$$

- ▶ Optimisation of h^* every 5 days based on 300-day-window.
- ▶ Out-of-sample effectiveness calculated on 100 days.

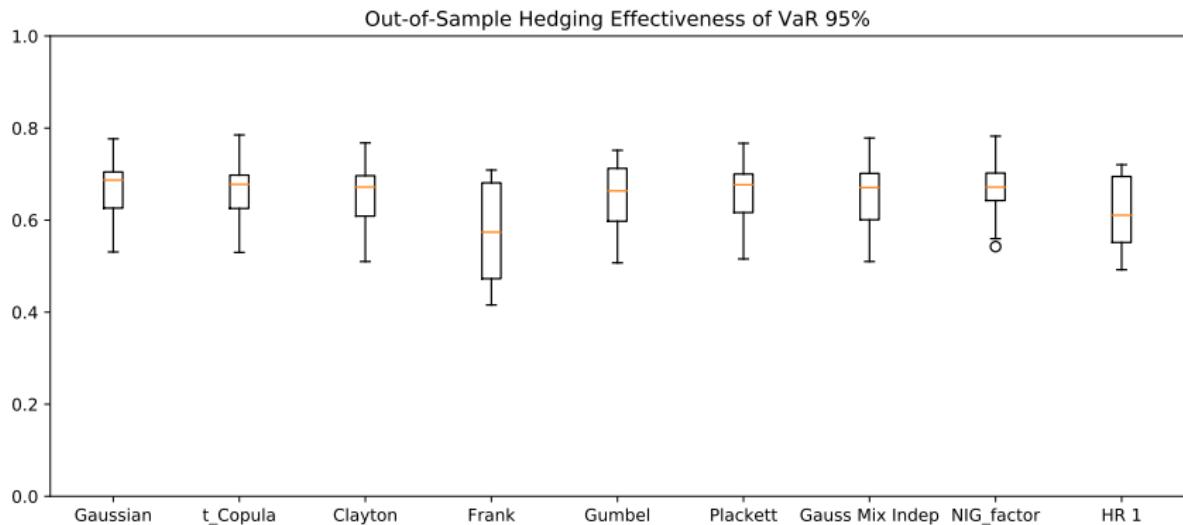
Hedge effectiveness



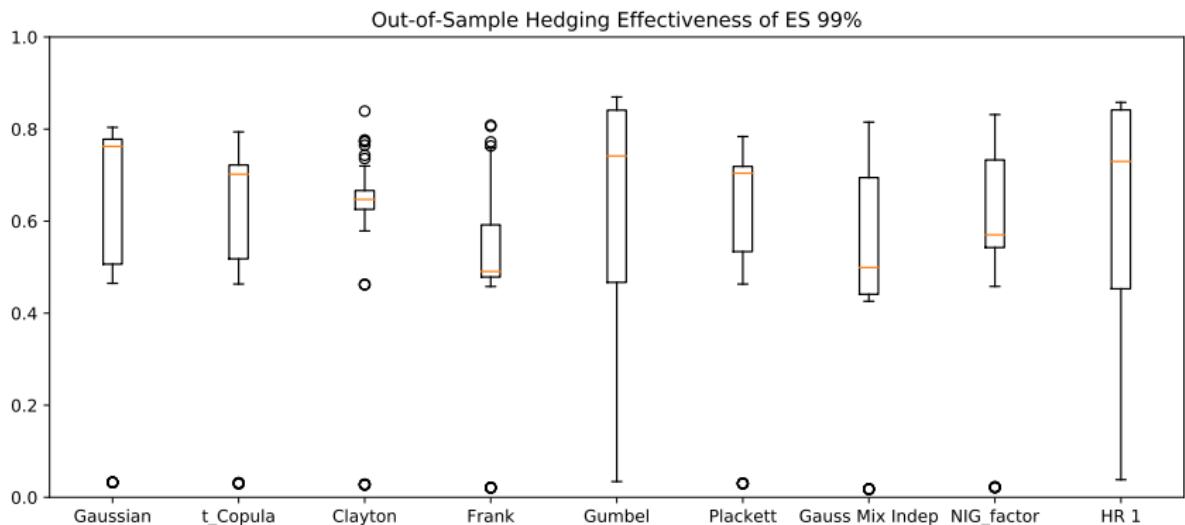
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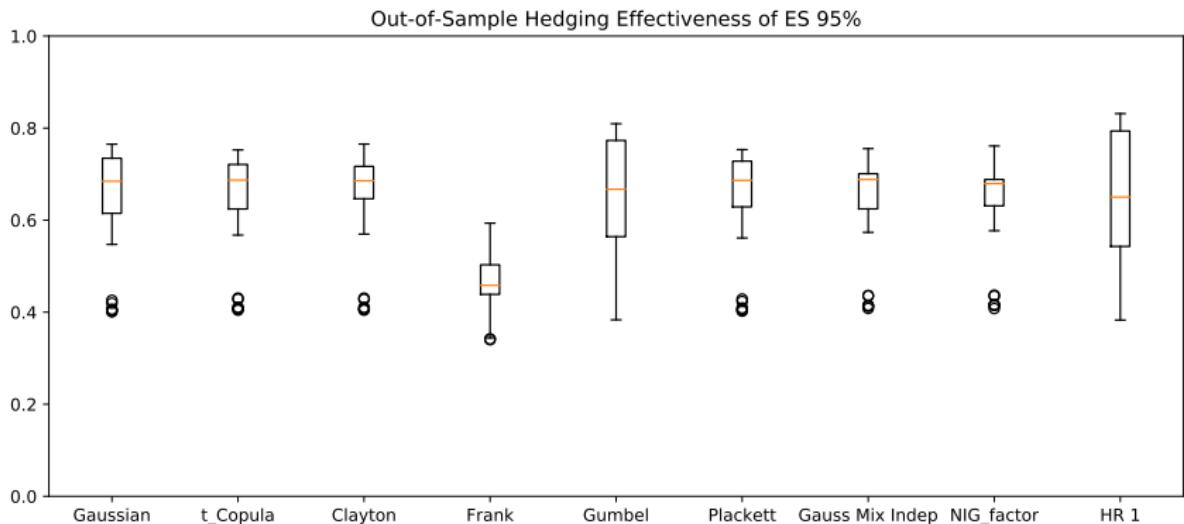
Hedge effectiveness



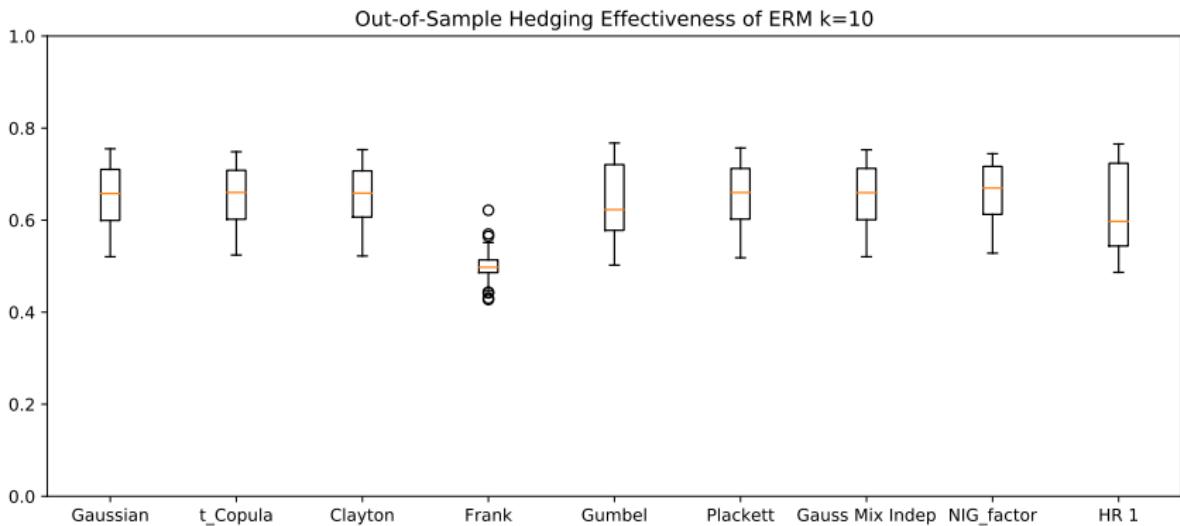
Hedge effectiveness



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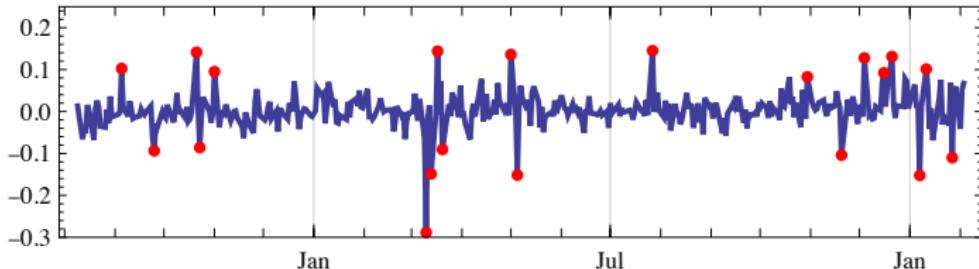


Hedge effectiveness

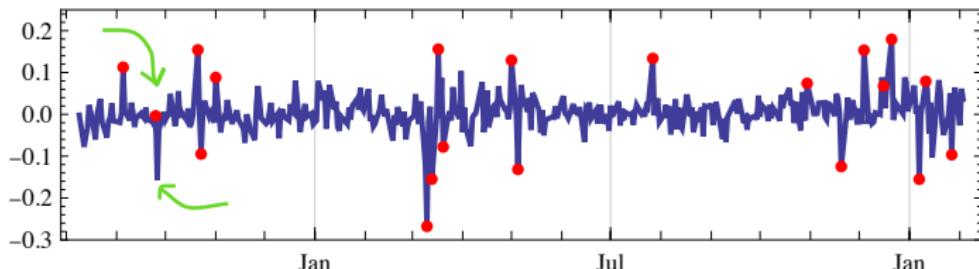


P&L (dynamic)

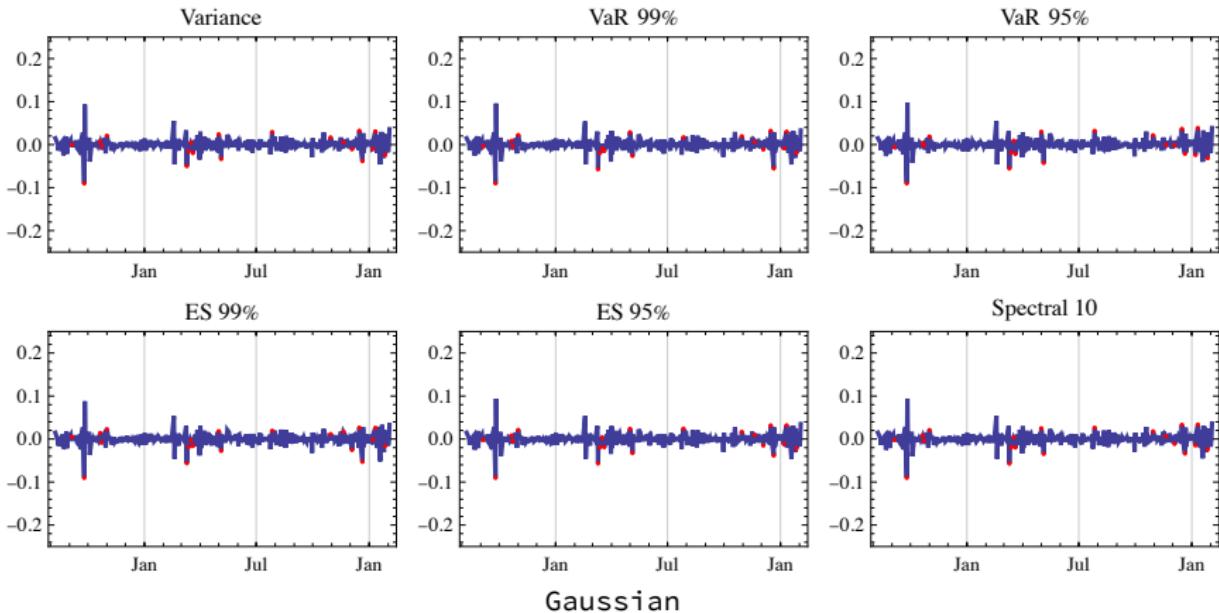
BTC spot returns with 20 most extreme observations



BTC future returns with 20 most spot extreme observations



P&L (dynamic)



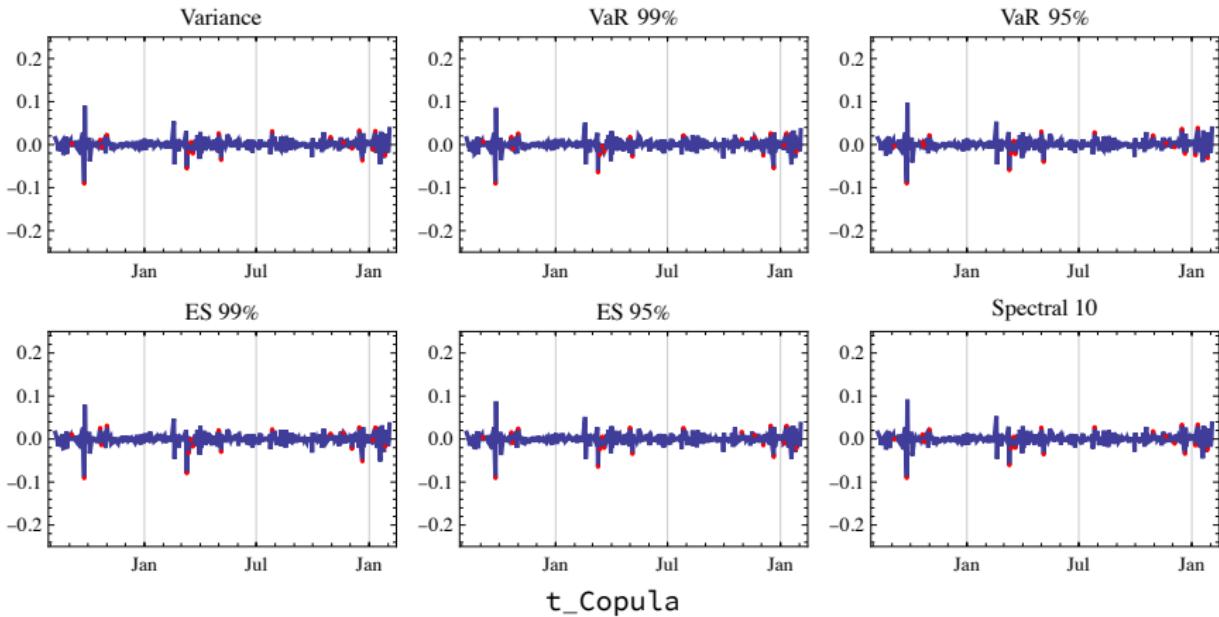
- ▶ Daily return from hedge, out-of-sample
- ▶ Recalibration every 5 days

Results

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P&L (dynamic)



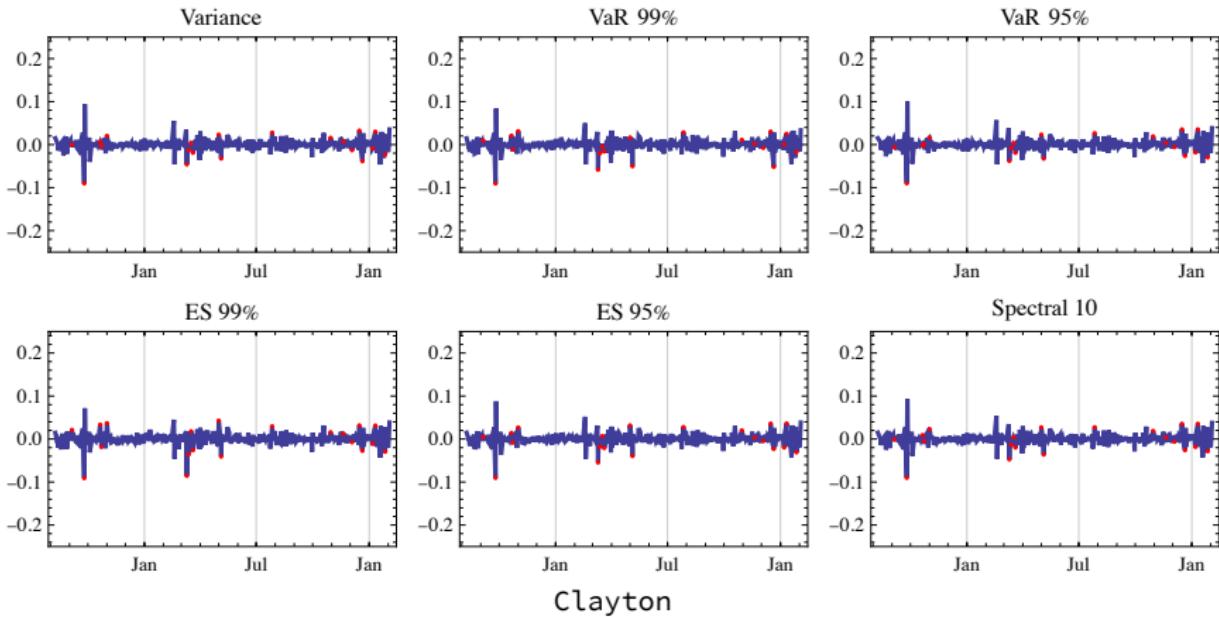
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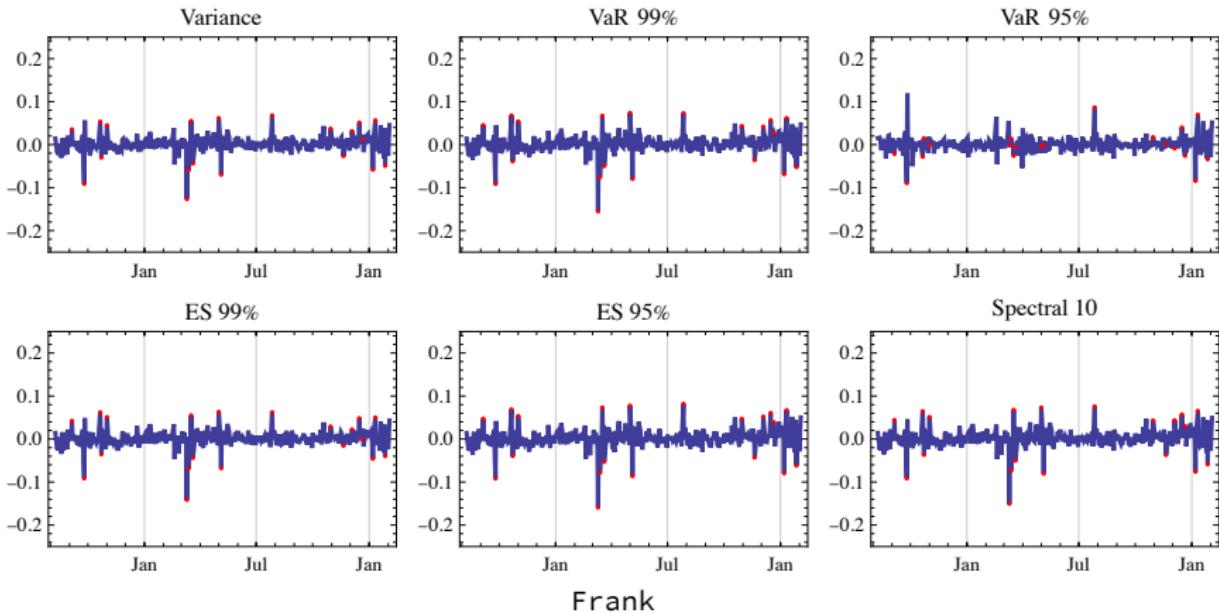
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N. Packham

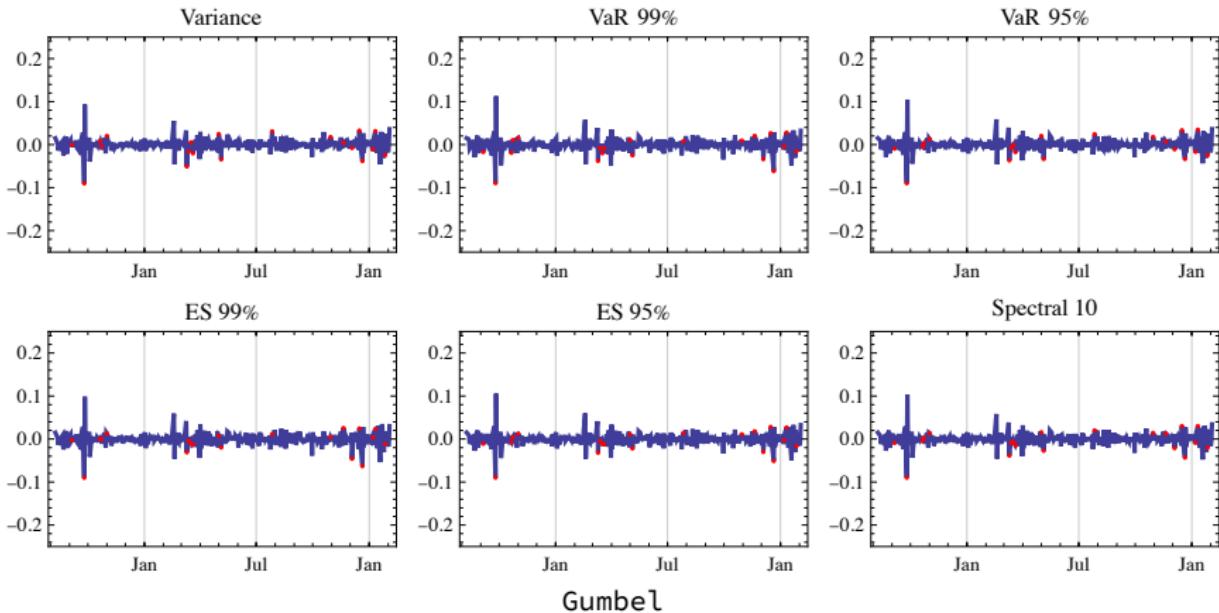
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P&L (dynamic)



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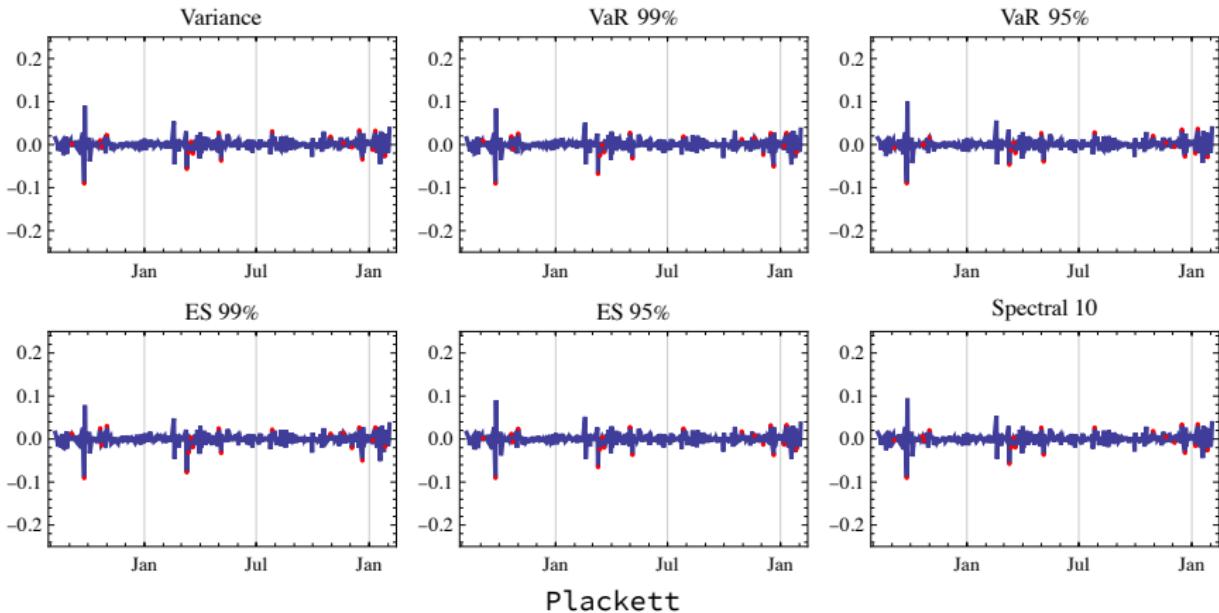
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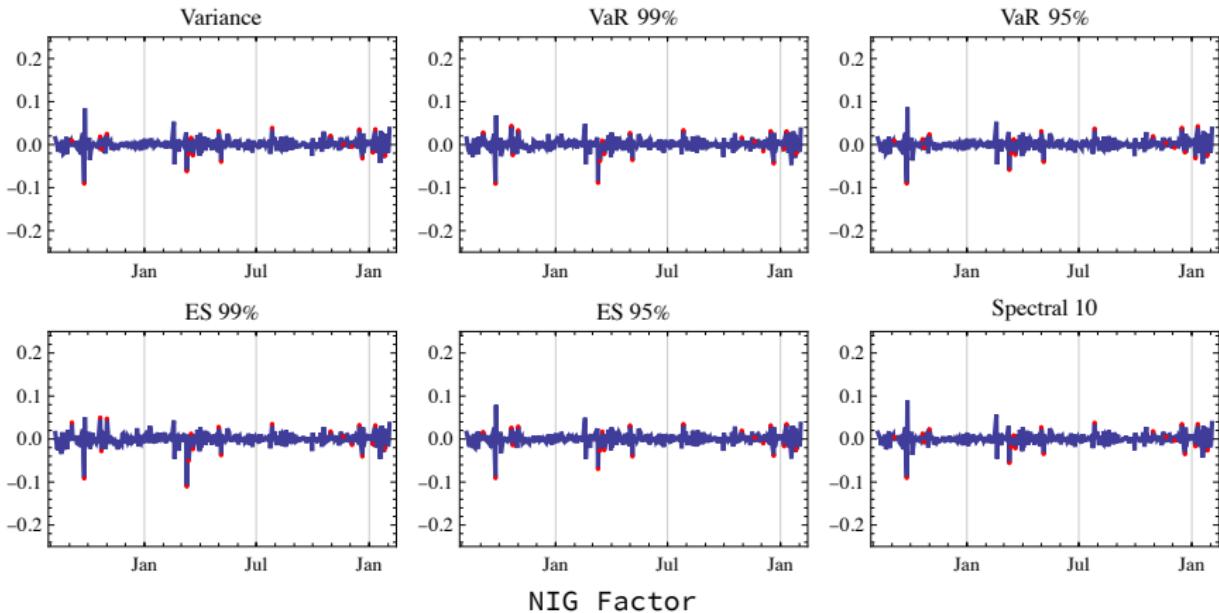
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Results

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P&L (dynamic)



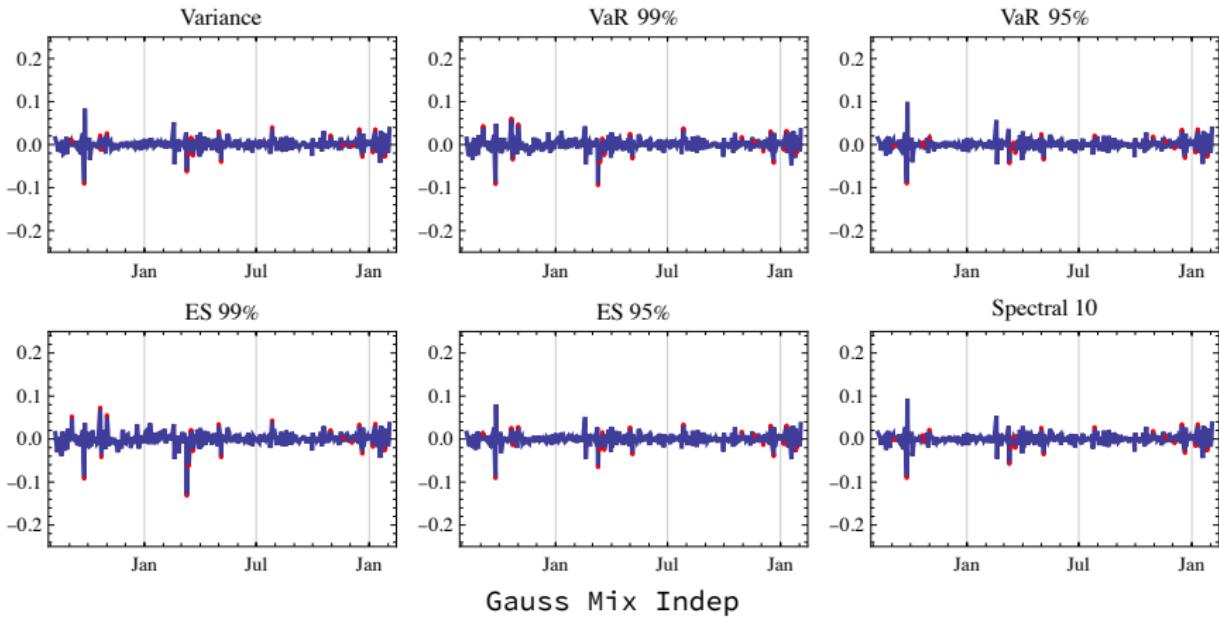
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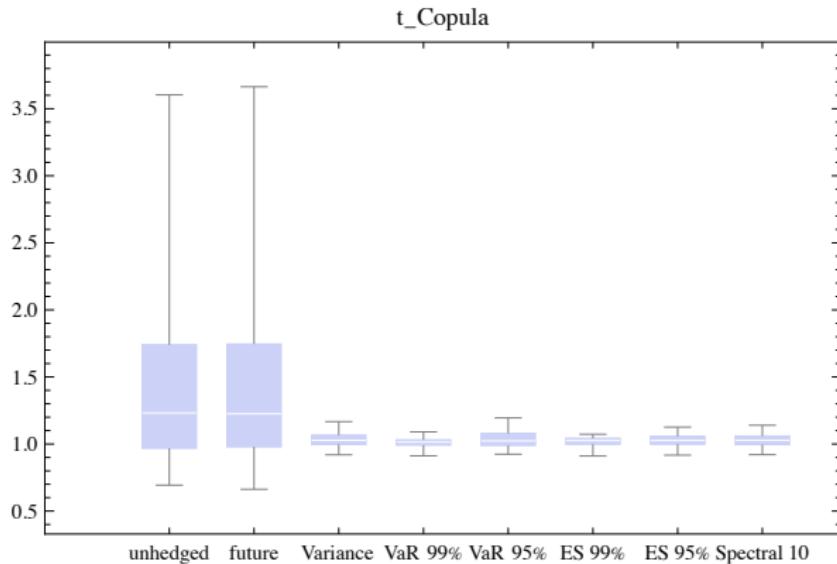
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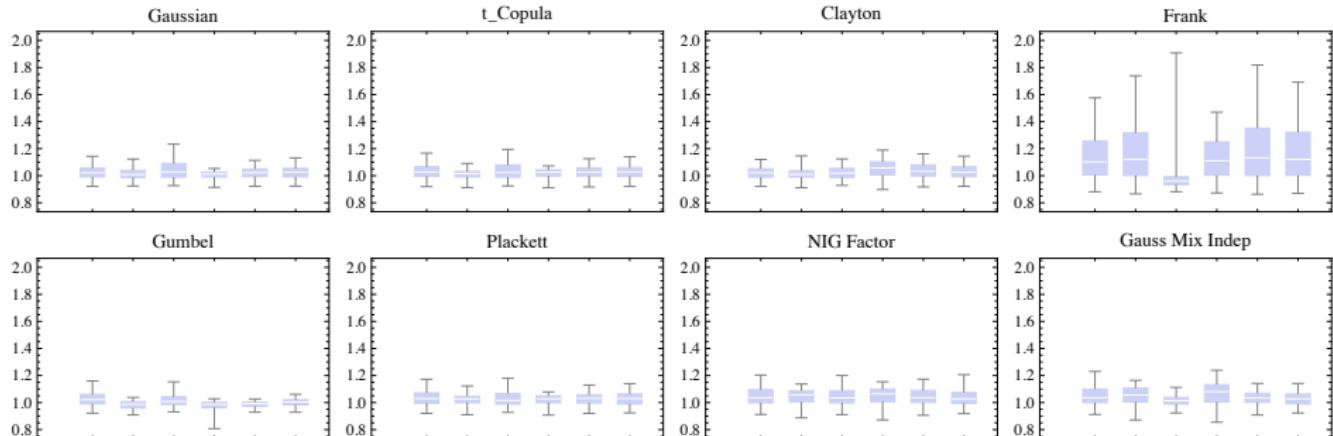
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P&L (static)



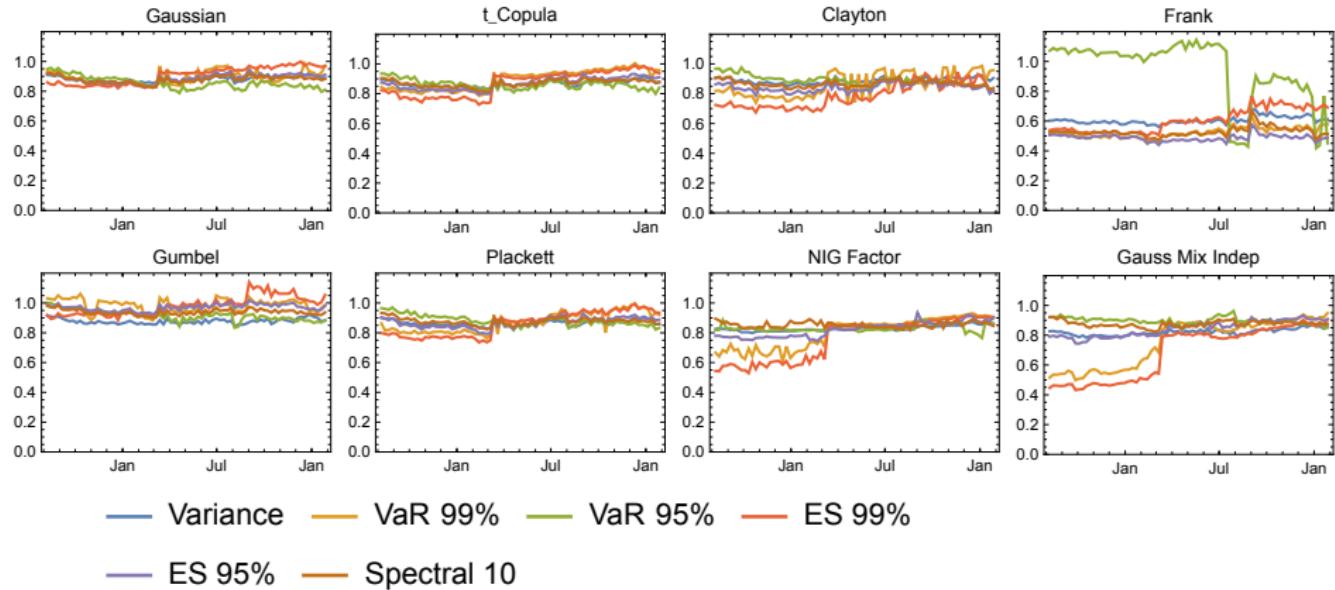
- ▶ P&L from static hedge, out-of-sample, 100 days, rolling every 5 days

P&L (static)



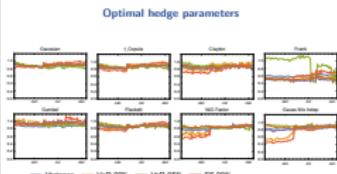
- ▶ P&L from static hedge, out-of-sample, rolling every 5 days
- ▶ From left to right: Variance, VaR 99%, VaR 95%, ES 99%, ES 95%, Spectral 10

Optimal hedge parameters



Copula-based hedging of cryptocurrencies

└ Results



- Because of 300 day burn-in period, the data show end-of 2019, 2020 and beginning of 2021
- NIG and Gaussian Mix, VaR 99% and ES 99%, show jump at beginning of Covid-19 crisis
- Frank copula aside, they have the lowest hedge parameters pre-Covid 19

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Thank you!



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