

Copula-based hedging of cryptocurrencies

Natalie Packham

joint work with

Francis Liu, Meng-Jou Lu, Wolfgang K. Härdle

Mathfinance Conference

16 March 2021



Hochschule für
Wirtschaft und Recht Berlin
Berlin School of Economics and Law



Overview

Motivation

Copula-based hedging

Data

Results

Digital assets are here to stay

- ▶ Markets for cryptocurrencies are maturing:
 - Institutional investors are buying into it.
 - Regulators are working hard to make stablecoins safe (e.g. resolve issues of jurisdiction, financial stability).
 - Exchanges (e.g. CME) are issuing futures and options.
- ➡ We are in the middle of the digitalisation of financial markets...

Digital assets are here to stay

- ▶ Markets for cryptocurrencies are maturing:
 - Institutional investors are buying into it.
 - Regulators are working hard to make stablecoins safe (e.g. resolve issues of jurisdiction, financial stability).
 - Exchanges (e.g. CME) are issuing futures and options.
- ➡ We are in the middle of the digitalisation of financial markets...
(... and it's progressing rapidly.)

Digital assets are here to stay

The image shows a screenshot of the GARP (Global Association of Risk Professionals) website. At the top, there's a navigation bar with links for Home, COVID-19 Hub, Technology, Culture & Governance, Energy, Operational, Credit, Market, and More. A search icon is also present. Below the navigation is a main article thumbnail. The thumbnail features a dark background with a digital circuit board pattern and several candlestick charts representing price movements. The title of the article is "As Bitcoin Rises, Institutions Make Crypto Market Impact", with a subtitle "Barriers fall away but hedging remains a challenge; regulatory clarity will help". The date "Friday, February 26, 2021" and author "By John Hintze" are at the bottom of the thumbnail.

GARP Global Association of Risk Professionals

FRM ▾ ERP ▾ SCR ▾ Courses ▾ Membership ▾ Events ▾ Risk Intelligence About Us ▾

Home COVID-19 Hub Technology Culture & Governance Energy Operational Credit Market More

- Investment Management -

As Bitcoin Rises, Institutions Make Crypto Market Impact

Barriers fall away but hedging remains a challenge; regulatory clarity will help

Friday, February 26, 2021

By John Hintze

<https://www.garp.org/#!/risk-intelligence/market/investment-management/a1Z1W000005kZDGUA2>

Bitcoin futures

- ▶ CME launched Bitcoin Futures in December 2017 and options on futures in January 2020
- ▶ Bitcoin Future:
 - Underlying: Bitcoin Reference Rate (BRR), based on relevant bitcoin transaction on certain exchanges
 - Maturities: nearest two Decembers and nearest six consecutive months
 - Settlement: cash
- ▶ <https://www.cmegroup.com/trading/equity-index/us-index/bitcoin.html>

Hedging cryptos

- ▶ Hedging Bitcoin exposure with Bitcoin futures:
 - Basis risk
 - BRR not traded
 - Ability of futures to hedge tail risks?

Hedging cryptos

- ▶ Hedging Bitcoin exposure with Bitcoin futures:
 - Basis risk
 - BRR not traded
 - Ability of futures to hedge tail risks?
- ▶ Hedge other cryptos with Bitcoin?
 - High correlation
 - Tail risks, extreme events?

skew. 1M Correlation Matrix								
Index	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	88.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.57%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	88.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.81%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	95.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	89.87%	99.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	28.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

Hedging cryptos

- ▶ Hedging Bitcoin exposure with Bitcoin futures:
 - Basis risk
 - BRR not traded
 - Ability of futures to hedge tail risks?
- ▶ Hedge other cryptos with Bitcoin?
 - High correlation
 - Tail risks, extreme events?
- ▶ Two directions:
 - Copulas
 - Risk measures

Index	1M Correlation Matrix							
	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	88.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.57%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	88.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.81%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	95.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	89.87%	99.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	26.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

Overview

Motivation

Copula-based hedging

Data

Results

Hedging spot with futures

- ▶ Hedge portfolio return: $R_t^h = R_t^S - h R_t^F$, where
 - R_t^S : spot return at time t
 - R_t^F : futures return at time t
 - h : hedge ratio
- ▶ Goal: Find optimal hedge ratio h^*
- ▶ Minimum-variance hedge ratio, e.g. Ederington (1979), assumes variance as risk measure and elliptical return distribution
- ▶ Extensions: risk measures, copulas, e.g. (Harris and Shen, 2006; Barbi and Romagnoli, 2014)

Copulas

Definition

A (bivariate) **copula** is a distribution function on $[0, 1]^2$ with standard uniform marginals.

- ▶ Copulas differ only through the dependence between the marginals.
- ▶ Sklar's Theorem (below) captures that copulas allow to separate
 - modelling of the marginals, and
 - modelling of the dependence structure.

Copulas

Theorem (Sklar's Theorem)

Let F be a joint distribution function with margins F_1, F_2 . Then, there exists a copula $C : [0, 1]^2 \rightarrow [0, 1]$ such that, for all $x, y \in \mathbb{R}$

$$F(x, y) = C(F_1(x), F_2(y)). \quad (1)$$

If the margins are continuous, then C is unique; otherwise C is unique on the range of the margins.

Conversely, if C is a copula and F_1, F_2 are univariate distribution functions, then the function F defined by (1) is a joint distribution function with margins F_1, F_2 .

- ▶ Representation of C in terms of F and its margins:

$$C(u, v) = F(F_1^{(-1)}(u), F_2^{(-1)}(v)).$$

Examples of copulas

- ▶ Gaussian, Student t , NIG factor, Gumbel, Clayton, Frank, Plackett, Gaussian Mixture
- ▶ Add scatterplots here

Copula-based hedging

Proposition (Barbi and Romagnoli (2014))

Let R^S and R^F be two real-valued random variables with corresponding absolutely continuous copula C_{R^S, R^F} and continuous marginals F_{R^S} and F_{R^F} . Then, the distribution of R^h is given by

$$F_{R^h}(x) = 1 - \int_0^1 D_1 C_{R^S, R^F} \left\{ u, F_{R^F} \left\{ \frac{F_{R^S}^{-1}(u) - x}{h} \right\} \right\} du. \quad (2)$$

- ▶ Easy to show (e.g. McNeil et al. (2005)):

$$D_1 C_{X, Y}(F_X(x), F_Y(y)) = \frac{\partial}{\partial u} C(u, v) = \mathbf{P}(Y \leq y | X = x).$$

Risk measures

- ▶ **Variance:** $\text{Var}(R^h)$
- ▶ **Value-at-risk (VaR):** $\text{VaR}_\alpha = -F_{R^h}^{(-1)}(1 - \alpha)$
- ▶ **Expected Shortfall (ES):** $\text{ES}_\alpha = -\frac{1}{1 - \alpha} \int_{-\infty}^{\alpha} F_{R^h}^{(-1)}(p) dp.$

Risk measures

- ▶ **Spectral risk measures (SRM)** (Acerbi, 2002; Cotter and Dowd, 2006):

$$\rho_\phi = - \int_0^1 \phi(p) F_{R^h}^{(-1)} dp,$$

where q_p is the p -quantile of the return distribution and $\phi(s)$, $s \in [0, 1]$, is the so-called **risk aversion function**, a weighting function such that

- (i) $\phi(p) \geq 0$,
- (ii) $\int_0^1 \phi(p) dp = 1$,
- (iii) $\phi'(p) \leq 0$.

- ▶ SRM's are coherent risk measures.

Risk measures

- ▶ **Exponential spectral risk measure:** weighting function $\phi(p) = \lambda e^{-k(1-p)}$, where λ is an unknown positive constant, derived from exponential utility function:

$$\rho_\phi = \int_0^1 \phi(p) F^{(-1)}(p) dp = \frac{k}{1 - e^{-k}} \int_0^1 e^{-k(1-p)} F^{(-1)}(p) dp.$$

Optimal hedge ratio

- ▶ Hedge portfolio: $R_t^h = R_t^S - hR_t^F$, with h hedge ratio
- ▶ Optimal hedge ratio:

$$h^* = \operatorname{argmin}_h \rho(h),$$

where $\rho(h)$ is the risk of the hedge portfolio with hedge ratio h .

Overview

Motivation

Copula-based hedging

Data

Results

Data

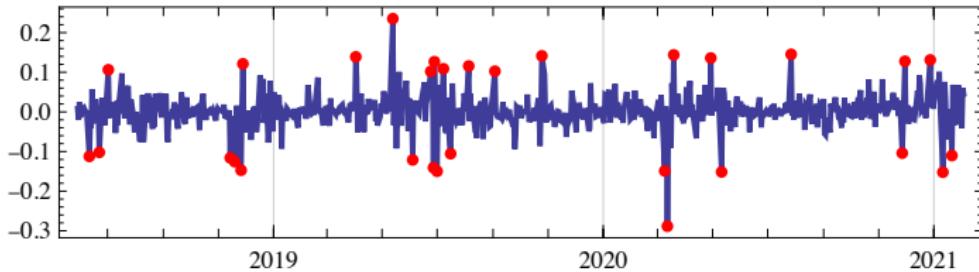
- ▶ Daily log returns, 23pm CET
- ▶ 29 May 2018 through 3 Feb 2021
- ▶ Spot: Coingecko Bitcoin / USDC
- ▶ Future: CME BTC Future

Summary statistics

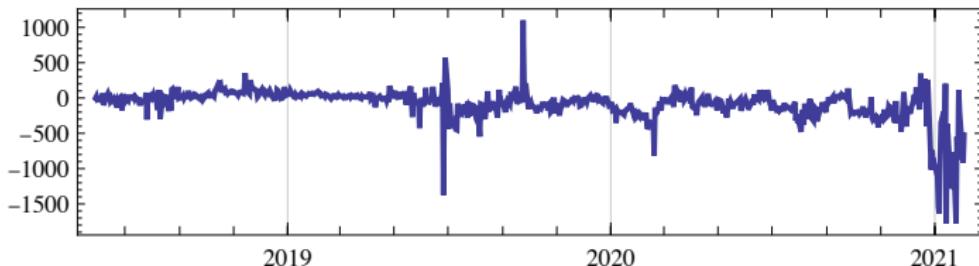
Variable	Mean	Median	Minimum	Maximum
BTC spot	0.0023782	0.0013855	-0.28846	0.23522
Future	0.0023970	0.0012540	-0.26773	0.22251
Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
BTC spot	0.043133	18.137	-0.27245	5.8658
Future	0.046288	19.311	-0.27321	5.4709
Variable	5% perc.	95% perc.	IQ Range	Missing obs.
BTC spot	-0.065328	0.072665	0.033819	0
Future	-0.066085	0.077146	0.037142	0

Time series

BTC spot returns with 30 extremes

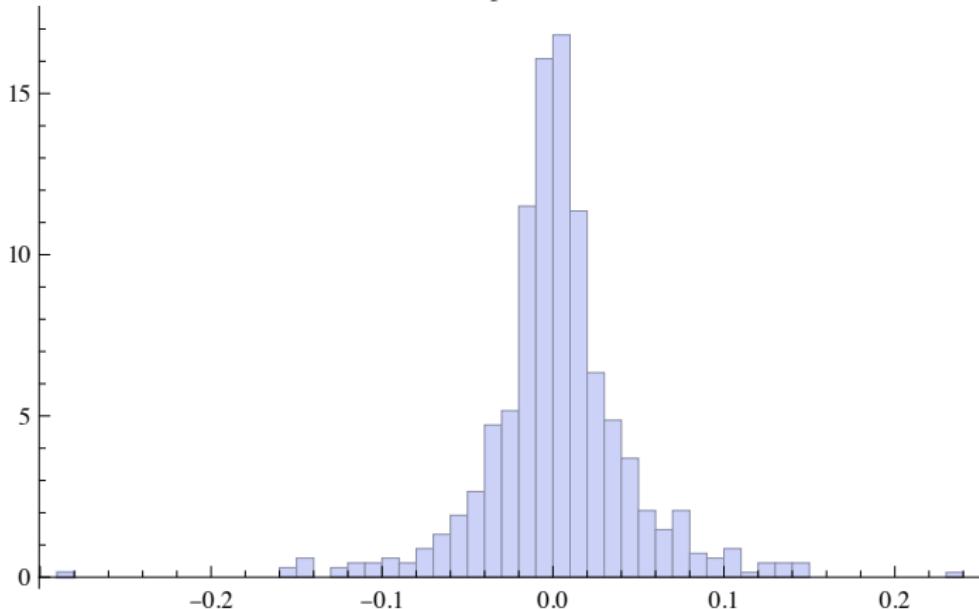


Difference BTC spot and future



Distribution

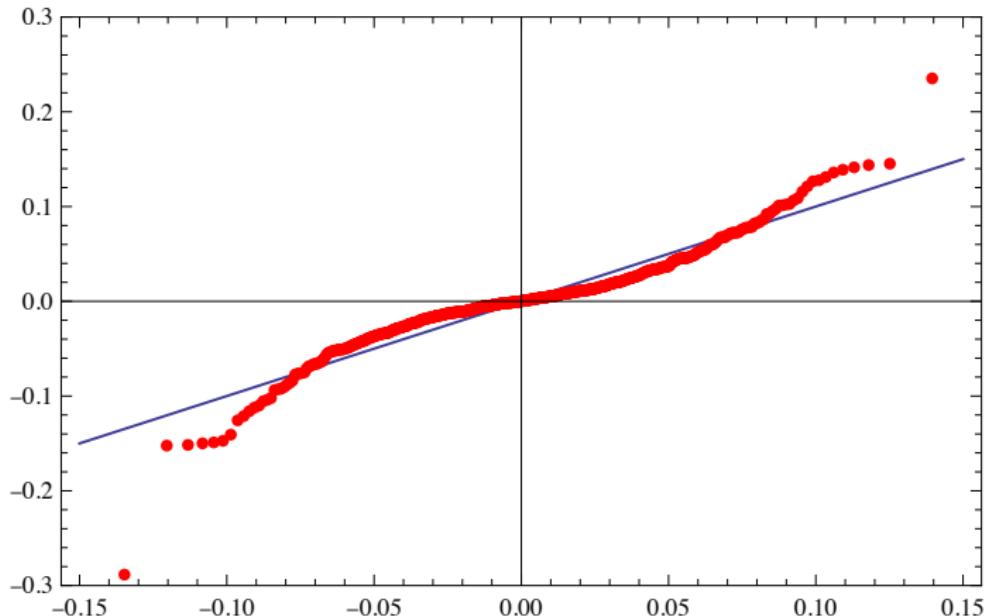
BTC spot return



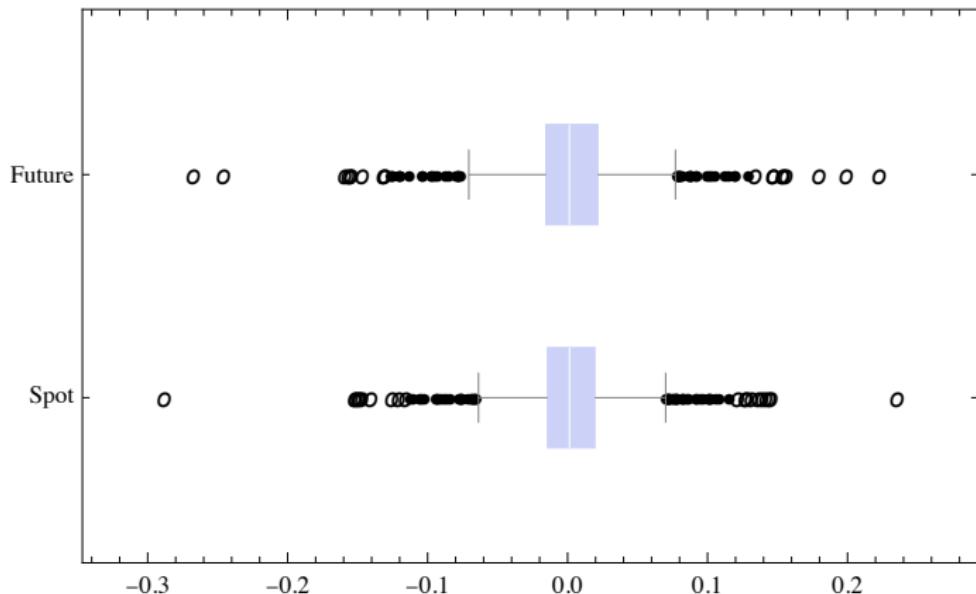
- ▶ Student t distribution: $\nu = 7.95$
- ▶ Tail / Generalised Pareto distribution: tail index $1/\xi = 4.92$

QQ-plot

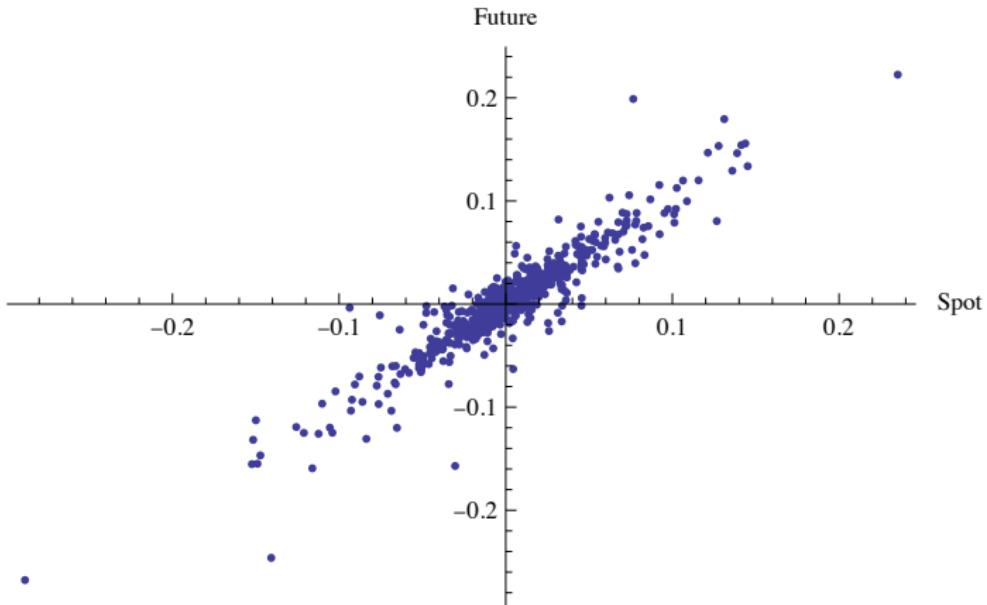
QQ-Plot of BTC returns against normal distribution



Spot and Future



Spot and Future



Overview

Motivation

Copula-based hedging

Data

Results

Hedge Effectiveness

- ▶ Hedge effectiveness (Ederington, 1979) captures percentage reduction in risk:

$$1 - \frac{\rho(R^h)}{\rho(R^S)}.$$

- ▶ Optimisation of h^* every 30 days based on 300-day-window.

Hedge effectiveness

P&L

Optimal hedge parameters

References |

- Acerbi, C. Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26(7):1505–1518, 2002.
- Barbi, M. and S. Romagnoli. A copula-based quantile risk measure approach to estimate the optimal hedge ratio. *Journal of Futures Markets*, 34(7):658–675, 2014.
- Cotter, J. and K. Dowd. Extreme spectral risk measures: an application to futures clearinghouse margin requirements. *Journal of Banking & Finance*, 30(12):3469–3485, 2006.
- Ederington, L. H. The hedging performance of the new futures markets. *The Journal of Finance*, 34(1):157–170, 1979.
- Harris, R. D. and J. Shen. Hedging and value at risk. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, 26(4):369–390, 2006.
- McNeil, A., R. Frey, and P. Embrechts. *Quantitative Risk Management*. Princeton University Press, Princeton, NJ, 2005.

Thank you!



Prof. Dr. Natalie Packham

Professor of Mathematics and Statistics
Berlin School of Economics and Law
Badensche Str. 52
10825 Berlin
natalie.packham@hwr-berlin.de



Hochschule für
Wirtschaft und Recht Berlin
[Berlin School of Economics and Law](http://www.hwr-berlin.de)

