Copula-based hedging of cryptocurrencies

Natalie Packham

joint work with
Francis Liu, Meng-Jou Lu, Wolfgang K. Härdle

Mathfinance Conference
16 March 2021

Hochschule für
Wirtschaft und Recht Berlin
Berlin School of Economics and Law

HUMBOLDT-UNIVERSITÄT ZU BERLIN

Overview

Motivation

Copula-based hedging

Data

Results

Digital assets are here to stay

- Markets for cryptocurrencies are maturing:
  - Institutional investors are buying into it.
  - Regulators are working hard to make stablecoins safe (e.g. resolve issues of jurisdiction, financial stability).
  - Exchanges (e.g. CME) are issuing futures and options.

$\Longrightarrow$ We are in the middle of the digitalisation of financial markets...

Digital assets are here to stay

- Markets for cryptocurrencies are maturing:
  - Institutional investors are buying into it.
  - Regulators are working hard to make stablecoins safe (e.g. resolve issues of jurisdiction, financial stability).
  - Exchanges (e.g. CME) are issuing futures and options.

$\Longrightarrow$ We are in the middle of the digitalisation of financial markets...

(... and it's progressing rapidly.)

Digital assets are here to stay

https://www.garp.org/#!/risk-intelligence/market/investment-management/a1Z1W000005kZDGUA2

Bitcoin futures

- CME launched Bitcoin Futures in December 2017 and options on futures in January 2020
- Bitcoin Future:
  - Underlying: Bitcoin Reference Rate (BRR), based on relevant bitcoin transaction onn certain exchanges
  - Maturities: nearest two Decembers and nearest six consecutive months
  - Settlement: cash
- https://www.cmegroup.com/trading/equity-index/us-index/bitcoin.html

Hedging cryptos

- Hedging Bitcoin exposure with Bitcoin futures:
  - Basis risk
  - BRR not traded
  - Ability of futures to hedge tail risks?

Hedging cryptos

- Hedging Bitcoin exposure with Bitcoin futures:
  - Basis risk
  - BRR not traded
  - Ability of futures to hedge tail risks?
- Hedge other cryptos with Bitcoin?
  - High correlation
  - Tail risks, extreme events?

skew.

1M Correlation Matrix

index	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	86.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.67%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	86.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.61%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	95.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	69.87%	59.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	26.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

Hedging cryptos

- Hedging Bitcoin exposure with Bitcoin futures:
  - Basis risk
  - BRR not traded
  - Ability of futures to hedge tail risks?
- Hedge other cryptos with Bitcoin?
  - High correlation
  - Tail risks, extreme events?
- Two directions:
  - Copulas
  - Risk measures

skew. 1M Correlation Matrix

index	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	86.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.67%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	86.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.61%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	95.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	69.87%	59.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	26.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

Overview

Motivation

Copula-based hedging

Data

Results

Hedging spot with futures

- Hedge portfolio return: $R_t^h = R_t^S - h\, R_t^F$, where
  - R_t^S: spot return at time t
  - R_t^F: futures return at time t
  - h: hedge ratio
- Goal: Find optimal hedge ratio $h^\star$
- Minimum-variance hedge ratio, e.g. Ederington (1979), assumes variance as risk measure and elliptical return distribution
- Extensions: risk measures, copulas, e.g. (Harris and Shen, 2006; Barbi and Romagnoli, 2014)

Copulas

Definition

A (bivariate) **copula** is a distribution function on $[0, 1]^2$ with standard uniform marginals.

- Copulas differ only through the dependence between the marginals.
- Sklar's Theorem (below) captures that copulas allow to separate
  - modelling of the marginals, and
  - modelling of the dependence structure.

Copulas

Theorem (Sklar's Theorem)

Let F be a joint distribution function with margins F_1, F_2. Then, there exists a copula $C : [0,1]^2 \to [0,1]$ such that, for all $x, y \in \mathbb{R}$

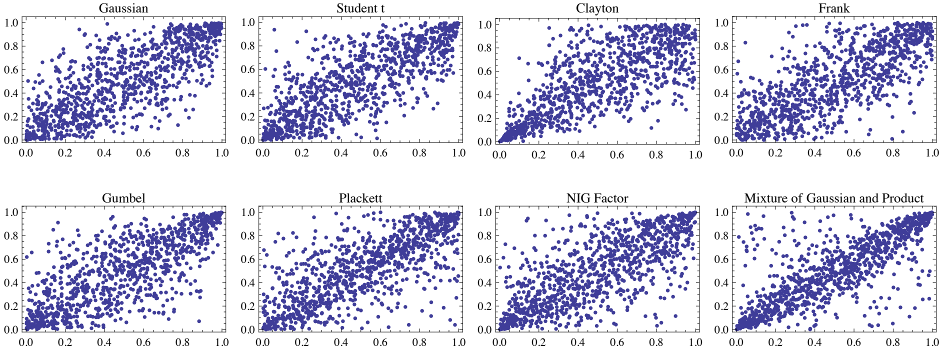
$$F(x,y) = C(F_1(x), F_2(y)). \tag{1}$$

If the margins are continuous, then C is unique; otherwise C is unique on the range of the margins.
Conversely, if C is a copula and F_1, F_2 are univariate distribution functions, then the function F defined by (1) is a joint distribution function with margins F_1, F_2.

- Representation of C in terms of F and its margins:

$$C(u,v) = F(F_1^{(-1)}(u), F_2^{(-1)}(v)).$$

Examples of copulas



- All copulas are calibrated to a Spearman's Rho of 0.75.

Copula-based hedging

Proposition (Barbi and Romagnoli (2014))

Let X and Y be two real-valued random variables with corresponding absolutely continuous copula C and continuous marginals F_X and F_Y. Then, the distribution of of $Z = X - hY$ is given by

$$F_Z(x) = 1 - \int_0^1 D_1 C \left\{ u, F_Y \left\{ \frac{F_X^{(-1)}(u) - x}{h} \right\} \right\} \mathrm{d}u. \tag{2}$$

- Easy to show (e.g. McNeil *et al.* (2005)):

$$D_1 C(F_X(x), F_Y(y)) = \frac{\partial}{\partial u} C(u, v) = \mathbf{P}(Y \leq y | X = x).$$

Risk measures

- **Variance**: $\mathrm{Var}(Z)$
- **Value-at-risk (VaR)**: $\mathrm{VaR}_\alpha = -F_Z^{(-1)}(1-\alpha)$
- **Expected Shortfall (ES)**: $\mathrm{ES}_\alpha = -\frac{1}{1-\alpha}\int_{-\infty}^{\alpha} F_Z^{(-1)}(p)\mathrm{d}p.$

Risk measures

- **Spectral risk measures (SRM)** (Acerbi, 2002; Cotter and Dowd, 2006):

$$\rho_\phi = -\int_0^1 \phi(p)\, F_Z^{(-1)}(p)\, \mathrm{d}p,$$

  where q_p is the p-quantile of the return distribution and $\phi(s)$, $s \in [0,1]$, is the so-called **risk aversion function**, a weighting function such that

$$\begin{aligned} &\text{(i)} && \phi(p) \geq 0, \\ &\text{(ii)} && \textstyle\int_0^1 \phi(p)\, \mathrm{d}p = 1, \\ &\text{(iii)} && \phi'(p) \leq 0. \end{aligned}$$

- SRM's are coherent risk measures.

Risk measures

- **Exponential spectral risk measure**: weighting function $\phi(p) = \lambda \mathbf{e}^{-k(1-p)}$, where λ is an unknown positive constant, derived from exponential utility function:

$$\rho_\phi = \int_0^1 \phi(p)\, F_Z^{(-1)}(p)\,\mathrm{d}p = \frac{k}{1-\mathbf{e}^{-k}} \int_0^1 \mathbf{e}^{-k(1-p)}\, F_Z^{(-1)}(p)\,\mathrm{d}p.$$

Optimal hedge ratio

- Hedge portfolio: $R_t^h = R_t^S - hR_t^F$, with h hedge ratio
- Optimal hedge ratio:

$$h^* = \operatorname*{argmin}_h \rho(h),$$

where $\rho(h)$ is the risk of the hedge portfolio with hedge ratio h.

Overview

Motivation

Copula-based hedging

Data

Results

Data

- Daily log returns, 23pm CET
- 29 May 2018 through 3 Feb 2021
- Spot: Coingecko Bitcoin / USD
- Future: CME BTC Future
- Sources: Bloomberg, coingecko

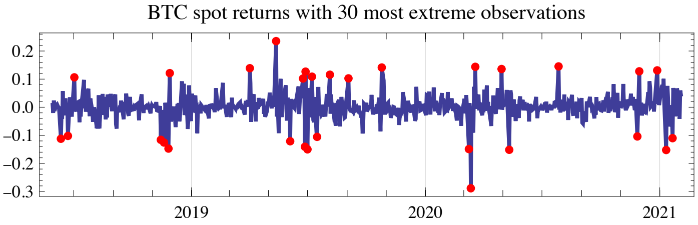
Summary statistics

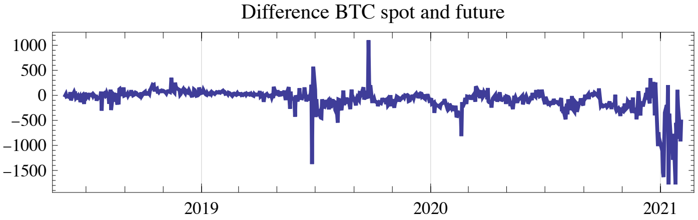
Variable	Mean	Median	Minimum	Maximum
BTC spot	0.0023782	0.0013855	−0.28846	0.23522
Future	0.0023970	0.0012540	−0.26773	0.22251

Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
BTC spot	0.043133	18.137	−0.27245	5.8658
Future	0.046288	19.311	−0.27321	5.4709

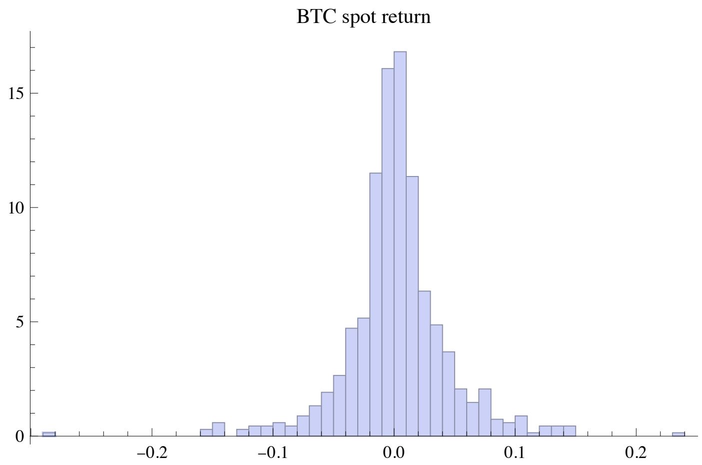
Variable	5% perc.	95% perc.	IQ Range	Missing obs.
BTC spot	−0.065328	0.072665	0.033819	0
Future	−0.066085	0.077146	0.037142	0

Time series


BTC spot returns with 30 most extreme observations
0.2
0.1
0.0
−0.1
−0.2
−0.3
2019
2020
2021

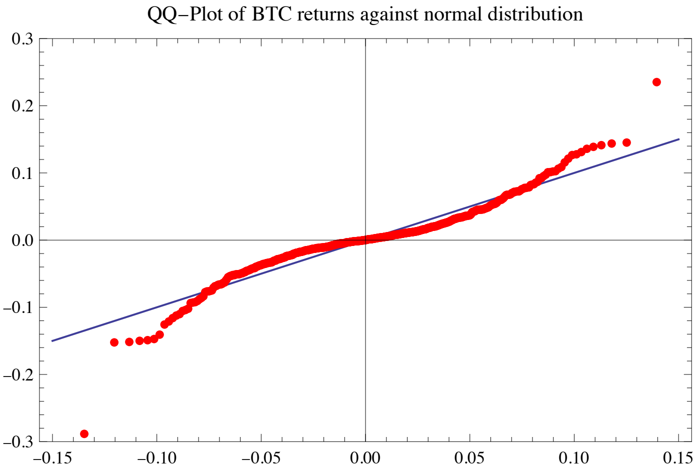

Difference BTC spot and future
1000
500
0
−500
−1000
−1500
2019
2020
2021

Distribution



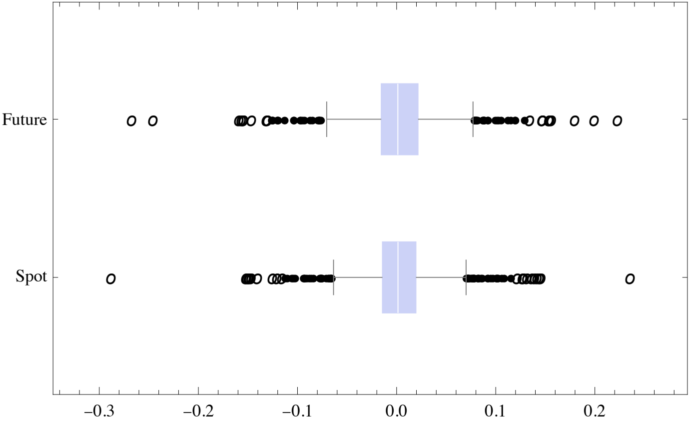
- Student t distribution: $\nu = 7.95$
- Tail / Generalised Pareto distribution: tail index $1/\xi = 4.92$

QQ-plot

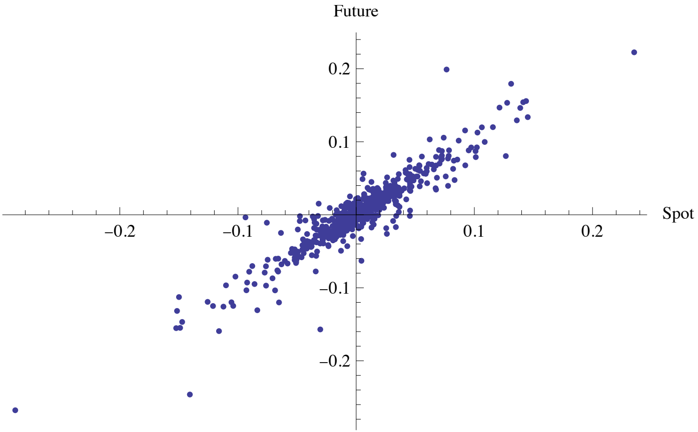


QQ−Plot of BTC returns against normal distribution
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15

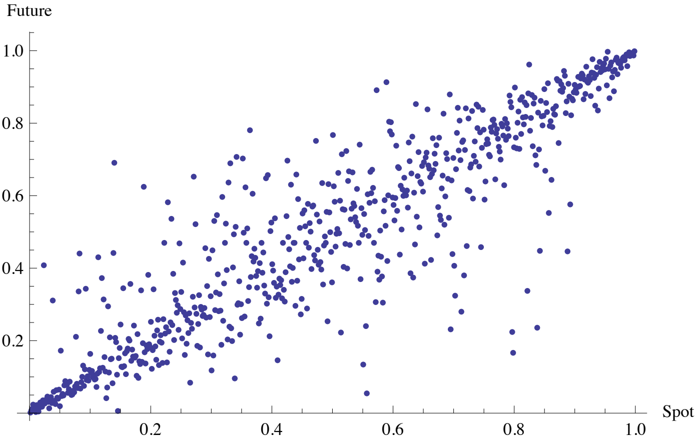
Spot and Future



Spot and Future



Spot and Future empirical copula



Overview

Motivation

Copula-based hedging

Data

Results

Hedge Effectiveness

- Hedge effectiveness (Ederington, 1979) captures percentage reduction in risk:
$$1 - \frac{\rho(R^h)}{\rho(R^S)}.$$
- Optimisation of h^* every 30 days based on 300-day-window.

Hedge effectiveness

P&L

Optimal hedge parameters

References I

Acerbi, C. Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26(7):1505–1518, 2002.

Barbi, M. and S. Romagnoli. A copula-based quantile risk measure approach to estimate the optimal hedge ratio. *Journal of Futures Markets*, 34(7):658–675, 2014.

Cotter, J. and K. Dowd. Extreme spectral risk measures: an application to futures clearinghouse margin requirements. *Journal of Banking & Finance*, 30(12):3469–3485, 2006.

Ederington, L. H. The hedging performance of the new futures markets. *The Journal of Finance*, 34(1):157–170, 1979.

Harris, R. D. and J. Shen. Hedging and value at risk. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, 26(4):369–390, 2006.

McNeil, A., R. Frey, and P. Embrechts. *Quantitative Risk Management*. Princeton University Press, Princeton, NJ, 2005.

Thank you!

Prof. Dr. Natalie Packham
Professor of Mathematics and Statistics
Berlin School of Economics and Law
Badensche Str. 52
10825 Berlin
natalie.packham@hwr-berlin.de

Hochschule für Wirtschaft und Recht Berlin
Berlin School of Economics and Law