

# Copula-based hedging of cryptocurrencies

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joint work with

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# Overview

Motivation

Copula-based hedging

Data

Results

## Digital assets are here to stay

- ▶ Markets for cryptocurrencies are maturing:
  - Institutional investors are buying into it.
  - Regulators are working hard to make stablecoins safe (e.g. resolve issues of jurisdiction, financial stability).
  - Exchanges (e.g. CME) are issuing futures and options.
- ➡ We are in the middle of the digitalisation of financial markets...

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- ➡ We are in the middle of the digitalisation of financial markets...  
(... and it's progressing rapidly.)

# Digital assets are here to stay

The image shows a screenshot of the GARP (Global Association of Risk Professionals) website. At the top, there's a navigation bar with links for Home, COVID-19 Hub, Technology, Culture & Governance, Energy, Operational, Credit, Market, and More. A search icon is also present. Below the navigation is a main article thumbnail. The thumbnail features a dark background with a digital circuit board pattern and several candlestick charts representing price movements. The title of the article is "As Bitcoin Rises, Institutions Make Crypto Market Impact", with a subtitle "Barriers fall away but hedging remains a challenge; regulatory clarity will help". The date "Friday, February 26, 2021" and author "By John Hintze" are at the bottom of the thumbnail.

GARP Global Association of Risk Professionals

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- Investment Management -

# As Bitcoin Rises, Institutions Make Crypto Market Impact

Barriers fall away but hedging remains a challenge; regulatory clarity will help

Friday, February 26, 2021

By John Hintze

<https://www.garp.org/#!/risk-intelligence/market/investment-management/a1Z1W000005kZDGUA2>

## Bitcoin futures

- ▶ CME launched Bitcoin Futures in December 2017 and options on futures in January 2020
- ▶ Bitcoin Future:
  - Underlying: Bitcoin Reference Rate (BRR), based on relevant bitcoin transaction on certain exchanges
  - Maturities: nearest two Decembers and nearest six consecutive months
  - Settlement: cash
- ▶ <https://www.cmegroup.com/trading/equity-index/us-index/bitcoin.html>

## Hedging cryptos

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  - BRR not traded
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- ▶ Hedge other cryptos with Bitcoin?
  - High correlation
  - Tail risks, extreme events?

skew. 1M Correlation Matrix								
Index	BTC	ETH	XRP	USDT	BCH	LTC	EOS	BNB
BTC	100.00%	91.17%	81.77%	-15.59%	88.69%	88.85%	90.70%	84.47%
ETH	91.17%	100.00%	78.50%	-24.51%	90.18%	93.10%	92.84%	88.57%
XRP	81.77%	78.50%	100.00%	-8.21%	81.90%	81.68%	84.96%	75.23%
USDT	-15.59%	-24.51%	-8.21%	100.00%	-16.64%	-18.96%	-15.91%	-20.18%
BCH	88.69%	90.18%	81.90%	-16.64%	100.00%	87.79%	87.02%	88.45%
LTC	88.85%	93.10%	81.68%	-18.96%	87.79%	100.00%	95.78%	78.81%
EOS	90.70%	92.84%	84.96%	-15.91%	87.02%	95.78%	100.00%	84.92%
BNB	84.47%	88.67%	75.23%	-20.18%	88.45%	78.61%	84.92%	100.00%
BSV	64.46%	89.87%	99.06%	-10.90%	72.47%	74.45%	72.54%	70.57%
XTZ	27.30%	21.79%	43.02%	13.80%	28.99%	17.18%	24.13%	34.09%

Source: skew.com, December 2019

# Hedging cryptos

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  - Tail risks, extreme events?
- ▶ Two directions:
  - Copulas
  - Risk measures

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## Hedging spot with futures

- ▶ Hedge portfolio return:  $R_t^h = R_t^S - h R_t^F$ , where
  - $R_t^S$ : spot return at time  $t$
  - $R_t^F$ : futures return at time  $t$
  - $h$ : hedge ratio
- ▶ Goal: Find optimal hedge ratio  $h^*$
- ▶ Minimum-variance hedge ratio, e.g. Ederington (1979), assumes variance as risk measure and elliptical return distribution
- ▶ Extensions: risk measures, copulas, e.g. (Harris and Shen, 2006; Barbi and Romagnoli, 2014)

# Copulas

## Definition

A (bivariate) **copula** is a distribution function on  $[0, 1]^2$  with standard uniform marginals.

- ▶ Copulas differ only through the dependence between the marginals.
- ▶ Sklar's Theorem (below) captures that copulas allow to separate
  - modelling of the marginals, and
  - modelling of the dependence structure.

# Copulas

## Theorem (Sklar's Theorem)

Let  $F$  be a joint distribution function with margins  $F_1, F_2$ . Then, there exists a copula  $C : [0, 1]^2 \rightarrow [0, 1]$  such that, for all  $x, y \in \mathbb{R}$

$$F(x, y) = C(F_1(x), F_2(y)). \quad (1)$$

If the margins are continuous, then  $C$  is unique; otherwise  $C$  is unique on the range of the margins.

Conversely, if  $C$  is a copula and  $F_1, F_2$  are univariate distribution functions, then the function  $F$  defined by (1) is a joint distribution function with margins  $F_1, F_2$ .

- ▶ Representation of  $C$  in terms of  $F$  and its margins:

$$C(u, v) = F(F_1^{(-1)}(u), F_2^{(-1)}(v)).$$

## Examples of copulas

- ▶ Gaussian, Student  $t$ , NIG factor, Gumbel, Clayton, Frank, Plackett, Gaussian Mixture
- ▶ Add scatterplots here

## Copula-based hedging

### Proposition (Barbi and Romagnoli (2014))

Let  $R^S$  and  $R^F$  be two real-valued random variables with corresponding absolutely continuous copula  $C_{R^S, R^F}$  and continuous marginals  $F_{R^S}$  and  $F_{R^F}$ . Then, the distribution of  $R^h$  is given by

$$F_{R^h}(x) = 1 - \int_0^1 D_1 C_{R^S, R^F} \left\{ u, F_{R^F} \left\{ \frac{F_{R^S}^{-1}(u) - x}{h} \right\} \right\} du. \quad (2)$$

- ▶ Easy to show (e.g. McNeil et al. (2005)):

$$D_1 C_{X, Y}(F_X(x), F_Y(y)) = \frac{\partial}{\partial u} C(u, v) = \mathbf{P}(Y \leq y | X = x).$$

## Risk measures

- ▶ **Variance:**  $\text{Var}(R^h)$
- ▶ **Value-at-risk (VaR):**  $\text{VaR}_\alpha = -F_{R^h}^{(-1)}(1 - \alpha)$
- ▶ **Expected Shortfall (ES):**  $\text{ES}_\alpha = -\frac{1}{1 - \alpha} \int_{-\infty}^{\alpha} F_{R^h}^{(-1)}(p) dp.$

## Risk measures

- ▶ **Spectral risk measures (SRM)** (Acerbi, 2002; Cotter and Dowd, 2006):

$$\rho_\phi = - \int_0^1 \phi(p) F_{R^h}^{(-1)} dp,$$

where  $q_p$  is the  $p$ -quantile of the return distribution and  $\phi(s)$ ,  $s \in [0, 1]$ , is the so-called **risk aversion function**, a weighting function such that

- (i)  $\phi(p) \geq 0$ ,
- (ii)  $\int_0^1 \phi(p) dp = 1$ ,
- (iii)  $\phi'(p) \leq 0$ .

- ▶ SRM's are coherent risk measures.

## Risk measures

- ▶ **Exponential spectral risk measure:** weighting function  $\phi(p) = \lambda e^{-k(1-p)}$ , where  $\lambda$  is an unknown positive constant, derived from exponential utility function:

$$\rho_\phi = \int_0^1 \phi(p) F^{(-1)}(p) dp = \frac{k}{1 - e^{-k}} \int_0^1 e^{-k(1-p)} F^{(-1)}(p) dp.$$

## Optimal hedge ratio

- ▶ Hedge portfolio:  $R_t^h = R_t^S - hR_t^F$ , with  $h$  hedge ratio
- ▶ Optimal hedge ratio:

$$h^* = \operatorname{argmin}_h \rho(h),$$

where  $\rho(h)$  is the risk of the hedge portfolio with hedge ratio  $h$ .

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# Data

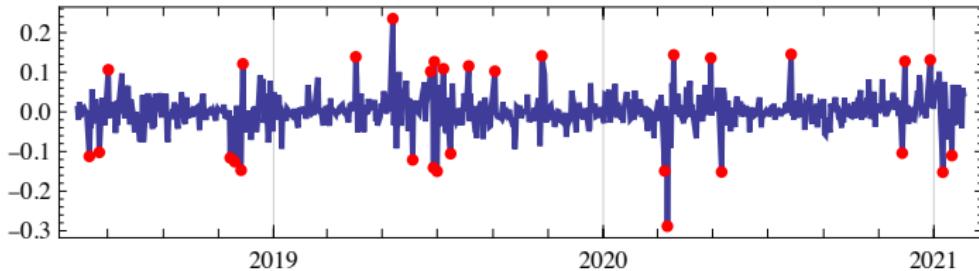
- ▶ Daily log returns, 23pm CET
- ▶ 29 May 2018 through 3 Feb 2021
- ▶ Spot: Coingecko Bitcoin / USDC
- ▶ Future: CME BTC Future

## Summary statistics

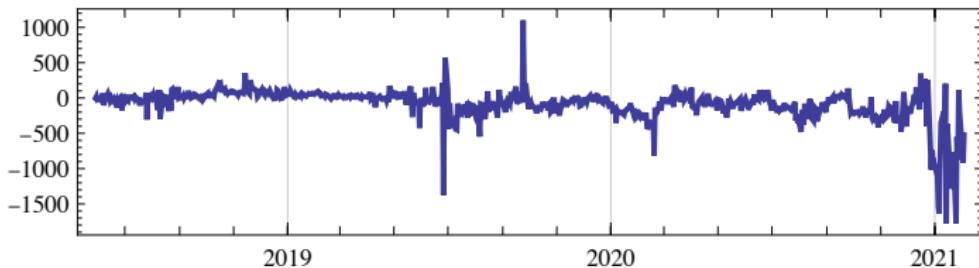
Variable	Mean	Median	Minimum	Maximum
BTC spot	0.0023782	0.0013855	-0.28846	0.23522
Future	0.0023970	0.0012540	-0.26773	0.22251
Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
BTC spot	0.043133	18.137	-0.27245	5.8658
Future	0.046288	19.311	-0.27321	5.4709
Variable	5% perc.	95% perc.	IQ Range	Missing obs.
BTC spot	-0.065328	0.072665	0.033819	0
Future	-0.066085	0.077146	0.037142	0

# Time series

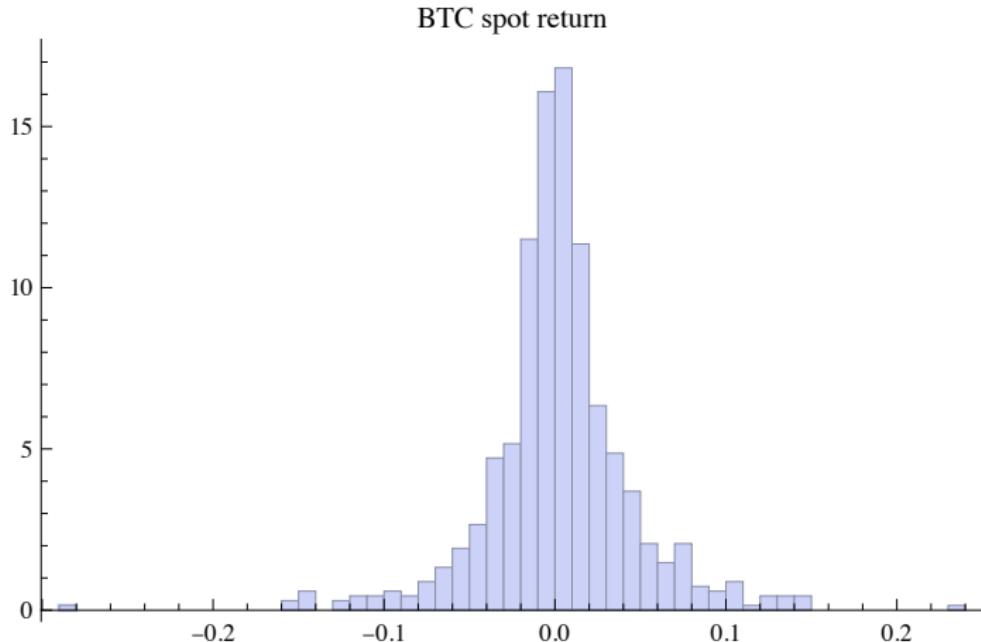
BTC spot returns with 30 most extreme returns



Difference BTC spot and future



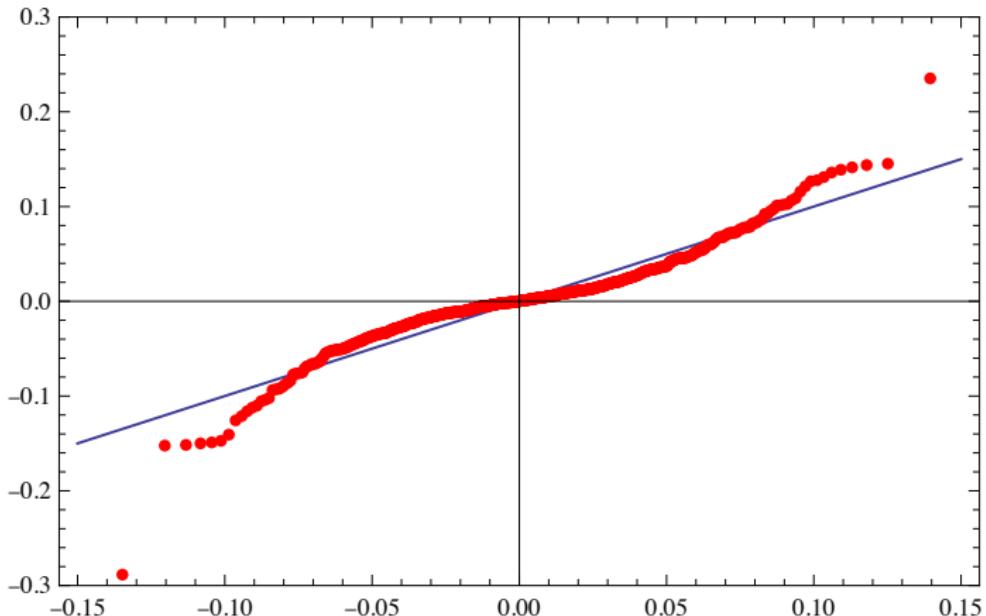
# Distribution



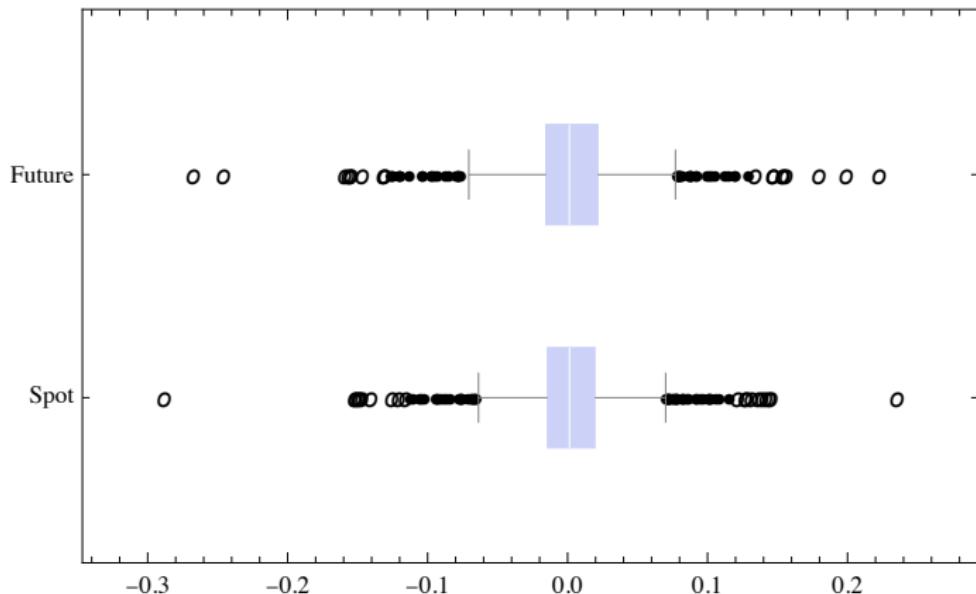
- ▶ Student  $t$  distribution:  $\nu = 7.95$
- ▶ Tail / Generalised Pareto distribution: tail index  $1/\xi = 4.92$

## QQ-plot

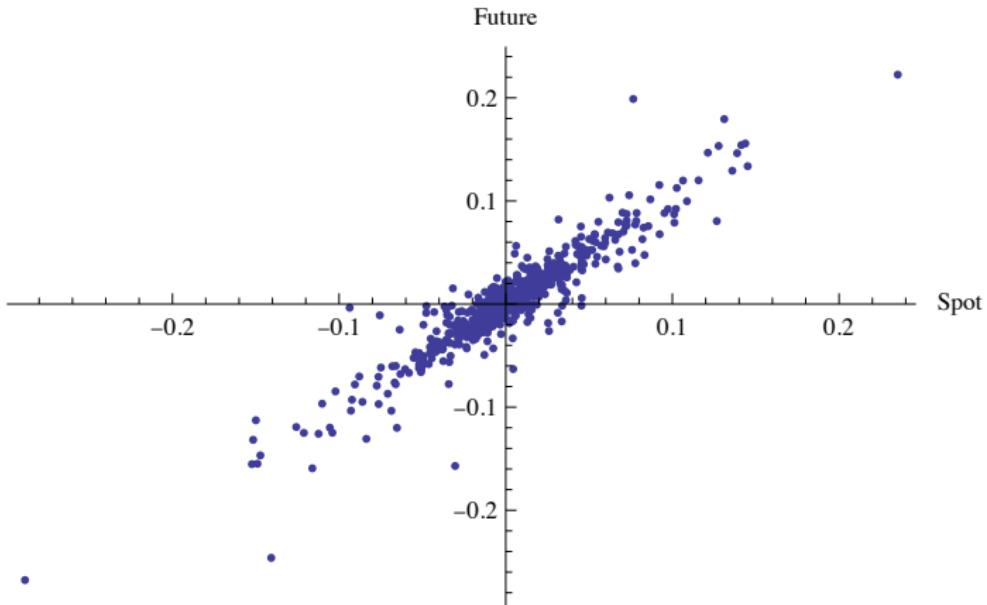
QQ-Plot of BTC returns against normal distribution



## Spot and Future



# Spot and Future



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## Hedge Effectiveness

- ▶ Hedge effectiveness (Ederington, 1979) captures percentage reduction in risk:

$$1 - \frac{\rho(R^h)}{\rho(R^S)}.$$

- ▶ Optimisation of  $h^*$  every 30 days based on 300-day-window.

## Hedge effectiveness

# P&L

## Optimal hedge parameters

## References |

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# Thank you!



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