FCI Fall 2013 Assignment IV. 50 pts.

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Electronic submission on Blackboard is due latest by 11 pm on Monday, Oct 14th. Submissions received after the deadline will be graded only for effort for a maximum of 70% of the total grade (Refer to class syllabus for detailed grading policy). **State any assumptions you make, justify your answers, show intermediate steps and explain your results for maximum credit**. You may leave quantitative answers in the form of expressions unless a numeric value is required to address the question. All answers should be in your own words with any sources you refer to cited at the appropriate places. Any knowledge you acquire from the Internet should be written in your own words and be appropriately referenced. Copying and pasting from the Internet, each other or any other source will not count as your effort (Refer to class syllabus for detailed policy on plagiarism).

You may submit this assignment in groups of two each. Write your names on this sheet and include it as the cover page for your submission. You are encouraged to use MATLAB for this assignment. You may base your code on the samples provided on the textbook website (http://www.dcs.gla.ac.uk/~srogers/firstcourseml/); clearly indicate which sections of your answer are not original.

Q1. (15) Plot a series of beta-density functions that maintain the same ratio of their parameters but with increasing magnitudes.

Q2. (15) Plot a series of beta-density functions that maintain the same magnitude of their parameters but with increasing skewness.

Q3. (10) Create a 3D plot of variance as a function of beta-density parameters (see Eq 3.7)

Q4. (10) Show an example of a beta-density where the probability of an event based on the best estimate of r is markedly different from the expectation of the probability of the same event (see pg 109-110)

Homework 4

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Exercise 4.1:
Here we want to exhibit five figures which keep the same ratio of their parameters but with increasing magnitudes. In order to keep the same ratio, the variables need to keep the same proportion.

Answer obtained:
Figure 01.

Exercise 4.2:
Here we want to exhibit five figures which do not keep the same ratio of their parameters but keep the same magnitudes. In order to diverge the ratio, the variables cannot keep same proportion.

Answer obtained:
Figure 02.

Exercise 4.3:
Here there is a plot of a 3D graph, which coordinates alpha (axis x), beta (axis y) and resulting variance (axis z).

Answer obtained:
Figure 03.

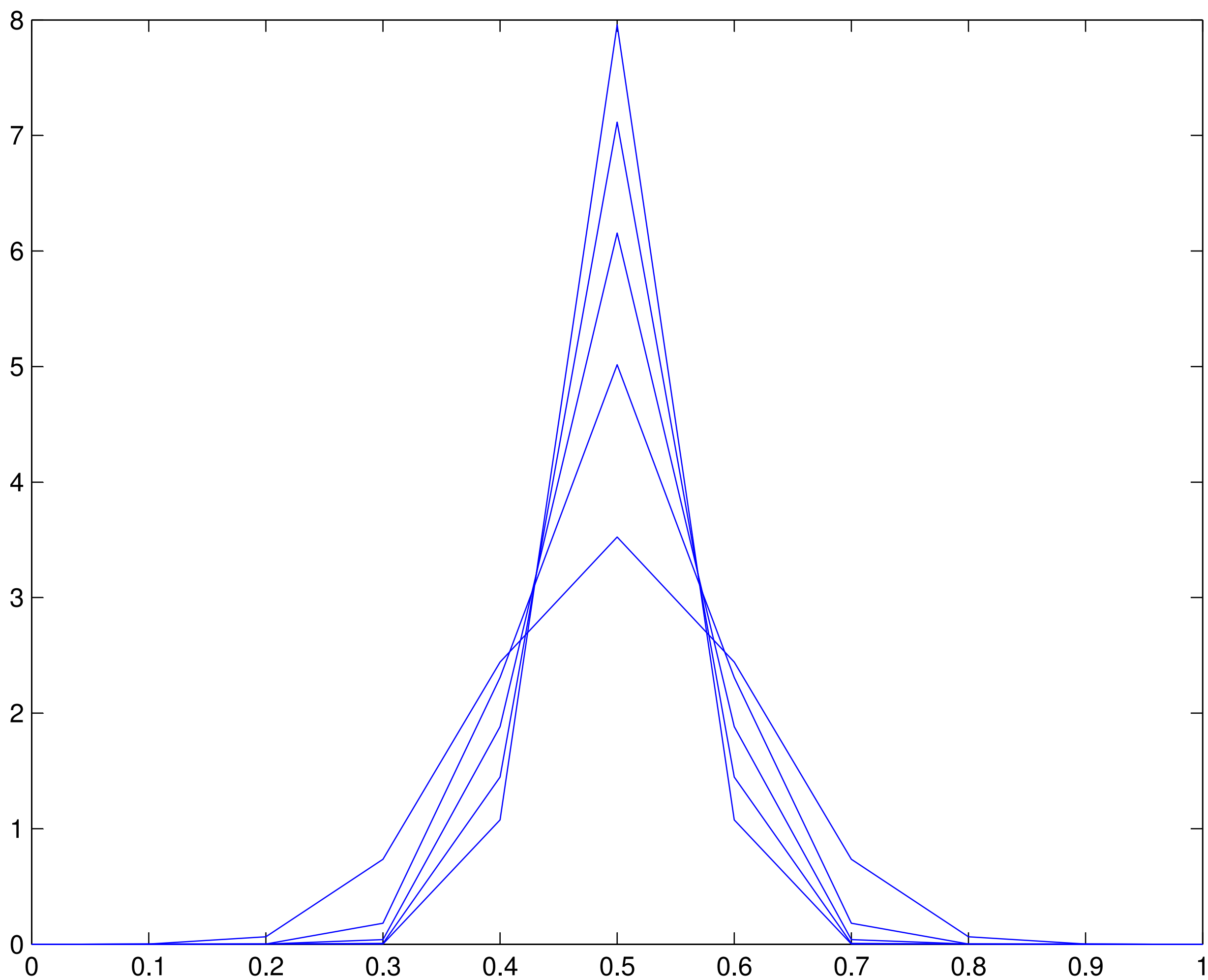
Exercise 4.4:
Here we realize that there is a big difference estimate of r comparing to the expectation of the probability in the first tosses, since we don't know “what to expect”.

Answer obtained:
“for yn = 0 N = 1
r: 0.333333
p: 0.980338
exp: 0.848485
difference:0.131854

for yn = 1 N = 2
r: 0.5
p: 0.828125
exp: 0.685315
difference:0.14281

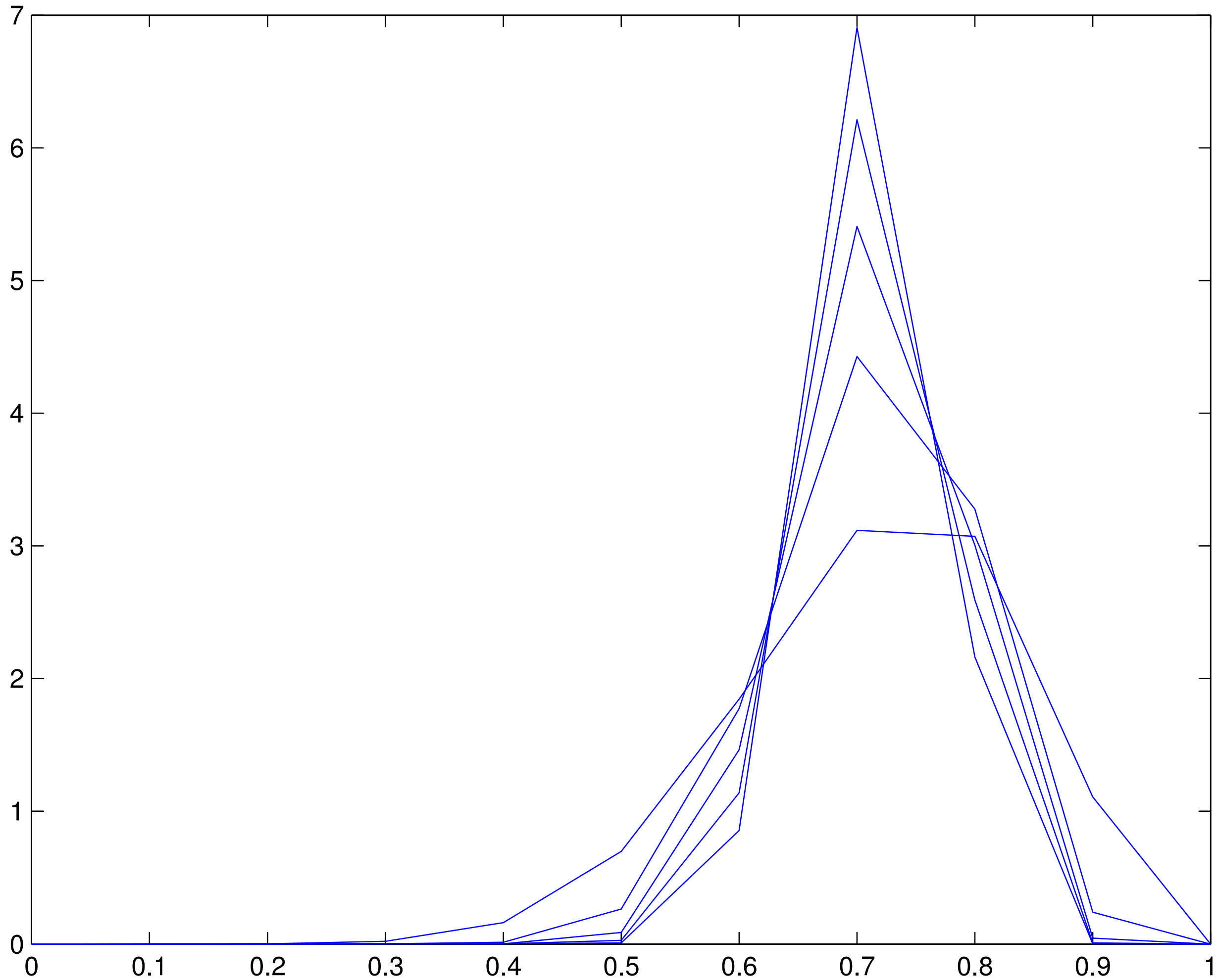
for yn = 2 N = 2
r: 0.75
p: 0.224125
exp: 0.293706
difference:0.0695814”

```
% Homework 4.1
% Francisco Nardi and Paulo Silva

% Cleaning the screen and variables as usual
clear all;
close all;

%it determines the range used
range = [0:0.1:1];

%we want to exhibit five figures which keep the same
%ratio of their parameters but with increasing magnitudes.
for i = 1:5

    %in order to keep the same ratio, the variables
    %need to keep the same proportion
    a = i * 10;
    b = i * 10;

    %here we calculate the first part of the equation, using gamma function
    part1 = ((gamma(a + b))/(gamma(a) * gamma(b)));

    %this is the second part
    part2 = (range .^ (a - 1)) .* ((1 - range) .^ (b - 1));

    %that is the result,
    %part1 multiplied by part2
    result = part1 * part2;
    figure(1)

    %here we plot the beta density in figure 1
    plot(range, result)
    hold on
end
```



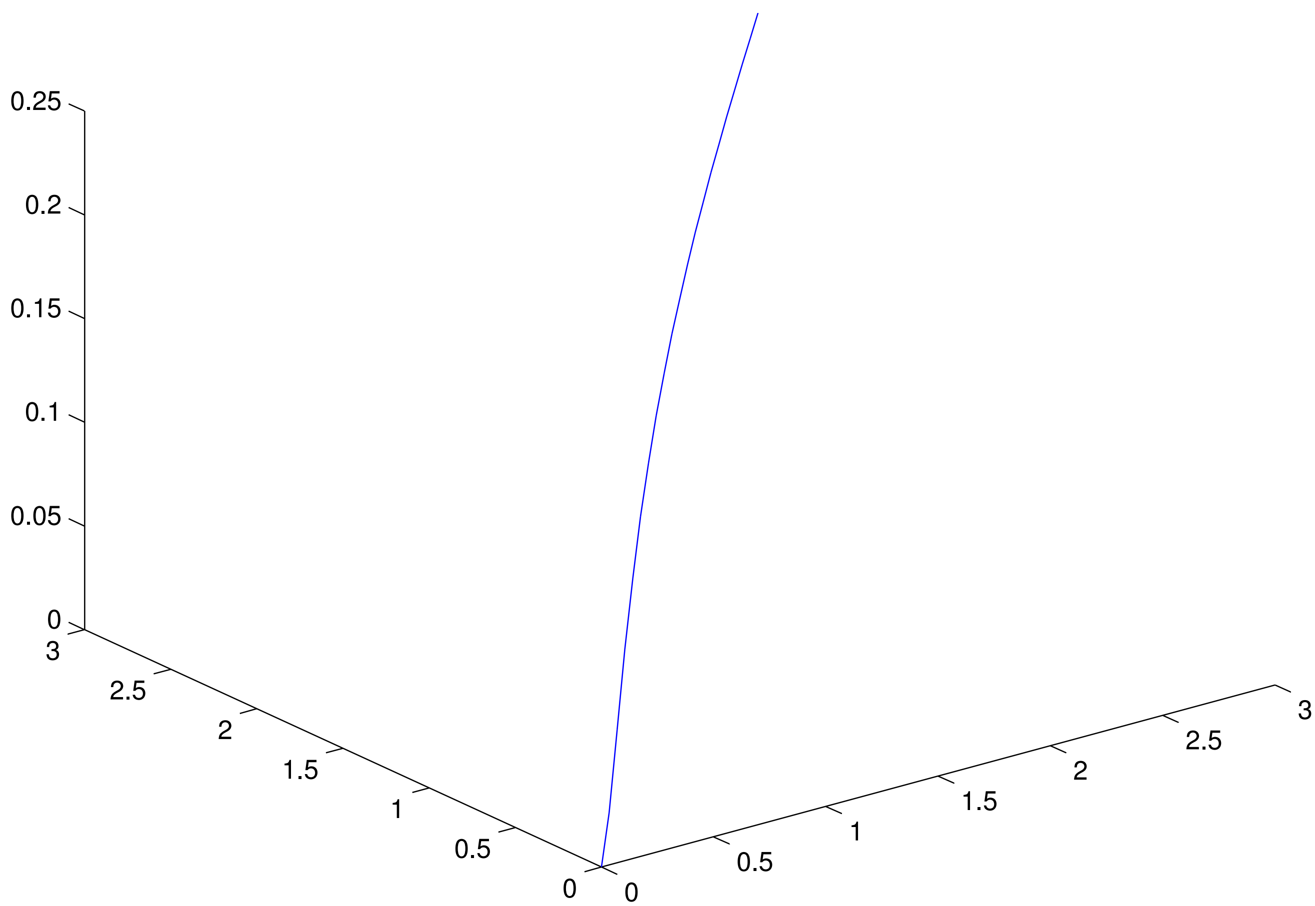
0
1
2
3
4
5
6
7
8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

```
% Homework 4.2
% Francisco Nardi and Paulo Silva

% Cleaning the screen and variables as usual
clear all;
close all;

%it determines the range used
range = [0:0.1:1];

%we want to exhibit five figures which do not keep the same
%ratio of their parameters but keep the same magnitudes.
for i = 1:5

    %in order to diverge the ratio, the variables
    %cannot keep same proportion
    a = i * 10;
    b = i * 4;

    %here we calculate the first part of the equation, using gamma function
    part1 = ((gamma(a + b))/(gamma(a) * gamma(b)));

    %this is the second part
    part2 = (range .^ (a - 1)) .* ((1 - range) .^ (b - 1));

    %that is the result,
    %part1 multiplied by part2
    result = part1 * part2;
    figure(1)

    %here we plot the beta density in figure 1
    plot(range, result)
    hold on
end
```



0
1
2
3
4
5
6
7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

```
% Homework 4.3
% Francisco Nardi and Paulo Silva

% Cleaning the screen and variables as usual
clear all;
close all;

% we need to define a value for alpha
a = [0:0.15:3];

% and for beta
b = [0:0.15:3];

% we calculate the function based on those values
result = (a .* b) ./ (((a + b) .^ 2) + (a + b + 1));

% and come up with the the 3d plot of the variables/result
figure(1)
plot3(a,b,result)
```


0.25
0.2
0.15
0.1
0.05
0
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3

```
% Homework 4.4
% Francisco Nardi and Paulo Silva

% Cleaning the screen and variables as usual
clear all;
close all;

% no prior knowledge
alp = 1;
bet = 1;

% we just have a big difference in the first tosses,
% since we don't know what to expect
yn = 0;
N = 1;
fprintf('for yn = %g N = %g\n',yn,N);
calculatePAndExp(alp,bet,yn,N);

% we just have a big difference in the first tosses,
% since we don't know what to expect
yn = 1;
N = 2;
fprintf('for yn = %g N = %g\n',yn,N);
calculatePAndExp(alp,bet,yn,N);

% we just have a big difference in the first tosses,
% since we don't know what to expect
yn = 2;
N = 2;
fprintf('for yn = %g N = %g\n',yn,N);
calculatePAndExp(alp,bet,yn,N);
```

```
function calculatePAndExp(alp,bet,yn,N)

% using formulas to define sigma and gamma
sig = alp + yn;
gam =  bet + N - yn;

summation = 0;

% using the formula of r

r = (sig)/(sig + gam);

% applying the binomial probability density function
for i = 7:10
    %result of summation for p
    summation = summation + binopdf(i,10,r);
end

% result of p
p = 1 - summation;


% new tosses
Nnew =  10;
summation = 0;

for ynew = 7:10
    % sharing the equation in order to better organization
    first = (factorial(Nnew)/(factorial(ynew)*factorial(Nnew - ynew)));
    second = gamma(sig+gam)/(gamma(sig)*gamma(gam));
    third = (gamma(sig+ynew)*gamma(gam+Nnew-ynew))/gamma(sig+gam+Nnew);

    %result of summation for exp
    summation = summation + (first * second * third);

end

exp = 1 - summation;

difference = abs(p-exp);

fprintf('r: %g\np: %g\nexp: %g\ndifference:%g\n\n',r,p,exp,difference);
```