<u>Approach</u>

I sought to model the system as a three dimensional surface using a 3x6 numpy array where months lay on the x axis, the departments lay on the y-axis and the z-axis are the array values. Array values are calculated as the number of employees available minus number of employees required. Positive values represent an employee surplus relative to requirements and negative values indicate a shortage of employees necessitating training new employees. The cost of understaffing for the system is 20% greater than training, so my implementation seeks to never be understaffed, which is achievable for the given parameters. My model assumes there will not be any extreme hiring requirements. This approach permitted me to flatten or zero the surface for any employment surplus values leaving only valleys or negative values. Negative values indicate a need for additional employees and accordingly the hiring of trainees.

With this approach, my objective is written simply to minimize the number of hires subject to the given constraints and additional constraints to adjust for training times. (My program requires install of PuLP solver which can be done with the following terminal command: *pip install pulp*)

Using a PuLP solver the model produced an optimal solution of hiring trainees as follows.

Department 1: 1 trainee hired in Jan, 2 trainees hired in Feb
Department 2: 1 trainee hired in Jan
Department 3: 4 trainees hired in Jan
Totalling 8 trainees hired which equates to $400,000 cost.

There is no understaffing cost because the existing employees and new hires satisfy the given hours required. I can quickly recognize this solution as optimal by comparing the visual representation of the hiring requirement and optimal solution numpy arrays which are conducive to visual analysis.

Hiring Requirements	Optimal Hires
[0 0 0 0 0 0]	[1 2 0 0 0 0]
[0 0 0 0 0 -1]	[1 0 0 0 0 0]
[-0.3 0 0 0 0 -4]	[4 0 0 0 0 0]

Generally speaking, allowing transfers between departments has the potential to lower training costs because an employee surplus in a particular department can be leveraged to offset employee shortages in other departments in lieu of training new employees. However, for the values provided in this test, there will not be training savings if transfers were permitted because the new hire requirements hits the minimum required of eight to satisfy the mandate that the total number of employees across all departments at the end of the planning should be equal or greater than the number of employees at the beginning of the planning.

Assuming 30% of employees are allowed to work in different departments can be modeled by adding a new integer variable in each department. These three variables would sum to zero and be subject to the ±30% boundary constraint.

Resignations are currently represented as variables within the model so can be adjusted at any point if not known prior to planning.

Parameters

$Departments\,for\,planning,\ dept\ =\ 1,\ 2,\ 3$
$Months\,for\,planning,\ months\ =\ JAN,\ FEB,\ MAR,\ APR,\ MAY,\ JUN$
$Hours\ required\ in\ department\ i\ in\ each\ month,\ Dept_iHR\ = (1900,\ 1700,\ 1600,\ 1900,\ 1500,\ 1800)$
$Number\ of\ employees\ in\ january\ in\ each\ department,\ Emp = (25,\ 20,\ 18)$
$Number\ of\ trainees\ hired\ month\ prior\ to\ planning\ for\ each\ department,\ T_0 = (2,\ 2,\ 2)$
$Resignations\ in\ the\ month\ in\ each\ department,\ 1 : (APR,\ 3),\ 2 : (MAY,\ 5),\ 3 : (JUN,\ 6)$
$Training\ capacity\ for\ any\ month,\ Train_{cap} = 6$
$Hours\ trainee\ can\ work\ per\ month,\ TH\ =\ 35$
$Hours\ employee\ can\ work\ per\ month,\ EH\ =\ 100$
$Trainees\ convert\ into\ an\ employee\ after\ two\ months$
$Cost\ to\ train\ a\ trainee =\ 50000$
$Understaffing\ cost = 60000$

Variables

$T_{ij} = integer\ number\ of\ trainees\ hired\ in\ department\ i\ in\ month\ j$

Objective

$Minimize\ \sum T_{ij}$

Constraints

$\sum T_{ij}\ \geq\ resignations\ - new\ trainees$

$\sum T_{1j} \geq max\ trainees\ required\ in\ the\ department\ for\ the\ planning\ period,\ where\ the\ limit\ of\ j\ is\ two\ months\ before\ max$

$\sum T_{2j} \geq max\ trainees\ required\ in\ the\ department\ for\ the\ planning\ period,\ where\ the\ limit\ of\ j\ is\ two\ months\ before\ max$

$\sum T_{3j} \geq max\ trainees\ required\ in\ the\ department\ for\ the\ planning\ period,\ where\ the\ limit\ of\ j\ is\ two\ months\ before\ max$

$\sum T_{i1} - \sum T_{i2}\ \geq\ 0$

$\sum T_{i2} - \sum T_{i3}\ \geq\ 0$

$\sum T_{i3} - \sum T_{i4}\ \geq\ 0$

$\sum T_{i4} - \sum T_{i5}\ \geq\ 0$

$\sum T_{i5} - \sum T_{i6}\ \geq\ 0$

$T_{1j} + T_{2j} + T_{3j} \leq Train_{cap}$

Python Program Output

```
Employee_Training_Planning_Problem: MINIMIZE
1*T_11 + 1*T_12 + 1*T_13 + 1*T_14 + 1*T_15 + 1*T_16 + 1*T_21 + 1*T_22 + 1*T_23 + 1*T_24 + 1*T_25 + 1*T_26 + 1*T_31 +
1*T_32 + 1*T_33 + 1*T_34 + 1*T_35 + 1*T_36 + 0
SUBJECT TO
_C1: T_11 + T_12 + T_13 + T_14 + T_15 + T_16 + T_21 + T_22 + T_23 + T_24 + T_25 + T_26 + T_31 + T_32 + T_33 + T_34 +
T_35 + T_36 >= 8
_C2: T_11 >= 0
_C3: T_21 + T_22 + T_23 >= 1
_C4: T_31 + T_32 + T_33 >= 4
_C5: T_11 - T_12 + T_21 - T_22 + T_31 - T_32 >= 0
_C6: T_12 - T_13 + T_22 - T_23 + T_32 - T_33 >= 0
_C7: T_13 - T_14 + T_23 - T_24 + T_33 - T_34 >= 0
_C8: T_14 - T_15 + T_24 - T_25 + T_34 - T_35 >= 0
_C9: T_15 - T_16 + T_25 - T_26 + T_35 - T_36 >= 0
_C10: T_11 + T_21 + T_31 <= 6
_C11: T_12 + T_22 + T_32 <= 6
_C12: T_13 + T_23 + T_33 <= 6
_C13: T_14 + T_24 + T_34 <= 6
_C14: T_15 + T_25 + T_35 <= 6
_C15: T_16 + T_26 + T_36 <= 6

VARIABLES
0 <= T_11 Integer
0 <= T_12 Integer
0 <= T_13 Integer
0 <= T_14 Integer
0 <= T_15 Integer
0 <= T_16 Integer
0 <= T_21 Integer
0 <= T_22 Integer
0 <= T_23 Integer
0 <= T_24 Integer
0 <= T_25 Integer
0 <= T_26 Integer
0 <= T_31 Integer
0 <= T_32 Integer
0 <= T_33 Integer
0 <= T_34 Integer
0 <= T_35 Integer
0 <= T_36 Integer

Status: Optimal
T_11 = 1.0
T_12 = 2.0
T_13 = 0.0
T_14 = 0.0
T_15 = 0.0
T_16 = 0.0
T_21 = 1.0
T_22 = 0.0
T_23 = 0.0
T_24 = 0.0
T_25 = 0.0
T_26 = 0.0
T_31 = 4.0
T_32 = 0.0
T_33 = 0.0
T_34 = 0.0
T_35 = 0.0
T_36 = 0.0
Total cost = $ 400000
```