Quasigeostrophic fluids and resonant interactions

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- Second order system

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- $\frac{d\vec{p}}{dt} = -\frac{\partial H}{\partial \vec{q}}, \frac{d\vec{q}}{dt} = \frac{\partial \mathcal{H}}{\partial \vec{p}}$
- First order, coupled system

Fluids

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Navier-Stokes Equations

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- $\rho \frac{d\vec{u}}{dt} = -\nabla p + \rho \nabla \Pi + F$
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Hamiltonian Fluid Dynamics

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System of PDEs

$$\mathbf{0} = F\left(\mathbf{q}, \frac{\partial}{\partial x_i}, \frac{\partial}{\partial t}\right)$$

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Hamiltonian

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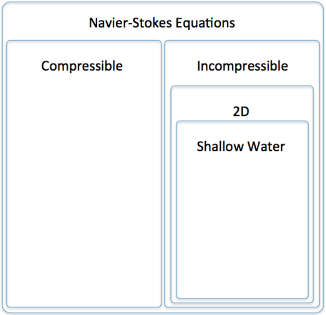
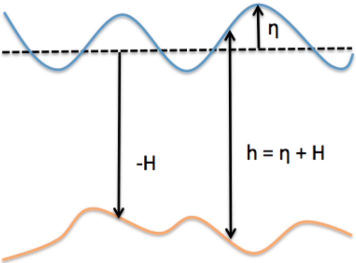
Canonical equations

$$\mathbf{q}_t = \mathbf{J}\frac{\delta\mathcal{H}}{\delta\mathbf{q}}$$

Shallow Water Model



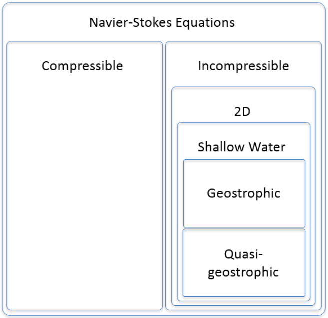
Shallow Water Model



$$0 = \nabla \cdot \vec{u}$$

$$0 = \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv)$$

Geostrophic Model



Geostropic Model

$$\frac{d\vec{u}}{dt} + 2\vec{\Omega} \times \vec{u} = -\frac{\nabla p}{\rho} + \nabla \Pi + \frac{F}{\rho}$$

$$\begin{aligned} \vec{u} &= \text{velocity field} \\ \vec{\Omega} &= \text{rotation vector} \\ p &= \text{pressure} \\ \Pi &= \text{scalar potential field} \\ F &= \text{viscous forces} \end{aligned}$$

Quasigeostrophic Model

$$
\begin{aligned}
0 &= \frac{\partial q}{\partial t} + J(\psi, q) \\
J(a, b) &= \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}
\end{aligned}
$$

References

1. Reference