Quasigeostrophic fluids and resonant interactions

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Contents

Classical Mechanics

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- Newtonian Dynamics

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- Solve second order system

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- Solve second order system

- Hamiltonian Dynamics
- $\frac{d\vec{p}}{dt} = -\frac{\partial\mathcal{H}}{\partial\vec{q}}$
- $\frac{d\vec{q}}{dt} = \frac{\partial\mathcal{H}}{\partial\vec{p}}$
- Often first order, coupled system

Fluids

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- Navier-Stokes Equations

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- $\rho \frac{d\vec{u}}{dt} = -\nabla p + \rho \nabla \Pi + F$
- Solve system of coupled, non-linear, PDEs

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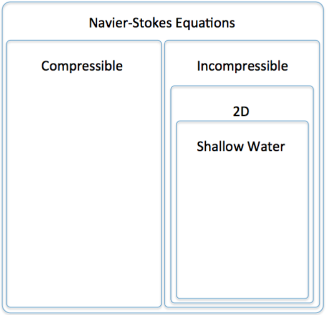
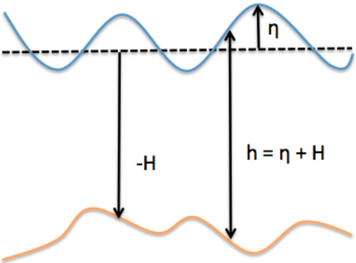
Shallow Water Model



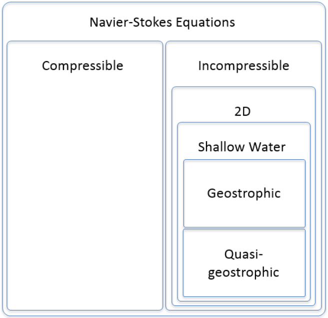
Shallow Water Model



$$0 = \nabla \cdot \vec{u}$$

$$0 = \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv)$$

Geostrophic Model



Geostropic Model

$$\frac{d\vec{u}}{dt} + 2\vec{\Omega} \times \vec{u} = -\frac{\nabla p}{\rho} + \nabla \Pi + \frac{F}{\rho}$$

$$\vec{u} = \text{velocity field}$$

$$\vec{\Omega} = \text{rotation vector}$$

$$p = \text{pressure}$$

$$\Pi = \text{scalar potential field}$$

$$F = \text{viscous forces}$$

Quasigeostrophic Model

$$0 = \frac{\partial q}{\partial t} + J(\psi, q)$$
$$J(a, b) = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$$

References

1. Reference