



Quasigeostrophic fluids and resonant interactions

James Hawley

University of Waterloo

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Classical Mechanics



Classical Mechanics

- Newtonian Dynamics



Classical Mechanics

- Newtonian Dynamics
- Solve system of ODEs or PDEs
- $\frac{d\vec{p}}{dt} = -\nabla\Pi$



Classical Mechanics

- Newtonian Dynamics
- Solve system of ODEs or PDEs
- $\frac{d\vec{p}}{dt} = -\nabla\Pi$
- Navier-Stokes Equations

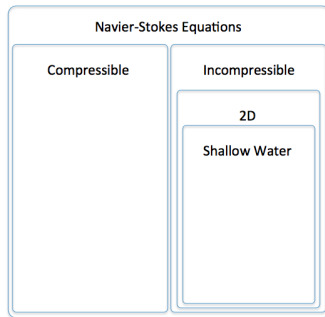


Classical Mechanics

- Newtonian Dynamics
- Solve system of ODEs or PDEs
- $\frac{d\vec{p}}{dt} = -\nabla\Pi$
- Navier-Stokes Equations
- Solve system of coupled, non-linear, PDEs
- $\rho \frac{d\vec{u}}{dt} = -\nabla p + \rho \nabla \Pi + F$

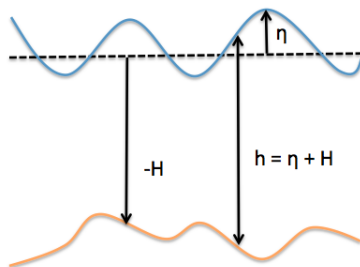


Shallow Water Model





Shallow Water Model

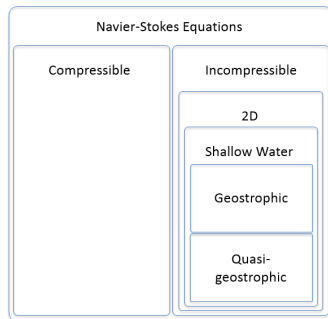


$$0 = \nabla \cdot \vec{u}$$

$$0 = \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv)$$



Geostrophic Model





Geostrophic Model

$$\frac{d\vec{u}}{dt} + 2\vec{\Omega} \times \vec{u} = -\frac{\nabla p}{\rho} + \nabla \Pi + \frac{F}{\rho}$$

\vec{u} = velocity field

$\vec{\Omega}$ = rotation vector

p = pressure

Π = scalar potential field

F = viscous forces



Quasigeostrophic Model

$$0 = \frac{\partial q}{\partial t} + J(\psi, q)$$

$$J(a, b) = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x}$$



References

1. Reference