x-independence:

$$\phi = \phi(y,t), \psi = \psi(y,t)$$

Green's first identity:

$$\int \psi \nabla^2 \phi + \nabla\psi \cdot \nabla\phi \, dV = \oint \psi \nabla\phi \cdot d\vec{S}$$

Channel BC's:

$$\begin{aligned}
\oint \psi \nabla\phi \cdot d\vec{S} &= \lim_{X\to\infty} \frac{1}{2X} \int_{-X}^{X} \psi(0,t)\phi_x(0,t)\,dx + \int_0^L \psi(y,t)\phi_y(y,t)\,dy \\
&\quad + \int_X^{-X} \psi(L,t)\phi_x(L,t)\,dx + \int_L^0 \psi(y,t)\phi_y(y,t)\,dy \\
&= \lim_{X\to\infty} \frac{1}{2X} \int_{-X}^{X} \psi(0,t)\phi_x(0,t)\,dx + \int_X^{-X} \psi(L,t)\phi_x(L,t)\,dx \\
&= \lim_{X\to\infty} \frac{1}{2X} \int_{-X}^{X} \psi(0,t)\phi_x(0,t) - \psi(L,t)\phi_x(L,t)\,dx \\
&= \psi(y,t)\phi_x(y,t)\big|_{y=L}^{0}
\end{aligned}$$

which is a function of time.

What I get by integrating the QG equation is

$$0 = \partial_t \left[\frac{1}{2} \iint \nabla\psi \cdot \nabla\psi + F\psi^2 \, dA + \frac{1}{2}\, \psi^2(y,t)\big|_{y=0}^{L} \right]$$

Am I able to neglect the last term? I know I can set ψ on one of the boundaries to be 0 at a certain time, but can I do it for all time? And if so, can I set ψ on the other boundary to be a constant for all time, too? If I can, then that last term is just a constant and can be neglected, which gives us the exact same Hamiltonian as the doubly periodic BC's, which will give the same stability criteria.

1 Zonal channel Boundary Conditions

$$
\begin{aligned}
&\oint \psi_i \vec{\nabla} \psi_i \cdot d\vec{S} \\
&= \lim_{X\to\infty} \frac{1}{2X} \oint_S \psi_i(y,t)\psi_{iy}(y,t)\,dy + \psi_i(y,t)\psi_{ix}(y,t)\,dx \\
&= \lim_{X\to\infty} \frac{1}{2X} \oint_S \psi_i(y,t)\psi_{iy}(y,t)\,dy \\
&= \lim_{X\to\infty} \frac{1}{2X} \left[\int_0^L \psi_i(y,t)\psi + iy(y,t)\,dy + \int_0^L \psi_i(y,t)\psi + iy(y,t)\,dy \right] \quad = 0
\end{aligned}
$$

2 D

oubly Periodic Boundary Conditions