

x -independence:

$$\phi = \phi(y, t), \psi = \psi(y, t)$$

Green's first identity:

$$\int \psi \nabla^2 \phi + \nabla \psi \cdot \nabla \phi dV = \oint \psi \nabla \phi \cdot d\vec{S}$$

Channel BC's:

$$\begin{aligned} \oint \psi \nabla \phi \cdot d\vec{S} &= \lim_{X \rightarrow \infty} \frac{1}{2X} \int_{-X}^X \psi(0, t) \phi_x(0, t) dx + \int_0^L \psi(y, t) \phi_y(y, t) dy \\ &\quad + \int_X^{-X} \psi(L, t) \phi_x(L, t) dx + \int_L^0 \psi(y, t) \phi_y(y, t) dy \\ &= \lim_{X \rightarrow \infty} \frac{1}{2X} \int_{-X}^X \psi(0, t) \phi_x(0, t) dx + \int_X^{-X} \psi(L, t) \phi_x(L, t) dx \\ &= \lim_{X \rightarrow \infty} \frac{1}{2X} \int_{-X}^X \psi(0, t) \phi_x(0, t) - \psi(L, t) \phi_x(L, t) dx \\ &= \psi(y, t) \phi_x(y, t) \Big|_{y=L}^0 \end{aligned}$$

which is a function of time.

What I get by integrating the QG equation is

$$0 = \partial_t \left[\frac{1}{2} \iint \nabla \psi \cdot \nabla \psi + F \psi^2 dA + \frac{1}{2} \psi^2(y, t) \Big|_{y=0}^L \right]$$

Am I able to neglect the last term? I know I can set ψ on one of the boundaries to be 0 at a certain time, but can I do it for all time? And if so, can I set ψ on the other boundary to be a constant for all time, too? If I can, then that last term is just a constant and can be neglected, which gives us the exact same Hamiltonian as the doubly periodic BC's, which will give the same stability criteria.