

1 Constant Zonal Flow

1.1 Basic States

$$\begin{aligned}\Psi_1 &= -U_1 y & \Psi_2 &= -U_2 y \\ Q_1 &= (U_1 - U_2 + \beta)y & Q_2 &= (U_2 - U_1 + \beta)y\end{aligned}$$

1.2 New States and Perturbations

$$\begin{aligned}\psi_1 &= \Psi_1 + \psi'_1 & \psi_2 &= \Psi_2 + \psi'_2 \\ q'_1 &= \nabla^2 \psi'_1 - F_1(\psi'_1 - \psi'_2) & q'_2 &= \nabla^2 \psi'_2 - F_2(\psi'_2 - \psi'_1)\end{aligned}$$

1.3 QG Equations

$$\begin{aligned}0 &= \partial_t q'_1 + (u'_1 + U_1)\partial_x q'_1 + v'_1 \partial_y (q'_1 + Q_1) \\ 0 &= \partial_t q'_2 + (u'_2 + U_2)\partial_x q'_2 + v'_2 \partial_y (q'_2 + Q_2)\end{aligned}$$

1.4 Normal Modes

$$\begin{aligned}\hat{q}'_1 &= -\hat{\psi}'_1(F_1 + K^2) + F_1 \hat{\psi}'_2 & \hat{q}'_2 &= F_2 \hat{\psi}'_1 - \hat{\psi}'_2(F_2 + K^2) \\ \hat{\psi}'_1 &= \frac{(F_2 + K^2)\hat{q}'_1 + F_1 \hat{q}'_2}{F_1 F_2 - (F_1 + K^2)(F_2 + K^2)} & \hat{\psi}'_2 &= \frac{F_2 \hat{q}'_1 + (F_1 + K^2)\hat{q}'_2}{F_1 F_2 - (F_1 + K^2)(F_2 + K^2)} \\ \hat{u}'_1 &= -ik \hat{\psi}'_1 & \hat{u}'_2 &= -ik \hat{\psi}'_2 \\ \hat{u}'_1 &= -il \hat{\psi}'_1 & \hat{u}'_2 &= -il \hat{\psi}'_2 \\ K^2 &= k^2 + l^2\end{aligned}$$

2 x -independent Zonal Flow

2.1 Basic States

$$\begin{aligned}\Psi_1 &= \Psi_1(y) & \Psi_2 &= \Psi_2(y) \\ Q_1 &= \partial_{yy}\Psi_1 - F_1(\Psi_1 - \Psi_2) + \beta y & Q_2 &= \partial_{yy}\Psi_2 - F_2(\Psi_2 - \Psi_1) + \beta y\end{aligned}$$

2.2 New States and Perturbations

$$\begin{aligned}\psi_1 &= \Psi_1 + \psi'_1 & \psi_2 &= \Psi_2 + \psi'_2 \\ q'_1 &= \nabla^2\psi'_1 - F_1(\psi'_1 - \psi'_2) & q'_2 &= \nabla^2\psi'_2 - F_2(\psi'_2 - \psi'_1)\end{aligned}$$

2.3 QG Equations

$$\begin{aligned}0 &= \partial_t q'_1 + (u'_1 + U_1)\partial_x q'_1 + v'_1\partial_y(q'_1 + Q_1) \\ 0 &= \partial_t q'_2 + (u'_2 + U_2)\partial_x q'_2 + v'_2\partial_y(q'_2 + Q_2)\end{aligned}$$

2.4 Normal Modes

$$\begin{aligned}\hat{q}'_1 &= -\hat{\psi}'_1(F_1 + K^2) + F_1\hat{\psi}'_2 & \hat{q}'_2 &= F_2\hat{\psi}'_1 - \hat{\psi}'_2(F_2 + K^2) \\ \hat{\psi}'_1 &= \frac{(F_2 + K^2)\hat{q}'_1 + F_1\hat{q}'_2}{F_1F_2 - (F_1 + K^2)(F_2 + K^2)} & \hat{\psi}'_2 &= \frac{F_2\hat{q}'_1 + (F_1 + K^2)\hat{q}'_2}{F_1F_2 - (F_1 + K^2)(F_2 + K^2)} \\ \hat{u}'_1 &= -ik\hat{\psi}'_1 & \hat{u}'_2 &= -ik\hat{\psi}'_2 \\ \hat{u}'_1 &= -il\hat{\psi}'_1 & \hat{u}'_2 &= -il\hat{\psi}'_2 \\ K^2 &= k^2 + l^2\end{aligned}$$