Drag Force Explanation
(FTSIM)

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In order to calculate the drag force on an individual particle we modify the equation (Greeley and Iverson 1985) (Shao 2008),

$$F_d = C_d \rho_a D_p^2 u_*^2, \tag{1}$$

in which drag force F_d is proportional to the the square of the wind shear velocity u_*, the square of the particle diameter D_p (representing area), the density of air ρ_a, and a drag coefficient C_d. However this equation is for understanding drag on a large scale, a more apropriote equation for an individual sand grain is one that uses the wind speed as opposed to the shear velocity, and the area exposed to the wind instead of the square diameter. Thus we modify equation (1) to be,

$$F_d = \rho_a C_d A_{exp} u^2. \tag{2}$$

Here we assume a left to right wind speed in only the positive x direction. In order for particles to have a component of area perpindicular to the wind, as is nessesary for drag force, the particles must posses projections of surface area in the yz plane greather than zero. Thus the particles are assumed spherical for the drag force calculation, with A_{exp} being the projection of the spherical particle on the yz plane, as is convention. In addition A_{exp} is calculated only for the particle's area that sticks into the flow, which is simplified as the area above the line $z = z_{bot}$, z_{bot} being the z coordinate of the point of contact between a particle and it's neighbor upstream (lift point), see figure (1). z_{bot} could also be the average height of the top row of particles (where $u = 0$) if it is higher than the lift point, because below that height there is no drag force.

Figure 1: yz Cross Section of Particle Protruding Into the Flow

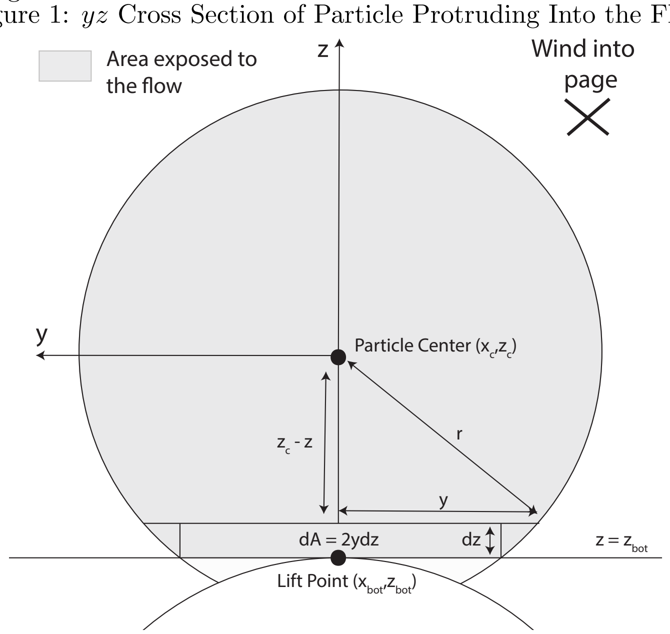

Figure 1: The imagined cross sectional area of the particle in the yz plane, perpindicular to the wind along the x axis. r is the radius of the circle, z is the variable vertical coordinate integrated over, z_c is the z coordinate of the center of the particle, and the differentials dA and dz are explained in the subsequent paragraph in accordance with equation (3) and (4)

It is important to note that the wind speed will vary across the area exposed, so that an integral is physically apropriote for this product. Thus we can rewrite equation (2) in terms of an infinitesimal area, dA, being multiplied by a perpindicular wind speed and summed over all the products,

$$F_d = \rho_a C_d \int_{z_{bot}}^{z_{top}} u^2 dA, \tag{3}$$

where $\int_{z_{bot}}^{z_{top}} dA$ would be the area of the particle protruding into the flow, with z_{top} being the z coordinate of the heighest point on the particle. From figure (1), a cross section it can be seen that $dA = 2ydz$, and $y = \sqrt{r^2 - (z_c - z)^2}$. Thus equation (3) can be rewritten,

$$F_d = 2\rho_a C_d \int_{z_{bot}}^{z_{top}} u^2 \sqrt{r^2 - (z_c - z)^2} dz, \tag{4}$$

A simplification is made in assuming the wind profile is logrithmic with height from the average height of the top particles ($z = 0$) upward. Taking wind speed from the law of the wall,

$$u(z) = \left(\frac{u_*}{\kappa}\right) \ln\left(\frac{z}{z_0}\right),$$

we can rewrite equation (4) as,

$$F_d = 2\rho_a C_d \left(\frac{u_*}{k}\right)^2 \int_{z_{bot}}^{z_{top}} \ln^2\left(\frac{z}{z_0}\right) \sqrt{r^2 - (z_c - z)^2} dz. \tag{5}$$

The integral in equation (5) is difficult to solve analyitically, so it is instead numericaly integrated with Matlab's global adaptive quadrature using default boundary tolerances.

Solving for the Fluid Threshold Shear Velocity

Assuming a moment balance of only gravity and drag forces,

$$F_g r_g = F_d r_d, \tag{6}$$

and a gravity force of,

$$F_g = \frac{\pi}{6}(\rho_p - \rho_a) g D_p^3, \tag{7}$$

the fluid threshold shear veloicty u_{*ft} can be solved for as the shear velocity that makes this moment balance true, i.e. when the lift forces are equal to the grounding forces. Substituting equations (5) and (7) into (6) and solving for u_{*ft} yeilds,

$$u_{*ft} = \left[\frac{r_g}{r_d} \frac{\pi(\rho_p - \rho_a)}{12\rho_a} \frac{g {D_p}^3 \kappa^2}{C_d} \left[\int_{z_{bot}}^{z_{top}} \ln^2\left(\frac{z}{z_0}\right) \sqrt{r^2 - (z_c - z)^2} dz \right]^{-1} \right]^{\frac{1}{2}}. \tag{8}$$