Game Theory: Homework 5

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Question 1

a

Every bidder submits a concealed bid to the auctioneer who then awards the item for sale to the highest bidder. This bidder then pays a price equal to the third-highest bid submitted.

b & c

A good strategy consists in submitting the highest bid as long as the third-highest bid is still less or equal to your true valuation. Differently from the best strategy for the second-price auction playing optimally thus involves reasoning about the other players' behaviour.
Moreover, truth-telling is no longer incentivized as it was in the second-price auction and strategizing becomes now possible. To see this consider an auction with three bidders: i, j and h where the true valuations are such that $v_j > v_i > v_h$. Say, j and h submit truthful bids, so that $\hat{v_j} = v_j$ and $\hat{v_h} = v_h$. Then i could profit from offering some bid $\hat{v_i} > \hat{v_j} > v_i$ and not just report her true valuation. Although i is not playing truthfully, for $\hat{v_i} \neq v_i$, her utility will be $v_i - \hat{v_h} > 0$, thus strictly larger than the payoff of zero she would have gotten for playing $\hat{v_i} = v_i$.
Therefore, a bidder's best bid depends on what the other players' true valuations are. This, in turn, implies that there is no dominant strategy.

d

When trying to maximize their utility bidders can submit bids in such a way as to incur a cost.
Consider the following example. Suppose player i anticipates mistakenly that her true valuation is higher than h's but lower than j and thinks correctly that h and j will bid truthfully. In reality the bid truthfully, however, $v_j > v_h > v_j$. Now, i will play $\hat{v_i} > \hat{v_i} = v_i$ which indeed garantuees her the win, yet since $\hat{v_h} = v_h > v_i$ she loses $v_h - v_i$ instead of winning the auction for a price below her valuation.
In such a situation it would be beneficial for i as well as for j to award the auctioned item to j for a price $p = v_h$. This would cancel i's loss, thus leaving her better off, and give j a payoff of $v_j - v_h$ which is an increase from the 0 she got before.
In brief, the third-price sealed bid auction is not Pareto efficient in the sense described.

Question 2

We have to define the elements of the tuple $< N, \boldsymbol{A}, \boldsymbol{\Theta}, p, \boldsymbol{u} >$.

$N = \{1,2\}$
$\boldsymbol{A} = A_1 \times A_2$, where $A_1 = A_2 = \mathbb{R}_{\geq 0}$. Note that this makes $\boldsymbol{A}$ infinite.
$\boldsymbol{\Theta} = \Theta_1 \times \Theta_2$, where $\Theta_1 = \Theta_2 = \mathbb{N} \cap [101, 200]$
$p : \boldsymbol{\Theta} \to [0,1]$ is a constant function assigning $p(\theta) = \frac{1}{|\boldsymbol{\Theta}|}$ to all $\theta \in \boldsymbol{\Theta}$
$\boldsymbol{u} = (u_1, u_2)$
$u_1 : \boldsymbol{A} \times \boldsymbol{\Theta} \to \mathbb{R}$, eg. $((a_1, a_2), (\theta_1, \theta_2)) \to r \in \mathbb{R}_{\geq 0}$
Concretely, with strict ordering of bidders' offers (no ties):
For some action profile $(a_1, a_2) \in \boldsymbol{A}$ and some type profile $(\theta_1, \theta_2) \in \boldsymbol{\Theta}$,

$$u_1 = \begin{cases} \text{if } a_1 > a_2 \text{ then } \theta_1 - a_1 \\ \text{otherwise } 0 \end{cases}$$

$$u_2 = \begin{cases} \text{if } a_2 > a_1 \text{ then } \theta_2 - a_2 \\ \text{otherwise } 0 \end{cases}$$

3

The mechanism described is not incentive-compatible. To see this consider the following auction with $n = 4$ and $k = 3$. The first table represents a situation where all bidders submit thruthful bids. Compare it with the second table where all the valuations remain as before and Bidder 1 obtains an advantage by submitting a false valuation. She manages to improve her payoff from 0 to 5. This goes to show that bidding thruthfully is not a dominant strategy.

Table 1: Situation with True Valuations

N	values:$(\alpha_1, \alpha_2, \alpha_3)$	bids	gets	pays	δ
1	$(8,4,2)$	8	-	0	0
2	$(32,16,8)$	32	α_2	$\frac{1}{2^1} * 12 = 6$	26
3	$(12,6,3)$	12	α_3	$\frac{1}{2^2} * 8 = 2$	10
4	$(60,30,15)$	60	α_1	$\frac{1}{2^0} * 32 = 32$	28

Table 2: Situation where Bidder 1 lies

N	values:$(\alpha_1, \alpha_2, \alpha_3)$	bids	gets	pays	δ
1	$(8,4,2)$	13	α_3	$\frac{1}{2^2} * 12 = 3$	5
2	$(32,16,8)$	32	α_2	$\frac{1}{2^1} * 13 = 6.5$	25.5
3	$(12,6,3)$	12	-	0	0
4	$(60,30,15)$	60	α_1	$\frac{1}{2^0} * 32$	28