Facility Location Problem - 5G tower placement in The Netherlands

$$\begin{aligned}
\text{minimize} \quad & \sum_{j=1}^{m} f_j y_j + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \\
\text{subject to:} \quad & \sum_{j=1}^{m} x_{ij} = d_i && \text{for } i = 1, \cdots, n \\
& \sum_{i=1}^{n} x_{ij} \le M_j y_j && \text{for } j = 1, \cdots, m \\
& x_{ij} \le d_i y_j && \text{for } i = 1, \cdots, n; j = 1, \cdots, m \\
& x_{ij} \ge 0 && \text{for } i = 1, \cdots, n; j = 1, \cdots, m \\
& y_j \in \{0, 1\} && \text{for } j = 1, \cdots, m
\end{aligned}$$

f - cost of building tower j

y - boolean; tower j built or not

c - cost of transporting 1unit from i to j

n - number of cities

x - units from tower i to city j

d - demand of city i

M - supply of tower j

m - number of (potential) towers

1. the problem was solved by solving the linear programming equation using Excel Solver
2. the relevant variables were:
    - the boolean y_j variables are indicators of whether the 5G tower is built or not
    - X_iP_j (initially described as X_{ij}) is the amount of units of 5G signal “transported from tower J to city X)
3. the relevant constants are:
    - the cost matrix, constructed by proportional factors corresponding to the distance from the facilities to the cities (C_{ij} = distance in meters/1000 rounded up) in a case of distance more than 50km, the cost was always rounded down to 50, since:
        - 1. the cities would never reach a site located further away at a sensible cost in various simulations
        - 2. in such a particular case they would connect to a nearby city that already has signal provided, which would end up in multiple different scenarios that were significantly too complex to consider in the algorithm
    - d_i - the demand for 5G signal values, corresponding to the population of the cities i (population/1000 rounded up)
    - M_j - the supply values (same as capacity) of a 5G towers j
    - f_j - the price of building one tower j

4. The objective function was constructed by
   - adding the costs of building the facilities accordingly with their boolean values being 0 or 1, corresponding to whether the factory was built or not. (tower building is averaged at 375000$, and therefore the cost of building larger or smaller ones (with larger or bigger capacity) is proportional to that particular value)
   - adding the cross product of the cost matrix values C_{ij} and X_iP_j, which are corresponding values of how many units of signal went from facility A to location B

5. constrained by two types of constraints - from the supply side and the demand side :
   - The supply side constraint - indicating that the total amount of signal supplied from tower j does not exceed its capacity M_j
   - Demand side - indicating that the total amount of signal inflowing into a city X_i is equal to the demand of 5G signal of a city d_i

The cost matrix and demand of cites

Demand = Population/1000	Cost = Distance/1000	Transport COST	PS1	PS2	PS3	PS4	PS5	PS6	PS7	PS8	PS9	PS10
102	Alkmaar	X1	22	30	11	50	50	50	50	50	50	50
70	Alphen aan den Rijn	X2	50	50	50	10	16	33	50	50	7	50
153	Amersfoort	X3	50	50	50	50	50	50	10	14	50	50
1115	Amsterdam	X4	15	9	25	50	50	50	50	50	21	24
221	Dordrecht	X5	50	50	50	34	22	8	50	50	50	50
205	Haarlem	X6	6	5	30	50	50	50	50	50	21	11
85	Hilversum	X7	50	50	50	50	50	50	7	23	50	50
74	Hoofddorp	X8	14	6	36	50	50	50	50	50	13	9
73	Hoorn	X9	34	39	11	50	50	50	50	50	50	50
117	Ijmuiden	X10	6	13	25	50	50	50	50	50	50	18
270	Leiden	X11	50	50	50	5	27	36	50	50	17	15
62	Nieuwegein	X12	50	50	50	50	19	50	16	15	50	50
82	Purmerend	X13	20	23	9	50	50	50	50	50	50	50
830	Rotterdam	X14	50	50	50	21	25	8	50	50	50	50
804	s-Gravenhage	X15	50	50	50	15	35	30	50	50	50	29
71	Spijkenisse	X16	50	50	50	32	38	14	50	50	50	50
383	Utrecht	X17	50	50	50	50	50	50	8	15	50	50
65	Veenendaal	X18	50	50	50	50	50	50	28	14	50	50
124	Zoetermeer	X19	50	50	50	6	21	25	50	50	22	50

The Supply values (Capacity of towers)

TOWERS	build cost	capacity
PS1	175000	500
PS2	350000	1000
PS3	140000	400
PS4	262500	750
PS5	175000	500
PS6	437500	1250
PS7	175000	500
PS8	350000	1000
PS9	175000	500
PS10	350000	1000

Problem formulation

Obj Function

175000*Y1 + 350000*Y2 + 140000*Y3 + 262500Y4+ 175000*Y4 + … + 350000*Y10 +
X1P1*22 + X1P2*30 + X1P3*11 + X1P4*50 + ... + X1P10*50 + ...
X2P1*50 + X2P2*50 + X2P3*50 + X2P4*10 + … + X2P10*50 + …
X3P1*50 + X3P2*50 + X3P3*50 + X3P4*50 + ... + X2P10*50 + ...
X4P1*15 + X4P2*9 + X4P3*25 + X4P4*50 + ... + X4P10*24 + ...
+ ... +
X10P1*6 + X10P2*13 + X10P3*25 + X10P4*50 + ... + X10P10*18 + ...
+ ... +
X19P1*50 + X19P2*50 + X19P3*50 + X19P4*6 + ... + X19P10*50

Constraints

Signal Delivered (Demand)

X1P1 + X1P2 + X1P3 + … + X1P10 = 102
X2P1 + X2P2 + X2P3 + … + X2P10 = 70
X3P1 + X3P2 + X3P3 + … + X3P10 = 153
X4P1 + X4P2 + X4P3 + ... + X4P10 = 1115
…
X19P1 + X19P2 + X19P3 + … + X19P10 = 124

Supply side

X1P1 + X2P1 + X3P1 + … + X19P1 <= 500*Y1
X1P2 + X2P2 + X3P2 + ... + X19P2 <= 1000*Y2
X1P3 + X2P3 + X3P3 + … + X19P3 <= 400*Y3
X1P4 + X2P4 + X3P4 + … + X19P4 <= 750*Y4
….
X1P10 + X2P10 + X3P10 + … + X19P10 <= 1000*Y10

The solutions are the following:

OBJECTIVE FUNCTION	1815287	dollars

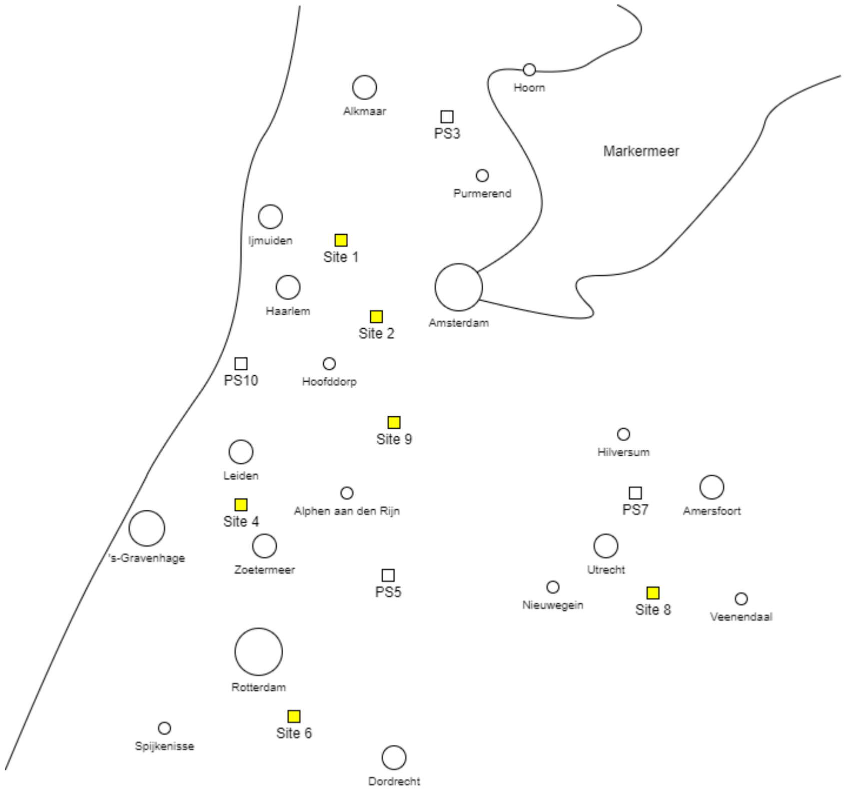
XiPj - signal from tower j to city i		PS1	PS2	PS3	PS4	PS5	PS6	PS7	PS8	PS9	PS10
Alkmaar	X1	67	0	0	0	0	0	0	35	0	0
Alphen aan den Rijn	X2	0	0	0	0	0	0	0	0	70	0
Amersfoort	X3	0	0	0	0	0	0	0	153	0	0
Amsterdam	X4	29	1000	0	0	0	0	0	0	86	0
Dordrecht	X5	0	0	0	0	0	221	0	0	0	0
Haarlem	X6	205	0	0	0	0	0	0	0	0	0
Hilversum	X7	0	0	0	0	0	0	0	85	0	0
Hoofddorp	X8	0	0	0	0	0	0	0	0	74	0
Hoorn	X9	0	0	0	0	0	0	0	73	0	0
Ijmuiden	X10	117	0	0	0	0	0	0	0	0	0
Leiden	X11	0	0	0	0	0	0	0	0	270	0
Nieuwegein	X12	0	0	0	0	0	0	0	62	0	0
Purmerend	X13	82	0	0	0	0	0	0	0	0	0
Rotterdam	X14	0	0	0	0	0	830	0	0	0	0
s-Gravenhage	X15	0	0	0	626	0	128	0	50	0	0
Spijkenisse	X16	0	0	0	0	0	71	0	0	0	0
Utrecht	X17	0	0	0	0	0	0	0	383	0	0
Veenendaal	X18	0	0	0	0	0	0	0	65	0	0
Zoetermeer	X19	0	0	0	124	0	0	0	0	0	0
	SUM	500	1000	0	750	0	1250	0	906	500	0

Constraints			
	LHS	Constraint	RHS
Signnal to X1	102	=	102
Signnal to X2	70	=	70
Signnal to X3	153	=	153
Signnal to X4	1115	=	1115
Signnal to X5	221	=	221
Signnal to X6	205	=	205
Signnal to X7	85	=	85
Signnal to X8	74	=	74
Signnal to X9	73	=	73
Signnal to X10	117	=	117
Signnal to X11	270	=	270
Signnal to X12	62	=	62
Signnal to X13	82	=	82
Signnal to X14	830	=	830
Signnal to X15	804	=	804
Signnal to X16	71	=	71
Signnal to X17	383	=	383
Signnal to X18	65	=	65
Signnal to X19	124	=	124
Supply From P1	500	<=	500
Supply From P2	1000	<=	1000
Supply From P3	2,8422E-14	<=	0
Supply From P4	750	<=	750
Supply From P5	0	<=	0
Supply From P6	1250	<=	1250
Supply From P7	5,6843E-14	<=	0
Supply From P8	906	<=	1000
Supply From P9	500	<=	500
Supply From P10	0	<=	0

TOWERS	Yi
PS1	1
PS2	1
PS3	0
PS4	1
PS5	0
PS6	1
PS7	0
PS8	1
PS9	1
PS10	0

The key results are the minimized objective function value - namely 1 815 287 $ and that we will build six 5G towers in potential sites number : 1,2,4,6,8 and 9. Furthermore, we can look at which site provides signal to which cities, and vice versa - which cities have signal provided from which towers. For example, we can see that our biggest city has signals delivered from 3 sites : 29 units of signal from site 1 , 1000 units from site 2, and 86 units from site 9. From the same table we see which sites have used all of its capacity, and which ones did not. We see that Site 8 is supplying 906 units of signal out of the total 1000 it has to offer. The cost of building a site is proportional to the size of it, therefore downsizing this potential site could be taken into consideration in order to reduce the costs of the whole operation. Moreover we see that all the constraints are fulfilled - with all the cities receiving the exact amount of units of 5G signal they need, as well as the towers providing at most the limit of their capacity. The general conclusion however is that a majority of the cities have their signal provided from 1 site only, though the number of sites varies between 1 2 or 3.

Graphical visualisation on an abstract map



What is most interesting to observe is the layout of the sites we decided to build on the map in comparison to the other potential ones that the algorithm omitted in the optimal solution. There are a few interesting occurrences that tell us about which features were more relevant than others. For example - PS5 was never built, despite being centered between a lot of cities, implying that the cost of increased distance was indeed a valuable factor. PS3 however was never built, despite being clearly the closest site to Alkmaar Hoorn and Purmerend, which. implies that the cost of building this new site was simply not that worth it, and it was cheaper to deliver signal from further sites. Therefore we see it is technically the opposite conclusion of the first observation - meaning that in this particular part of the map the cost of building a new site was more significant than the costs of delivering the signal from much further origins. Having different features play a more or less significant role in various areas of the map, which implies that the transportation cost and building costs were appropriately balanced out and almost equally valuable. On the contrary, we could imagine a scenario where the building costs are really low - the algorithm would decide to build all the sites, or if the building costs would be incredibly high - the decision would be to build fewer sites, and provide signal very far.

Overall, I found the facility location algorithm to be a highly efficient method to resolve this particular matter. The model ended up providing us with interesting insights that were not fully predictable. The 5G towers placement would overall be a much more complex task, as it would have to cover all the small towns as well, with a much larger set of potential sites, since In this particular project we only considered the 19 cities with the highest population. However as complex as the complete problem would be, I believe the facility location algorithm would succeed in finding the optimal solution yet again.

I have learned that the most vital aspect to take into consideration is an appropriate assignment/calculation of the parameters like costs of transportation and cost of building a facility. If one has a more substantial impact than the other, the whole algorithm focuses one way - which is either building all the facilities, or very little of them, and the solution should be somewhat balanced (just like in our final outcome). The second thing I have learned is that there usually would be more external factors influencing the variables, like whether the signal tower was built at a high altitude or not. However by properly adjusting the cost matrix as well as the fixed costs of building the potential sites, obtaining a realistic optimal solution is highly probable by using this model. In general, the Facility Location Problem algorithm was highly successful in finding the optimal solution for the placement of potential 5G towers across North Holland, South Holland and Utrecht.

1] - Putman, G., 2020. SPATIAL ANALYSIS OF THE INFRASTRUCTURAL DEMANDS OF ANDS OF INTEGRATING 5G TECHNOLOGY: A A CASE STUDY IN THE CITY OF DETROIT.

[2] - Guevara, L. and Auat Cheein, F., 2020. The Role of 5G Technologies: Challenges in Smart Cities and Intelligent Transportation Systems. Sustainability, 12(16), p.6469.

[3] - Mathematical Optimization: Solving Problems using Gurobi and Python. [online] Available at: <https://scipbook.readthedocs.io/en/latest/flp.html> [Accessed 11 January 2022

[4] -Facility Location Problem. (2020). Optimization.Cbe.Cornell.Edu.https://optimization.cbe.cornell.edu/index.php?title=Facility_location_proble