

AIDI 1000: AI Algorithms and Mathematics – Practice Midterm

1. A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B. At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours. The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

- Formulate the problem of deciding how much of each product to make in the current week as a linear program.
- Solve this linear program graphically.

Solution Link: <http://people.brunel.ac.uk/~mastjjb/jeb/or/morelp.html>

2. Let $f(x) = x^2 + 4x + 5$

- 2.1 Compute $f'(x)$, $f''(x)$, and determine whether f convex, concave, or neither.

Solution: Take the double derivative of f(x) and follow the concepts discussed in lecture.

- 2.2 Optimize the following function by finding the critical value(s) at which the function is optimized. (**Hint :To find the critical values put $f'(x) = 0$**)

3. Consider the matrix M and answer the following questions:

$$M_{2 \times 2} = \begin{pmatrix} -3 & 2 \\ 2 & 5 \end{pmatrix} \quad (1)$$

- 3.1 Find the inverse of matrix M.

Solution: It can refer from the lecture video. I solved a similar example.

- 3.2 Find the eigen values if possible and then find the find the eigen vectors.

4. Let the random variable X denote the time a person waits for an elevator to arrive. Suppose the longest one would need to wait for the elevator is 2 minutes, so that the possible values of X (in minutes) are given by the interval [0,2]. A possible pdf for X is given by:

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the CDF of X?
- Draw the graph of CDF for all the possible values in the range of X?

Solution Link: [https://stats.libretexts.org/Courses/Saint_Mary%27s_College_Notre_Dame/DSCI_500B_Essential_Probability_Theory_for_Data_Science_\(Kuter\)/04%3A_Continuous_Random_Variables/4.01%3A_Probability_Density_Functions_\(PDFs\)_and_Cumulative_Distribution_Functions_\(CDFs\)_for_Continuous_Random_Variables](https://stats.libretexts.org/Courses/Saint_Mary%27s_College_Notre_Dame/DSCI_500B_Essential_Probability_Theory_for_Data_Science_(Kuter)/04%3A_Continuous_Random_Variables/4.01%3A_Probability_Density_Functions_(PDFs)_and_Cumulative_Distribution_Functions_(CDFs)_for_Continuous_Random_Variables)