

## AIDI 1000: AI Algorithms and Mathematics – Assignment - 2

Due Date : October 24, 2022, 11:59 PM

Note : Submit only one pdf file showing all your work. (File name : *Assignment\_2\_firstname\_lastname.pdf*)

1. (25 points) There are three types of coins which have different probabilities of landing heads when tossed:

- Type A coins are fair, with probability 0.5 of heads
- Type B coins are bent and have probability 0.6 of heads
- Type C coins are bent and have probability 0.9 of heads

Suppose I have a drawer containing 5 coins: 2 of type A, 2 of type B, and 1 of type C. I reach into the drawer and pick a coin at random. Without showing you the coin I flip it once and get heads (i.e. event  $D$ ). What is the probability it is type A (i.e.  $P(H=A|D)$ )? Type B (i.e.  $P(H=B|D)$ )? Type C (i.e.  $P(H=C|D)$ )? Fill out table below with your answers.

hypothesis	prior	likelihood	posterior
$H$	$P(H)$	$P(D H)$	$P(H D)$
A			
B			
C			

2. (25 points - Written) Consider the following dataset of four rows and three features (Malicious, Viagara, Meet) with class labels (ham and spam). Suppose we see a message having these features  $M_5 = (\text{Malicious} = \text{'yes'}, \text{Viagara} = \text{'no'}, \text{Meet} = \text{'yes'})$ , What is the probability that it is a spam or ham? Using Naive Bayes algorithm.

S.No	Malicious	Viagara	Meet	class
$M_1$	yes	yes	yes	spam
$M_2$	no	no	yes	ham
$M_3$	yes	no	yes	spam
$M_4$	no	yes	no	ham

3. (30 points - Written) Use the below gradient descent algorithm to find the  $\mathbf{x}$  value that minimizes the function  $f(\mathbf{x}) = 6x_1^2 - 3x_1x_2 + 2x_2^2$ . Choose the starting point as  $(x_1^{(0)}, x_2^{(0)}) = (2, 3)$  and use the  $\eta_k$  as 0.1 and  $\epsilon$  as 0.0001 Consider the following gradient descent algorithm (which is a variant of the directional search algorithm) that aims to find the  $x$  value that minimizes the function  $f(x)$ .

- Guess  $x^{(0)}$ , set  $k \leftarrow 0$
- while  $\|\nabla f(x^{(k)})\| \geq \varepsilon$  do
- $x^{(k+1)} = x^{(k)} - \eta_k \nabla f(x^{(k)})$
- $k \leftarrow k + 1$
- end while
- return  $x^{(k)}$

Perform only 3 iterations of this algorithm and report the values of  $(x_1, x_2)$  after performing gradient descent each time

Hint : since the  $f(x)$  is in two variable you need to apply partial differentiation

4. (20 points - Written) Find the eigen values and eigen vectors for the following matrix.
- $$\begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix}$$