Last Name: _____ First Name: _____ Student ID: _____

AIDI 1000: AI Algorithms and Mathematics – Midterm

Nov 4, 2022, 6:00 p.m. - 8:30 p.m.

1. (20 points) Answer the following questions as follows:

   1.1 TRUE or FALSE? and Why? Naive Bayes algorithm assumes the feature independence.

   1.2 The functions f and g are both concave functions of a single variable. Neither function is necessarily differentiable. Is the function h defined by $h(x) = f(x) + g(x)$ necessarily concave, necessarily convex, or not necessarily either ?

   1.3 Find the angle between these two vectors, $u = (6,2)$, and $v = (5,-2)$.

   1.4 Suppose we were interested in determining if there were differences in the average prices among two automobile companies. We randomly pick ten cars to compare at both companies. Which statistical procedure would be best to use for this study and why?

2. (20 points) Assume that when deciding whether an applicant is low-risk (c_0), medium-risk (c_1), or high-risk (c_2) for bank loans, the bank only considers the amount of money in the saving accounts of the applicants. Historic data indicate following distribution about the savings (in thousand dollars) of the applicant based on their classes:

   $$(x|c_0) \sim Uniform(35,60), \quad (x|c_1) \sim Uniform(15,45), \quad (x|c_2) \sim Uniform(0,20)$$

   Furthermore, we know that 20% and 30% of the previous applicants are low-risk and medium-risk, respectively. If an applicant has $48,000 in her savings account, predict the applicant's class (based on Bayes decision rules).

   Hint: pdf for uniform distribution — if $x \sim Uniform(a,b)$, $f(x) = \frac{1}{b-a}$ $for\ a <= x <= b$.

   Answer (SHOW YOUR WORK!):

3. (20 points) Let $f(x) = x^3 + 6x + 5$

   3.1 Compute $f'(x)$, $f''(x)$, and determine whether f convex, concave, or neither.

   3.2 Optimize the following function by finding the critical value(s) at which the function is optimized. (Hint :To find the critical values put $f'(x) = 0$)

4. (20 points) Consider the following table as a training data for Naive Bayes classifier. Find the following probabilities. Apply Laplace correction if required.

   4.1 $P(Label = Yes|A = 1, B = 1, C = 0)$

   4.2 $P(Label = No|A = 1, B = 0, C = 0)$

A	B	C	LABEL
1	0	1	Yes
1	1	1	Yes
0	1	1	No
1	1	0	No
1	0	1	No

5. (20 points) Consider the pdf of $f(x)$ and calculate the following :
   - find the value of c
   - CDF of $f_X(x)$
   - find $P(X \geq 1/2)$

$$f_X(x) = \begin{Bmatrix} 4cx^3 & 0 \leq x \leq 1 \\ 0 & otherwise \end{Bmatrix} \tag{1}$$

where c is a positive constant.

Required Formulas :

$$\frac{\mathrm{d}x^n}{\mathrm{d}x} = nx^{n-1} \tag{2}$$

$$\int x^n = \frac{x^{n+1}}{n+1} \tag{3}$$