- **Logic Gates**
  - Basic Gates (AND, OR, NOT)
  - Derived Gates (e.g., NAND, XOR, IMPLY)
- **Emulating Logic Gates with MP Neuron**
  - NOT, AND, OR
  - Extension to N-Input Gates
- **Graphical Interpretation**

Recap: Binary Decision Making

All Binary Inputs

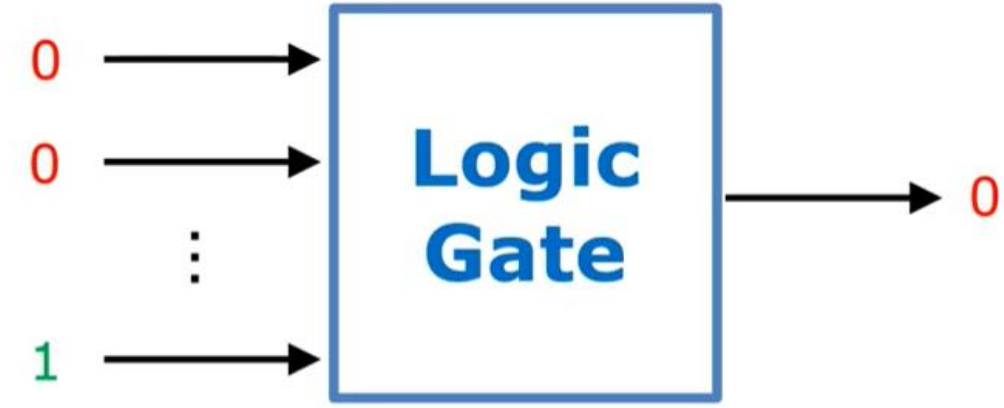
- x_1 ☑ Fantastic weather ?
- x_2 ☑ Diverse culture ?
- x_3 ☑ Gorgeous beaches ?
- x_4 ☒ High cost of living ?
- x_5 ☒ Island fever ?

A Binary Output

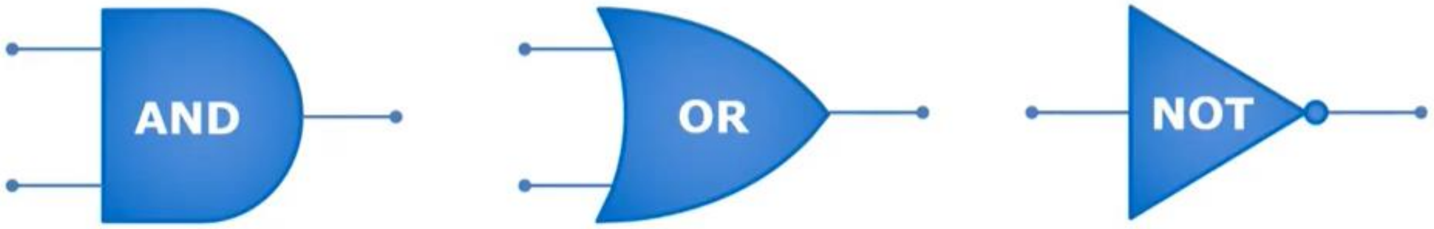
y ☑ **Living in Hawaii ?**

The decision-making elements that process binary values can be represented by **logic gates**.

Logic Gates

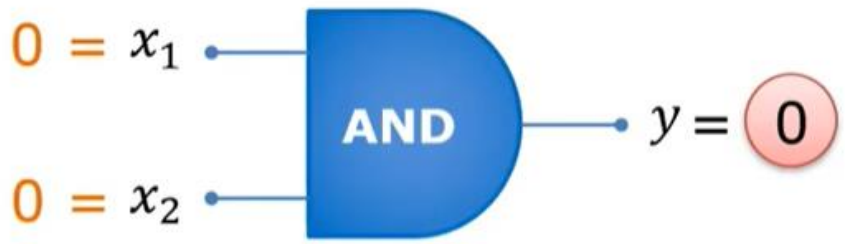


Three Basic Logic Gates

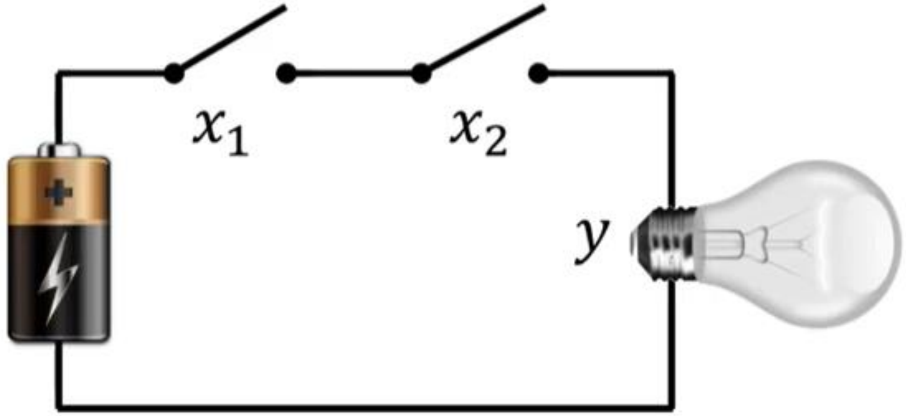


Circuit & Truth Table for AND Gate

x_1 **AND** $x_2 = y$



AND Gate



Truth Table

x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

A truth table shows how each possible input of a logic gate relates to its output.

Basic vs. Derived Gates

Basic Gates

x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

x_1	x_2	y
1	1	1
0	1	1
1	0	1
0	0	0

x	y
1	0
0	1

Derived Gates

x_1	x_2	y
1	1	0
0	1	1
1	0	1
0	0	1

x_1	x_2	y
1	1	0
0	1	1
1	0	1
0	0	0

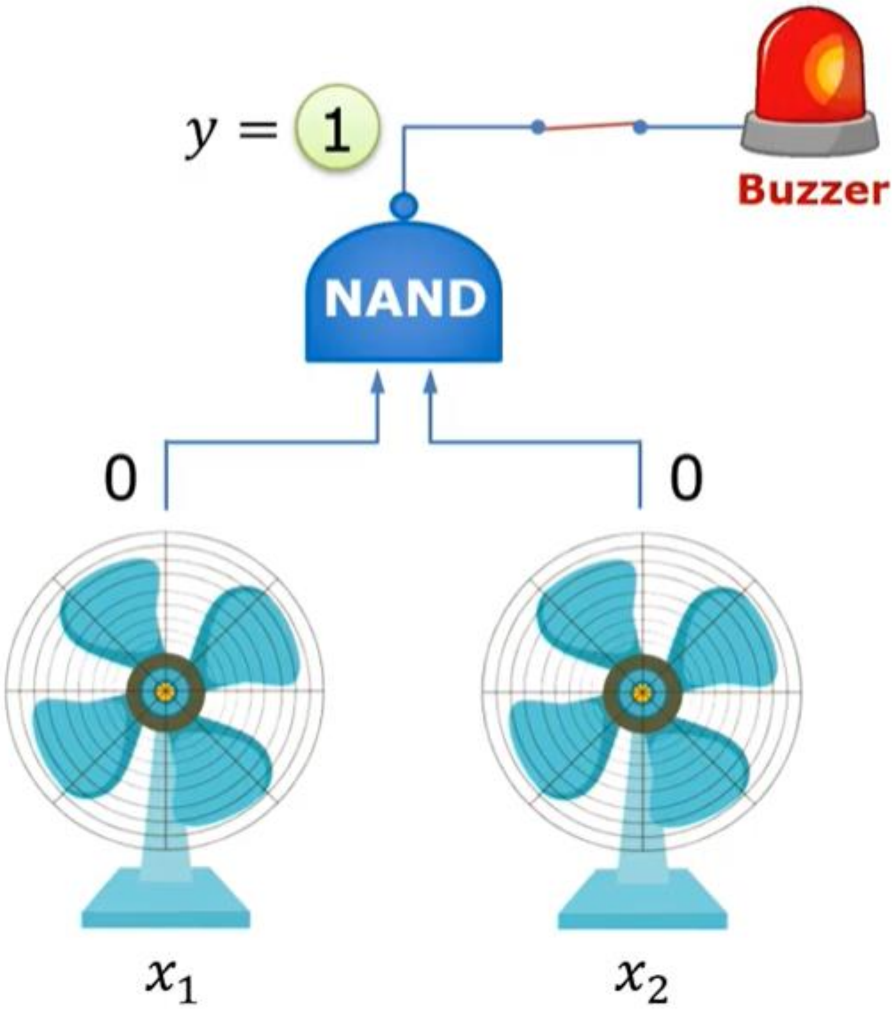
x_1	x_2	y
1	1	1
1	0	0
0	1	1
0	0	1

NAND Gate

x_1 **NAND** $x_2 = y$

Truth Table

x_1	x_2	y	x_1 AND x_2
1	1	0	1
1	0	1	0
0	1	1	0
0	0	1	0

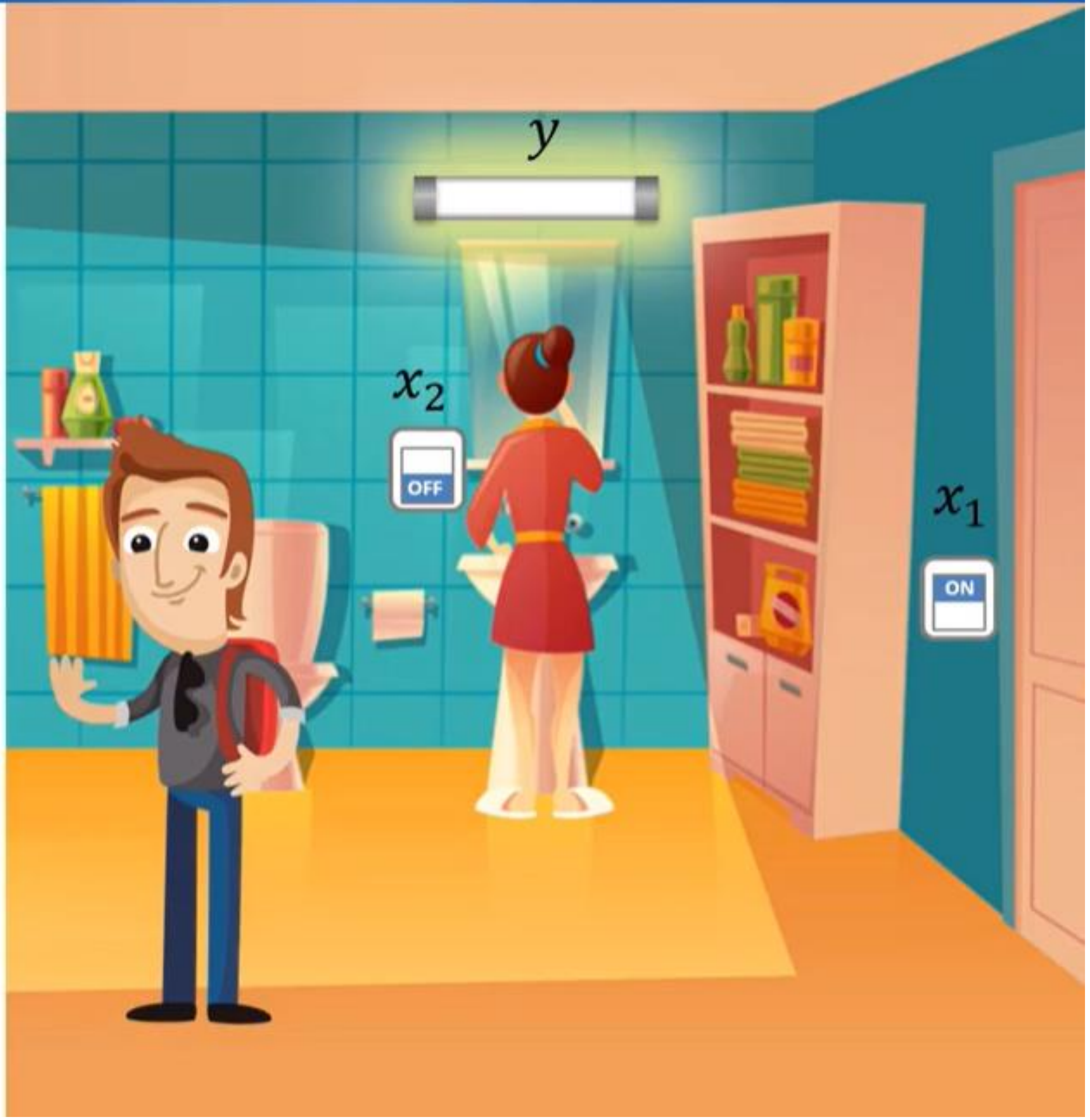


XOR Gate

x_1 **XOR** $x_2 = y$

Truth Table

x_1	x_2	y
0	0	0
0	1	1
1	1	0
1	0	1



Basic vs. Derived Gates

Basic Gates

x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

x_1	x_2	y
1	1	1
0	1	1
1	0	1
0	0	0

x	y
1	0
0	1

How to mimic different logic gates (NOT, AND, OR) with MP neuron?

Derived Gates

x_1

x_2

y

x_1	x_2	y
1	1	0
0	1	1
1	0	1
0	0	1

x_1	x_2	y
1	1	0
0	1	1
1	0	1
0	0	0

x_1	x_2	y
1	1	1
1	0	0
0	1	1
0	0	1

NOT Gate

Question

Emulate **NOT** Gate with MP neuron.

$$\text{NOT}\ (x) = y$$

Predict Output y

If any inhibitory input is 1,
Output: $y = 0$
Else
Sum: $z = x_1 + \cdots + x_n$
Threshold: $y = \begin{cases} 1, & (z \geq \theta) \\ 0, & (z < \theta) \end{cases}$
Output: y

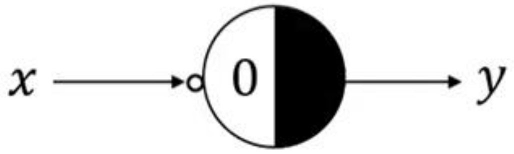
Solution

1. Write the truth table
2. Find threshold θ
3. Depict Rojas diagram

x	y
0	1
1	0

When $x = 0$, $y = 1$. ➔ Set threshold $\theta = 0$.

When $x = 1$, $y = 0$. ➔ Set x as inhibitory.



Rewrite Logic Operators as Functions

x_1 AND $x_2 = y$ → $\boxed{\text{AND}\ (x_1, x_2) = y}$

2-INPUT Gate

x_1 AND x_2 AND $x_3 = y$ → AND $(x_1, x_2, x_3) = y$

3-INPUT Gate

AND Gate

Question

- Emulate 2-Input AND Gate with MP neuron.

$$\text{AND}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Find threshold θ

AND neuron fires
only when all the inputs are 1s.

When $x_1 = x_2 = 1,\ y = 1.$

$x_1 + x_2 = \boxed{2\ \geq \theta} \Rightarrow \theta \leq 2$

Inputs		Output
x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

AND Gate

Question

- Emulate 2-Input AND Gate with MP neuron.

$$\text{AND}\,(x_1, x_2) = y$$

Solution

1. Write the truth table
2. Find threshold θ

When $(x_1, x_2) = (0, 1)$, $y = 0$.

$$x_1 + x_2 = \boxed{1 < \theta} \Rightarrow \theta > 1$$

	Inputs x_1	x_2	Output y
$\theta \leq 2$	1	1	1
$\theta > 1$	0	1	0
	1	0	0
	0	0	0

AND Gate

Question

- Emulate 2-Input AND Gate with MP neuron.

$$\text{AND}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Find threshold θ

When $(x_1, x_2) = (1, 0)$, $y = 0$.

$$x_1 + x_2 = \boxed{1 < \theta} \Rightarrow \theta > 1$$

	Inputs x_1	x_2	Output y
$\theta \le 2$	1	1	1
$\theta > 1$	0	1	0
$\theta > 1$	1	0	0
	0	0	0

AND Gate

Question

- Emulate 2-Input AND Gate with MP neuron.

$$\text{AND } (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Find threshold θ

When $(x_1, x_2) = (0,0)$, $y = 0$.

$$x_1 + x_2 = \boxed{0 < \theta} \Rightarrow \theta > 0$$

	Inputs x_1	x_2	Output y
$\theta \leq 2$	1	1	1
$\theta > 1$	0	1	0
$\theta > 1$	1	0	0
$\theta > 0$	0	0	0

AND Gate

Question

- Emulate 2-Input AND Gate with MP neuron.

$$\text{AND}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Find threshold $\theta = 2$

$1 < \theta \leq 2$

	Inputs		Output
	x_1	x_2	y
$\theta \leq 2$	1	1	1
$\theta > 1$	0	1	0
$\theta > 1$	1	0	0
$\theta > 0$	0	0	0

AND Gate

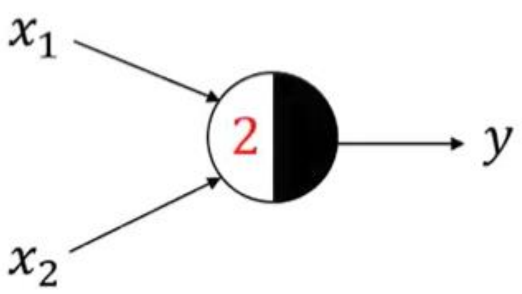
Question

- Emulate 2-Input AND Gate with MP neuron.

$$\text{AND } (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Find threshold $\theta = 2$
3. Depict Rojas diagram



Inputs		Output
x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

3-Input AND Gate

x_1 AND $x_2 = y$ → AND $(x_1, x_2) = y$

2-INPUT Gate

x_1 AND x_2 AND $x_3 = y$ → AND $(x_1, x_2, x_3) = y$

3-INPUT Gate

3-Input AND Gate

Question

- Emulate 3-Input AND Gate with MP neuron.

$$\text{AND}\ (x_1, x_2, x_3) = y$$

Solution

1. Write the truth table
2. Find threshold θ

An AND neuron fires
only when all the inputs are 1s.

$y = 1$ if $x_1 + x_2 + x_3 = 3 \geq \theta$

$y = 0$ if $x_1 + x_2 + x_3 = 2 < \theta$

$y = 0$ if $x_1 + x_2 + x_3 = 1 < \theta$

$y = 0$ if $x_1 + x_2 + x_3 = 0 < \theta$

$\Rightarrow 2 < \theta \leq 3$

$x_1 + x_2 + x_3$	Inputs x_1	x_2	x_3	Output y
3	1	1	1	1
2	0	1	1	0
2	1	0	1	0
1	0	0	1	0
2	1	1	0	0
1	0	1	0	0
1	1	0	0	0
0	0	0	0	0

3-Input AND Gate

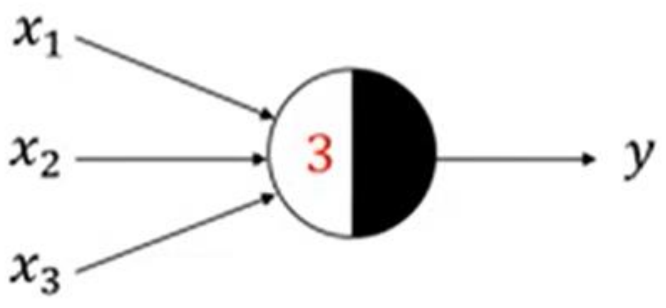
Question

- Emulate 3-Input AND Gate with MP neuron.

$$\text{AND}\ (x_1, x_2, x_3) = y$$

Solution

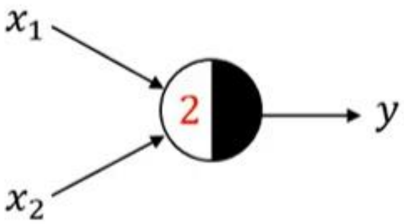
1. Write the truth table
2. Find threshold $\theta = 3$
3. Depict Rojas diagram



$x_1 + x_2 + x_3$	Inputs x_1	x_2	x_3	Output y
3	1	1	1	1
2	0	1	1	0
2	1	0	1	0
1	0	0	1	0
2	1	1	0	0
1	0	1	0	0
1	1	0	0	0
0	0	0	0	0

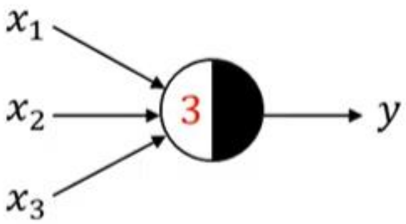
N-Input AND Gate

2 INPUTS



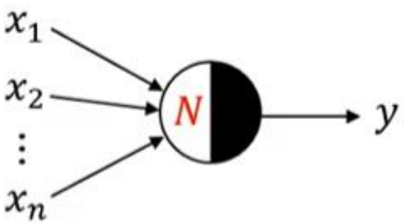
AND$(x_1, x_2) = y$

3 INPUTS



AND$(x_1, x_2, x_3) = y$

***N* INPUTS**



AND$(x_1, x_2, \cdots, x_N) = y$

AND neuron fires only when all the inputs are 1s.

$$y = \begin{cases} 1 & \text{if } \boxed{x_1 = x_2 = \cdots = x_N = 1} \\ 0 & \text{otherwise} \end{cases} \Rightarrow y = \begin{cases} 1 & \text{if } \boxed{x_1 + x_2 + \cdots + x_N \geq N} \\ 0 & \text{otherwise} \end{cases}$$

OR Gate

Question

- Emulate 2-Input OR Gate with MP neuron.

$$\text{OR}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Set threshold θ

OR neuron fires
when any input is 1.

When $x_1 = x_2 = 1$, $y = 1$.

$x_1 + x_2 = \boxed{2 \geq \theta}$ ⇨ $\theta \leq 2$

	x_1	x_2	y
$\theta \leq 2$	1	1	1
	0	1	1
	1	0	1
	0	0	0

OR Gate

Question

- Emulate 2-Input OR Gate with MP neuron.

$$\text{OR}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Set threshold θ

When $(x_1, x_2) = (0, 1)$, $\boxed{y = 1.}$

$x_1 + x_2 = \boxed{1 \geq \theta} \Rightarrow \theta \leq 1$

	Inputs		Output
	x_1	x_2	y
$\theta \leq 2$	1	1	1
$\theta \leq 1$	0	1	1
	1	0	1
	0	0	0

OR Gate

Question

- Emulate 2-Input OR Gate with MP neuron.

$$\text{OR}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Set threshold θ

When $(x_1, x_2) = (1, 0)$, $\boxed{y = 1.}$

$x_1 + x_2 = \boxed{1 \geq \theta}$ ⇨ $\theta \leq 1$

	Inputs		Output
	x_1	x_2	y
$\theta \leq 2$	1	1	1
$\theta \leq 1$	0	1	1
$\theta \leq 1$	1	0	1
	0	0	0

OR Gate

Question

- Emulate 2-Input OR Gate with MP neuron.

$$\text{OR}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Set threshold θ

When $(x_1, x_2) = (0, 0)$, $y = 0$.

$x_1 + x_2 = \boxed{0 < \theta} \Rightarrow \theta > 0$

	Inputs x_1	x_2	Output y
$\theta \leq 2$	1	1	1
$\theta \leq 1$	0	1	1
$\theta \leq 1$	1	0	1
$\theta > 0$	0	0	0

OR Gate

Question

- Emulate 2-Input OR Gate with MP neuron.

$$\text{OR}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Set threshold $\theta = 1$

$0 < \theta \leq 1$

	Inputs		Output
	x_1	x_2	y
$\theta \leq 2$	1	1	1
$\theta \leq 1$	0	1	1
$\theta \leq 1$	1	0	1
$\theta > 0$	0	0	0

OR Gate

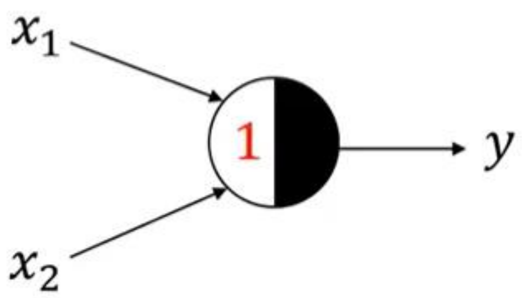
Question

- Emulate 2-Input OR Gate with MP neuron.

$$\text{OR}\ (x_1, x_2) = y$$

Solution

1. Write the truth table
2. Set threshold $\theta = 1$
3. Depict Rojas diagram



Inputs		Output
x_1	x_2	y
1	1	1
0	1	1
1	0	1
0	0	0

3-Input OR Gate

x_1 OR $x_2 = y$ → OR $(x_1, x_2) = y$

2-INPUT Gate

x_1 OR x_2 OR $x_3 = y$ → OR $(x_1, x_2, x_3) = y$

3-INPUT Gate

3-Input OR Gate

Question

- Emulate 3-Input OR Gate with MP neuron.

$$\text{OR}\,(x_1, x_2, x_3) = y$$

Solution

1. Write the truth table
2. Find threshold θ

OR neuron fires
when any input is 1.

$$y = 1 \quad \text{if } x_1 + x_2 + x_3 = 3 \geq \theta$$
$$y = 1 \quad \text{if } x_1 + x_2 + x_3 = 2 \geq \theta$$
$$y = 1 \quad \text{if } x_1 + x_2 + x_3 = 1 \geq \theta$$
$$y = 0 \quad \text{if } x_1 + x_2 + x_3 = 0 < \theta$$

$$\Rightarrow 0 < \theta \leq 1$$

$x_1 + x_2 + x_3$	Inputs x_1	x_2	x_3	Output y
3	1	1	1	1
2	0	1	1	1
2	1	0	1	1
1	0	0	1	1
2	1	1	0	1
1	0	1	0	1
1	1	0	0	1
0	0	0	0	0

3-Input OR Gate

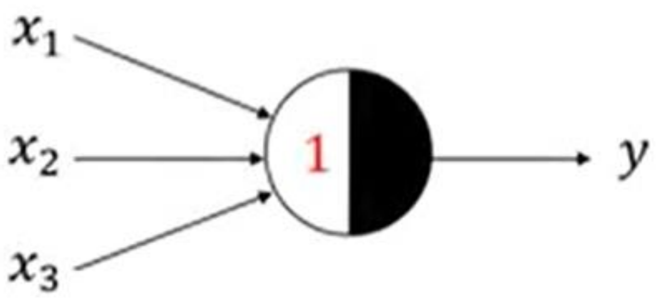
Question

- Emulate 3-Input OR Gate with MP neuron.

$$\text{OR}\,(x_1, x_2, x_3) = y$$

Solution

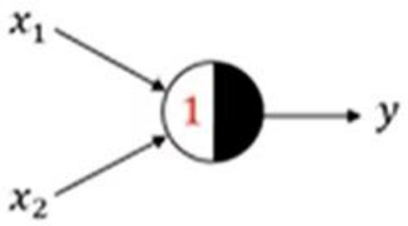
1. Write the truth table
2. Find threshold $\theta = 1$
3. Depict Rojas diagram



$x_1 + x_2 + x_3$	Inputs x_1	x_2	x_3	Output y
3	1	1	1	1
2	0	1	1	1
2	1	0	1	1
1	0	0	1	1
2	1	1	0	1
1	0	1	0	1
1	1	0	0	1
0	0	0	0	0

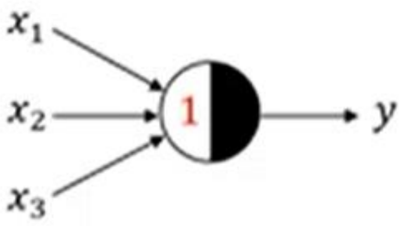
OR Gate

2 INPUTS



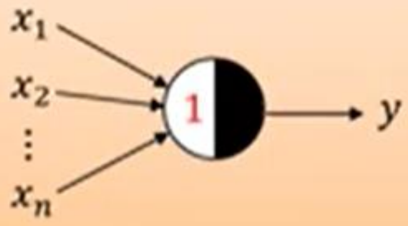
$\mathrm{OR}(x_1, x_2) = y$

3 INPUTS



$\mathrm{OR}(x_1, x_2, x_3) = y$

***N* INPUTS**



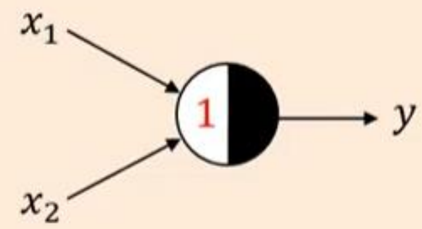
$\mathrm{OR}(x_1, x_2, \cdots, x_N) = y$

OR neuron fires as long as any input is 1.

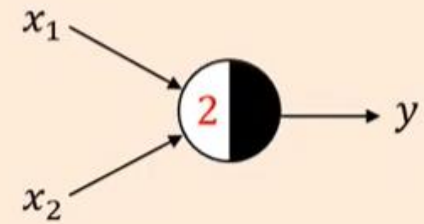
$$y = \begin{cases} 1 & \text{if } \boxed{x_1 = 1 \text{ or } x_2 = 1 \text{ or } \ldots\ x_N = 1} \\ 0 & \text{otherwise} \end{cases} \Rightarrow y = \begin{cases} 1 & \text{if } \boxed{x_1 + x_2 + \cdots + x_N \geq 1} \\ 0 & \text{otherwise} \end{cases}$$

Comparison btw OR and AND Gate

2 INPUTS

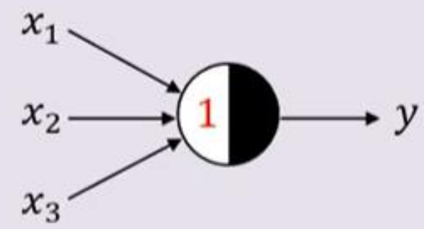


$\mathrm{OR}(x_1, x_2) = y$

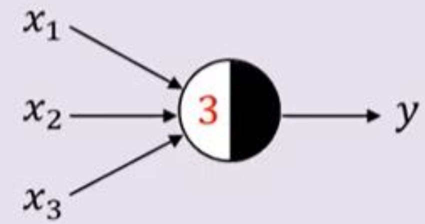


$\mathrm{AND}(x_1, x_2) = y$

3 INPUTS

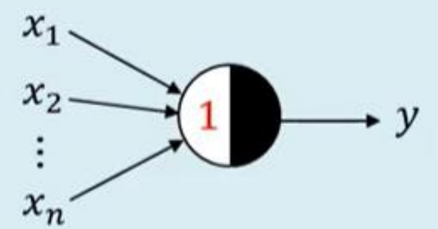


$\mathrm{OR}(x_1, x_2, x_3) = y$

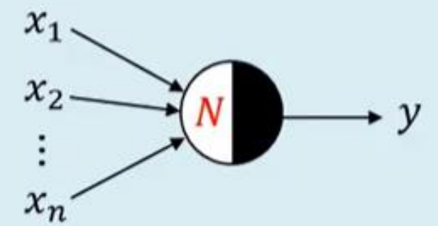


$\mathrm{AND}(x_1, x_2, x_3) = y$

N INPUTS

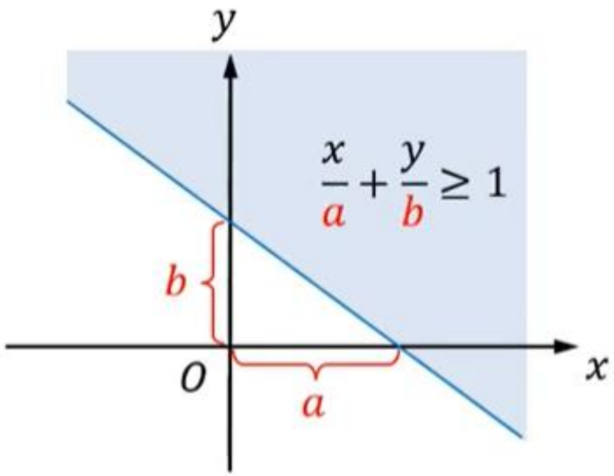


$\mathrm{OR}(x_1, x_2, \cdots, x_N) = y$



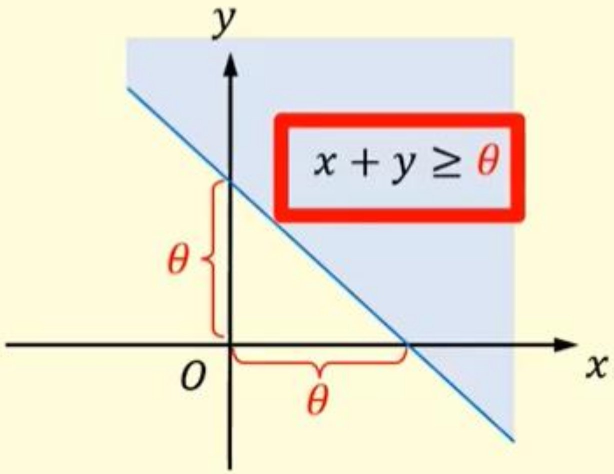
$\mathrm{AND}(x_1, x_2, \cdots, x_N) = y$

Recap: Straight Line Equation



The equation of a line,
whose x-intercept is a
and y-intercept is b, is

$$\frac{x}{a} + \frac{y}{b} = 1$$

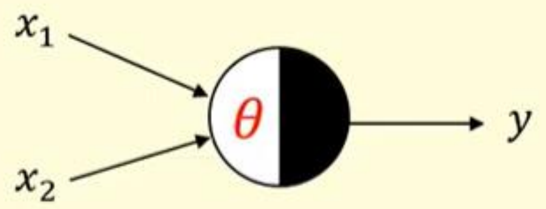


$$\frac{x}{\theta} + \frac{y}{\theta} = 1$$

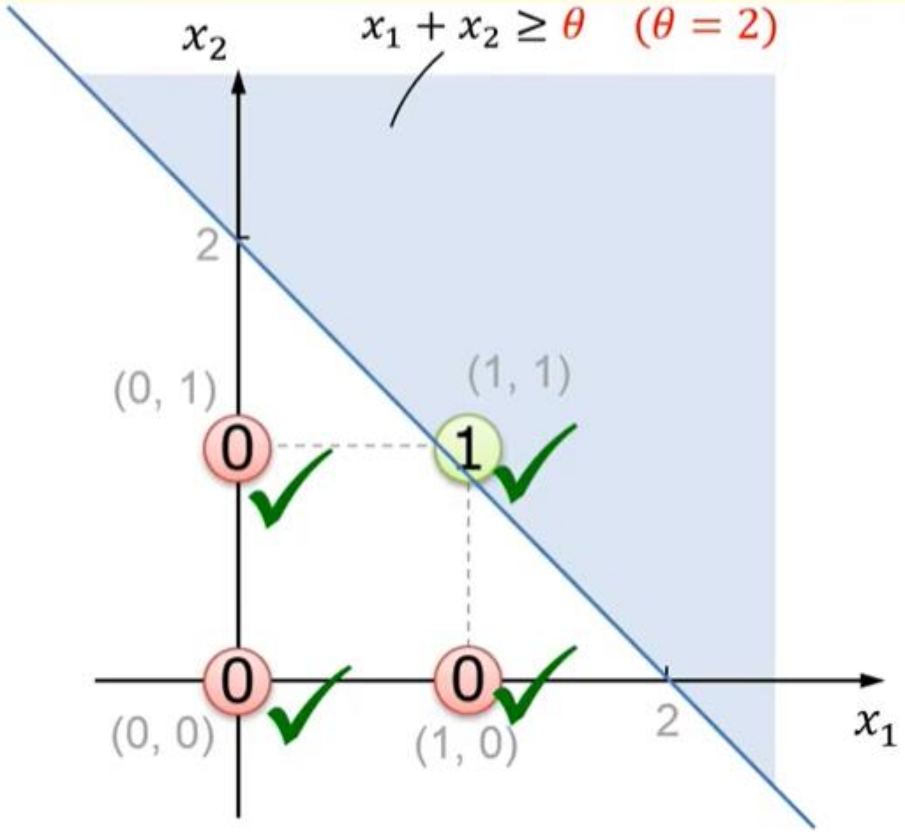
$$x + y = \theta$$

Graphical Interpretation of AND Gate

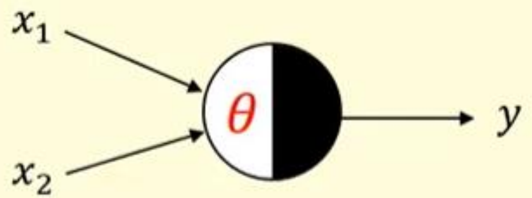
$$\mathrm{AND}(x_1, x_2) = y$$



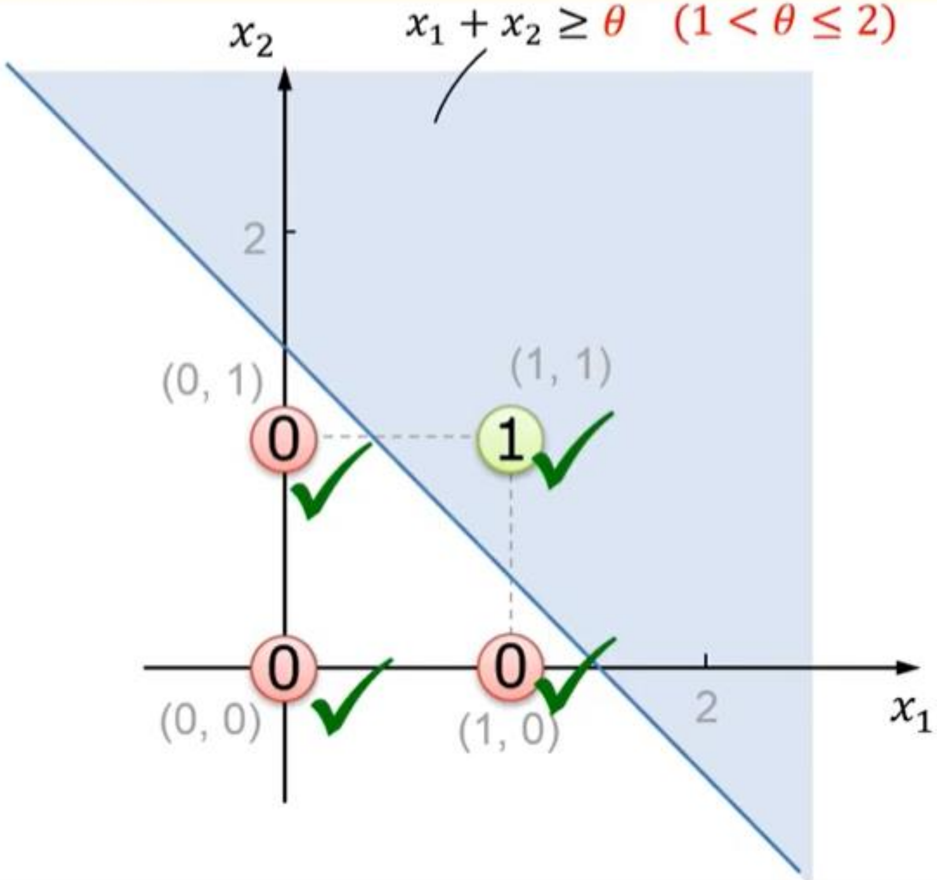
x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0



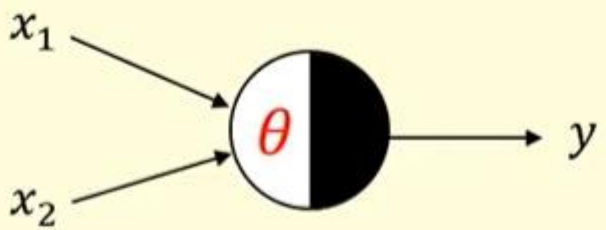
$$\mathrm{AND}(x_1, x_2) = y$$



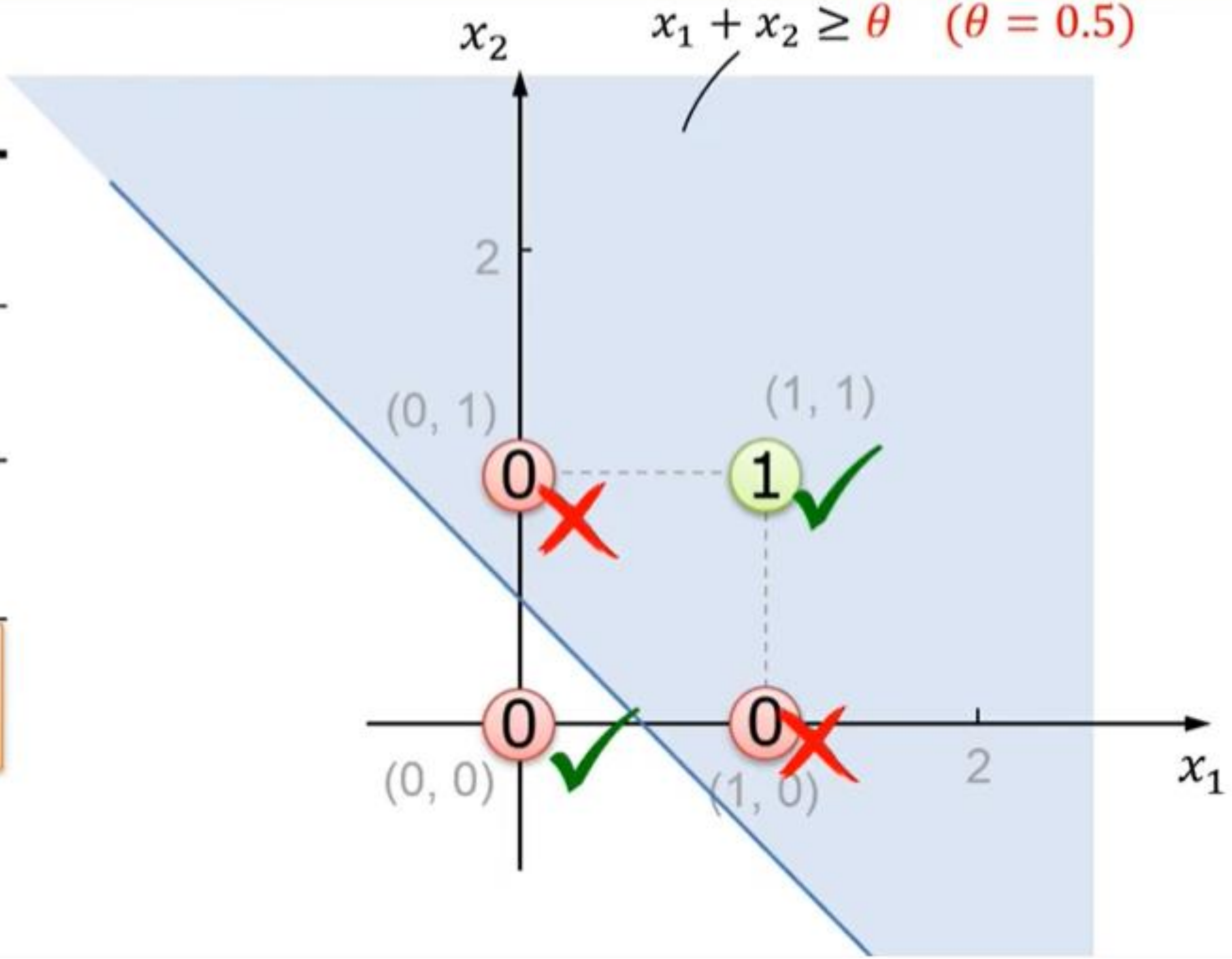
x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0



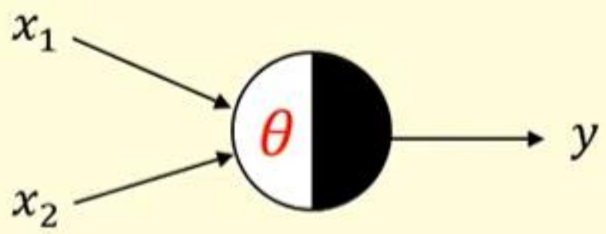
$$\mathrm{AND}(x_1, x_2) = y$$



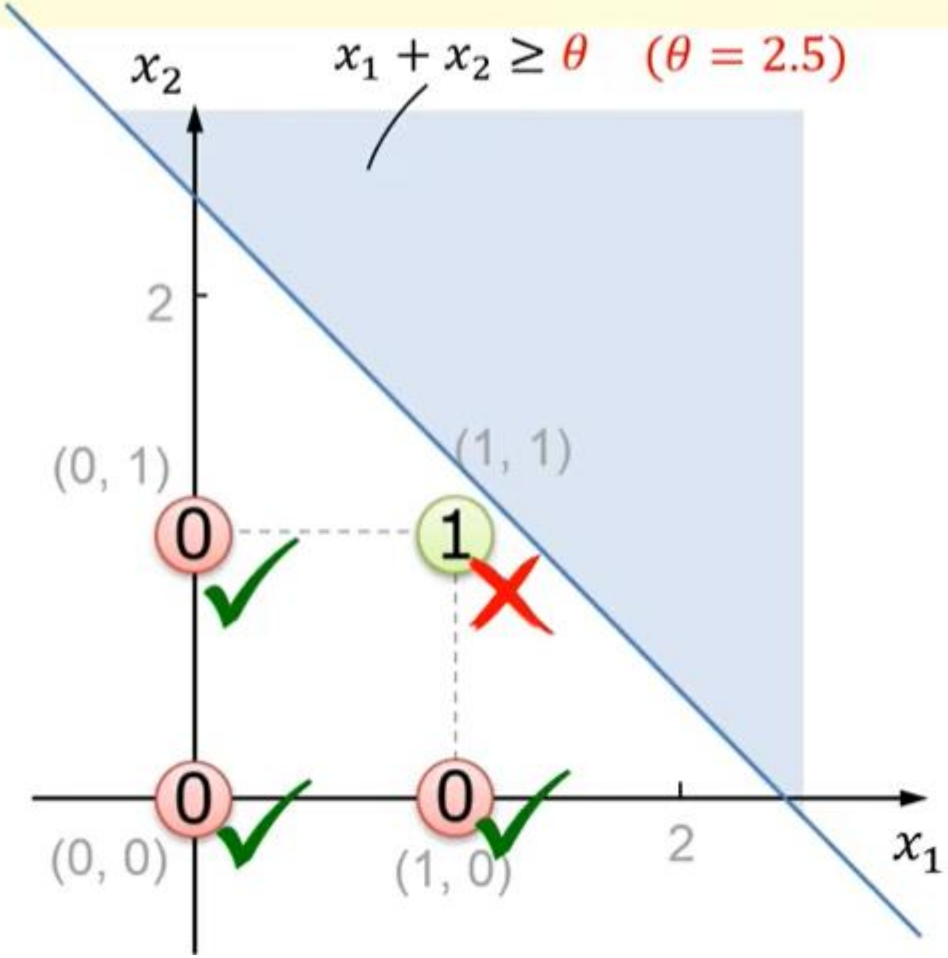
x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0



$$\text{AND}(x_1, x_2) = y$$

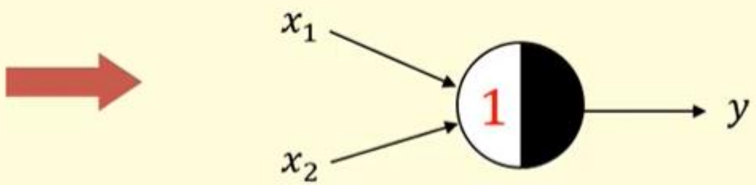


x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

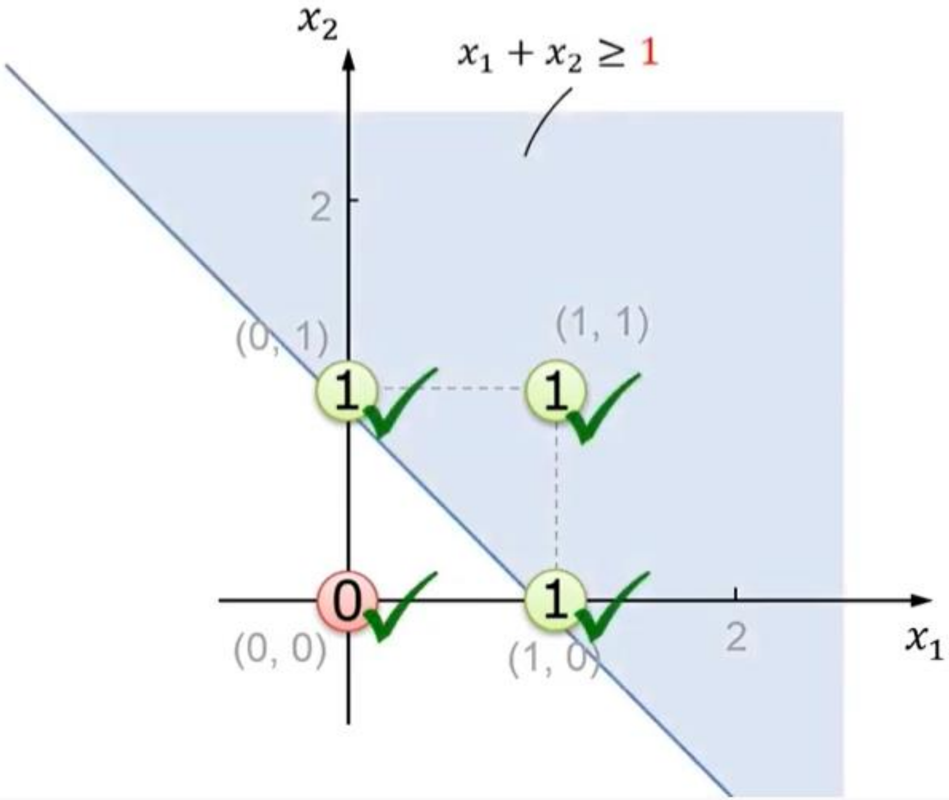


Graphical Interpretation of OR Gate

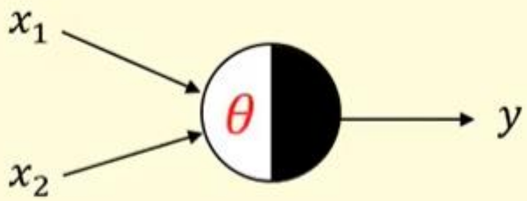
$\mathrm{OR}(x_1, x_2) = y$



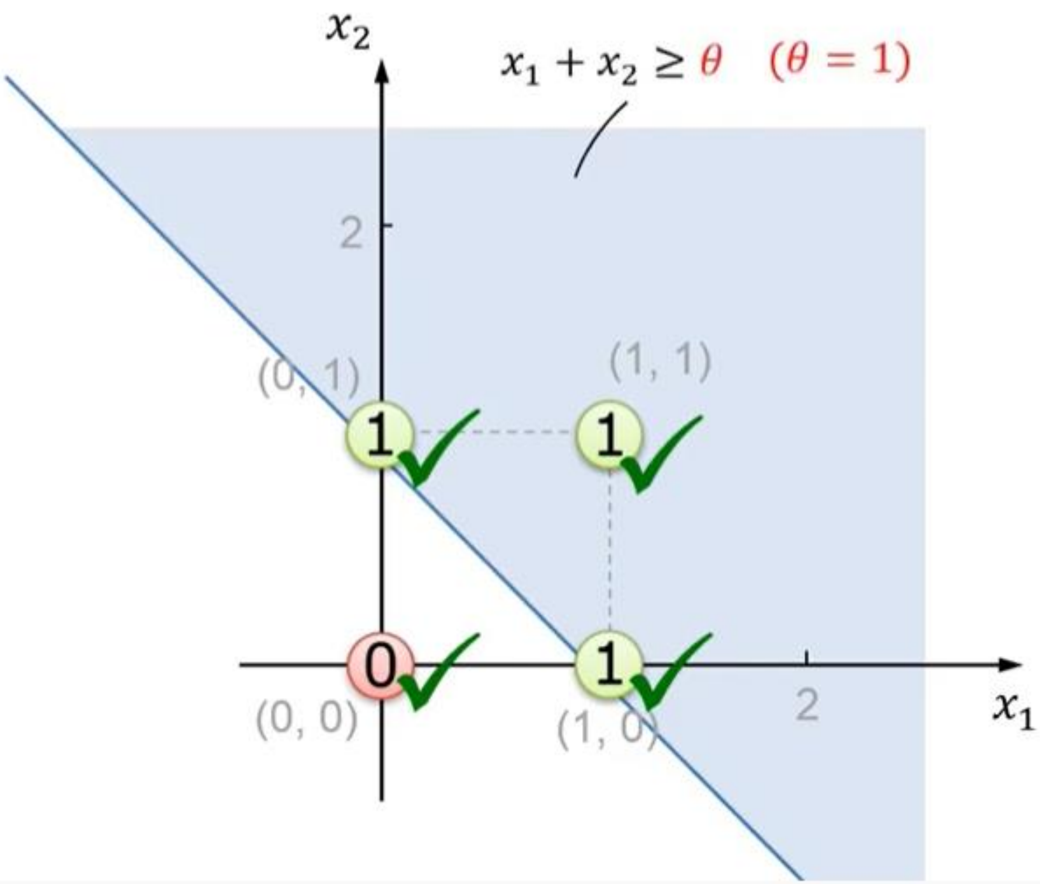
x_1	x_2	y
1	1	1
0	1	1
1	0	1
0	0	0



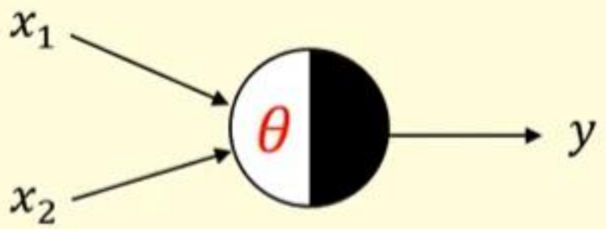
$$\mathrm{OR}(x_1, x_2) = y$$



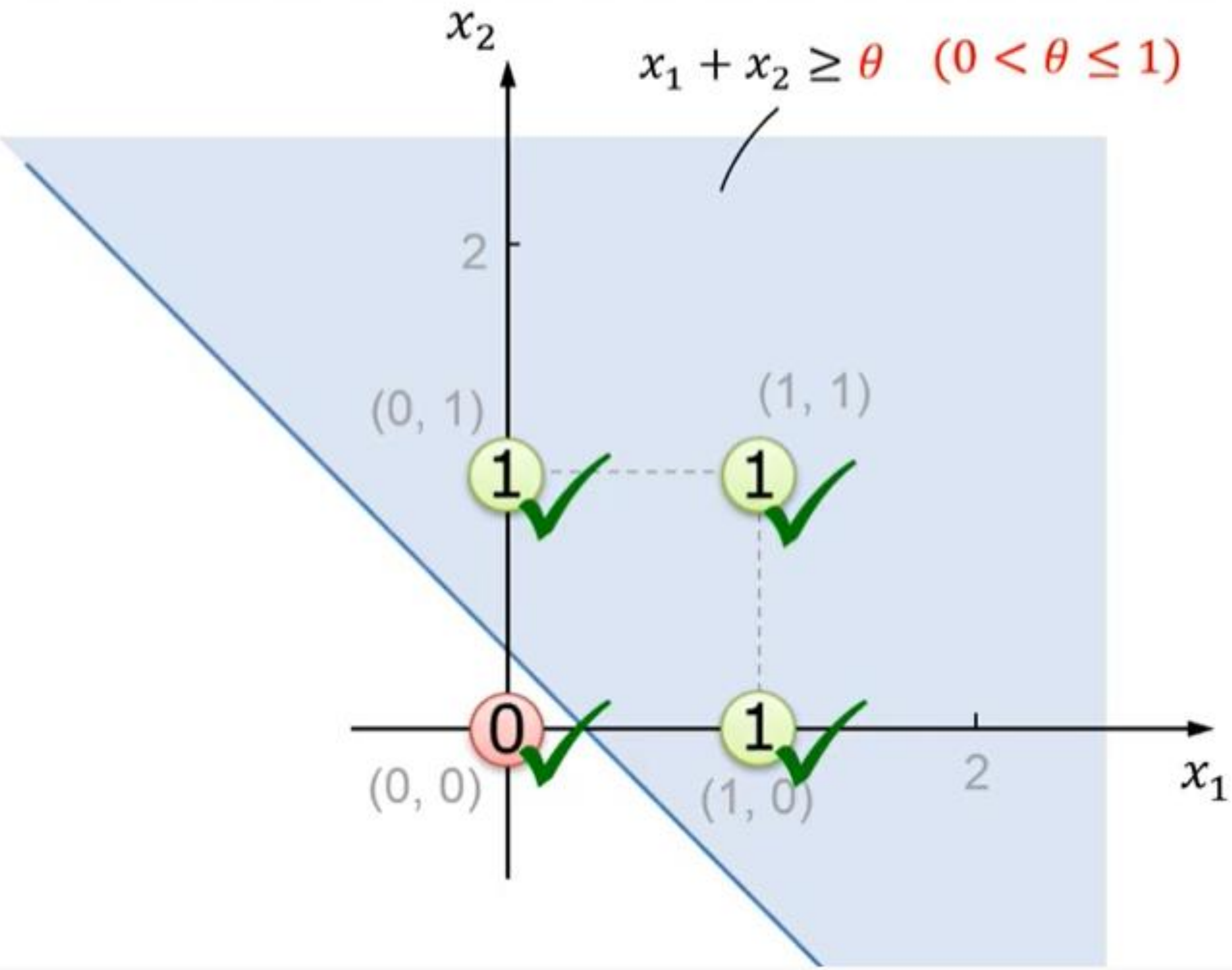
x_1	x_2	y
1	1	1
0	1	1
1	0	1
0	0	0



$$\mathrm{OR}(x_1, x_2) = y$$

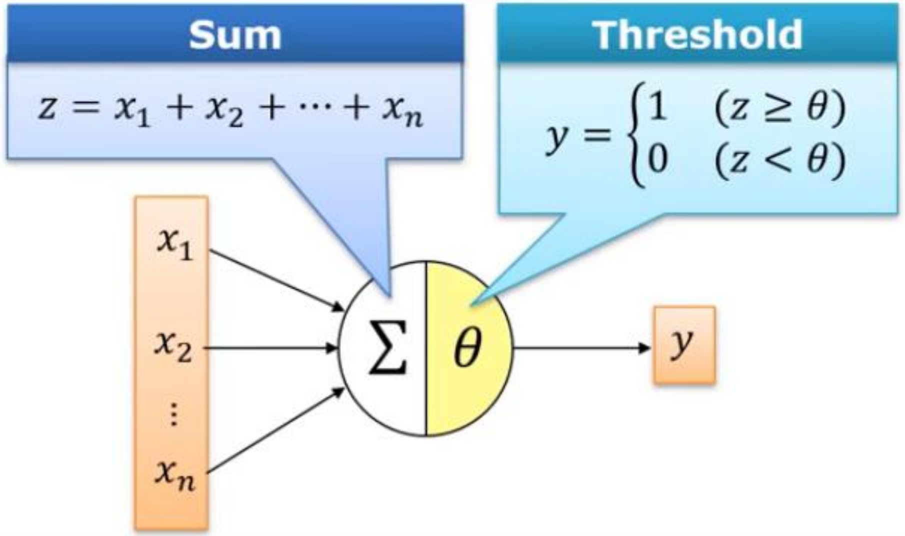


x_1	x_2	y
1	1	1
0	1	1
1	0	1
0	0	0



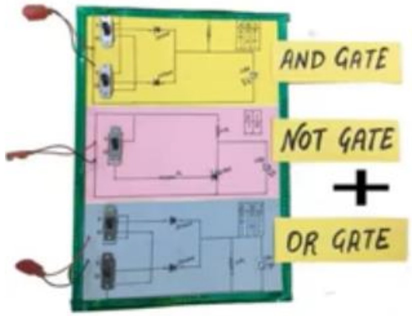
- **Limitations of MP Neuron**
- **Single-Layer Perceptron**
  - A Bio-Inspired Binary Classifier
  - Weights & Bias
  - Computational Model
- **Two Vector Forms of Perceptron**

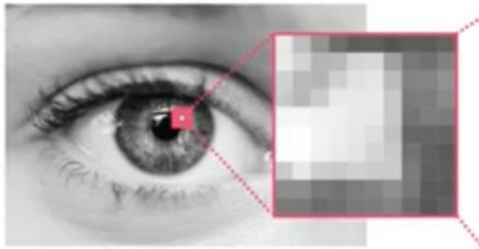
Limitations of MP Neuron



Limitations of MP Neuron

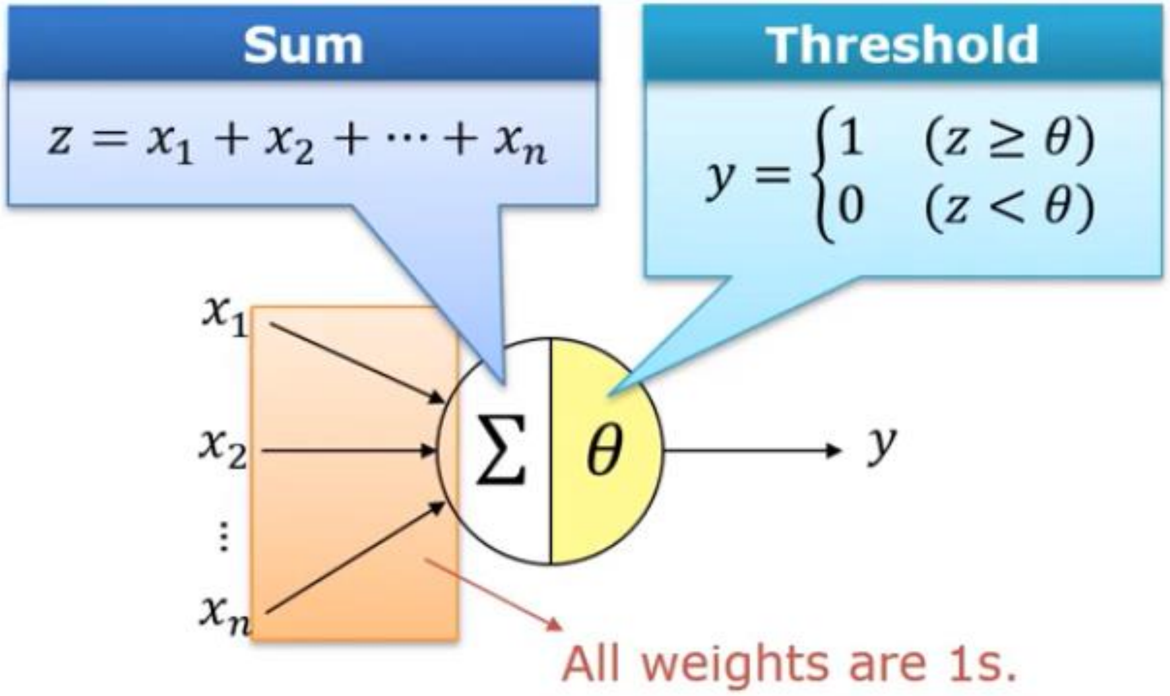
- Inputs and output are limited to binary values only.





230	194	147	108	90	98	84	96	91	101
237	206	188	195	207	213	163	123	116	128
210	183	180	205	224	234	188	122	134	147
198	189	201	227	229	232	200	125	127	135
249	241	237	244	232	226	202	116	125	126
251	254	241	239	230	217	196	102	103	99
243	255	240	231	227	214	203	116	95	91
204	231	208	200	207	201	200	121	95	95
144	140	120	115	125	127	143	118	92	91
121	121	108	109	122	121	134	106	86	97

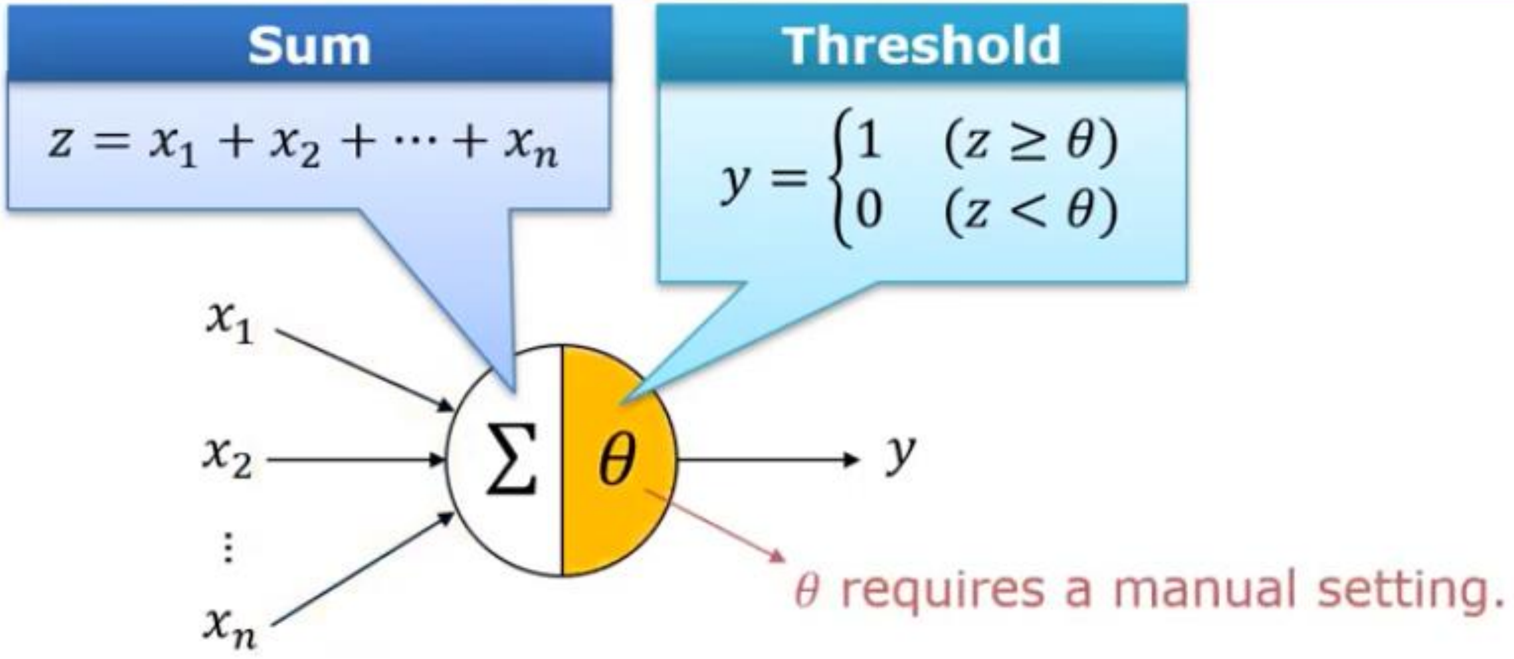
Limitations of MP Neuron



Limitations of MP Neuron

- Inputs and output are limited to binary values only.
- All inputs are treated as equally important. There is no chance to assign more importance to some inputs.

Limitations of MP Neuron



Limitations of MP Neuron

- Inputs and output are limited to binary values only.
- All inputs are treated as equally important. There is no chance to assign more importance to some inputs.
- A manual setting of threshold θ is always required.

Perceptron

Frank Rosenblatt
Psychologist
Cornell University

- Rosenblatt invented Perceptron, a learning model for a single-layer neural net, which was inspired by biological principles and showed an ability to learn.

FIG. 1 — Organization of a biological brain. (Red areas indicate active cells, responding to the letter X.)

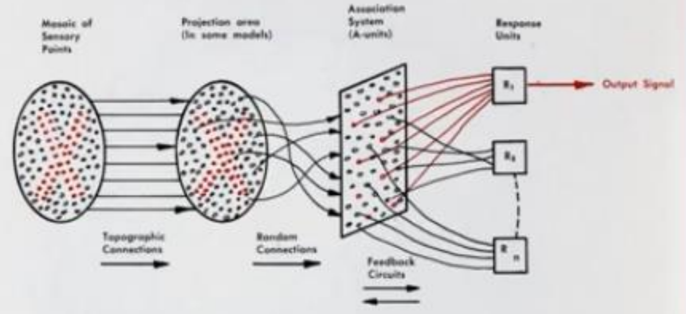
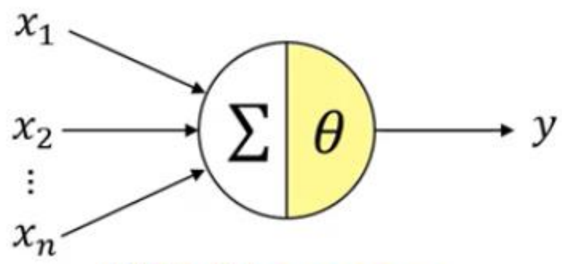


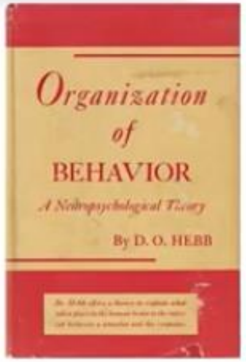
FIG. 2 — Organization of a perceptron.

An image of the perceptron from Rosenblatt's "The Design of an Intelligent Automation", which paved the way for NC.



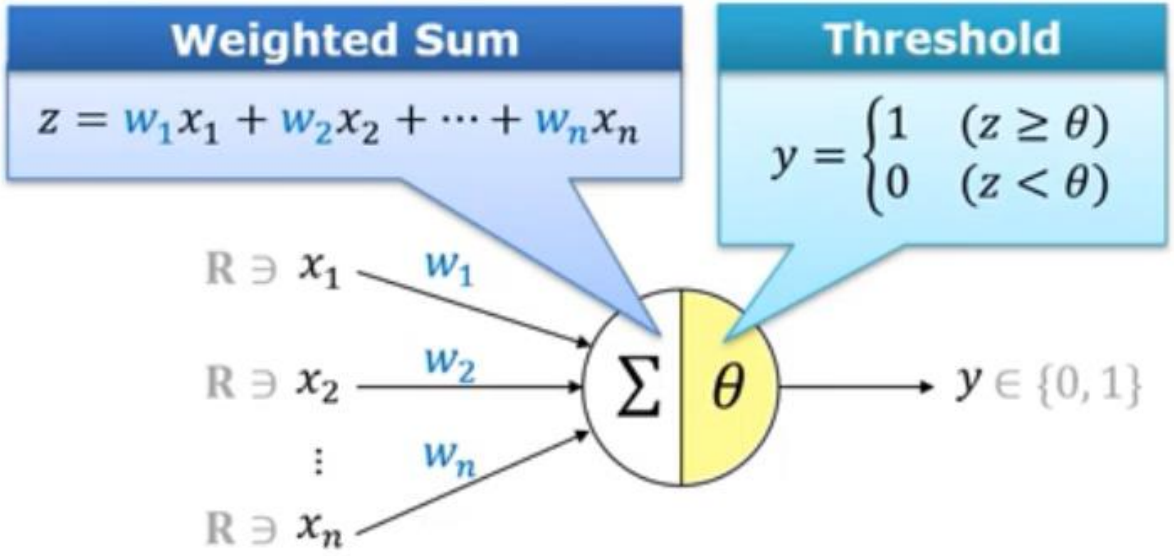
MP Neuron

+



Hebbian Learning Rule

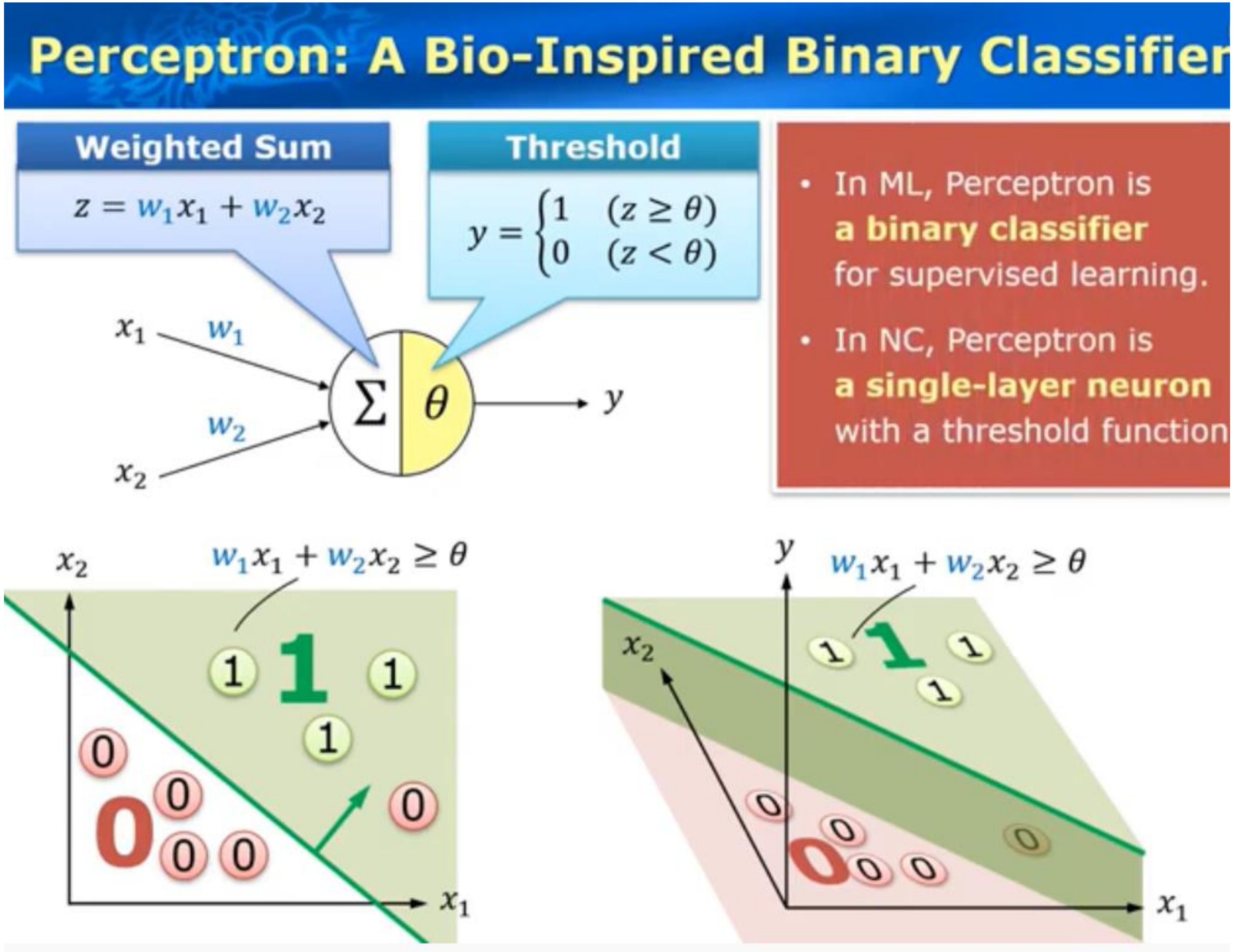
Perceptron vs MP Neuron



Perceptron covers MP neuron as a special case when

- all $x_i \in \{0,1\}$
- all $w_i = 1$
- θ is set manually

	Perceptron	MP Neuron
Inputs	Real numbers	Binary (0 or 1)
Weights	Each input carries a weight (which can be learned)	All inputs are equally important.
Threshold	Can be learned automatically	Manually set by users


Perceptron: A Bio-Inspired Binary Classifier
Weighted Sum
$z = w_1x_1 + w_2x_2$
Threshold
$y = \begin{cases} 1 & (z \geq \theta) \\ 0 & (z < \theta) \end{cases}$
• In ML, Perceptron is a binary classifier for supervised learning.
• In NC, Perceptron is a single-layer neuron with a threshold function
x_1 w_1 x_2 w_2 Σ θ y
x_2 $w_1x_1 + w_2x_2 \geq \theta$ x_1
1 1 1 1
0 0 0 0 0 0 0
y $w_1x_1 + w_2x_2 \geq \theta$ x_2 x_1
1 1 1 1
0 0 0 0 0 0

Bias

Weighted Sum

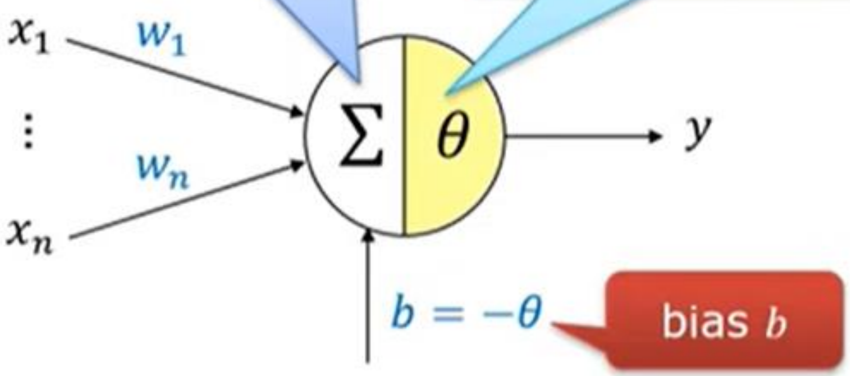
$$z = w_1 x_1 + \cdots + w_n x_n$$

Threshold

$$y = \begin{cases} 1 & (z \geq \theta) \\ 0 & (z < \theta) \end{cases}$$

In a Perceptron:

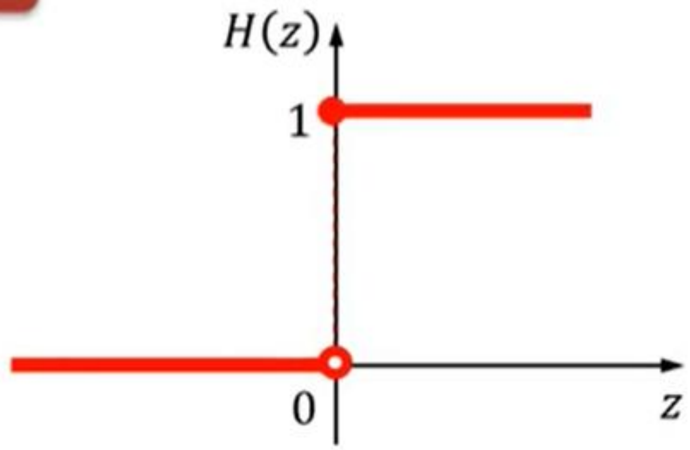
- $(x_1, \cdots, x_n)$ is real-valued;
- $(w_1, \cdots, w_n)$ is real-valued
- The neuron "fires" if the weighted sum $\geq \theta$.



Heaviside Function

$$H(z) = \begin{cases} 1, & (z \geq 0) \\ 0, & (z < 0) \end{cases}$$

$$z = w_1 x_1 + \cdots + w_n x_n - \theta$$



Bias

Weighted Sum

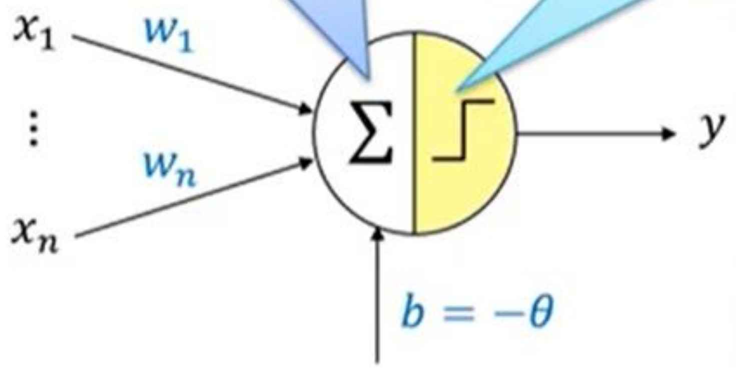
$$z = w_1x_1 + \cdots + w_nx_n + b$$

Threshold

$$y = H(z)$$

In a Perceptron:

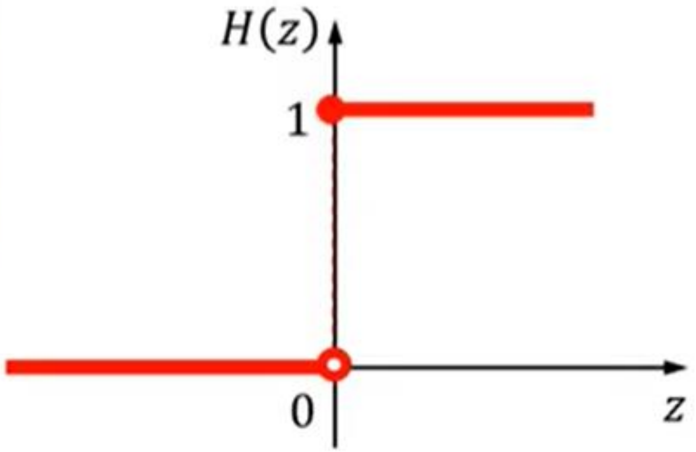
- $(x_1, \cdots, x_n)$ is real-valued;
- $(w_1, \cdots, w_n)$ is real-valued
- The neuron "fires" if the weighted sum $\geq \theta$.



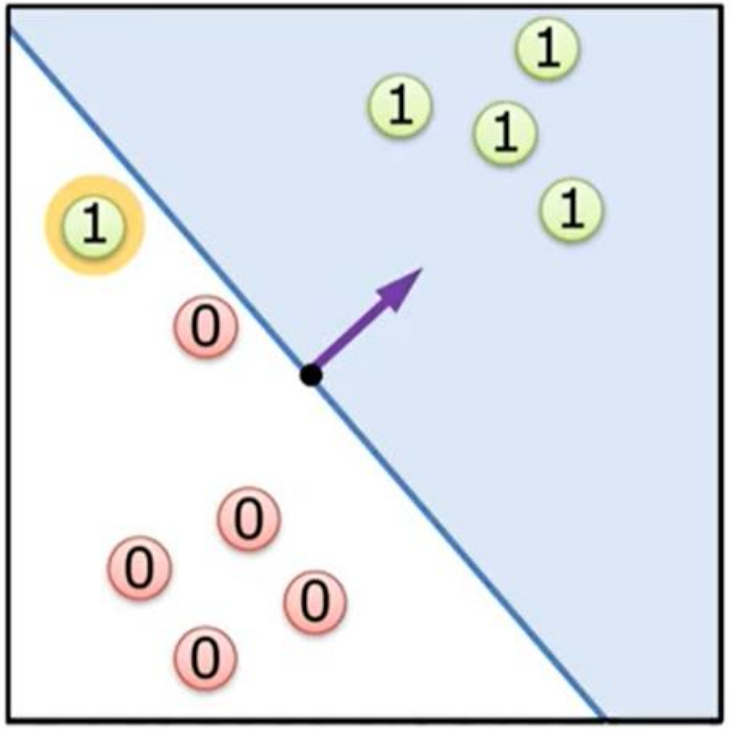
Heaviside Function

$$H(z) = \begin{cases} 1, & (z \geq 0) \\ 0, & (z < 0) \end{cases}$$

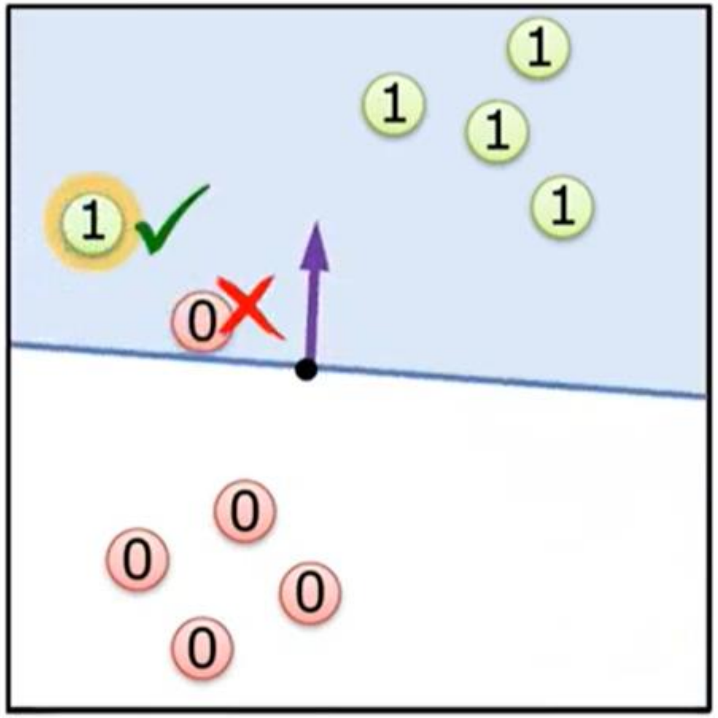
$$z = w_1x_1 + \cdots + w_nx_n - \theta$$



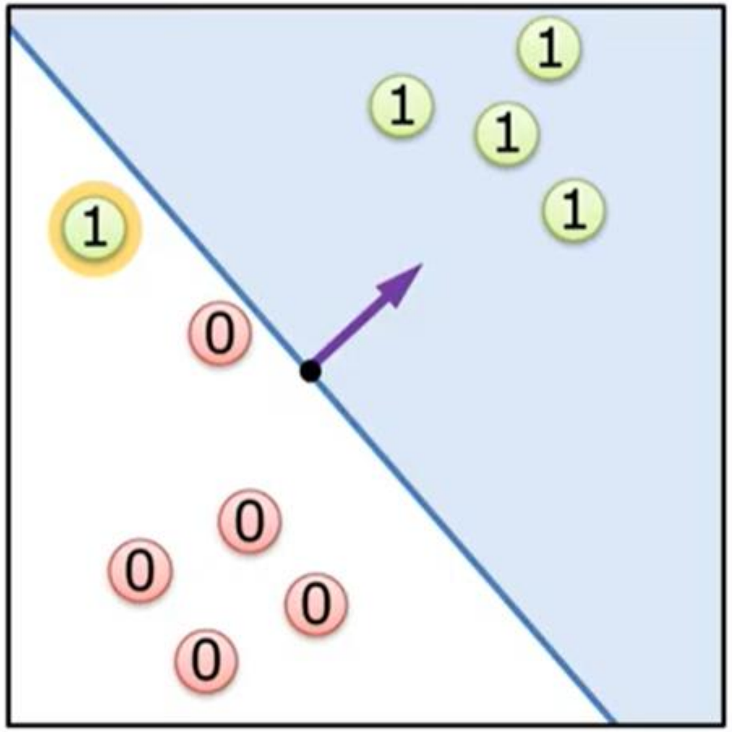
Why Bias is Important



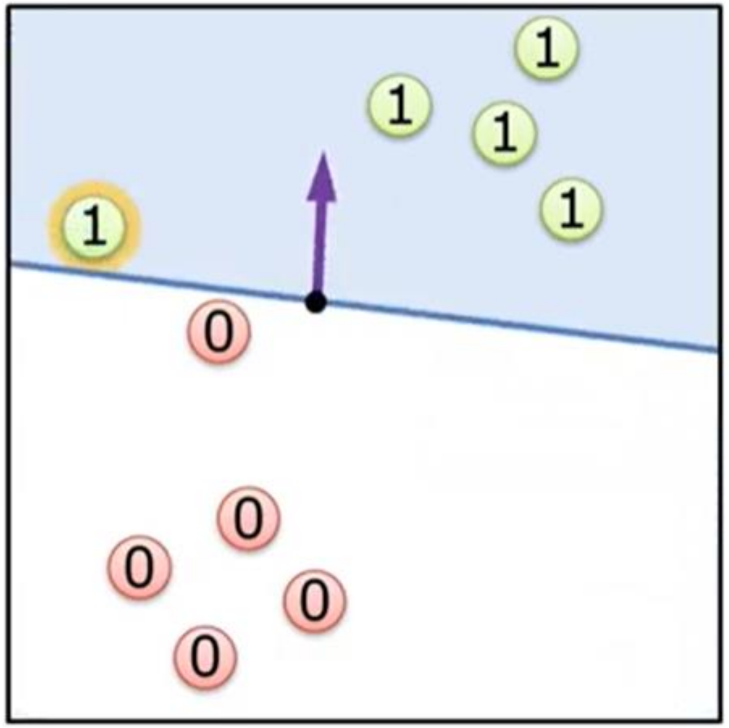
Without Bias



Without Bias



With Bias



With Bias

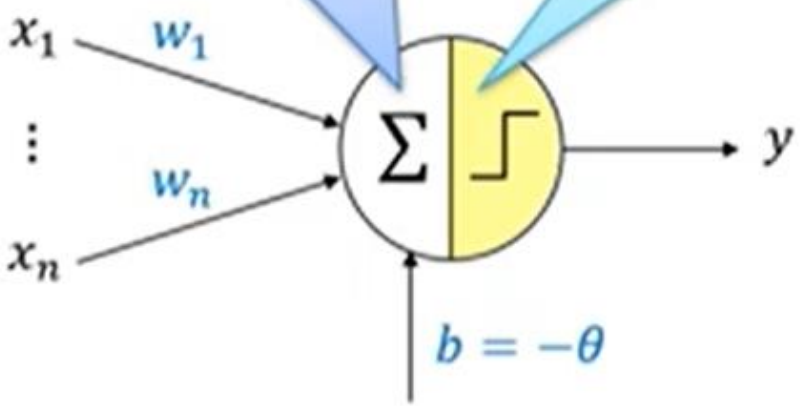
Two Vector Forms of Perceptron

Weighted Sum

$z = \boldsymbol{w} \cdot \boldsymbol{x}$

Threshold

$y = H(z)$



Vector Form 1

Let $\boldsymbol{x} = [x_1, \cdots, x_n]^T$

$\boldsymbol{w} = [w_1, \cdots, w_n]$

Then $y = H(\boldsymbol{w} \cdot \boldsymbol{x} + b)$

Vector Form 2

Let $\boldsymbol{x} = [x_1, \cdots, x_n, b]^T$

$\boldsymbol{w} = [w_1, \cdots, w_n, 1]$

Then $\boxed{y = H(\boldsymbol{w} \cdot \boldsymbol{x})}$

Explain Form 1

Let $\boldsymbol{w} = [w_1, \cdots, w_n]$ $\quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

Thus $\boldsymbol{w} \cdot \boldsymbol{x} + b$

$= w_1 x_1 + \cdots + w_n x_n + b$

Explain Form 2

Let $\boldsymbol{w} = [w_1, \cdots, w_n, b]$ $\quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ 1 \end{bmatrix}$

Thus $\boldsymbol{w} \cdot \boldsymbol{x}$

$= w_1 x_1 + \cdots + w_n x_n + b \times 1$

Exercise

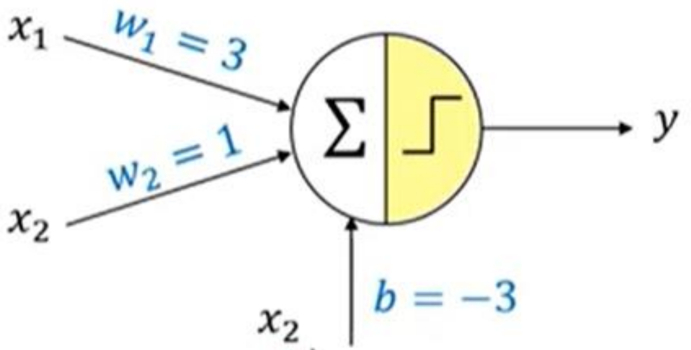
A perceptron has two inputs (x_1, x_2). Given weights $(w_1, w_2) = (3, 1)$ and bias $b = -3$, depict the diagram of this perceptron, and predict its output when $\boxed{(x_1, x_2) = (0, 1)}$ and $(x_1, x_2) = (1, 1)$, respectively.

Vector Form 1

Let $\boldsymbol{x} = [x_1, \cdots, x_n]^T$

$\boldsymbol{w} = [w_1, \cdots, w_n]$

Then $y = H(\boldsymbol{w} \cdot \boldsymbol{x} + b)$

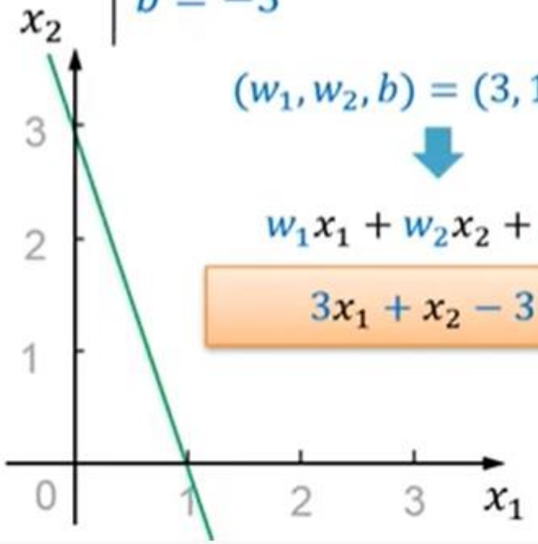


When $\boldsymbol{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\because\ \boldsymbol{w} \cdot \boldsymbol{x} + b = [3, 1] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 3$

$= 3 \times 0 + 1 \times 1 - 3 = -2 < 0$

$\therefore\ y = H(-2) = 0$



$(w_1, w_2, b) = (3, 1, -3)$

$w_1 x_1 + w_2 x_2 + b = 0$

$\boxed{3x_1 + x_2 - 3 = 0}$

Exercise

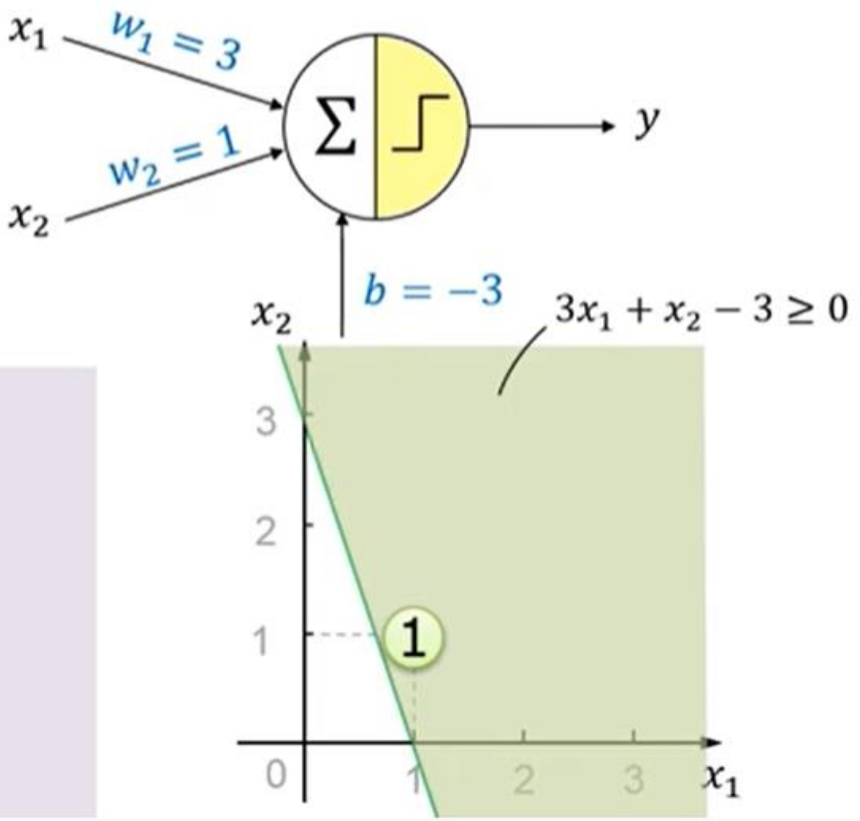
A perceptron has two inputs (x_1, x_2). Given weights $(w_1, w_2) = (3, 1)$ and bias $b = -3$, depict the diagram of this perceptron, and predict its output when $(x_1, x_2) = (0, 1)$ and $\boxed{(x_1, x_2) = (1, 1)}$, respectively.

Vector Form 1

Let $\boldsymbol{x} = [x_1, \cdots, x_n]^T$

$\boldsymbol{w} = [w_1, \cdots, w_n]$

Then $y = H(\boldsymbol{w} \cdot \boldsymbol{x} + b)$



When $\boxed{\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$$\because \ \boldsymbol{w} \cdot \boldsymbol{x} + b = [3, 1] \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 3$$

$$= 3 \times 1 + 1 \times 1 - 3 = 1 \ \geq 0$$

$$\therefore y = H(1) = 1 \ ✓$$