Lecture 4: Model-Free Prediction

Outline

Model-Free Reinforcement Learning

- Last lecture:
  - Planning by dynamic programming
  - Solve a *known* MDP
- This lecture:
  - Model-free prediction
  - Estimate the value function of an *unknown* MDP
- Next lecture:
  - Model-free control
  - Optimise the value function of an *unknown* MDP

Monte-Carlo Reinforcement Learning

- MC methods learn directly from episodes of experience
- MC is *model-free*: no knowledge of MDP transitions / rewards
- MC learns from *complete* episodes: no bootstrapping
- MC uses the simplest possible idea: value = mean return
- Caveat: can only apply MC to *episodic* MDPs
  - All episodes must terminate

Monte-Carlo Policy Evaluation

- Goal: learn v_π from episodes of experience under policy π

$$S_1, A_1, R_2, ..., S_k \sim \pi$$

- Recall that the *return* is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T$$

- Recall that the value function is the expected return:

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s \right]$$

- Monte-Carlo policy evaluation uses *empirical mean* return instead of *expected* return

First-Visit Monte-Carlo Policy Evaluation

- To evaluate state s
- The first time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- By law of large numbers, $V(s) \rightarrow v_{\pi}(s)$ as $N(s) \rightarrow \infty$

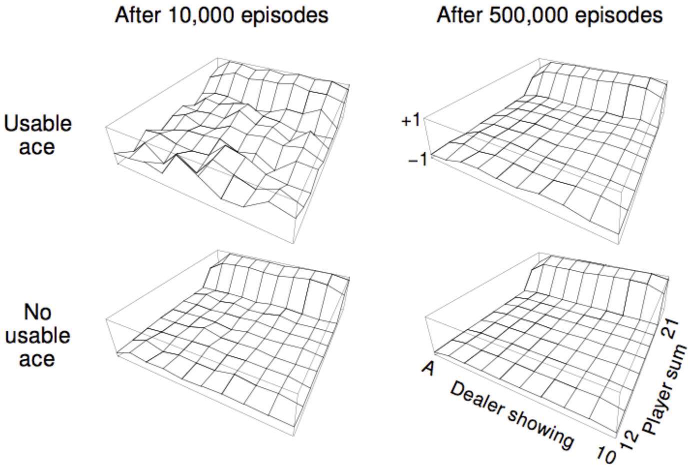
Every-Visit Monte-Carlo Policy Evaluation

- To evaluate state s
- Every time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- Again, $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

Blackjack Example

- States (200 of them):
    - Current sum (12-21)
    - Dealer's showing card (ace-10)
    - Do I have a "useable" ace? (yes-no)
- Action stick: Stop receiving cards (and terminate)
- Action twist: Take another card (no replacement)
- Reward for stick:
    - +1 if sum of cards > sum of dealer cards
    - 0 if sum of cards = sum of dealer cards
    - -1 if sum of cards < sum of dealer cards
- Reward for twist:
    - -1 if sum of cards > 21 (and terminate)
    - 0 otherwise
- Transitions: automatically twist if sum of cards < 12

Blackjack Value Function after Monte-Carlo Learning



Policy: stick if sum of cards ≥ 20, otherwise twist

Incremental Mean

The mean $\mu_1, \mu_2, \ldots$ of a sequence $x_1, x_2, \ldots$ can be computed incrementally,

$$
\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j \\
&= \frac{1}{k}\left(x_k + \sum_{j=1}^{k-1} x_j\right) \\
&= \frac{1}{k}\left(x_k + (k-1)\mu_{k-1}\right) \\
&= \mu_{k-1} + \frac{1}{k}\left(x_k - \mu_{k-1}\right)
\end{aligned}
$$

Incremental Monte-Carlo Updates

- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, ..., S_T$
- For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$

$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

- In non-stationary problems, it can be useful to track a running mean, i.e. forget old episodes.

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

Algorithm 1: First-Visit MC Prediction

Input: policy π, positive integer *num_episodes*

Output: value function V ($\approx v_\pi$, if *num_episodes* is large enough)

Initialize $N(s) = 0$ for all $s \in \mathcal{S}$

Initialize Returns$(s) = 0$ for all $s \in \mathcal{S}$

for *episode* $e \leftarrow 1$ ***to*** $e \leftarrow$ *num_episodes* **do**

  Generate, using π, an episode $S_0, A_0, R_1, S_1, A_1, R_2 \ldots, S_{T-1}, A_{T-1}, R_T$

  $G \leftarrow 0$

  for *time step* $t = T - 1$ ***to*** $t = 0$ *(of the episode e)* **do**

    $G \leftarrow G + R_{t+1}$

    if *state* S_t *is* ***not*** *in the sequence* $S_0, S_1, \ldots, S_{t-1}$ **then**

      Returns$(S_t) \leftarrow$ Returns$(S_t) + G_t$

      $N(S_t) \leftarrow N(S_t) + 1$

  end

end

$V(s) \leftarrow \frac{\text{Returns}(s)}{N(s)}$ for all $s \in \mathcal{S}$

return V

Algorithm 2: Every-Visit MC Prediction

Input: policy π, positive integer *num_episodes*

Output: value function V ($\approx v_\pi$, if *num_episodes* is large enough)

Initialize $N(s) = 0$ for all $s \in \mathcal{S}$

Initialize $\text{Returns}(s) = 0$ for all $s \in \mathcal{S}$

for *episode* $e \leftarrow 1$ ***to*** $e \leftarrow$ *num_episodes* **do**

  Generate, using π, an episode $S_0, A_0, R_1, S_1, A_1, R_2 \ldots, S_{T-1}, A_{T-1}, R_T$

  $G \leftarrow 0$

  for *time step* $t = T-1$ ***to*** $t = 0$ *(of the episode e)* **do**

    $\boxed{\begin{array}{l} G \leftarrow G + R_{t+1} \\ \text{Returns}(S_t) \leftarrow \text{Returns}(S_t) + G_t \\ N(S_t) \leftarrow N(S_t) + 1 \end{array}}$

  end

end

$V(s) \leftarrow \frac{\text{Returns}(s)}{N(s)}$ for all $s \in \mathcal{S}$

return V

Temporal-Difference Learning

- TD methods learn directly from episodes of experience
- TD is *model-free*: no knowledge of MDP transitions / rewards
- TD learns from *incomplete* episodes, by *bootstrapping*
- TD updates a guess towards a guess

MC and TD

- Goal: learn v_π online from experience under policy π
- Incremental every-visit Monte-Carlo
  - Update value $V(S_t)$ toward *actual* return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$

- Simplest temporal-difference learning algorithm: TD(0)
  - Update value $V(S_t)$ toward *estimated* return $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$

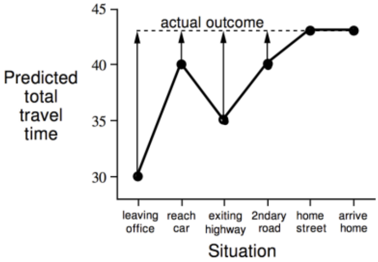
  - $R_{t+1} + \gamma V(S_{t+1})$ is called the *TD target*
  - $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called the *TD error*

Driving Home Example

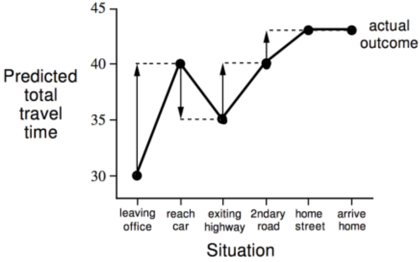
State	Elapsed Time (minutes)	Predicted Time to Go	Predicted Total Time
leaving office	0	30	30
reach car, raining	5	35	40
exit highway	20	15	35
behind truck	30	10	40
home street	40	3	43
arrive home	43	0	43

Driving Home Example: MC vs. TD

Changes recommended by Monte Carlo methods $(\alpha=1)$



Changes recommended! by TD methods $(\alpha=1)$



Advantages and Disadvantages of MC vs. TD

- TD can learn *before* knowing the final outcome
  - TD can learn online after every step
  - MC must wait until end of episode before return is known
- TD can learn *without* the final outcome
  - TD can learn from incomplete sequences
  - MC can only learn from complete sequences
  - TD works in continuing (non-terminating) environments
  - MC only works for episodic (terminating) environments

Bias/Variance Trade-Off

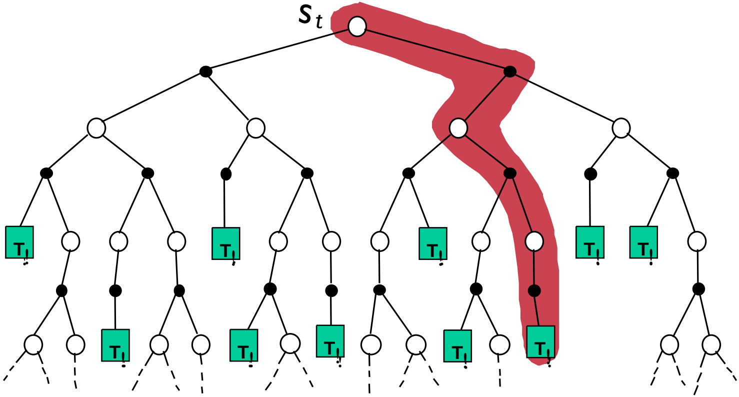
- Return $G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$ is *unbiased* estimate of $v_\pi(S_t)$
- True TD target $R_{t+1} + \gamma v_\pi(S_{t+1})$ is *unbiased* estimate of $v_\pi(S_t)$
- TD target $R_{t+1} + \gamma V(S_{t+1})$ is *biased* estimate of $v_\pi(S_t)$
- TD target is much lower variance than the return:
  - Return depends on *many* random actions, transitions, rewards
  - TD target depends on *one* random action, transition, reward

Advantages and Disadvantages of MC vs. TD (2)

- MC has high variance, zero bias
  - Good convergence properties
  - (even with function approximation)
  - Not very sensitive to initial value
  - Very simple to understand and use
- TD has low variance, some bias
  - Usually more efficient than MC
  - TD(0) converges to $v_\pi(s)$
  - (but not always with function approximation)
  - More sensitive to initial value

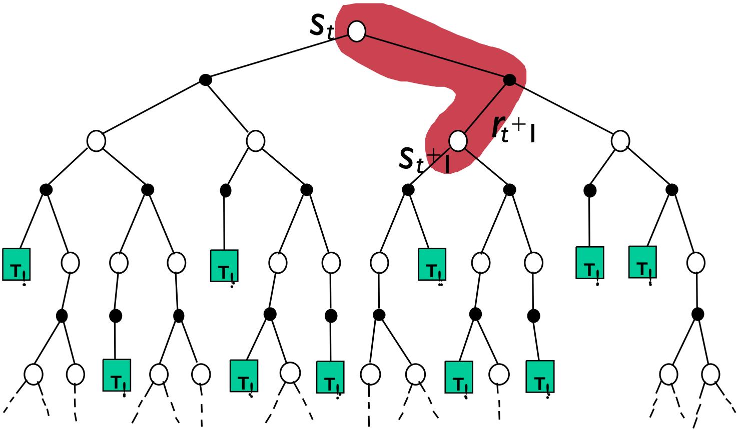
Monte-Carlo Backup

$$V(S_t) \leftarrow \mathbb{E}_\pi \left[R_{t+1} + \gamma V(S_{t+1})\right]$$



Temporal-Difference Backup

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$



Dynamic Programming Backup

$$V(S_t) \leftarrow \mathbb{E}_\pi \left[R_{t+1} + \gamma V(S_{t+1})\right]$$