Risk Management 1
46-954
Homework #1

Due: Monday, September 11, 2017, 5:30pm

Problem 1 Practice on VaR versus Shortfall VaR

Suppose we have a portfolio of n stocks, with price processes $\{S_{i,t},\ t \geq 0, 1 \leq i \leq n\}$. Assume that the stocks all follow a multivariate geometric Brownian motion with mean vector μ and covariance matrix Σ (although in parts of this problem the usual normal random variable is replaced by a t random variable, so it is not really GBM). Now suppose the portfolio consists of λ_i shares of stock i, $1 \leq i \leq n$. Then the portfolio values at time 0 and at time Δ are $V(0) = \sum_{i=1}^n \lambda_i S_{i,0}$ and $V(\Delta) = \sum_{i=1}^n \lambda_i S_{i,0} \exp(\mu_i \Delta + \sigma_i \sqrt{\Delta} Z_i)$ respectively, although the latter formula assumes both that a) the underlying stocks are independent and b) one needs only to multiply the standard normals Z_i by $\sqrt{\Delta}$. Actually, using 1-day time steps, the formula should have the factor $\sigma(Z_{i,1} + \ldots + Z_{i,\Delta})$ where the $Z_{i,j}$ random variables are i.i.d. standard normal. In the normal case, the sum would have the same distribution as $\sqrt{\Delta}$ times a standard normal random variable. However, if the 1-day increments had a t distribution, then the resulting Δ-day increment, $\sigma(T_1 + \ldots + T_\Delta)$, would not have the same distribution as a t random variable multiplied by $\sqrt{\Delta}$. Now the loss is given by $L = V(0) - V(\Delta) = \sum_{i=1}^n \lambda_i S_{i,0}(1 - \exp(\mu_i \Delta + \sigma_i \sqrt{\Delta} Z_i))$. Linearizing this expression gives the approximate loss to be $L = -\sum_{i=1}^n \lambda_i S_{i,0}(\mu_i \Delta + \sigma_i \sqrt{\Delta} Z_i)$. Use this linearization in the problems below. As mentioned in class, the sum of i.i.d. t random variables does not have any common distribution, hence for some of the problems below, simulation methods will need to be employed. Note that the formulas and problems can be easily generalized to non-zero mean return and non-diagonal covariance matrix. Note that when dealing with the Student t distribution, that standard version of this distribution is centered at 0; however, the variance is $\nu/(\nu - 2)$ rather than 1 where ν is the degree of freedom parameter. Hence if one wants to achieve a particular volatility parameter, one needs to choose σ so that $\sigma\nu/(\nu - 2)$ equals the required value.

a) Consider a single underlying security ($n = 1$) and suppose that you have a position worth \$10,000 in it. Assume that the log returns are 1) normally distributed or 2) t-distributed with 4 degrees of freedom. Assume in either case that the mean, μ, is 0 and the volatility is 20% per year, i.e. $.2/\sqrt{250}$ per day. Find VaR and the expected shortfall, E_S, for a one day loss for $\alpha = .90, .95, .975, .99, \text{and } .995$. Comment on the differences between normal and t and between VaR and expected shortfall.

b) Consider the same setup as part a); however, assume your portfolio consists of \$1,000 positions in 10 stocks, the returns for each are assumed to be independent (Σ is diagonal) with the same marginal distributions (normal and t_4) as the stock in described in part a). Using the same values for α, work out one day VaR and shortfall risk for the same values of α in part a). Compare these results with those of part a).

c) Some risk management books say that one can calculate the standard 10 day 99% VaR by taking the 1 day 99% VaR and multiplying it by $\sqrt{10}$. Using the same log return assumptions given in a) and assuming daily log returns are i.i.d calculate the 10 day 99% VaR and the associated expected shortfall assuming 1) a normal and 2) a t-distribution with 4 degrees of freedom. Note the comment at the start of the problem that the formula for the loss will need to be modified for the t distribution. Specifically the log returns will be of the form $\mu\Delta + \sigma(T_1 + \ldots + T_{10})$ where T_i are i.i.d. with appropriate t distribution and $\mu = 0$.

Problem 2 The asymptotic distribution of the sample quantile

For parameters N, n and α, generate N samples of n observations from a distribution with cdf F.

Imagine that each of these samples represents a set of n observations from the loss distribution F. For each of these samples, find the order statistic corresponding to VaR_α, i.e. $\lceil n\alpha \rceil$. With these data also find the estimate of the shortfall risk for each of the n samples.

a) Assume that $N = 1,000$, $\alpha = .99$, and $F = N(0,1)$. Generate N values of the estimate of VaR_α each based on a sample of size n. Calculate the mean and standard deviation of the N values and the theoretical mean and variance and give normal plots for $n = 100, 1,000, 10,000$

b) Repeat part a) assuming F is a t-distribution with $\nu = 5$.

c) Using the same data, give normal plots of the N values of the empirical shortfall risk for both the normal and t-distributions used in parts a) and b) also for $n = 100, 1,000, 10,000$. Give the sample mean and standard deviations, but don't worry about the theoretical values for the mean and standard deviation.

Problem 3 Practice on Delta-Gamma Approximation for VaR
Section 9.1.2 of Glasserman's text discusses the delta-gamma approximation to Var. Focus on the middle two paragraphs on page 488 from which Figure 9.1 on page 489 is generated.

Consider the 10 underlying assets and 15 options written on each as described. Assume all options are initially at-the-money with maturity of .10 year. Assuming that the underlying assets are uncorrelated and their price changes are normally distributed with mean 0 and volatility .40. Assume the riskless interest rate is .05, the current price of all the underlying stocks is 100, and all the options are at the money. Using the setup described by Glasserman, generate a set of 1000 scenarios and determine the actual loss over .04 years and the delta-gamma approximation to that loss using the Greeks computed from the Black Scholes formula. Plot a figure equivalent to Figure 9.1 and extimate VaR for $\alpha = .99$ from the empirical sample quartile.

Problem 4 (FRM Exam Question)
If stock returns are independently identically normally distributed and the annual volatility is 30%, then the daily VaR at the 99% confidence level of a stock market portfolio is approximately:

a) 2.41%

b) 3.11%

c) 4.4%

d) 1.89%

Problem 5 (FRM Exam Question)
Assume the true distribution of returns is leptokurtotic. If we assume normality when we calculate the VaR, then which of the following statements is true?

a) The 95% VaR is overstated.

b) The 95% VaR is understated.

c) The 95% VaR is appropriate

d) We cannot state the relationship between the true VaR and the calculated VaR.

Problem 6 (FRM Exam Question)
A trading book consists of the following two assets, with correlation of 0.2.

Asset	Expected Return	Annual Volatility	Value
A	10%	25%	$100
B	20%	20%	$50

How would the daily VaR at the 99% level change if the bank sells $50 worth of A and buys $50 worth of B? Assume a normal distribution and 250 trading days.

a) 0.2286

b) 0.4571

c) 0.7705

d) 0.7798

Problem 7 (FRM Exam Question)
A large international bank has a trading book whose size depends on the opportunities perceived by its traders. The market risk manager estimates the one-day VaR, at the 95% confidence level, to be $50M. You are asked to evaluate how good a job the manager is doing in estimating the one-day VaR. Which of the following would be the most convincing evidence that the manager is doing a poor job, assuming that losses are i.i.d.?

a) Over the past 250 days, there are eight exceptions.

b) Over the past 250 days, the largest loss is $500M.

c) Over the past 250 days, the mean loss is $60M.

d) Over the past 250 days, there is no exception.

Problem 8 (FRM Exam Question)
Backtesting routinely compares daily profits and losses with model-generated risk measures to gauge the quality and accuracy of their risk measurement systems. The 1996 Market Risk Amendment describes the backtesting framework that is to accompany the internal models capital requirement. This backtesting framework involves:

I) The size of outliers.

II) The use of a risk measure calibrated to a one-day holding period.

III) The size of outliers for a risk measure calibrated to a 10-day holding period.

IV) The number of outliers.

a) II and III

b) II only

c) I and II

d) II and IV