01NAEX - Lecture 08
The Fractional Factorial Design

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The Fractional Factorial Design (FrFD)

Learning objectives for this lesson:

- Understanding the application of fractional factorial designs.
- Becoming familiar with the terms: design generator, alias structure, and design resolution.
- Knowing how to analyze fractional factorial designs when there are not enough degrees of freedom for error estimation.

Note: FrFD save up to 75% of the experimental runs compared to full FD, making them a popular choice in process optimization.

The Fractional Factorial Design - Introduction

If the number k of factors of interest is very large, the size of the 2^k factorial design grows exponentially.

- Many variables are often involved when little is known about the system.
- The primary interest lies in identifying the factors with significant effects.
- These designs are frequently run as unreplicated factorials but may include center points to check for curvature.

Note:

- The FrFD helps address cost and resource constraints, enabling a systematic approach to screening important factors.
- FrFD helps reduce the number of experiments in industries, where resources are costly.

The Fractional Factorial Design - Key Principles

Fractional factorial design relies on three fundamental principles:

- **The Sparsity of Effects Principle:**
  Most systems are dominated by main effects and low-order interactions. Only a few factors significantly influence the outcome.
- **The Projection Property:**
  Each fractional factorial design contains full factorial designs for a subset of factors, allowing further detailed study if necessary.
- **Sequential Experimentation:**
  It is possible to add runs to a fractional factorial design to refine analysis or resolve ambiguities. It also supports adaptive learning during the experimental process.

The One-Half Fraction of the 2^k Factorial Design

The one-half fraction of the 2^k factorial design consists of $\frac{2^k}{2}$ runs and is represented as a 2^{k-1} design.

Example: Consider a 2^3 factorial design:

- Selecting combinations based on a defining generator, e.g., $ABC = I$.
- An alternative fraction uses the generator $ABC = -I$.

Treatment Combination	Factorial Effect							
	I	A	B	C	AB	AC	BC	ABC
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
c	+	−	−	+	+	−	−	+
abc	+	+	+	+	+	+	+	+
ab	+	+	+	−	+	−	−	−
ac	+	+	−	+	−	+	−	−
bc	+	−	+	+	−	−	+	−
(1)	+	−	−	−	+	+	+	−

- Generators helps identify which effects are aliased, meaning we cannot differentiate between certain main effects and interactions.
- Alias structures act like 'aliases' or alternative names in mathematics and coding, they represent the same underlying data.

The One-Half Fraction of the 2^3 Factorial Design

Treatment Combination	Factorial Effect							
	I	A	B	C	AB	AC	BC	ABC
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
c	+	−	−	+	+	−	−	+
abc	+	+	+	+	+	+	+	+

In a 2^{3-1} design, we have 4 observations, giving us 3 degrees of freedom for estimating effects:

$$
\begin{aligned}
A = BC &= \frac{1}{2}(a - b - c + abc) \\
B = AC &= \frac{1}{2}(b - a - c + abc) \\
C = AB &= \frac{1}{2}(c - a - b + abc)
\end{aligned}
$$

Alias Concept:

- It is impossible to separate the effects of A and BC, B and AC, or C and AB. This leads to 'aliasing' of main effects with two-way interactions.
- 2^{k-1} design is limited when higher-order interactions are significant, as we only estimate combined effects like $A + BC$.

The Defining Relation and Aliases in the 2^{3-1} Design

In the 2^{3-1} design, ABC serves as the defining relation: $I = ABC$ (or alternatively $I = -ABC$). This generator determines the alias structure in the design.

- With relation $I = ABC$, we can find aliases for each factor.
- For instance:

$$\begin{aligned} A \cdot I &= A \cdot ABC = A^2BC = BC \\ B \cdot I &= B \cdot ABC = AB^2C = AC \\ C \cdot I &= C \cdot ABC = ABC^2 = AB \end{aligned}$$

- Running the alternate fraction (e.g., generated by $ABC = -I$) allows us to combine the two designs into a full factorial, resolving aliasing between effects.
- Alternate fractions and sequential experimentation help clarify confounded effects, enabling more robust conclusions.

Design Resolution

Definition:
A design is of **resolution** R if no p-factor effect is aliased with any effect containing fewer than $R - p$ factors.

Resolution Classification:

- **Resolution III:** No main effects are aliased with each other, but they are aliased with two-factor interactions.
- **Resolution IV:** No main effect is aliased with other main effects or with two-factor interactions, but two-factor interactions are aliased with each other.
- **Resolution V:** No main effect or two-factor interaction is aliased with other main effects or two-factor interactions, though two-factor interactions may be aliased with three-factor interactions.

Design Resolution

Simple definition:
The resolution is called Resolution R Design, if the generator $I = ABCD\cdots$ has R letters. For example in 2^{3-1} design with $I = ABC$ generator we have Resolution III Design and the main effects are confounded with 2-way interactions.

- Higher resolutions provide clearer interpretation by minimizing aliasing of main effects and low-order interactions.
- Resolution III designs are common for initial screening, while Resolution V designs are preferred for more detailed studies.
- Resolution is often analogized with camera resolution - the higher it is, the clearer the 'picture' of factor relationships.
- Generally, we want the highest resolution possible, and construct fractional factorial with highest order interaction.

Constructing the One-Half Fraction of the 2^k Design

One method to construct a one-half fraction of the 2^k design with the highest resolution is by using a basic full 2^{k-1} design and then adding a kth factor with levels defined by the signs of the highest-order interaction term.

Example: To construct a one-half fraction of the 2^3 design:

	Full 2^2 Factorial (Basic Design)		2_{III}^{3-1}, $I = ABC$			2_{III}^{3-1}, $I = -ABC$		
Run	A	B	A	B	$C = AB$	A	B	$C = -AB$
1	−	−	−	−	+	−	−	−
2	+	−	+	−	−	+	−	+
3	−	+	−	+	−	−	+	+
4	+	+	+	+	+	+	+	−

- Choosing a different interaction for the generator will generally lower the design's resolution. The highest resolution is achieved when the generator is the highest-order interaction.
- When constructing a fractional factorial design, carefully choose the defining generator to maximize resolution and ensure that important effects are not aliased.

2^{4-1}_{IV} Design with the Defining Relation $I = ABCD$

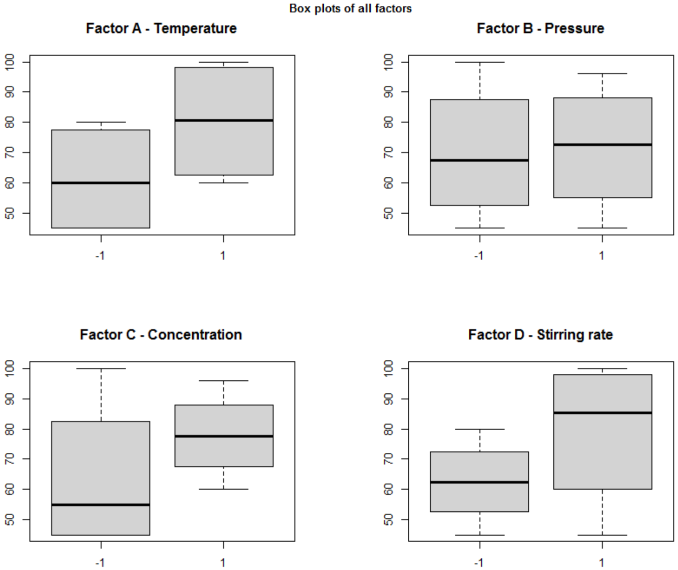
Consider a 2^{4-1}_{IV} design for a filtration rate experiment. If A, C, D, and interactions AC and AD are significant, then simulating a 2^{4-1}_{IV} design can reveal the alias structure.

	Basic Design					
Run	A	B	C	$D = ABC$	Treatment Combination	Filtration Rate
1	−	−	−	−	(1)	45
2	+	−	−	+	*ad*	100
3	−	+	−	+	*bd*	45
4	+	+	−	−	*ab*	65
5	−	−	+	+	*cd*	75
6	+	−	+	−	*ac*	60
7	−	+	+	−	*bc*	80
8	+	+	+	+	*abcd*	96

- The design has Resolution IV, as the main effects A, B, C, and D are aliased with three-way interactions. This makes it useful for studying main effects and two-factor interactions.
- The 'IV' in 2^{4-1}_{IV} means this design achieves a balance between economy and accuracy.
- What is the generator and alias structure of this design?

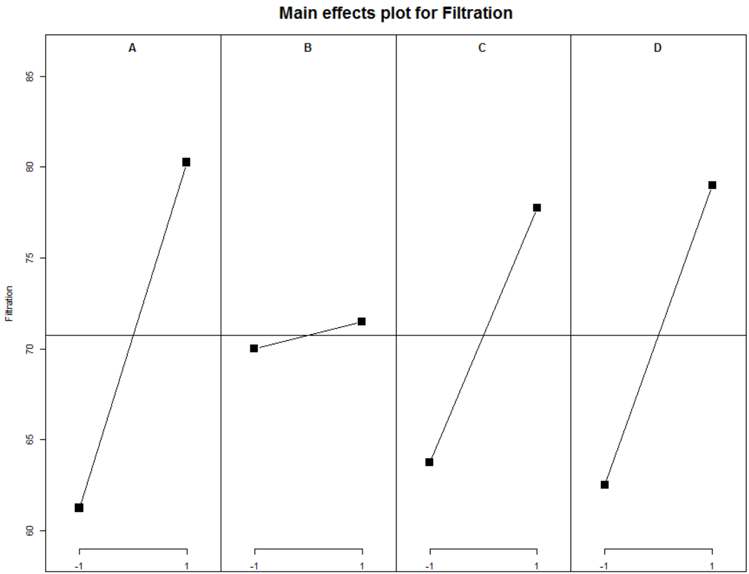
One-half 2^4 filtration rate experiment

Plot of effects:



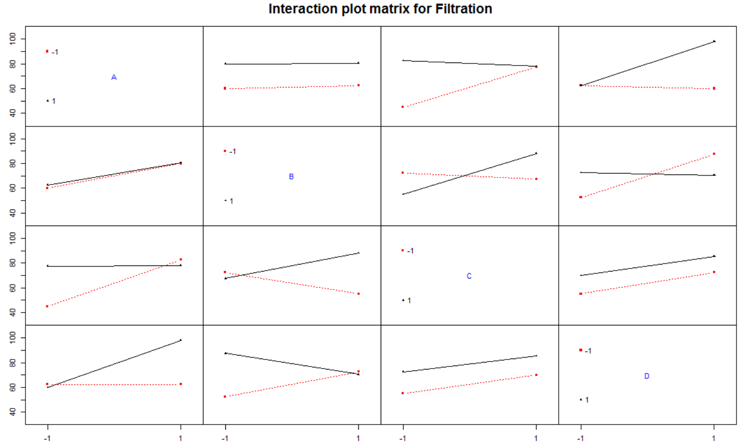
One-half 2^4 filtration rate experiment

Plot of effects:



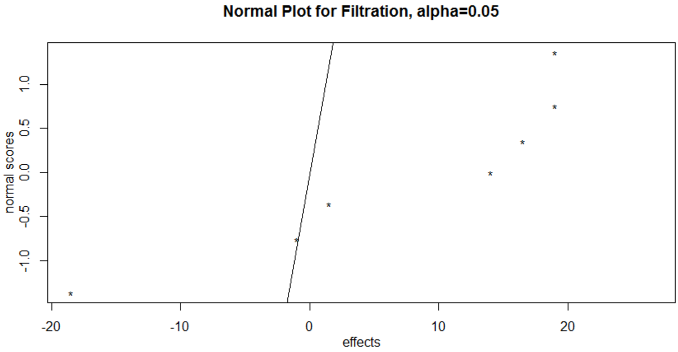
One-half 2^4 filtration rate experiment

Plot of effects:



One-half 2^4 filtration rate experiment

Plot of effects:



2^{4-1}_{IV} design with the defining relation $I = ABCD$

```
> k = 4
> design8_1    <-  FrF2(2^(k-1), k, replications = 1,
                   randomize = FALSE,factor.names = c("A", "B", "C", "D"))
> Filtration   <-  c(45,100,45,65,75,60,80,96)
> design8_1    <-  add.response(design8_1, Filtration)
> summary(design8_1)
Experimental design of type  FrF2 - 8  runs
Responses:         [1] Filtration
generators:        [1] D=ABC
Alias structure: [1] AB=CD AC=BD AD=BC
The design itself:
   A  B  C  D Filtration
1 -1 -1 -1 -1         45
2  1 -1 -1  1        100
3 -1  1 -1  1         45
4  1  1 -1 -1         65
5 -1 -1  1  1         75
6  1 -1  1 -1         60
7 -1  1  1 -1         80
8  1  1  1  1         96
class=design, type= FrF2
```

2_{IV}^{4-1} design with the defining relation $I = ABCD$

One-half 2^4 filtration rate experiment design with generator $I = ABCD$

- A(ABCD) = BCD $[A] \rightarrow A + BCD$
- B(ABCD) = ACD $[B] \rightarrow B + ACD$
- C(ABCD) = ABD $[C] \rightarrow C + ABD$
- D(ABCD) = ABC $[D] \rightarrow D + ABC$
- AB(ABCD) = CD $[AB] \rightarrow AB + CD$
- AC(ABCD) = BD $[AC] \rightarrow AC + BD$
- AD(ABCD) = BC $[AD] \rightarrow AD + BC$

We have seven degree of freedom and the alias structure is a four letter word, therefore this is a Resolution *IV* design, *A*, *B*, *C* and *D* are each aliased with a 3-way interaction, (so we can't estimate them any longer), and the two way interactions are aliased with each other.

2^{4-1}_{IV} filtration rate experiment design

ANOVA analysis results
- model without all third and fourth order interactions:

```
> summary(aov(Filtration ~ A*C*D + B  - A:C:D,
                data = design8_1 ))
            Df  Sum Sq  Mean Sq
A            1   722.0    722.0
C            1   392.0    392.0
D            1   544.5    544.5
B            1     4.5      4.5
A:C          1   684.5    684.5
A:D          1   722.0    722.0
C:D          1     2.0      2.0
```

Still, not enough free degrees of freedom.

2_{IV}^{4-1} filtration rate experiment design

ANOVA analysis results
- model without all third order interactions and without AB=CD interaction :

```
> summary(aov(Filtration ~ A*C*D + B - C:D - A:C:D,
              data = design8_1 ))
             Df Sum Sq Mean Sq F value Pr(>F)
A             1  722.0   722.0  361.00 0.0335 *
C             1  392.0   392.0  196.00 0.0454 *
D             1  544.5   544.5  272.25 0.0385 *
B             1    4.5     4.5    2.25 0.3743
A:C           1  684.5   684.5  342.25 0.0344 *
A:D           1  722.0   722.0  361.00 0.0335 *
Residuals     1    2.0     2.0
```

Because factor *B* is not significant, we can dropped it.

2^{4-1}_{IV} filtration rate experiment design

Analysis results:

```
summary(design8_1_numeric.lm)
Call:
lm.default(formula = Filtration ~ -1+A+B+C+D+
           A:B+A:C+A:D,data = design8_1)
Coefficients:
        Estimate Std. Error t value Pr(>|t|)
A        9.50        70.75    0.134     0.915
B        0.75        70.75    0.011     0.993
C        7.00        70.75    0.099     0.937
D        8.25        70.75    0.117     0.926
A:B     -0.50        70.75   -0.007     0.996
A:C     -9.25        70.75   -0.131     0.917
A:D      9.50        70.75    0.134     0.915
```

If A, C, and D are important main effects, it is logical to conclude that the two interactions alias chains $AC + BD$ and $AD + BC$ have large effects because the AC and AD interactions are also significant.

2^{4-1}_{IV} filtration rate experiment design

Final regression model:

```
summary(design8_1.lm2 )
lm.default(formula = Filtration ~ A + C + D +
         A:C + A:D, data = design8_1)
Residuals:
    1     2     3     4     5     6     7     8
-1.25 -0.25  1.25  0.25 -1.25 -0.25  1.25  0.25
Coefficients:
     Estimate Std.Error  t value Pr(>|t|)
1     70.7500    0.6374  111.00 8.11e-05 ***
A      9.5000    0.6374   14.90  0.00447 **
C      7.0000    0.6374   10.98  0.00819 **
D      8.2500    0.6374   12.94  0.00592 **
A:C   -9.2500    0.6374  -14.51  0.00471 **
A:D    9.5000    0.6374   14.90  0.00447 **

Residual standard error: 1.803 on 2 degrees of freedom
Multiple R-squared: 0.9979,     Adjusted R-squared: 0.9926
F-statistic: 188.6 on 5 and 2 DF,  p-value: 0.005282
```

Compare this model with results that we obtained in previous lectures.

2_{IV}^{4-1} filtration rate experiment design

Because factor B is not significant, we dropped it and projected the 2^{4-1} design into a single replicate of the 2^3 design.

Final regression model is:

$$\hat{y} = 70.75 + 9.5x_1 + 7x_3 + 8.25x_4 - 9.25x_1x_3 + 9.5x_1x_4,$$

where x_1, x_3, x_4 are coded variables $(-1 \leq x_i \leq +1)$ that represents A, C, D. Remember that the intercept $\hat{\beta}_0$ is the average of all responses at the eight runs in the design.

If we can run the alternate fraction to complete the 2^4 design we will block the two fractions and confounding $ABCD$ with blocks.

2^{5-1} design used for Process Improvement Example

Five factors in a manufacturing process for an integrated circuit were investigated in a 2^{5-1} design with the objective of improving the process yield.

Run	Basic Design A	B	C	D	$E = ABCD$	Treatment Combination	Yield
1	−	−	−	−	+	*e*	8
2	+	−	−	−	−	*a*	9
3	−	+	−	−	−	*b*	34
4	+	+	−	−	+	*abe*	52
5	−	−	+	−	−	*c*	16
6	+	−	+	−	+	*ace*	22
7	−	+	+	−	+	*bce*	45
8	+	+	+	−	−	*abc*	60
9	−	−	−	+	−	*d*	6
10	+	−	−	+	+	*ade*	10
11	−	+	−	+	+	*bde*	30
12	+	+	−	+	−	*abd*	50
13	−	−	+	+	+	*cde*	15
14	+	−	+	+	−	*acd*	21
15	−	+	+	+	−	*bcd*	44
16	+	+	+	+	+	*abcde*	63

$I = ABCDE$ is the generator. Consequently, every main effect is aliased with a four-factor interaction and every two-factor interaction is aliased with a three-factor interaction.
The design is of resolution V and we have 15 degree of freedom.

2^{5-1} design used for Process Improvement Example

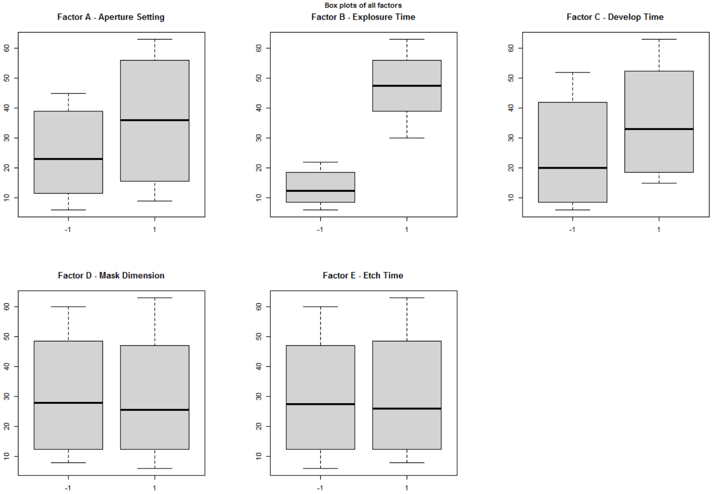
```
k = 5
summary(design8_2)
FrF2(2^(k - 1), k, replications = 1, randomize = FALSE,
factor.names = c("A", "B", "C", "D", "E"))

Experimental design of type  FrF2
16  runs
Generators: E=ABCD
Alias structure:
no aliasing among main effects and 2fis

The design itself:
   A  B  C  D  E yield     A  B  C  D  E yield
1 -1 -1 -1 -1  1    8   9 -1 -1 -1  1 -1    6
2  1 -1 -1 -1 -1    9   10  1 -1 -1  1  1   10
3 -1  1 -1 -1 -1   34   11 -1  1 -1  1  1   30
4  1  1 -1 -1  1   52   12  1  1 -1  1 -1   50
5 -1 -1  1 -1 -1   16   13 -1 -1  1  1  1   15
6  1 -1  1 -1  1   22   14  1 -1  1  1 -1   21
7 -1  1  1 -1  1   45   15 -1  1  1  1 -1   44
8  1  1  1 -1 -1   60   16  1  1  1  1  1   63
class=design, type= FrF2
```

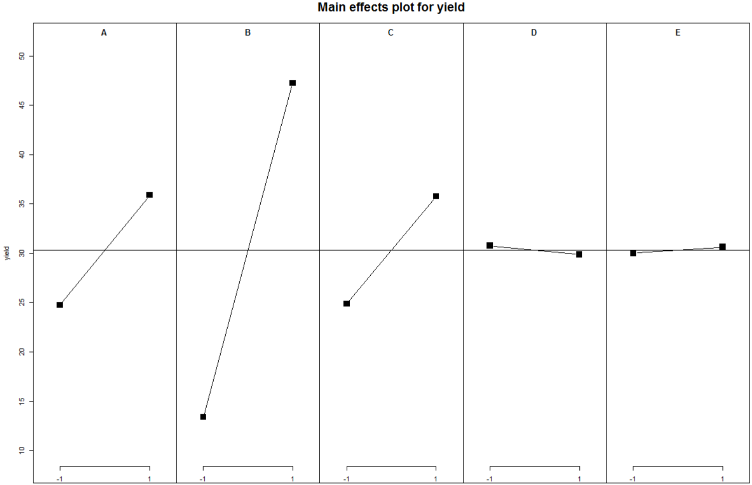
2^{5-1} design used for Process Improvement Example

Plot of effects:



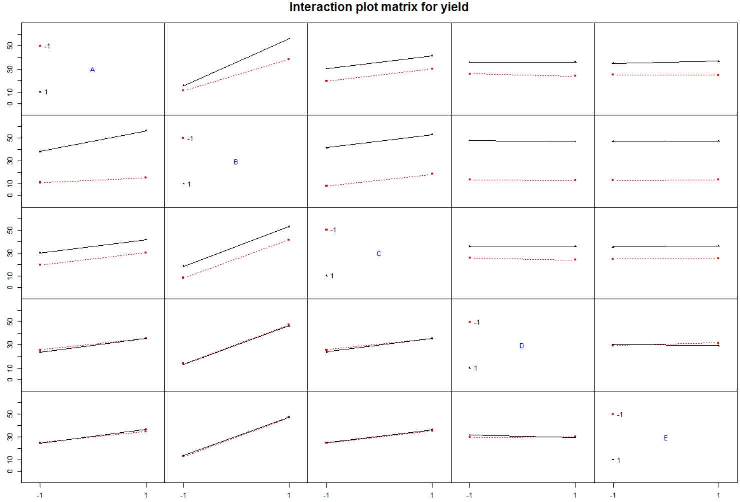
2^{5-1} design used for Process Improvement Example

Plot of effects:



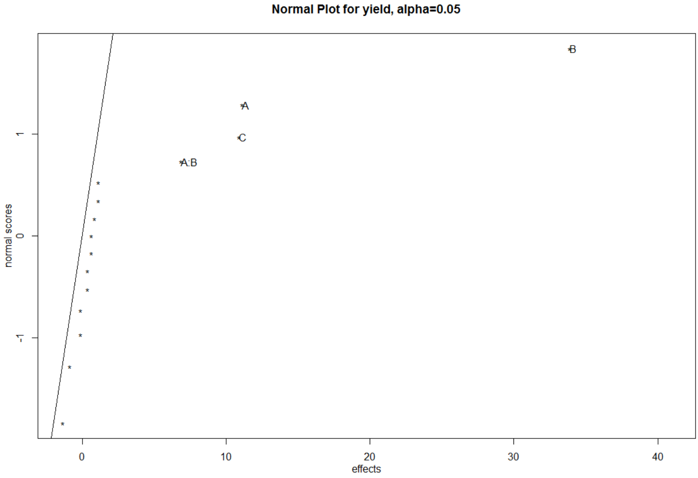
2^{5-1} design used for Process Improvement Example

Plot of effects:



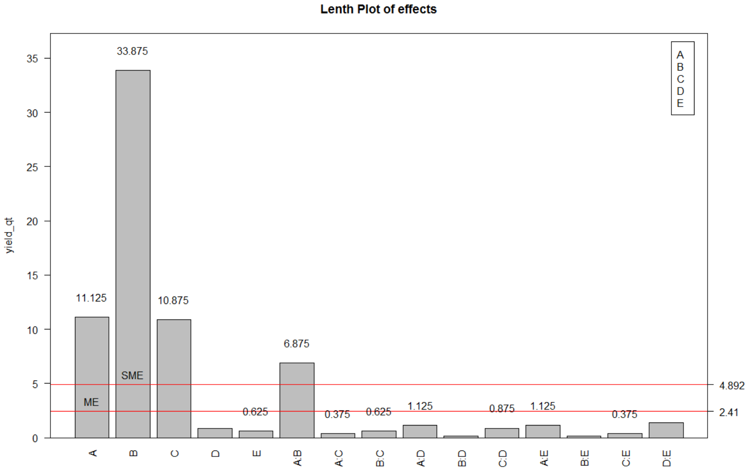
2^{5-1} design used for Process Improvement Example

Plot of effects:



2^{5-1} design used for Process Improvement Example

Plot of effects:



2^{5-1} design used for Process Improvement Example

Effects, regression coefficients and sum of squares:

Variable	Name	−1 Level	+1 Level
A	Aperture	Small	Large
B	Exposure time	−20%	+20%
C	Develop time	30 s	40 s
D	Mask dimension	Small	Large
E	Etch time	14.5 min	15.5 min

Variable	Regression Coefficient	Estimated Effect	Sum of Squares
Overall Average	30.3125		
A	5.5625	11.1250	495.062
B	16.9375	33.8750	4590.062
C	5.4375	10.8750	473.062
D	−0.4375	−0.8750	3.063
E	0.3125	0.6250	1.563
AB	3.4375	6.8750	189.063
AC	0.1875	0.3750	0.563
AD	0.5625	1.1250	5.063
AE	0.5625	1.1250	5.063
BC	0.3125	0.6250	1.563
BD	−0.0625	−0.1250	0.063
BE	−0.0625	−0.1250	0.063
CD	0.4375	0.8750	3.063
CE	0.1875	0.3750	0.563
DE	−0.6875	−1.3750	7.563

2^{5-1} design used for Process Improvement Example

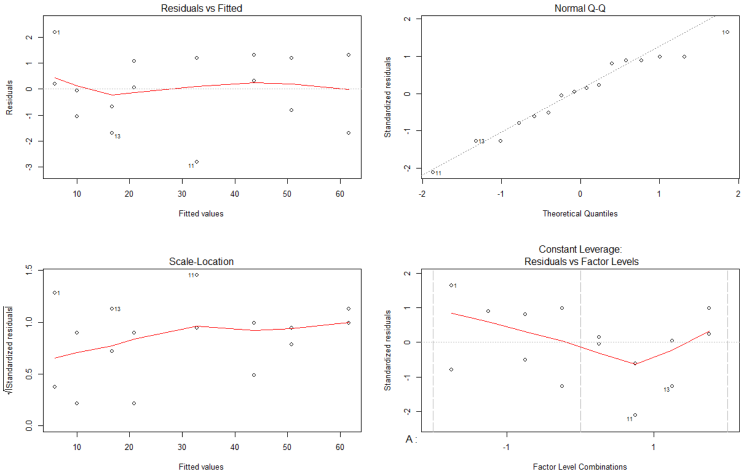
R - results

```
> summary(aov(yield~A*B +C, data = design8_2) )
             Df Sum Sq Mean Sq F value   Pr(>F)
A             1    495     495  193.19 2.53e-08 ***
B             1   4590    4590 1791.24 1.56e-13 ***
C             1    473     473  184.61 3.21e-08 ***
A:B           1    189     189   73.78 3.30e-06 ***
Residuals    11     28       3
```

The 2^{5-1} design has collapsed into two replicates of a 2^3 design, because factors *D* and *E* have little effect on average process yield.

2^{5-1} design used for Process Improvement Example

Model adequacy checking:



The one-quarter fraction of the 2^k design

The one-quarter fraction of the 2^k design contains 2^{k-2} runs and is usually called a 2^{k-2} **fractional factorial.**

One-quarter fraction of the 2^k design has two generators. If P and Q represents the generators chosen, then $I = P = Q = PQ$ are called the generating relations. We call the elements P, Q, and PQ the defining relation **words**. The aliases of any effect are produced by the multiplication of the column for that effect by each word in the defining relation.

Each effect has three aliases.

Be careful in choosing the generators so that potentially important effects are not aliased each other.

Example of one-quarter fraction - the 2^{6-2}_{IV} design

Construction of the 2^{6-2}_{IV} design with the generators $I = ABCE$ and $I = BCDF$.

	Basic Design					
Run	A	B	C	D	$E = ABC$	$F = BCD$
1	−	−	−	−	−	−
2	+	−	−	−	+	−
3	−	+	−	−	+	+
4	+	+	−	−	−	+
5	−	−	+	−	+	+
6	+	−	+	−	−	+
7	−	+	+	−	−	−
8	+	+	+	−	+	−
9	−	−	−	+	−	+
10	+	−	−	+	+	+
11	−	+	−	+	+	−
12	+	+	−	+	−	−
13	−	−	+	+	+	−
14	+	−	+	+	−	−
15	−	+	+	+	−	+
16	+	+	+	+	+	+

Example of one-quarter fraction - the 2_{IV}^{6-2} design

Alias structure for the 2_{IV}^{6-2} design with the generators $I = ABCE = BCDF = ADEF$.

$$A = BCE = DEF = ABCDF$$
$$B = ACE = CDF = ABDEF$$
$$C = ABE = BDF = ACDEF$$
$$D = BCF = AEF = ABCDE$$
$$E = ABC = ADF = BCDEF$$
$$F = BCD = ADE = ABCEF$$

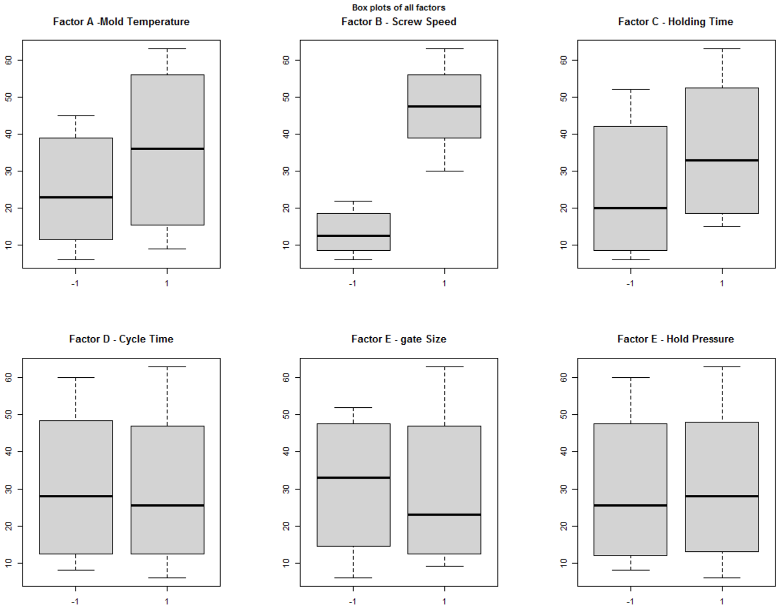
$$ABD = CDE = ACF = BEF$$
$$ACD = BDE = ABF = CEF$$

$$AB = CE = ACDF = BDEF$$
$$AC = BE = ABDF = CDEF$$
$$AD = EF = BCDE = ABCF$$
$$AE = BC = DF = ABCDEF$$
$$AF = DE = BCEF = ABCD$$
$$BD = CF = ACDE = ABEF$$
$$BF = CD = ACEF = ABDE$$

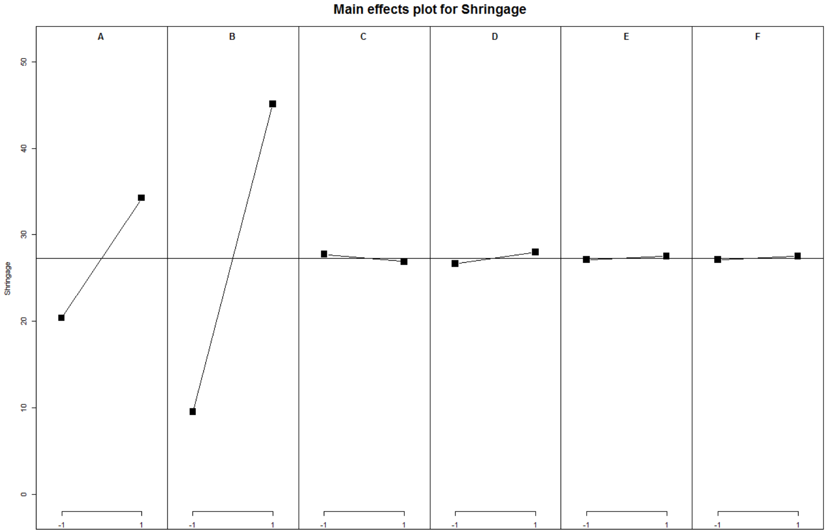
Example of the 2_{IV}^{6-2} design - Injection Molding Experiment

```
design8_4            <-  FrF2(2^(6-2), 6, replications = 1,
             randomize = FALSE,generators = c("ABC","BCD") ,
          factor.names = c("A", "B", "C", "D", "E", "F"))
Shringage            <-  c(6,10,32,60,4,15,26,60,8,12,34,60,16,5,37,52)
design8_4            <-  add.response(design8_4, Shringage)
summary(design8_4)
FrF2(2^(k - 2), k, replications = 1, randomize = FALSE,generators =
    c("ABC","BCD"), factor.names = c("A", "B", "C", "D", "E", "F"))
Generators:
E=ABC F=BCD
Alias structure:
AB=CE, AC=BE, AD=EF, AE=BC=DF, AF=DE, BD=CF, BF=CD
The design itself:
    A  B  C  D  E  F Shringage      A  B  C  D  E  F Shringage
1  -1 -1 -1 -1 -1 -1         6   9  -1 -1 -1  1 -1  1         8
2   1 -1 -1 -1  1 -1        10   10  1 -1 -1  1  1  1        12
3  -1  1 -1 -1  1  1        32   11 -1  1 -1  1  1 -1        34
4   1  1 -1 -1 -1  1        60   12  1  1 -1  1 -1 -1        60
5  -1 -1  1 -1  1  1         4   13 -1 -1  1  1  1 -1        16
6   1 -1  1 -1 -1  1        15   14  1 -1  1  1 -1 -1         5
7  -1  1  1 -1 -1 -1        26   15 -1  1  1  1 -1  1        37
8   1  1  1 -1  1 -1        60   16  1  1  1  1  1  1        52
```

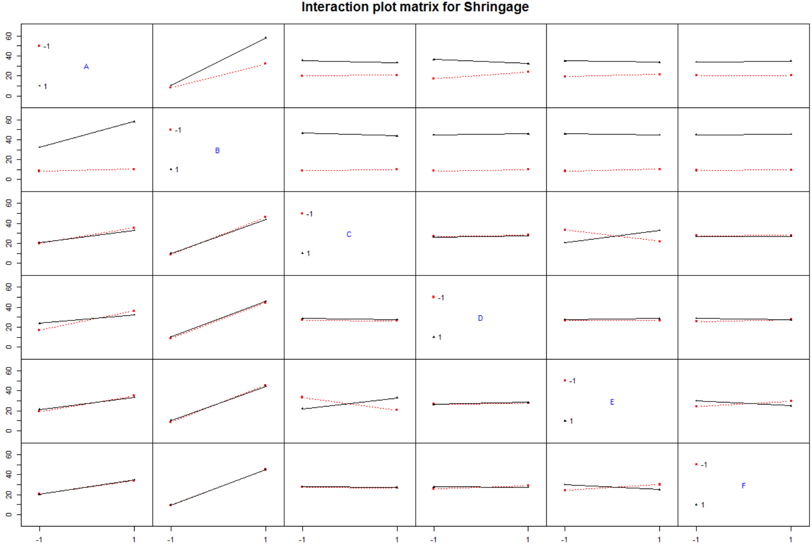
Example of the 2_{IV}^{6-2} design - Injection Molding Experiment



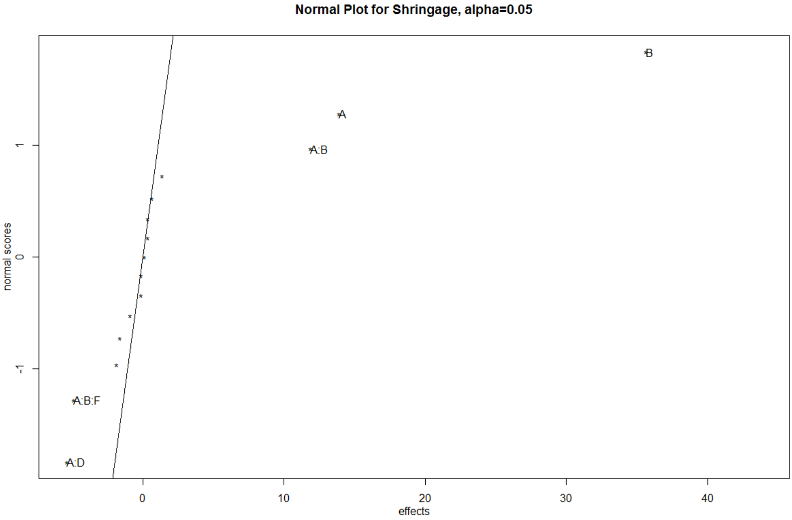
Example of the 2_{IV}^{6-2} design - Injection Molding Experiment



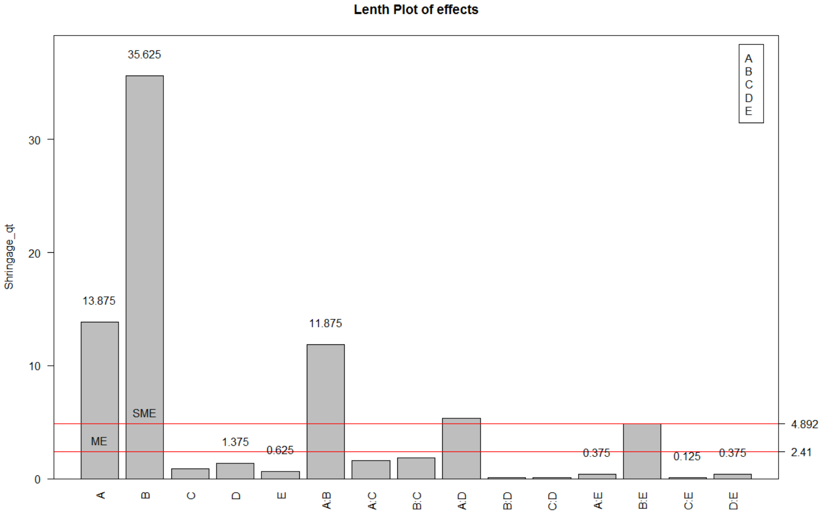
Example of the 2^{6-2}_{IV} design - Injection Molding Experiment



Example of the 2^{6-2}_{IV} design - Injection Molding Experiment



Example of the 2^{6-2}_{IV} design - Injection Molding Experiment



Example of the 2_{IV}^{6-2} design - Injection Molding Experiment

Anova for final model for Injection Molding Experiment:

```
> summary(aov(yield~A*B, data = design8_4) )
            Df Sum Sq Mean Sq F value   Pr(>F)
A            1    495     495  11.852  0.00487 **
B            1   4590    4590 109.887 2.15e-07 ***
A:B          1    189     189   4.526  0.05480 .
Residuals   12    501      42
```

Example of the 2^{6-2}_{IV} design - Injection Molding Experiment

Final linear regression model for Injection Molding Experiment:

```
Call: lm.default(formula =
       Shringage ~ A.num * B.num, data = design8_4)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   27.312      1.138  23.996 1.65e-11 ***
A.num          6.938      1.138   6.095 5.38e-05 ***
B.num         17.812      1.138  15.649 2.39e-09 ***
A.num:B.num    5.938      1.138   5.216 0.000216 ***
---
Residual standard error: 4.553 on 12 degrees of freedom
Multiple R-squared: 0.9626,     Adjusted R-squared: 0.9533
F-statistic: 103.1 on 3 and 12 DF,  p-value: 7.837e-09
```

Final regression model is:

$$\hat{y} = 27.312 + 6.938x_1 + 17.812x_2 + 5.938x_1x_2,$$

where x_1, x_2 are coded variables ($-1 \leq x_i \leq +1$) that represents A, B.

The general 2^{k-p} fractional factorial design

Highest possible resolution and minimum aberration design:
it is important to select the p generators for 2^{k-p} fractional design in such a way that we obtain the best possible alias relationships.

1) we want to obtain the highest possible resolution.

Design *A* Generators: $F = ABC$, $G = BCD$ $I = ABCF = BCDG = ADFG$	Design *B* Generators: $F = ABC$, $G = ADE$ $I = ABCF = ADEG = BCDEFG$	Design *C* Generators: $F = ABCD$, $G = ABDE$ $I = ABCDF = ABDEG = CEFG$
Aliases (Two-Factor Interactions)	Aliases (Two-Factor Interactions)	Aliases (Two-Factor Interactions)
$AB = CF$	$AB = CF$	$CE = FG$
$AC = BF$	$AC = BF$	$CF = EG$
$AD = FG$	$AD = EG$	$CG = EF$
$AG = DF$	$AE = DG$	
$BD = CG$	$AF = BC$	
$BG = CD$	$AG = DE$	
$AF = BC = DG$		

2) we want to obtain minimum number of words in the defining relation that are of minimum length. We call such a design a **minimum aberration design**.

The general 2^{k-p} fractional factorial design

Number of Factors, k	Fraction	Number of Runs	Design Generators
3	2^{3-1}_{III}	4	$C = \pm AB$
4	2^{4-1}_{IV}	8	$D = \pm ABC$
5	2^{5-1}_{V}	16	$E = \pm ABCD$
	2^{5-2}_{III}	8	$D = \pm AB$
			$E = \pm AC$
6	2^{6-1}_{VI}	32	$F = \pm ABCDE$
	2^{6-2}_{IV}	16	$E = \pm ABC$
			$F = \pm BCD$
	2^{6-3}_{III}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
7	2^{7-1}_{VII}	64	$G = \pm ABCDEF$
	2^{7-2}_{IV}	32	$F = \pm ABCD$
			$G = \pm ABDE$
	2^{7-3}_{IV}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
	2^{7-4}_{III}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
			$G = \pm ABC$
8	2^{8-2}_{V}	64	$G = \pm ABCD$
			$H = \pm ABEF$
	2^{8-3}_{IV}	32	$F = \pm ABC$
			$G = \pm ABD$
			$H = \pm BCDE$
	2^{8-4}_{IV}	16	$E = \pm BCD$
			$F = \pm ACD$
			$G = \pm ABC$
			$H = \pm ABD$
9	2^{9-2}_{VI}	128	$H = \pm ACDFG$
			$J = \pm BCEFG$
	2^{9-3}_{IV}	64	$G = \pm ABCD$
			$H = \pm ACEF$
			$J = \pm CDEF$
	2^{9-4}_{IV}	32	$F = \pm BCDE$
			$G = \pm ACDE$
			$H = \pm ABDE$
			$J = \pm ABCE$

Number of Factors, k	Fraction	Number of Runs	Design Generators
	2^{9-5}_{III}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
			$H = \pm ABD$
			$J = \pm ABCD$
10	2^{10-3}_{V}	128	$H = \pm ABCG$
			$J = \pm ACDE$
			$K = \pm ACDF$
	2^{10-4}_{IV}	64	$G = \pm BCDF$
			$H = \pm ACDF$
			$J = \pm ABDE$
			$K = \pm ABCE$
	2^{10-5}_{IV}	32	$F = \pm ABCD$
			$G = \pm ABCE$
			$H = \pm ABDE$
			$J = \pm ACDE$
			$K = \pm BCDE$
	2^{10-6}_{III}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
			$H = \pm ABD$
			$J = \pm ABCD$
			$K = \pm AB$
11	2^{11-5}_{IV}	64	$G = \pm CDE$
			$H = \pm ABCD$
			$J = \pm ABF$
			$K = \pm BDEF$
			$L = \pm ADEF$
	2^{11-6}_{IV}	32	$F = \pm ABC$
			$G = \pm BCD$
			$H = \pm CDE$
			$J = \pm ACD$
			$K = \pm ADE$
			$L = \pm BDE$
	2^{11-7}_{III}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
			$H = \pm ABD$
			$J = \pm ABCD$
			$K = \pm AB$

Number of Factors, k	Fraction	Number of Runs	Design Generators
			$L = \pm AC$
12	2^{12-8}_{III}	16	$E = \pm ABC$
			$F = \pm ABD$
			$G = \pm ACD$
			$H = \pm BCD$
			$J = \pm ABCD$
			$K = \pm AB$
			$L = \pm AC$
			$M = \pm AD$
13	2^{13-9}_{III}	16	$E = \pm ABC$
			$F = \pm ABD$
			$G = \pm ACD$
			$H = \pm BCD$
			$J = \pm ABCD$
			$K = \pm AB$
			$L = \pm AC$
			$M = \pm AD$
			$N = \pm BC$
14	2^{14-10}_{III}	16	$E = \pm ABC$
			$F = \pm ABD$
			$G = \pm ACD$
			$H = \pm BCD$
			$J = \pm ABCD$
			$K = \pm AB$
			$L = \pm AC$
			$M = \pm AD$
			$N = \pm BC$
			$O = \pm BD$
15	2^{15-11}_{III}	16	$E = \pm ABC$
			$F = \pm ABD$
			$G = \pm ACD$
			$H = \pm BCD$
			$J = \pm ABCD$
			$K = \pm AB$
			$L = \pm AC$
			$M = \pm AD$
			$N = \pm BC$
			$O = \pm BD$
			$P = \pm CD$

Blocking Fractional Factorials

In general choose some aliases (preferably with longest words) and generate block by the help of (+) and (-) signs.

Example of the 2_{IV}^{6-2} design - Two blocks with *ABD* confounded:

- **Block 1:** (1), abf, cef, abce,abef,bde,acd,bcdf
- **Block 2:** ae,acf,bef,bc,df,abd,cde,abcdef

Remember: the generator of blocks *ABD* is aliased

$$ABD = CDE = ACF = BEF.$$

Homework 02

Design and analyze **Reaction Time Experiment**.

Form groups (prefere 4 students in each), measure data according to instructions in the file 01NAEX_HW2_reaction_time_experiment_2025.ipynb

Submit a jupyter notebook with commented code, solutions, and data file till December the 7th 2025.