**Reading: Chapters 1 of Binney & Tremaine, Chapters 1-2 of Binny & Merrifield**

**Number Densities and Luminosity Functions:**

A simple tool for studying the *space density* of a group of objects is the $\log N$—$\log S$ diagram. It is simply a plot of the numbers of objects in different flux intervals.

Take the simplest case of a **spherical** distribution of **identical objects**.

Define $S$ as

$$S \equiv F = \frac{L}{4\pi R^2}$$

For a spherical distribution, the number of sources within a distance $R$ is proportional to $R^3$, as in

$$N \propto R^3$$

This implies

$$N^{2/3} \propto R^2$$

which in turn implies

$$S \propto N^{-2/3}$$

**sidebar:**

$$
\begin{aligned}
S &= N^x \\
d\log S &= \frac{dS}{S} \\
d\log N &= \frac{dN}{N} \\
\frac{d\log S}{d\log N} &= \frac{N}{S} \\
&= \left(\frac{N}{N^x}\right) x N^{x-1} \\
&= x
\end{aligned}
$$

**For a spherical distribution, the slope of the $\log N$—$\log S$ plot will be $-2/3$.**

Some people prefer to plot the numbers on the $y$-axis, and fluxes on the $x$-axis. In this case, the slope of the line for a uniform spherical distribution will be -3/2.

What if the spherical distribution has an edge?

Then there is a minimum flux, and the numbers of sources grows more slowly as the flux threshold goes down.

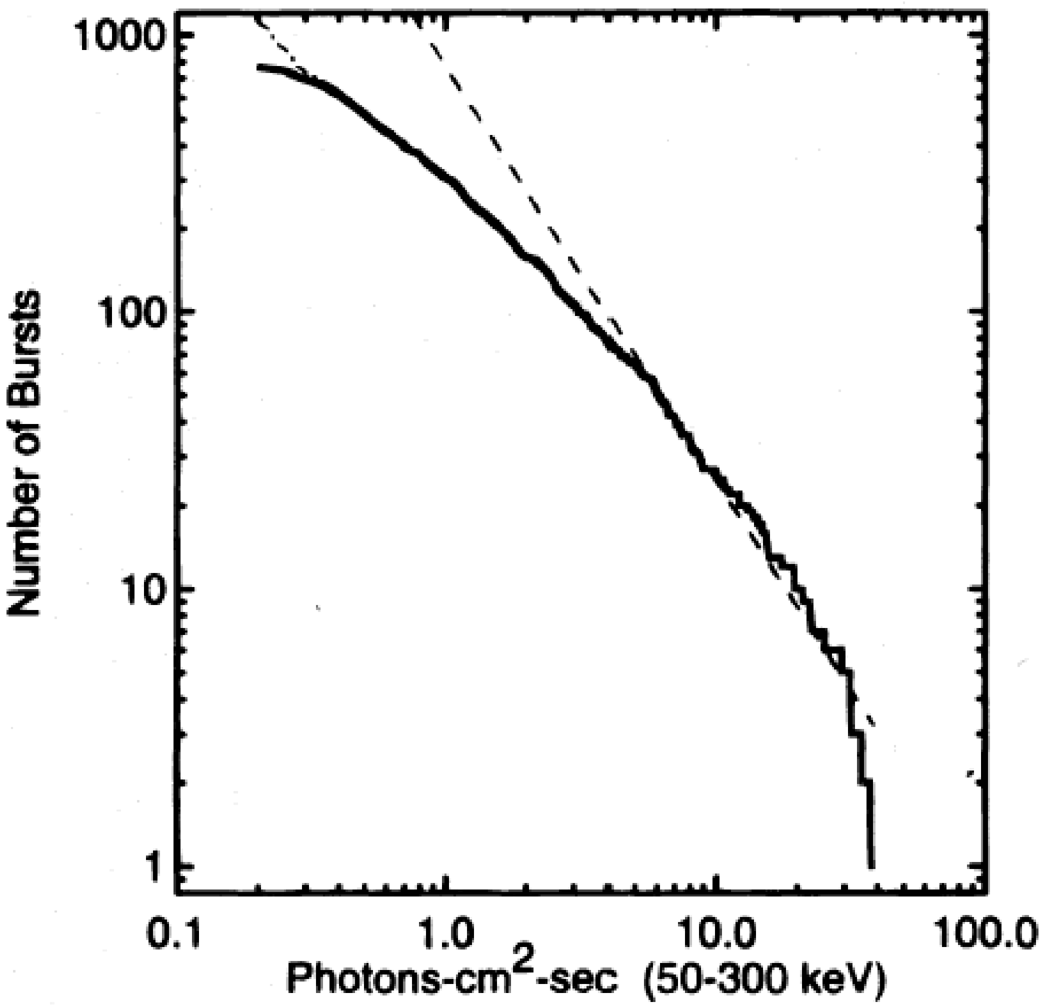


FIG. 10c

The $V/V_{\mathrm{max}}$ plot for **gamma ray bursts** flattens at faint fluxes, indicating the distribution of gamma ray bursts has an edge (this turns out to be the edge of the universe). From Pendleton et al. 1996, ApJ, 464, 606.

Consider a distribution of objects confined to a plane. In this case,

$$N \propto R^2,$$

which implies

$$S \propto N^{-1}.$$

The slope of the $\log N$—$\log S$ curve is steeper.

Now consider a "hockey puck" distribution, i.e. a slice through a sphere of thickness $2z$.

Find the volume:

For radii of $R < z$,

$$V = \frac{4\pi R^3}{3}$$

For radii $R \geq z$,

$$\begin{aligned}
V &= \int_0^R r^2 dr \int_0^{2\pi} d\phi \int_{90-\Theta}^{90+\Theta} \sin\theta d\theta \\
&= \left(\frac{2\pi R^3}{3}\right)\left(-\cos\theta|_{90-\Theta}^{90+\Theta}\right) \\
&= \left(\frac{2\pi R^3}{3}\right)(-\cos(90+\Theta)+\cos(90-\Theta)) \\
&= \left(\frac{2\pi R^3}{3}\right)(2\sin\Theta) \\
&= \left(\frac{4\pi R^3}{3}\right)\left(\frac{z}{R}\right)
\end{aligned}$$

For small $R$

$$N \propto R^3$$
$$\rightarrow S \propto N^{-2/3}$$

For large $R$,

$$N \propto R^2$$
$$\Rightarrow S \propto N^{-1}$$

The slope of the $\log N$—$\log S$ curve is -1 for large $N$ (small $S$), and -2/3 for small $N$ (large $S$). In between, there is a gradual transition.

What about dust? Suppose there is an interstellar extinction of a factor of 2 per kpc in all directions.

The apparent **flux** of an object depends on its distance via the inverse square law **and** through the effects of the absorbing material:

$$S = \left(\frac{1}{2}\right)^R \frac{L}{4\pi R^3}$$

if $R$ is in kpc and $L$ and $S$ adjusted accordingly.

For a spherical distribution,

$$N \propto R^3$$

which implies

$$N^{1/3} \propto R.$$

Hence

$$S \propto \left(\frac{1}{2}\right)^{N^{1/3}} N^{-2/3}$$

$$\log S = -N^{1/3} \log 2 - \frac{2 \log N}{3}$$

For small $N$, the $\log N$ term dominates.

For large $N$, $N^{1/3}$ grows faster than $\log N$.

If you did not know about interstellar extinction, you might conclude your infinite spherical distribution had an edge.

**What is the shape of our galaxy?**

William Herschel counted stars in 683 different regions. He recorded the number of stars he could see to various limiting brightnesses.

He assumed all stars had the same intrinsic brightness, and concluded the galaxy at was a roughly cylindrical in shape, with the Sun near the center.

Using **photographic plates** Kapteyn studied 200 selected areas in great detail, including compiling star counts, proper motions, spectral types, and radial velocities.

His picture of the galaxy was similar to that of Hershel's. The galaxy was a flattened distribution that extended about 5 times further out in the direction of the plane than in the perpendicular direction. The radius of the plane was 8500 pc.

Kapteyn considered the effects of an absorbing medium between the stars, but concluded that it was not important.

Shapley looked at the distribution of **globular clusters** and concluded that the Sun was 15 kpc from the center of the distribution. The whole globular cluster distribution was estimated to be 100 kpc across.

Kapteyn's Universe, and Shapley's model.

**Distributions of different stellar types:**

OR

**Lies, Damn Lies, and Statistics:**

Among the stars in our Galaxy, what is the **distributions of different stellar types**?

Look at the Henry Draper (HD) catalog for distribution of stellar types:

| spectral type | percent |
|---|---|
| O | 1 |
| B | 10 |
| A | 22 |
| F | 19 |
| G | 14 |
| K | 31 |
| M | 3 |

Also, here is the distribution of luminosity classes:

| spectral | Luminosity class | | | | |
|---|---|---|---|---|---|
| type | V | VI | III | II | I |
| O, B | 10 | 3 | 6 | 2 | 3 |
| A, F | 14 | 3 | 5 | 1 | 4 |
| G, K, M | 1 | 4 | 25 | 6 | 4 |

Now look at the **closest stars**. The MK types are dominated by mid M stars, not A stars. What gives?

**Samples and observational bias:**

The HD catalog and the catalog of nearby stars are constructed in a completely different way.

The HD catalog is a **magnitude limited sample**, in this case all stars with **apparent** magnitudes of $V \leq 6.5$.

The Gliese catalog (the nearby stars) is a **distance or volume limited** sample, in this case all known stars closer than 20 pc.

Look at basic stellar data to see why these two are so different...

Consider a magnitude limited sample with all stars brighter than $V = 5$. The distance at which an O5V star has an **apparent** magnitude of $V = 5$ is

$$\begin{aligned} m - M_v &= 5\log d - 5 \\ 5 - -5.7 &= 5\log d - 5 \\ \rightarrow \log d &= (10 + 5.7)/5 \\ d &= 1380\,\mathrm{pc} \end{aligned}$$

Likewise, the distance at which an M5V star has an apparent magnitude of $V = 5$ is

$$\begin{aligned} m - M_v &= 5\log d - 5 \\ 5 - 12.3 &= 5\log d - 5 \\ \rightarrow \log d &= (10 - 12.3)/5 \\ d &= 0.35\,\mathrm{pc} \end{aligned}$$

The **Malmquist Bias** is well known in many areas of astronomy. Simply put, it states that a **magnitude limited** sample will be dominated by objects that are more intrinsically luminous.

Clearly, volume-limited surveys usually be more useful when studying the number density of objects (either stars, clusters, or galaxies). How does one go about constructing a **complete** volume-limited sample?

For faint, nearby stars, one often uses **proper motion surveys**.

The idea is that stars have small random motions in addition to the general motion around the galaxy. The proper motion of a star as observed on Earth depends on the intrinsic velocity and the distance. For a fixed intrinsic velocity, closer stars give larger proper motions.

One is biased in several ways.

For example, it is hard to survey near the galactic plane owing to the crowded fields.

Also, a nearby star may just happen to have a small proper motion.

In that same vein, more distant and luminous stars may have intrinsically large velocities.

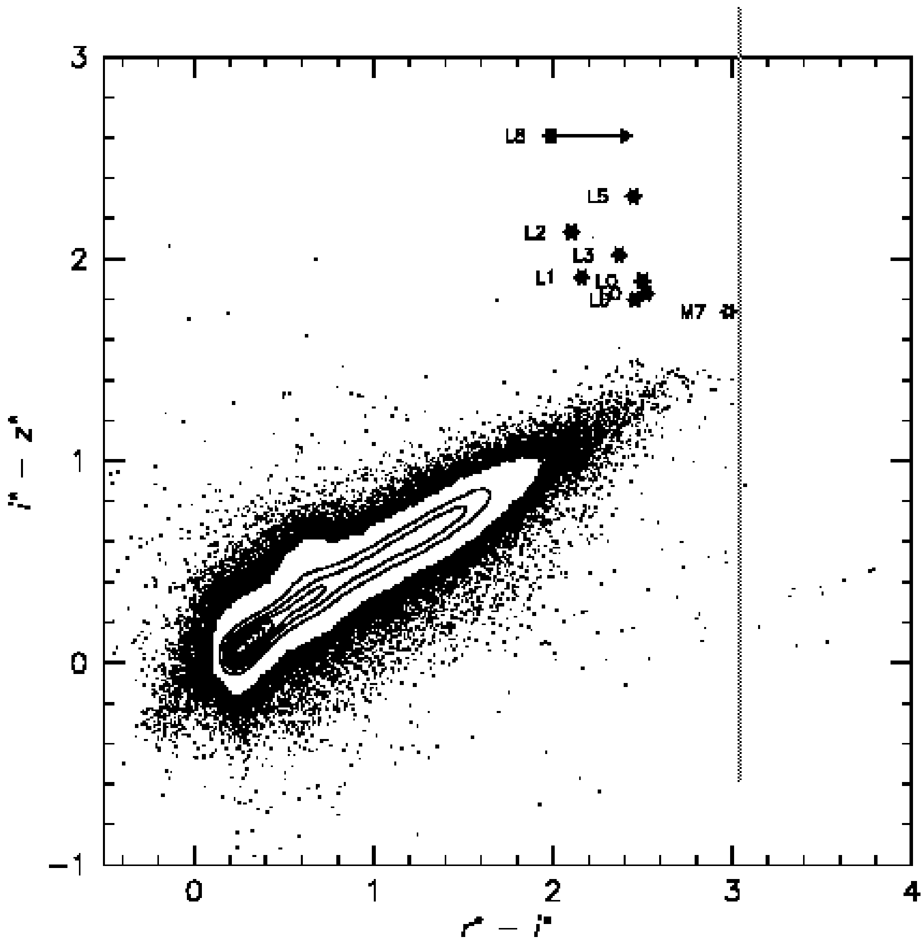


FIG. 1.—Color-color ($i^* - z^*$ vs. $r^* - i^*$) plot for 366,859 point sources brighter than $i^* = 20.2$ from run 94, selected to be unblended (i.e., the image does not overlap the image of another object). The contours are drawn every 10% of the peak density of points. The bump at ($r^* - i^*$, $i^* - z^*$) ≈ (0.7, 0.4) is due to unresolved faint galaxies. The colors of the eight objects identified as late-type or substellar objects in this paper, plus an additional such SDSS object spectroscopically identified at the Hobby-Eberly Telescope (Schneider et al. 2000), are for spectral type M (*open symbols*) and spectral type L (*filled symbols*). The L8 dwarf is undetected in $r^*$.

(from Fan et al. 2000, AJ, 119 928)

Intrinsically faint stars tend to be **red**. Large **color** surveys have had success in finding intrinsically faint stars by looking for objects with unusual colors.

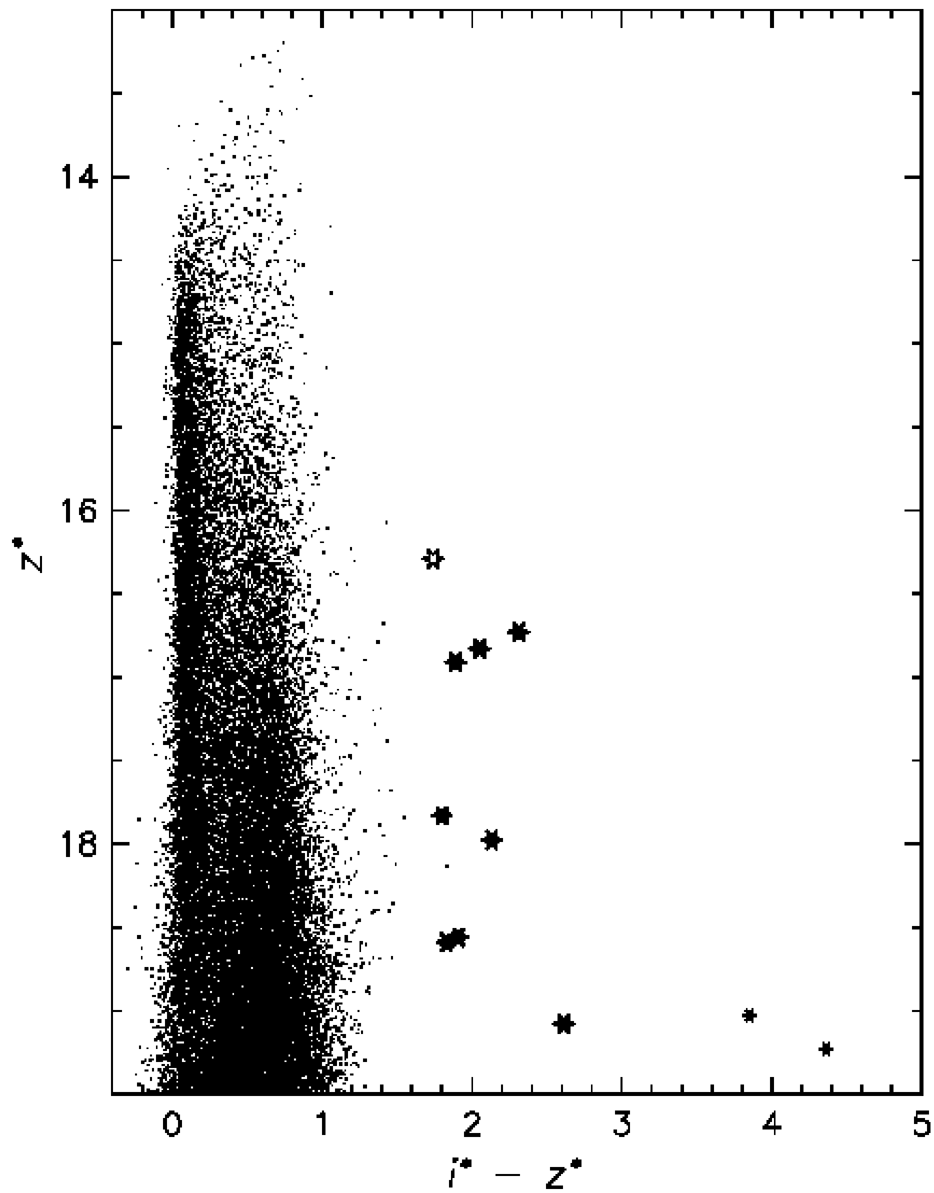


FIG. 2.—Color-magnitude diagram ($z^*$ vs. $i^* - z^*$) for 50,000 objects from **run 94** plus the late-type dwarfs identified by SDSS: M7 dwarf (*large open symbols*), L dwarfs by (*large filled symbols*), and T dwarfs (*small symbols*).

(from Fan et al. 2000, AJ, 119 928)

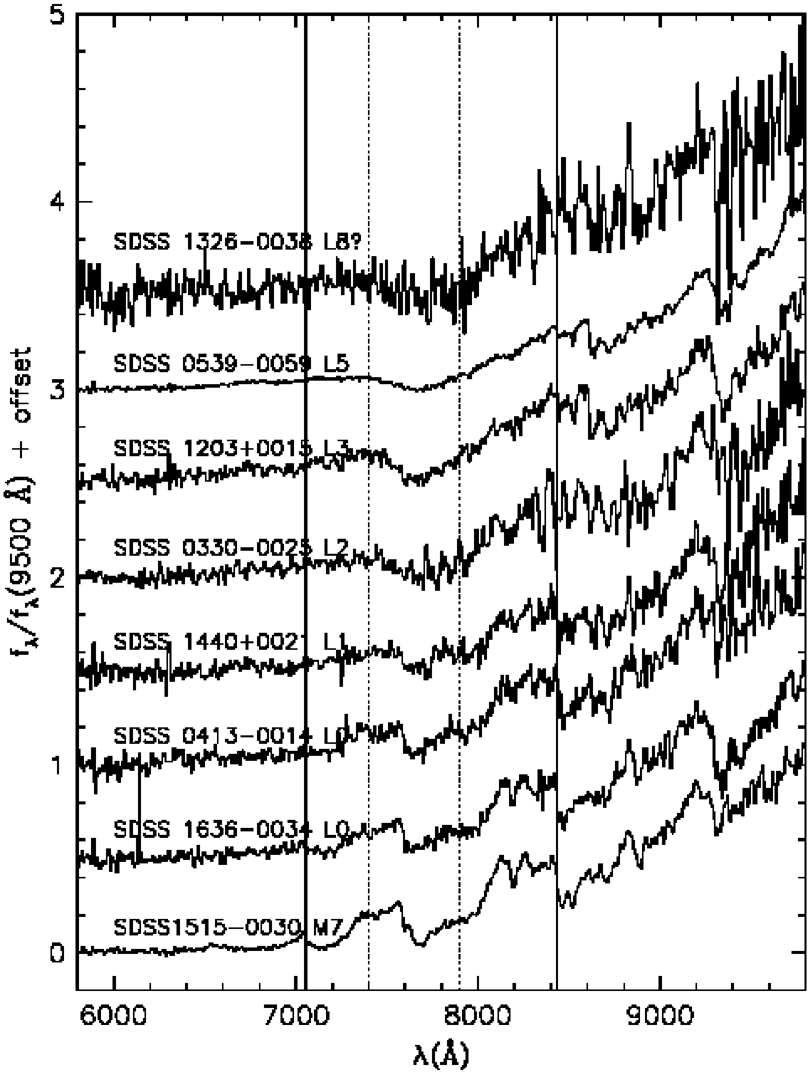


FIG. 4.—Far-red spectra of the M and L dwarfs, observed by the Double Imaging Spectrograph (DIS) on the APO 3.5 m telescope. The vertical scale for each object has been approximately normalized to the flux density of the object at 9500 Å (see Table 3), plus an offset. *Top to bottom*: Objects in decreasing spectral type and increasing effective temperature. Vertical lines show the wavelengths of the TiO 7053 and 8432 Å band heads (*solid lines*) and the VO 7400 and 7900 Å band heads (*dotted lines*).

(from Fan et al. 2000, AJ, 119 928)

## The Stellar luminosity function.