

Is there a Size Difference between Red and Blue Globular Clusters?

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ABSTRACT

Blue (metal-poor) globular clusters are observed to have half-light radii that are $\sim 20\%$ larger than their red (metal-rich) counterparts. The origin of this enhancement is not clear and differences in either the luminosity function or in the actual size of the clusters have been proposed. I analyze a set of dynamically self-consistent Monte Carlo globular cluster simulations to determine the origin of this enhancement. I find that my simulated blue clusters have larger half-light radii due to differences in the luminosity functions of metal-poor and metal-rich stars. I find that the blue clusters can also be physically larger, but only if they have a substantial number of black holes heating their central regions. In this case the difference between half-light radii is significantly larger than observed. I conclude that the observed difference in half-light radii between red and blue globular clusters is due to differences in their luminosity functions and that half-light radius is not a reliable proxy for cluster size.

Key words: galaxies: star clusters: general – globular clusters: general – stellar dynamics – methods: N-body

1 INTRODUCTION

Globular cluster (GC) systems are common in disk and elliptical galaxies and in the past decade significant observational progress has been made in understanding them. These globular clusters are frequently used to help understand the formation and evolution of their host galaxies. However, the observational properties of globular clusters are not fully understood. A case in point is the half-light radius (r_{hl}). Several models show that the half-light radius of a cluster should remain fairly constant during its lifetime (Spitzer & Thuan 1972; Aarseth & Heggie 1998) so it is often used to compare the structures of globular clusters of different ages. Several observational studies, however, show that metal-poor (blue) globular clusters have half-light radii that are systematically larger (by $\sim 20\%$) than their metal-rich (red) counterparts (Kundu & Whitmore 1998, 2001; Kundu et al. 1999; Puzia et al. 1999; Larsen et al. 2001; Larsen, Forbes & Brodie 2001; Barmby, Holland & Huchra 2002; Harris et al. 2002; Jordán et al. 2004; Harris 2009).

It is not clear what the origin of this discrepancy is nor is it certain that it truly represents a difference in the sizes of the clusters as measured by their half-mass radii (r_{hm}). Larsen & Brodie (2003) proposed that the observed enhancement in the size of the blue clusters could be due to a

projection effect. They note both that blue and red globular clusters follow different radial distributions and that in the Milky Way there is a relationship between cluster size and galactocentric distance (van den Bergh, Morbey & Pazder 1991). They argue that if such a relationship exists in all galaxies, the differing radial distributions of red and blue clusters could account for the observed difference in sizes. A detailed survey by Harris (2009), however, showed that the ratio of r_{hl} between blue and red clusters does not depend on galactocentric distance, as would be predicted by the Larsen & Brodie (2003) model. Furthermore, Larsen & Brodie (2003) give no reason why a galactocentric distance-globular cluster size trend should exist in other galaxies.

Jordán (2004) explained the difference as a result of differing stellar evolution processes in metal-poor and metal-rich stellar populations. Metal poor stars lose less mass and have longer main-sequence lifetimes than metal-rich stars. Assuming that the difference between red and blue stellar populations is metallicity, this leads to differing luminosity functions in blue and red GCs. Using multi-mass Michie-King models with fixed half-mass radii and a stellar population with an age of 13 Gyrs, Jordán (2004) was able to re-produce the observed r_{hl} enhancement in blue globular clusters. However, these models are not dynamical simulations of globular cluster evolution and the results are valid only if r_{hm} is the same in red and blue globular clusters.

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By contrast Schulman, Glebbeek & Sills (2012) used direct N -body simulations to investigate the differences in dynamical evolution between blue and red open clusters. They also assumed that the colour of a cluster reflects its metallicity and argued that, because metal-poor stars lose mass more slowly than metal-rich stars, blue clusters will lose less mass to stellar evolution over their lifetimes than will red clusters. This reduces the gravitational potential of red clusters, causing them to expand and become larger than blue clusters. However, scattering interactions between the more massive metal-poor stars will be more energetic than those between less massive metal-rich stars and the blue clusters will experience stronger two-body heating. This will cause them to expand relative to the red clusters once the initial phase of rapid mass-loss is concluded. If the effect of two-body heating is stronger, blue clusters will be physically larger than red clusters. Schulman, Glebbeek & Sills (2012) showed that the half-mass radii of their simulated blue clusters were indeed $\sim 20\%$ larger after several initial half-mass relaxation times (t_{rh}) than the half-mass radii of their simulated red clusters. They also found little difference between the ratio of r_{hm} and r_{hl} between blue and red clusters. Therefore they conclude that the enhancement in r_{hl} in blue globular clusters is due to an actual size enhancement and has a dynamical origin. The Schulman, Glebbeek & Sills (2012) models, however, contain 10-100 times fewer stars than are present in globular clusters and consequently relax and dissolve when they are only a few hundred Myr old, far younger than the 10-13 Gyr age of many globular cluster systems. The relationship between stellar evolution timescales and dynamical timescales is also quite different in these simulations than in GCs.

In this paper I re-visit the problem of size differences between blue and red globular clusters by analyzing a set of Monte Carlo star cluster models. These are self-consistent dynamical simulations that contain a similar number of stars to globular clusters, include parametrized stellar evolution and primordial binaries. I will investigate whether my blue globular clusters are larger than the red ones, as was reported by Schulman, Glebbeek & Sills (2012). If they are, I will determine if r_{hl} is enhanced to the same degree as r_{hm} and if the processes reported by Jordán (2004) have a significant effect. I will also be able to determine whether or not r_{hl} is a good observational proxy for r_{hm} .

2 NUMERICAL MODELS

The simulations in this paper were performed using a Monte Carlo method to self-consistently simulate the dynamical evolution of a star cluster in the Fokker-Planck two-body relaxation limit. The primary advantage to this method over direct N -body is speed; the Monte Carlo code scales with $\mathcal{O}(N^1) - \mathcal{O}(N^2)$, where N is the number of stars in the system, as opposed to $\mathcal{O}(N^3) - \mathcal{O}(N^4)$ for direct N -body codes. Thus it is possible to run globular cluster-sized simulations over a full Hubble time of dynamical evolution, a task currently impossible to do using a direct N -body code. Unlike standard Fokker-Planck codes, however, the Monte Carlo method treats the cluster as an ensemble of particles and can provide the same star-by-star information as a direct N -body simulation. This

Table 1. Parameters of the simulations. The first column gives the identifying label, the second gives the metallicity, the third the total mass of the cluster, the fourth the initial half-mass radius and the fifth the initial half-mass relaxation time.

Simulation Parameters				
Simulation	Z	$M [M_{\odot}]$	$r_{hm} [\text{pc}]$	$t_{rh} [\text{Myr}]$
red21	0.02	3.61×10^5	7.14	3.54×10^3
red37	0.02	3.63×10^5	4.05	1.51×10^3
red75	0.02	3.62×10^5	2.00	5.25×10^2
blue21	0.001	3.60×10^5	7.14	3.55×10^3
blue37	0.001	3.62×10^5	4.05	1.51×10^3
blue75	0.001	3.62×10^5	2.00	5.25×10^2

also makes it much simple to include prescriptions for stellar evolution and strong few-body interactions. The Monte Carlo code is described in detail by Giersz (1998, 2001, 2006) while the strong few-body interactions are calculated as described in Giersz & Spurzem (2003). The code computes parametrized evolution for each star and binary using the SSE (Hurley, Pols & Tout 2000) and BSE (Hurley, Tout & Pols 2002) stellar evolution prescriptions Giersz, Heggie & Hurley (2008). These are essentially the same prescriptions as used by Schulman, Glebbeek & Sills (2012). The code has been compared to, and provides excellent agreement with, both direct N -body simulations and observations (Giersz, Heggie & Hurley 2008; Heggie & Giersz 2008; Giersz & Heggie 2009; Heggie & Giersz 2009).

The simulations used in this paper are a subset of a collection of simulations that were originally performed to investigate the dynamical creation of black hole binaries in globular clusters. They are described in some detail in Downing et al. (2010). All simulations are initialized as 5×10^5 particle Plummer spheres with 10% primordial binaries. Initial binary parameters are chosen according to the eigenvalue evolution and feeding algorithms of Kroupa (1995). The initial stellar density is controlled by the ratio of the tidal radius (r_t) to r_{hm} . For all simulations $r_t = 150$ pc and I use $r_t/r_{hm} \in \{21, 37, 75\}$, corresponding to initial stellar number densities within r_{hm} of $\sim 10^2, 10^3$ and 10^4 respectively. The masses are between 0.1 and 150 M_{\odot} and are drawn from a Kroupa, Tout & Gilmore (1993) initial mass function with a low-mass slope of $\alpha_l = 1.3$, a high mass slope of $\alpha_h = 2.3$ and a break mass of $M_{\text{break}} = 0.5 M_{\odot}$.

The simulations come in two metallicities, $Z = 0.02$ and $Z = 0.001$. Throughout the paper I will refer to the $Z = 0.02$ clusters as the “red” clusters and the $Z = 0.001$ clusters as the “blue” clusters. One significant difference between the red and blue simulations is the treatment of black holes (BHs). Metal-poor stars lose less mass to line-driven winds than do metal-rich stars and experience more matter fallback after supernovae (Belczynski, Kalogera & Bulik 2002; Belczynski et al. 2006). This produces both more and more massive BHs in the blue clusters. BHs also receive natal kicks drawn from a Maxwellian distribution with a peak at 190 km/s (Hansen & Phinney 1997), significantly higher than the escape velocity of a GC. This kick is then reduced according to the amount of fallback on to the BH during the supernova (Belczynski, Kalogera & Bulik 2002). Because the amount of fallback is larger in the blue clusters, the BHs in these clusters receive a larger kick reduction and

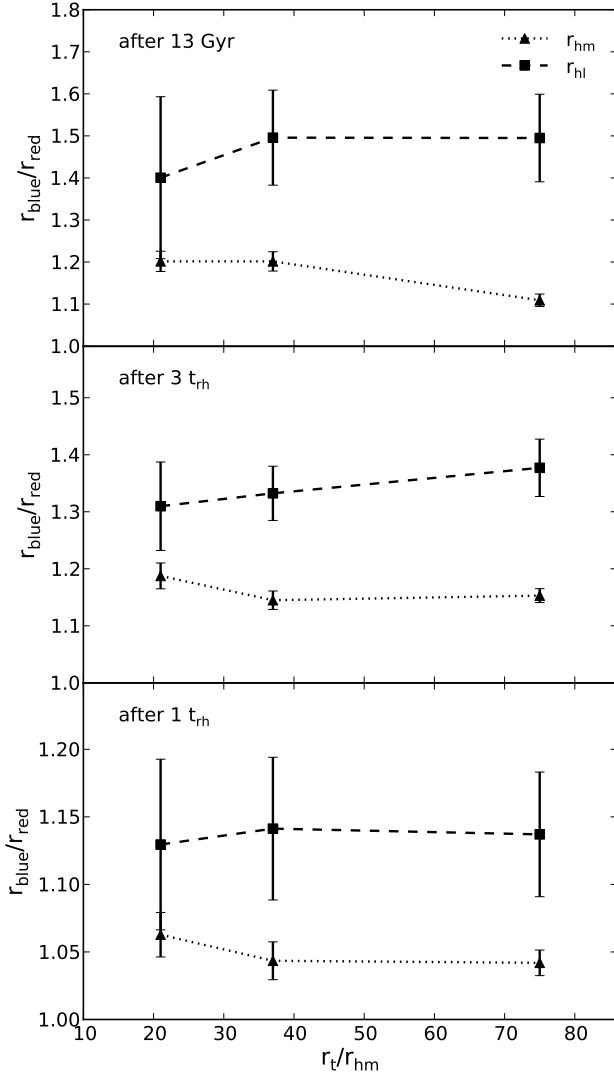


Figure 1. Ratio of r_{hl} and r_{hm} in blue globular clusters as compared to red globular clusters for clusters with different initial concentrations. The top plot gives the ratio after 13 Gyr, the middle the ratio after three initial half-mass relaxation times, and the bottom the ratio after one initial half-mass relaxation time. One sigma error bars are given. Half-mass and half-light radii are clearly larger in blue clusters.

are less likely to escape. The combination of the larger BH masses and higher BH retention rates leads to significant differences in the BH populations of blue and red clusters.

The combination of initial concentration and metallicity give 6 different sets of initial conditions, each of which is then independently realized 10 times to constrain the stochasticity inherent in collisional stellar dynamics. All values in this paper are an average over all ten realizations unless specifically noted otherwise. This yields a set of 60 simulations. The basic parameters are given in Table 1. The initial half-mass relaxation times are calculated according to Spitzer (1987):

$$t_{rh} = 0.138 \frac{N^{1/2} r_{hm}^{3/2}}{\langle m \rangle^{1/2} G^{1/2} \ln \gamma N}, \quad (1)$$

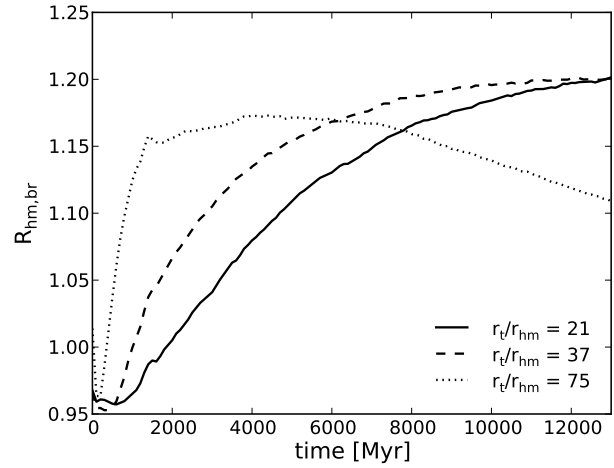


Figure 2. The evolution of $R_{hm,br}$ as a function of time for all three different initial concentrations.

where $\langle m \rangle$ is the average mass of stars in the cluster. I note that each of these simulations takes only ~ 1 day to run as opposed to the months needed for direct N -body simulations. It is this advantage in speed that makes this investigation possible. I have performed several further simulations to investigate specific effects and these will be described later.

3 THE RELATIVE SIZES OF RED AND BLUE GCS

In figure 1 I show the ratios of r_{hm} and r_{hl} between blue and red GCS. r_{hl} is calculated by projecting the luminosity of all stars into annular rings and summing the total luminosity in each ring radially outwards until finding the radius containing half the total luminosity. The ratio of the value of r_{hm} in blue over the value of r_{hm} in red clusters will be referred to as $R_{hm,br}$ while the ratio of the value of r_{hl} in blue over the value of r_{hl} in red cluster will be referred to as $R_{hl,br}$. It is clear that both r_{hm} and r_{hl} are enhanced in blue clusters. After 13 Gyr the value of r_{hm} is $\sim 20\%$ greater in blue clusters, in rough agreement with the value found after ~ 250 Myr by Schulman, Glebbeek & Sills (2012). By contrast the value of r_{hl} is some ~ 40 - 50% greater, a larger enhancement than observed in real clusters. The difference in $R_{hm,br}$ and $R_{hl,br}$ indicates that the enhancement of r_{hl} in blue GCS is not simply the result of a difference in size. That $R_{hl,br}$ is larger in my simulations than in the observations suggests that my simulations over-predict the difference between blue and red GCS.

The time evolution of $R_{hm,br}$, shown in figure 2, is rather similar to the behaviour found by Schulman, Glebbeek & Sills (2012) – r_{hm} starts off smaller in the blue clusters but then grows rapidly so the blue clusters become larger than the red ones within an initial relaxation time. Schulman, Glebbeek & Sills (2012) suggest that this pattern reflects stronger two-body relaxation between the more massive stars in the blue clusters. Both clusters experience strong initial mass-loss due to the rapid evolution of massive stars and because this effect is stronger in the red clusters they initially expand faster. However, the

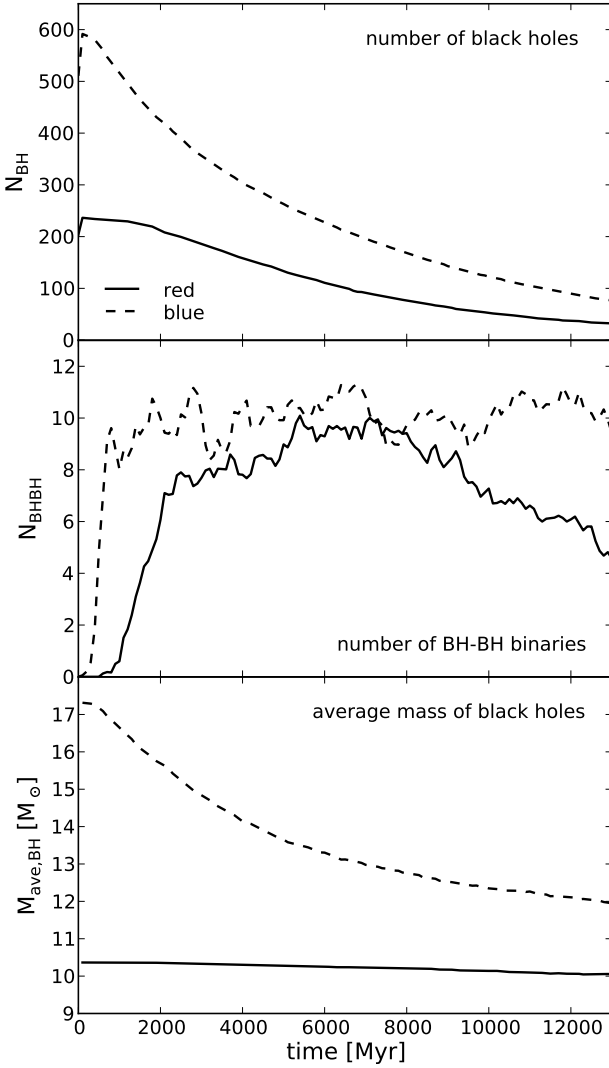


Figure 3. The evolution of the BH population in blue and red GCs with an initial concentration of $r_t/r_{hm} = 37$. The top panel gives the number of single BHs, the middle panel the number of BH-BH binaries and the bottom panel the average mass of single BHs.

more energetic two-body interactions between the massive stars in the blue clusters generate more dynamical heat and cause them to expand faster than the red clusters once the initial phase of rapid mass-loss is complete. Thus after the first few hundred Myrs the blue clusters become larger than the red ones.

Schulman, Glebbeek & Sills (2012) do not determine whether the dominant source of the two-body heating in the blue clusters is the interactions between a large number of stars, all of which are only slightly more massive than the stars in the red clusters, or if a small population of very massive stars is responsible for the effect. A comparison of the time evolution of $R_{hm,br}$ in figure 2 for different initial concentrations provides a clue. For the lowest density (dynamically youngest) clusters the value of $R_{hm,br}$ slowly grows to ~ 1.2 and then remains constant whereas in the densest (dynamically oldest) cluster $R_{hm,br}$ grows rapidly but

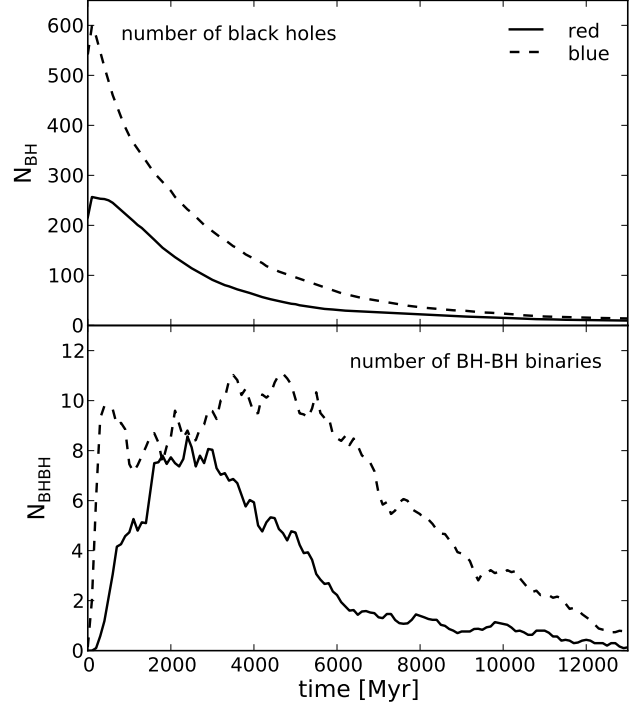


Figure 4. The BH and BH-BH binary population in clusters with $r_t/r_{hm} = 75$. The top panel gives the number of single BHs while the bottom panel gives the number of BH-BH binaries.

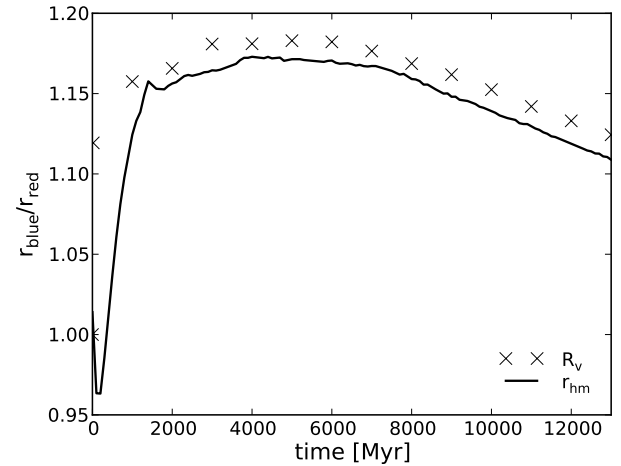


Figure 5. A comparison of ratios of R_v and r_{hm} between blue and red clusters with $r_t/r_{hm} = 75$.

then begins to decline. This indicates that the densest blue clusters start to re-contract compared to the red clusters because the population of stars responsible for the expansion has been lost. Such behaviour suggests a small population of massive objects that can pump a large amount of kinetic energy into the blue clusters through interactions but are also scattered out of the cluster and depleted.

BHs are excellent candidates for this population: after the first few 100 Myrs they are significantly more massive than the average mass in the cluster; they are expected to

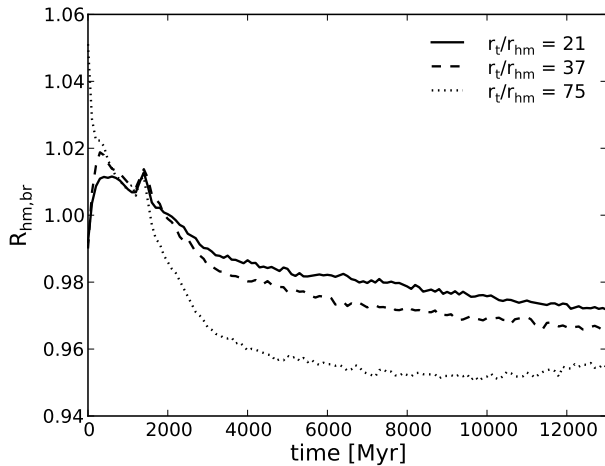


Figure 6. The same as for figure 2 but for clusters where all BHs have been removed by giving them 1000 km/s natal kicks.

mass-segregate to the core, become Spitzer-unstable (Spitzer 1987; Sigurdsson & Hernquist 1993), and interact strongly; they are also few in number so they can be rapidly depleted by interactions. MacKey et al. (2008) presented simulations that show a population of BHs can lead to a significant growth in core radius compared to simulations without BHs. Figure 8 of the same paper suggests the same may be true for r_{hm} . Specifically, MacKey et al. (2008) propose that the BHs mass-segregate to the centres of the clusters where they strongly interact and form BH-BH binaries. Downing et al. (2010) show that this also occurs in my simulations. These binaries scatter single BHs from the cluster centre to its outer regions. From there these BHs sink back to the centre of the cluster due to dynamical friction. This process does $m_{BH}|\phi|$ work on the cluster per BH where m_{BH} is the mass of the BH and ϕ is the gravitational potential of the cluster. This work dynamically heats the cluster and causes it to expand.

As discussed in § 2, there can be major differences between the BH populations in my red and blue simulations. I show the population of BHs in the $r_t/r_{hm} = 37$ simulations in figure 3. I choose to analyze this set of simulations because there are the least extreme set of initial conditions and should be the most generally representative. Figure 3 confirms that the blue clusters indeed have a larger population of BHs and that these BHs are significantly more massive than those in the red clusters. The blue clusters also produce slightly larger numbers of BH-BH binaries slightly earlier than the red clusters but the difference is not nearly as great. This is in line with the model of MacKey et al. (2008) where the BH-BH binaries do not interact directly with the rest of the cluster but only with other BHs. It is these scattered single BHs that are responsible for the dynamical heating of the luminous stellar population. Although the red clusters have not insubstantial BH-BH binary populations compared to the blue clusters, they do not have a sufficient number of sufficiently massive single BHs to heat the rest of the cluster and cause it to expand.

The BH hypothesis also explains the evolution of the most concentrated clusters. Figure 4 shows that the both

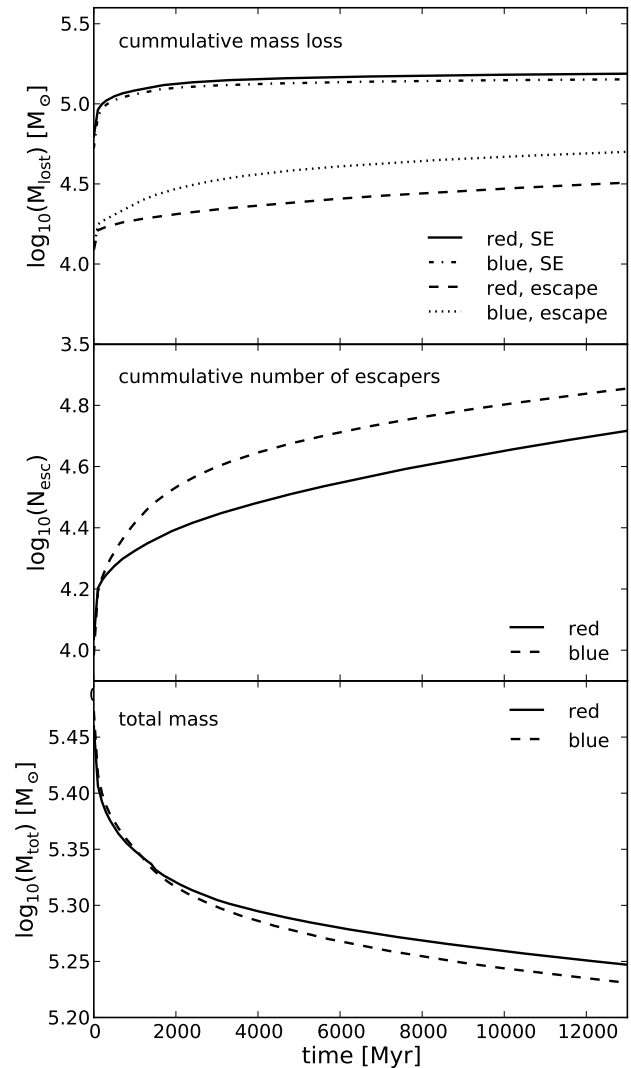


Figure 7. Mass-loss and number of escapers in blue vs. red clusters with BHs and with $r_t/r_{hm} = 37$. The top panel gives the cumulative mass-loss due to stellar evolution and escapers, the middle the cumulative number of escapers and the bottom the total mass as a function of time.

BHs and BH-BH binaries are rapidly depleted in the $r_t/r_{hm} = 75$ clusters. Consequently these clusters lose this source of dynamical heat. The Monte Carlo models remain in virial equilibrium to within a fraction of a per cent by construction so this loss of heat must be compensated for by a change in the structure of the cluster. The virial radius,

$$R_v = \frac{GM^2}{2K}, \quad \left(K = \frac{1}{2}M\sigma^2 \right) \quad (2)$$

where M is the total mass of the cluster, σ the velocity dispersion and K the kinetic energy, is a good measure of this change. Figure 5 shows that the ratio of R_v between the blue and red clusters increases while there are many BHs and BH-BH binaries and then decreases when these objects are depleted. Indeed the drop in the ratio of R_v and a sharp drop in the number of BH-BH binaries both occur at ~ 6 Gyr. The ratio of R_v also tracks the evolution of $R_{hm,br}$ very

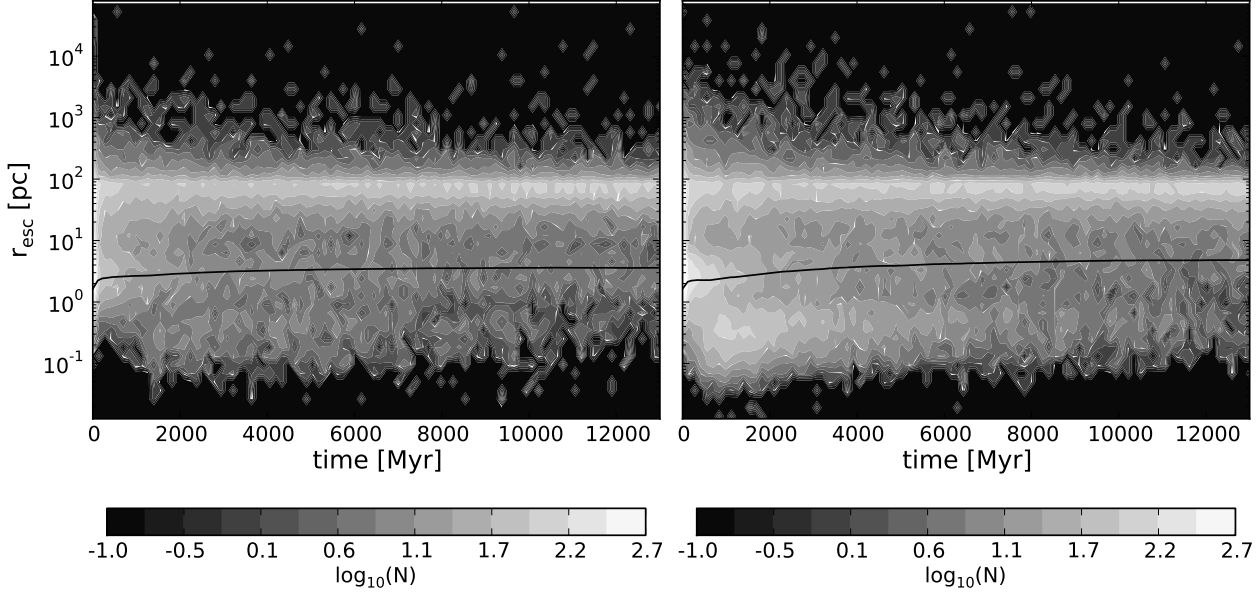


Figure 8. Position of last interaction before escape relative to the cluster centre for escapers from a single red (left) and single blue (right) cluster with $r_t/r_{hm} = 37$ as a function of time. The black line is the 10% Lagrangian radius.

closely. This strongly supports the case that heating by BHs is responsible for the size differences between these clusters.

To confirm that the difference in r_{hm} between my blue and red clusters is a result of the different BH populations I have run a set of simulations without BHs. For each set of initial conditions listed in table 1 I have performed an additional five simulations, identical up to and including the random seed, but where all BHs are given a kick of 1000 km/s upon formation. This instantaneously removes all black holes while allowing the cluster to continue its evolution. Values of $R_{hm,br}$ for the clusters without BHs are given in figure 6 and can be compared to the results for clusters with BHs given in figure 2. The results are dramatic – the enhancement in r_{mh} of blue clusters completely disappears! Indeed the trend observed in figure 2 is reversed: the blue clusters are slightly larger at early times – probably due to the fact that BHs are more numerous and massive in these clusters, resulting in more mass loss when they are ejected – and slightly smaller at late times. The effect in figure 6 is only $\sim 2 - 4\%$, much smaller than the $\sim 20\%$ differences observed in figure 2. Because the only difference between these two sets of simulations is the BH population, this proves that the enhancement in r_{hm} observed in my first set of blue clusters is due to the dynamical activity of BHs.

One effect I observe in my simulations with BHs that was not extensively discussed by MacKey et al. (2008) is a difference in the total escape rate of stars from the blue and red clusters (MacKey et al. (2008) examine only the escape rate of BHs). In figure 7 I show that the blue clusters lose more stars, both by number and by mass, to escape than do the red clusters. By contrast the red clusters lose slightly more mass to stellar evolution than do the blue clusters. The difference in mass lost to escapers is larger and the blue clusters lose more mass overall than do the red clusters.

Figure 8 shows that the majority of these additional escapers come from the central regions of the cluster, which suggests they are stars that have gained additional energy through interactions with the BH sub-system. This additional mass-loss from the cluster centre will cause the inner Lagrangian radii to expand compared to the overall radius of the cluster and enhance the expansion of r_{hm} . Figure 9 gives the total energy of each escaping star ($E_* = K_* + m_*\phi$ where K_* is the kinetic energy of the star and m_* is its mass) as a function of the position of its last interaction before escaping from the cluster. Most of the additional escapers in the inner region do not have particularly high energies. Therefore they are unlikely to have been ejected in an interaction with a binary. Rather they have gained their energy due to strong two-body relaxation in the inner regions of the cluster. This is fully consistent with the results of MacKey et al. (2008) who found that the BH-BH binaries do not interact with other cluster stars directly but only with the single BHs. It is two-body heating from these single BHs that produce the expansion of the cluster and the higher escape rate.

In figure 10 I compare the mass-loss and escaper rates in blue and red clusters without BHs. The difference in total number of escapers almost completely disappears. There is still a difference between the mass lost due to escapers but the majority of this occurs early and can be attributed to the instantaneous ejection of BHs from the clusters. Because the blue clusters form more BHs and these BHs are more massive, it follows that the blue clusters will lose more mass when they are ejected. This may also explain the slightly larger values of r_{hm} in blue clusters without BHs at early times. This difference has little effect on the total mass of the clusters and, unlike the case with BHs, the blue and red clusters have approximately the same mass at 13 Gyrs of age.

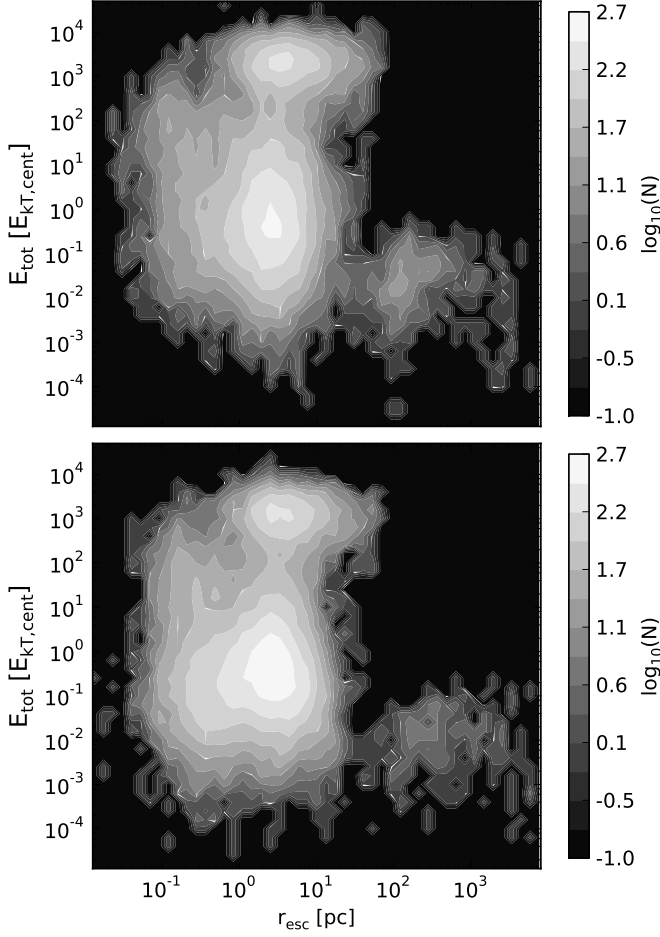


Figure 9. The total energy of escaping stars as a function of the position of their last interaction before escape for a single red (top) and single blue (bottom) cluster with $r_t/r_{hm} = 37$. The total energy of each star is given in units of the core kinetic energy of the cluster. The low-energy peak is primarily ejections due to two-body relaxation while the high-energy peak is ejections due to interactions with binaries.

4 THE HALF-LIGHT RADII OF RED AND BLUE GCS

Figure 1 shows that the value of $R_{hl,br}$ is larger both than $R_{hm,br}$ and than the value that is observed in real GCS. Therefore it is clear from these simulations that $R_{hl,br}$ is not necessarily a good predictor of $R_{hm,br}$. To understand the difference between $R_{hl,br}$ and $R_{hm,br}$ I compare the time evolution of $R_{hl,br}$ in clusters with and without BH in figure 11. The half-light radii are clearly larger in blue clusters for all sets of initial conditions, regardless of whether r_{hm} is larger or not. The enhancement in r_{hl} in the clusters without BHs is smaller but is actually in much better agreement with the observed enhancement of $\sim 20\%$, at least for the two most initially concentrated clusters. The difference between r_{hm} in the blue and red clusters with BHs contributes to the difference in r_{hl} but clearly there are other processes at work.

In figure 12 I compare the fraction of the total luminosity in the most massive 10% of stars for the blue and red

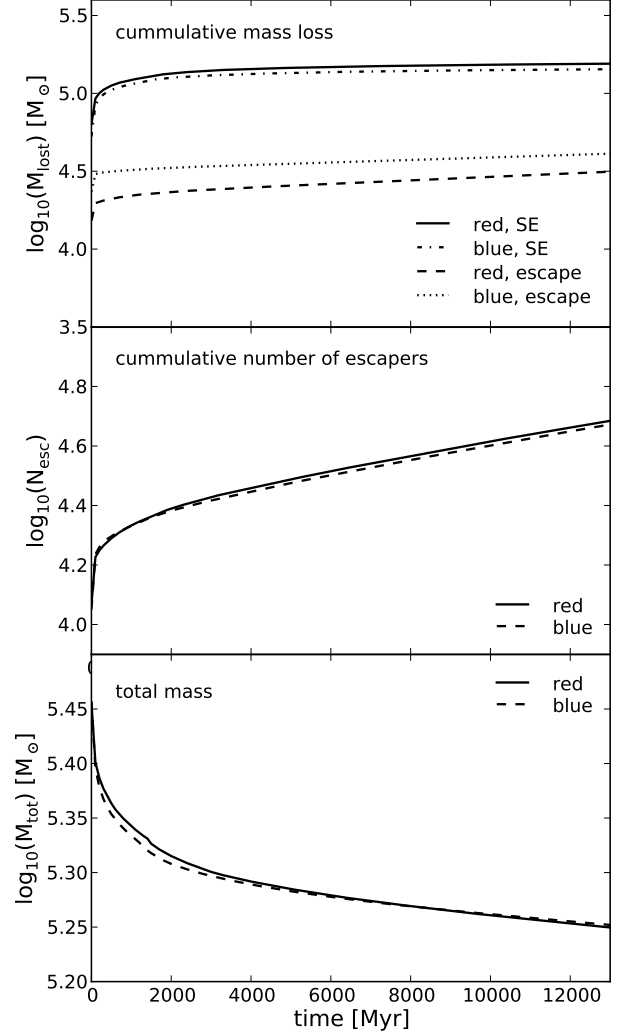


Figure 10. The same as for figure 7 but for clusters without BHs.

clusters while in figure 13 I compare the mass-segregation in these clusters using the average masses within selected Lagrangian radii. The red clusters have a steeper luminosity function with a significantly larger proportion of their luminosity concentrated in their most massive stars. The average stellar mass is higher for the inner Lagrangian radii in both blue and red clusters so both sets of clusters are mass-segregated. For a given Lagrangian radius, however, the average mass is similar in both the blue and the red clusters. Taking figures 12 and figure 13 together, this means that the red clusters will have a larger fraction of their luminosity located within their innermost Lagrangian radii than will the blue clusters and thus they will have a smaller value of r_{hl} compared to r_{hm} . Consequently the value of $R_{hl,br}$ will be larger than the value of $R_{hm,br}$. This explains why $R_{hl,br} > 1$ in clusters of the same size according to r_{hm} . The

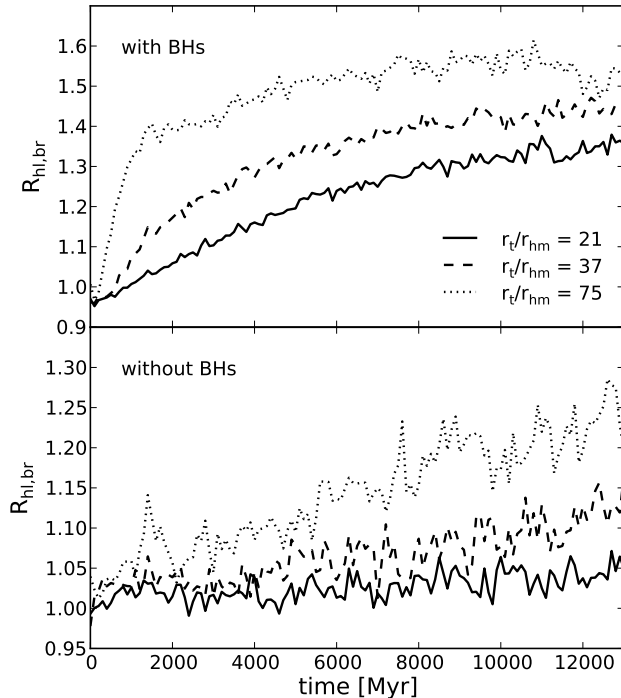


Figure 11. The ratio of r_{hl} between blue and red clusters for all initial concentrations with and without BHs. The top panel gives the values for clusters with BHs while the bottom panel gives the values for clusters without BHs.

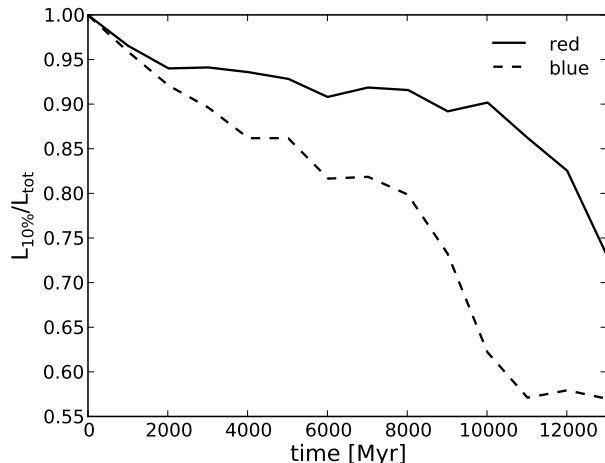


Figure 12. The fraction of the total luminosity contained within the most massive 10% of stars for blue and red clusters without BHs and with $r_t/r_{hm} = 37$.

effect will be further enhanced if the blue clusters are also larger than the red clusters and explains the greater value of $R_{hl,br}$ in clusters with BHs. However it is not necessary for a cluster with a larger value of r_{hl} to have a larger value of r_{hm} . This suggests an additional corollary: r_{hl} cannot be used to measure the relative sizes of GCs, at least not in a straightforward way.

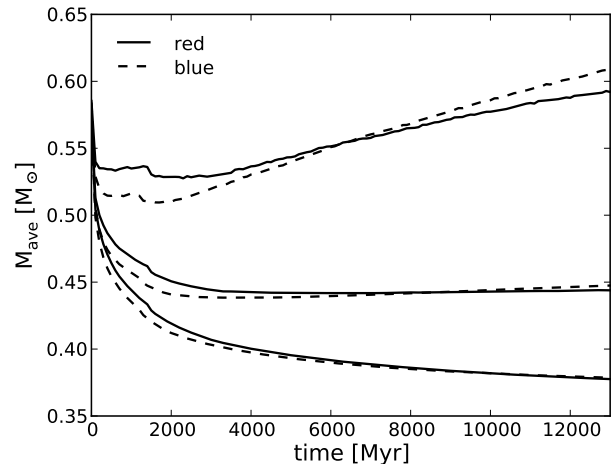


Figure 13. From top to bottom: the average mass within the 10%, 50% and 100% Lagrangian radii as a function of time for red and blue clusters without BHs and with $r_t/r_{hm} = 37$.

5 DISCUSSION

My simulations show that blue GCs can be larger than red GCs but only if they have a substantial population of BHs. Blue and red clusters without BHs will be roughly the same size but the red clusters will still have smaller values of r_{hl} due to their steeper luminosity functions. I find that the enhancement in r_{hl} for blue clusters with BHs is significantly larger than that observed value. In the clusters without BHs the enhancement in r_{hl} roughly matches the observations, at least for the clusters with the highest initial concentrations. This finding supports the results of Jordán (2004) – that the difference in half-light radii between blue and red GCs is the result of different luminosity functions in clusters with similar sizes – rather than the results of Schulman, Glebbeek & Sills (2012). It may also indicate that there is little difference between the BH population in blue and red GCs. This will have consequences for understanding effect of metallicity on the late-time evolution and supernovae of massive stars.

The simulations of Schulman, Glebbeek & Sills (2012) used the initial mass function of Kroupa (2001), which has a high-mass slope of ~ -2.3 , and used a maximum Mass of $50 M_{\odot}$ (Glebbeek, private communication). Such a mass function will yield few BHs so a question remains as to why my simulations with BHs behave in a qualitatively similar way to those of Schulman, Glebbeek & Sills (2012) while the ones without BHs do not. I speculate that the similarity is a result of the differing relationship between the stellar evolution and the relaxation timescales. In my simulations t_{rh} is significantly longer than the evolution timescale for massive stars, even in the most concentrated clusters. Thus by the time relaxation becomes important for the structure of the cluster the only remaining population of stars significantly more massive than average are the BHs. By contrast the relaxation timescales of the simulations of Schulman, Glebbeek & Sills (2012) are ~ 10 -50 Myr, shorter than the evolution timescales of moderately massive stars. Therefore these stars remain when two-body relaxation starts to drive cluster evolution. Furthermore, the

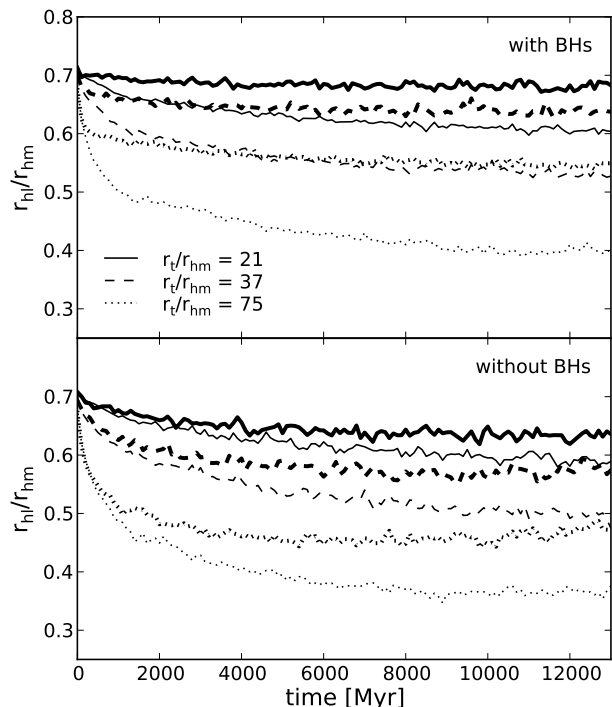


Figure 14. The ratio of r_{hl} to r_{hm} for all simulated GCs. Thin lines are red GCs while thick lines are blue GCs. The top panel gives the values for GCs with BHs while the bottom panel gives values for GCs without BHs.

short relaxation time will allow these stars to mass-segregate before they lose a significant amount of their mass to winds and nuclear burning. Interactions between these stars will be able to act as a heat source in a similar way to the BH subsystem in my simulations. I hypothesize that small clusters, with relaxation times shorter than the evolution timescale for massive stars, have a variety of objects massive enough to drive cluster expansion through two-body heating while in GCs, where the relaxation timescale is much longer than the stellar evolution timescale, BHs are the only remaining objects massive enough to produce this effect.

It is also apparent that the value of $R_{hl,br}$ is not the same as that of $R_{hm,br}$ and thus it cannot be used as a direct estimate of the relative sizes of GCs. Indeed a comparison of figures 6 and 11 shows that a cluster can have a smaller value of r_{hm} than another cluster but still have a larger value of r_{hl} . In figure 14 I show the ratio of r_{hl} to r_{hm} for all of my cluster models. The ratio varies considerably both with time and with the initial conditions used. The ratio does seem to attain a fairly stable value for each individual cluster, indicating that r_{hl} is not completely independent of r_{hm} . Since this value always lies somewhere between 0.35 and 0.7 it is, in principle, possible to use r_{hl} to estimate the value of r_{hm} to within a factor of ~ 2 . It is not possible, however, to use $R_{hl,br}$ to estimate the relative size of two clusters unless (at minimum) their metallicities, initial concentrations, and ages are known.

6 CONCLUSIONS

I have analyzed a suite of Monte Carlo globular cluster simulations to determine the origin of the difference in half-light radii between blue and red GCs. I find that, provided they are mass-segregated, the value of r_{hl} is larger in blue than in red GCs simply due to differences in the luminosity function between metal-poor and metal rich stellar populations. Depending on the initial conditions this effect is sufficient to explain the observed differences between r_{hl} in blue and red GCs. I find that it is also possible for blue clusters to be physically larger than red clusters as measured by their half-mass radii. However this only occurs if there are significant differences in the number and properties of BHs between metal-poor and metal-rich stellar populations. A difference in r_{hm} can certainly enhance the difference in r_{hl} but it is not a necessary condition for such a difference to exist. Furthermore, the enhancement in r_{hl} in blue GCs when there is significant difference in the BH population is larger than observed. This leads me to conclude that blue and red GCs probably do not have significant differences in their BH populations but further simulations and more careful comparisons with observations will be necessary to confirm this. Further simulations are also necessary to determine if it is number or mass that determines whether BHs can cause a GC to expand. Finally, I find that $R_{hl,br}$ does not directly predict $R_{hm,br}$ so differences in r_{hl} cannot be used to infer differences in size between clusters of different metallicities.

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REFERENCES

- Aarseth, S. J. and Heggie, D. C., 1998, MNRAS, 297, 794
- Barmby, P., Holland, S. T. and Huchra, J. P., 2002, AJ, 123, 1937
- Belczynski, K., Kalogera, V. and Bulik, T., 2002, ApJ, 572, 407
- Belczynski, K., Sadowski, S., Rasio, F. A. and Bulik, T., 2006, ApJ, 650, 303
- Downing, J. M. B., Benacquista, M. J., Giersz, M. and Spurzem, R., 2010, MNRAS, 407, 1946
- Giersz, M., 1998, MNRAS, 298, 1239
- Giersz, M., 2001, MNRAS, 324, 218
- Giersz, M., 2006, MNRAS, 371, 484
- Giersz, M. and Heggie, D. C., 2009, MNRAS, 395, 1173
- Giersz, M., Heggie, D. C. and Hurley, J. R., 2008, MNRAS, 388, 429
- Giersz, M. and Spurzem, R., 2003, MNRAS, 343, 781
- Hansen, B. M. S. and Phinney, E. S., MNRAS, 291, 569
- Harris, W. E., Harris, G. L. H., Holland, S. T. and McLaughlin, D. E., 2002, AJ, 124, 1435

- Harris, W. E., 2009, *ApJ*, 699, 254
- Heggie, D. C. and Giersz, M., 2008, *MNRAS*, 389, 1858
- Heggie, D. C. and Giercz, M., 2009, *MNRAS*, 397, 46
- Hurley, J. R., Pols, O. R. and Tout, C. A., 2000, *MNRAS*, 315, 543
- Hurley, J. R., Tout, C. A. and Pols, O. R., 2002, *MNRAS*, 329, 897
- Jordán, A., 2004, *ApJ*, 613, 117
- Jordán, A., Côte, P., Ferrarese, L., Blakeslee, J. P., Mei, S., Merritt, D., Milosavljević, M., Peng, W. E., Tonry, J. L. and West, M. J., 2004, *ApJ*, 613, 279
- Kroupa, P., 1995, *MNRAS*, 277, 1507
- Kroupa, P., 2001, in *ASP Conf. Ser.*, eds. Deiters, S., Fuchs, B., Just, A., Spurzem, R., Wielan, R., 228, 187
- Kroupa, P., Tout, C. A. and Gilmore, G., 1993, *MNRAS*, 251, 293
- Kundu, A. and Whitmore, B. C., 1998, *AJ*, 116, 2841
- Kundu, A. and Whitmore, B. C., 2001, *AJ*, 121, 2950
- Kundu, A., Whitmore, B. C., Sparks, W. B., Macchetto, F. D., Zepf, S. E. and Ashman, K. M., 1999, *ApJ*, 513, 733
- Larsen, S. S. and Brodie, J. P., 2003, *ApJ*, 593, 340
- Larsen, S. S., Brodie, J. P., Huchra, J. P., Forbes, D. A. and Grillmair, C., 2001a, *AJ*, 121, 2974
- Larsen, S. S., Forbes, D. A. and Brodie, J. P., 2001b, *MNRAS*, 327, 1116
- MacKey, A. D., Wilkinson, M. I., Davies, M. B. and Gilmore, G. F., 2008, *MNRAS*, 386, 65
- Puzia, T. H., Kissler-Patig, M., Brodie, J. P., and Huchra, J. P., 1999, *AJ*, 118, 2734
- Schulman, R. D., Glebbeek, E. and Sills, A., 2012, *MNRAS*, 420, 651
- Sigurdsson, S. and Hernquist, L., 1993, *Nature*, 364, 423
- Spitzer, L., Jr., 1987, *Dynamical Evolution of Globular Clusters*, Princeton University Press, Princeton, NJ, USA
- Spitzer, L., Jr. and Thuan, T. X., 1972, *ApJ*, 175, 31
- van den Bergh, S., Morbey, C. and Pazder, J., 1991, *ApJ*, 375, 594