**The Initial Mass Function:**

There are two functions related to star formation that have fundamental importance:

1. The stellar birth rate.

2. The frequency distribution of stellar masses at birth, aka the **initial mass function** or IMF.

There are two standard papers on the IMF:

Salpeter, E. E. 1955, ApJ, 121, 161

Miller, G. E., & Scalo, J. M. 1979, ApJS, 41, 513

**Notation:**

Suppose there is a burst of star formation. Let the number of stars with masses between $\mathcal{M}$ and $\mathcal{M} + d\mathcal{M}$ be

$$dN = N_0 \xi(\mathcal{M}) d\mathcal{M}$$

The function $\xi(\mathcal{M})$ is the IMF. This function gives the relative number of stars formed with masses between $\mathcal{M}$ and $\mathcal{M} + d\mathcal{M}$

Normalize things such that

$$\int d\mathcal{M} \mathcal{M} \xi(\mathcal{M}) = \mathcal{M}_\odot.$$

Given this normalization, $N_0$ is the total mass of the stars in the starburst in solar masses.

Finding the functional form of the IMF is a very long process. Among other things, one must know:

- The number density of stars at a given luminosity (the luminosity function);
- The number density of stars at a given mass that are **presently** observed (the present day mass function or PDMF);
- The relation between stellar mass and stellar luminosity;
- The scale height of various populations in the Galactic disk;
- The contribution to stars not on the main sequence.

Is the IMF universal? Does this function depend on metallicity or other factors (e.g. local mass density, ...)

Is there any reason why $\xi(\mathcal{M})$ should have a simple functional form?

For simplicity, it is usually **assumed** that the IMF has the form of a power-law:

$$\xi(\mathcal{M}) \propto \mathcal{M}^{\alpha}$$

The **present day mass function** is fundamental. This is defined as the number of main-sequence stars per unit logarithmic mass interval per square parsec in the solar neighborhood. The PDMF is integrated perpendicular to the Galactic disk to account for the fact that high mass stars are near the plane and low mass stars are found further out.

In equation form, the PDMF is

$$\Phi_{\mathrm{MS}}(\log \mathcal{M}) = \Phi(M_V) \left| \frac{dM_V}{d \log \mathcal{M}} \right| 2H(M_V) f_{\mathrm{MS}}(M_V)$$

where $dM_V/d\log\mathcal{M}$ is the slope of the (absolute magnitude, mass)-relation, $2H(M_V)$ accounts for the integrated scale heights, and $f_{\mathrm{MS}}(M_V)$ corrects for the stars not on the main sequence.

As there is no good theory, all of these factors must be observationally determined.

To find the **luminosity function**, simply count the number of stars as a function of the apparent magnitude and find their distances.

There are lots of potential problems and biases:

- The **Malmquist bias**, whereby stars that are intrinsically brighter tend to dominate magnitude-limited samples;
- Errors in stellar distance determinations, notably parallax measurements, and other indirect distance indicators;
- Uncertain corrections for interstellar extinction;
- Uncertain **bolometric corrections**.

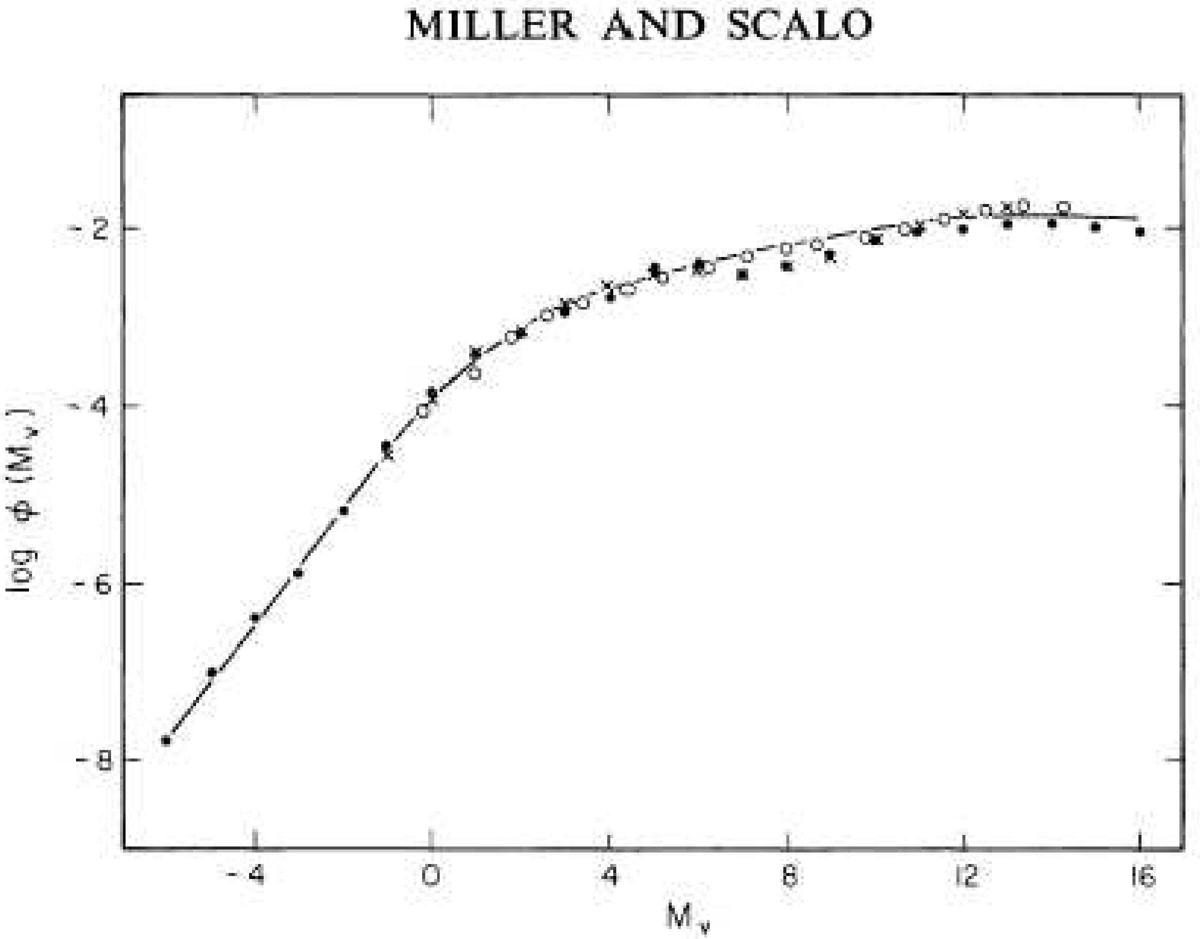
The general luminosity function (per $10^4$ pc$^3$), from Binney & Merrifield:

| $M_V$ | $\Phi(M_V)$ | $\delta L/L_\odot$ | $\delta\mathcal{M}/\mathcal{M}$ | $M_V$ | $\Phi(M_V)$ | $\delta L/L_\odot$ | $\delta\mathcal{M}/\mathcal{M}$ |
|---|---|---|---|---|---|---|---|
| −6 | 0.0001 | 2.6 | 0.005 | 7 | 29 | 4.0 | 21.3 |
| −5 | 0.0006 | 5.1 | 0.020 | 8 | 33 | 1.8 | 21.8 |
| −4 | 0.0029 | 9.4 | 0.060 | 9 | 42 | 0.90 | 24.2 |
| −3 | 0.013 | 17.1 | 0.17 | 10 | 70 | 0.60 | 35.0 |
| −2 | 0.05 | 28.2 | 0.05 | 11 | 90 | 0.30 | 36.0 |
| −1 | 0.25 | 53.9 | 1.6 | 12 | 127 | 0.17 | 36.3 |
| 0 | 1 | 95.8 | 4.0 | 13 | 102 | 0.055 | 20.8 |
| 1 | 3 | 111 | 7.4 | 14 | 102 | 0.022 | 16.3 |
| 2 | 5 | 64 | 8.7 | 15 | 127 | 0.011 | 16.3 |
| 3 | 12 | 66 | 17.3 | 16 | 102 | 0.0035 | 10.5 |
| 4 | 17 | 36 | 19.4 | 17 | 51 | 0.0007 | 4.3 |
| 5 | 29 | 25 | 28.1 | 18 | 22 | 0.0001 | 1.6 |
| 6 | 30 | 10 | 24.7 | 19 | 13 | 0.0000 | 0.07 |

The luminosity function measures the relative frequency of stars in different magnitude intervals.

It is worthwhile to recall these 4 points:

1. Most stars in the solar neighborhood are intrinsically faint.
2. Nearly all of the light is emitted by a few intrinsically bright stars (even though they are rare).
3. Most of the stellar mass density is contributed by the large number of intrinsically faint stars.
4. The average mass-to-light-ratio is about $0.67\,\mathcal{M}_\odot/L_\odot$, although this is a lower limit owing to the presumably large number of white dwarfs missed in surveys.




Luminosity function from Miller & Scalo.

This function spans more than 16 magnitudes in $V$, which corresponds to a factor of $10^8$ in luminosity.

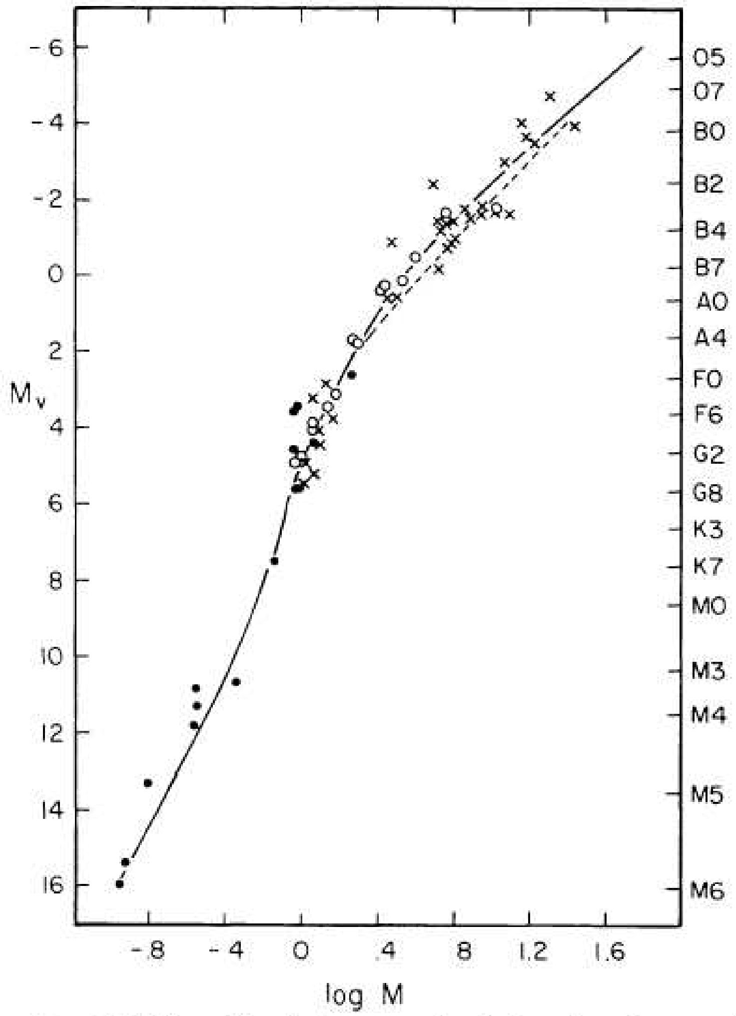



FIG. 2.—Relationship between absolute visual magnitude $M_v$ and mass $M$ ($M_\odot$) for main-sequence stars. ●, visual binaries (Lacy 1977); ○, eclipsing binaries (Lacy 1977); ×, spectroscopic binaries (Cester 1965); *dashed line*, theoretical mass-luminosity relation (Stothers 1974); *solid line*, adopted relation.

$M(\mathcal{M})$ from Miller & Scalo.

The most reliable masses come from binary stars. In many cases, the data are sparse (i.e. there are very few eclipsing double-lined high mass binaries, etc.)

The stellar masses range from the hydrogen burning limit of $\approx 0.08\,M_\odot$ to about 50.

Getting reliable data at these extreme ends is difficult:

Very low mass stars are very faint and very red.

Very high mass stars are very rare.

Binaries contaminate the samples at high rates (i.e. what you may think is a single star is actually two close stars).

**Scale heights:**

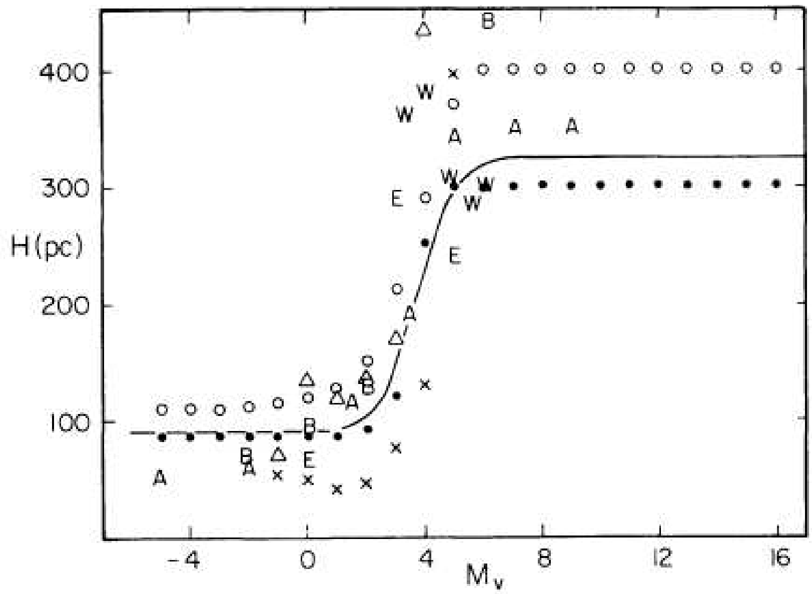



FIG. 3.—Relationship between scale height $H$ (pc) and absolute visual magnitude $M_v$. ●, Schmidt (1963); ○, Schmidt (1959); ×, Upgren (1963); ▲, McCuskey (1966); *W*, Weistrop (1972); *B*, Bok and MacRae (1941); *E*, Elvius (1951, 1962); *A*, Allen (1973); *solid line*, adopted relation.

(From Miller & Scalo).

The **distribution** of stars perpendicular to the galactic disk varies with spectral type. O stars are concentrated near the plane, whereas K and M stars are found at much further distances.

Finding the scale heights of low-mass stars is difficult because the stars are faint.

for a distance of 400 pc,

$$\begin{aligned} m_V - M_V &= 5\log d - 5 \\ &= 5\log 400 - 5 \\ &= 8 \end{aligned}$$

If $M_V = 14$ (corresponding to a mass of $\approx 0.2\,\mathcal{M}_\odot$), then the apparent $V$ magnitude is 22.

**Stars off the main sequence:**

TABLE 2

FRACTION ($f_{ms}$) OF STARS OF A GIVEN ABSOLUTE MAGNITUDE ON THE MAIN SEQUENCE

| $M_v$ | Salpeter 1955 | Sandage 1957 | Schmidt 1959 | Upgren 1963 | McCuskey 1966 |
|---|---|---|---|---|---|
| −6...... | ... | 0.46 | ... | ... | 0.40 |
| −5...... | ... | 0.48 | 0.41 | ... | 0.42 |
| −4...... | 0.18 | 0.48 | 0.41 | ... | 0.43 |
| −3...... | 0.36 | 0.50 | 0.46 | ... | 0.44 |
| −2...... | 0.50 | 0.51 | 0.48 | ... | 0.45 |
| −1...... | 0.47 | 0.53 | 0.52 | 0.12 | 0.47 |
| 0...... | 0.41 | 0.56 | 0.46 | 0.29 | 0.51 |
| +1...... | 0.47 | 0.62 | 0.33 | 0.62 | 0.56 |
| +2...... | 0.65 | 0.71 | 0.69 | 0.91 | 0.66 |
| +3...... | 0.76 | 0.86 | 0.87 | 1.00 | 0.82 |
| +4...... | 0.95 | 1.00 | 1.00 | 1.00 | 0.98 |
| +5...... | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

(From Miller & Scalo).

Both an A0V star and a K2III star have $M_V \approx 0$, so one has to account for this.

The corrections are not well-known for the most luminous stars, but there are few stars at these luminosities.

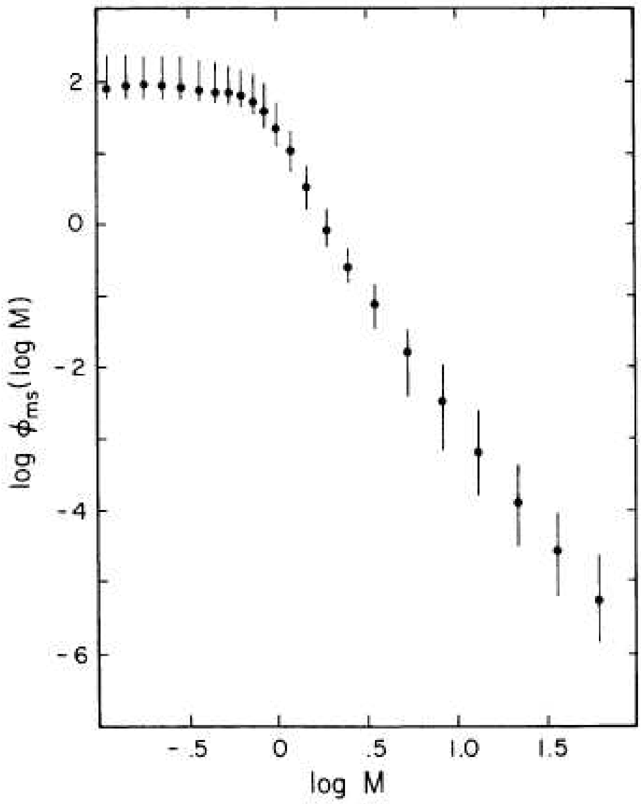



FIG. 4.—The PDMF of main-sequence field stars $\phi_{ms}(\log M)$ given as the number of stars $pc^{-2} \log M^{-1}$ in the solar neighborhood. Uncertainty bars are from Table 5.

Adopted PDMF, from Miller & Scalo.

The units give the number of main sequence stars $\mathrm{pc}^{-2}(\log \mathcal{M})^{-1}$.

The masses range from $0.11\,\mathcal{M}_\odot$ to $62\,\mathcal{M}_\odot$, and the number densities span 7 orders of magnitude.

To get the IMF from the PDMF, we have to correct the for the effects of stellar evolution. If the star formation rate has been roughly constant over time, then

$$\xi(\mathcal{M}) = \Phi(\mathcal{M}) \times \begin{cases} t/\tau_{\mathrm{MS}}(M) & \tau_{\mathrm{MS}}(M) < t \\ 1 & \text{otherwise} \end{cases}$$

$t$ is the age of the system, and $\tau_{\mathrm{MS}}(M)$ is the main sequence lifetime of a star of absolute magnitude $M$.

The fusion of hydrogen into helium gives

$$E \approx 0.0067 \Delta \mathcal{M} c^2$$

of useful energy.

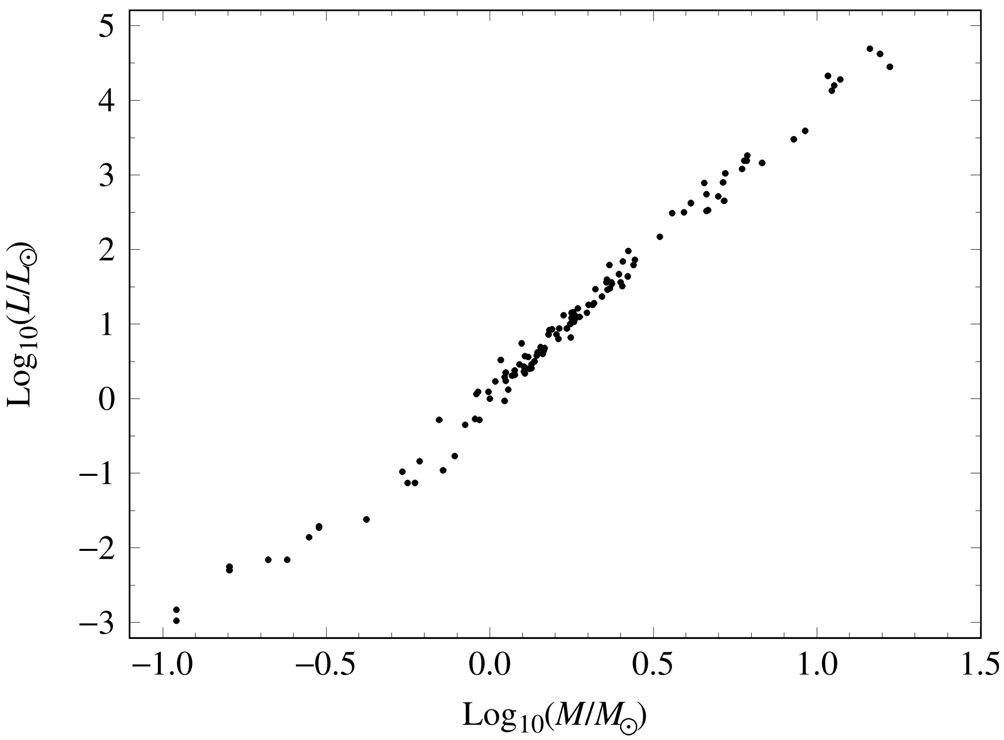
If a fraction $\alpha$ of the total mass of a star can be converted, then

$$\tau_{\mathrm{MS}}(\mathcal{M}) \approx \frac{0.0067 \alpha \mathcal{M} c^2}{L}$$

where $L$ is the luminosity.

Stellar evolution models give $\alpha \approx 0.1$, so that

$$\tau_{\mathrm{MS}}(\mathcal{M}) \approx 10 \left(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}\right) \left(\frac{L}{L_{\odot}}\right)^{-1} \text{ Gyr}$$




Mass-luminosity relation using data from Popper (1990, ARAA, 18, 115). Figure from Ostlie & Carroll.

The most reliable masses come from binary stars. In many cases, the data are sparse (i.e. there are very few eclipsing double-lined high mass binaries, etc.)

The luminosity on the zero-age main sequence depends on the mass:

$$\frac{L_{\rm MS}}{L_\odot} \propto \begin{cases} 81(\mathcal{M}/\mathcal{M}_\odot)^{2.14} & \text{for } \mathcal{M} \gtrsim 20\,\mathcal{M}_\odot \\ 1.78(\mathcal{M}/\mathcal{M}_\odot)^{3.5} & \text{for } 2 \lesssim \mathcal{M} \lesssim 20\,\mathcal{M}_\odot \\ 0.75(\mathcal{M}/\mathcal{M}_\odot)^{4.8} & \text{for } \mathcal{M} \lesssim 2\,\mathcal{M}_\odot \end{cases}$$

Examples:

If $\mathcal{M} = 100\,\mathcal{M}_\odot$, then $L_{\rm MS} = 5.1 \times 10^6\,L_\odot$ and $\tau_{\rm MS} = 2.0 \times 10^5$ years.

If $\mathcal{M} = 60\,\mathcal{M}_\odot$, then $L_{\rm MS} = 5.2 \times 10^5\,L_\odot$ and $\tau_{\rm MS} = 1.2 \times 10^6$ years.

If $\mathcal{M} = 0.6\,\mathcal{M}_\odot$, then $L_{\rm MS} = 0.065\,L_\odot$ and $\tau_{\rm MS} = 93$ billion years.

If $\mathcal{M} = 0.1\,\mathcal{M}_\odot$, then $L_{\rm MS} = 1.2 \times 10^{-5}\,L_\odot$ and $\tau_{\rm MS} = 84$ trillion years.

Salpeter (1955) concluded

$$\xi(\mathcal{M}) \propto \mathcal{M}^{-2.35}$$

Note that the integral of $\mathcal{M}\xi(\mathcal{M})d\mathcal{M}$ diverges...

Recently, Scalo (1986) found

$$\xi(\mathcal{M}) \propto \begin{cases} \mathcal{M}^{-2.45} & \text{for } \mathcal{M} > 10\,\mathcal{M}_\odot \\ \mathcal{M}^{-3.27} & \text{for } 1 < \mathcal{M} < 10\,\mathcal{M}_\odot \\ \mathcal{M}^{-1.83} & \text{for } \mathcal{M} < 0.2\,\mathcal{M}_\odot \end{cases}$$

Note that $\xi(\mathcal{M})$ is poorly constrained at the low mass end owing to theoretical difficulties and observational difficulties.

In general, more low-mass than high mass stars form when a cloud condenses and fragments.

The theory of cloud fragmentation is a difficult subject: