Discrete Time Hedging Error

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1 Numerical experiment

We choose the parameters to do simulate the term M_T^n and compare the simulation with the bound we have obtained.
Under the conditions that $r = 0$, $\sigma(X_t) = \sigma = constant > 0$, $f(x) = (x-K)^+$recall that

$$
\begin{aligned}
M_T^n &= \int_0^T \frac{\partial u}{\partial x}(t, X_t) d\widetilde{X_t} - \int_0^T \frac{\partial u}{\partial x}(\varphi(t), X_{\varphi(t)}) d\widetilde{X_t} \\
&= (X_T - K)^+ - \mathbb{E}[(X_T - K)^+] - \int_0^T \frac{\partial u}{\partial x}(\varphi(t), X_{\varphi(t)}) dX_t \\
&= (X_T - K)^+ - x_0 N(d_1(0)) + K N(d_2(0)) - \sum_{i=0}^{n-1} N(d_1(t_i))(X_{t_{i+1}} - X_{t_i}) \\
&= (X_T - K)^+ - \frac{x_0}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\frac{x_0}{K}) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}} e^{-\frac{v^2}{2}} dv + \frac{K}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\frac{x_0}{K}) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}} e^{-\frac{v^2}{2}} dv \\
&- \sum_{i=0}^{n-1} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\frac{X_{t_i}}{K}) + \frac{1}{2}(T - t_i)}{\sigma\sqrt{T - t_i}}} e^{-\frac{v^2}{2}} dv (X_{t_{i+1}} - X_{t_i})
\end{aligned}
$$

What we need to simulate are the price X_{t_i}, with $i = 1, 2, ..., n$. The algorithm is as follows:

Algorithm 1 Simulation of Geometric Brownian Motion

Step1: Generate $2 \times n \times m$ independent random variables that are uniformly distributed on the interval [0,1], here m is the Monte Carlo samples.
Step2: Generate a normal distributed random variable from two independent uniformly distributed random variables using Box Muller transform. Form a matrix $NRV[m][n]$ of size $m \times n$. formed by $m \times n$ independent normal distributed random variables.
Step3: Construct the Brownian motion matrix $BM[m][n]$ by setting $BM[i][j] = BM[i][j-1] + \sqrt{\frac{T}{n}} NRV[i][j]$ with the initial condition $BM[i][1] = \sqrt{\frac{T}{n}} NRV[i][1]$, $1 \le i \le m, 1 \le j \le n$.
Step4: Construct the matrix $PX[m][n]$ of price X_t by $PX[i][j] = x_0 exp(-\frac{1}{2}\sigma^2 j \frac{T}{n} + \sigma BM[i][j])$, $1 \le i \le m, 1 \le j \le n$.

Step5: For each i, calculate $M_T^n[i]$ using the expression above. Count the numbers s that $M_T^n[i]$ is greater or equals to γ, calculate the simulated probability as $P_1 = prob = \frac{s}{m}$.
Step6: Compare $P_1 = prob$ with $P_2 = L(p*)$.