University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.520 Computer Aided Engineering Analysis
Problem Set 4

1. Consider the solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

for over the interval $0 \le x \le 1$ for the boundary conditions where $y(0) = 1.0$ and $y(1) = 2.0$. The approach to solution will be based on you spatially discretizing the domain and approximating the derivative operators. Define $x_j = j\delta x$, $y(x_j) = y_j$ and $\delta x = 1/N$ where $j = (0, N)$. At x_j

$$\frac{d^2y}{dx^2} = \frac{y_{j-1} - 2y_j + y_{j+1}}{(\delta x)^2}$$

$$\frac{dy}{dx} = \frac{y_{j+1} - y_{j-1}}{(2\delta x)}$$

a. Analytically determine the exact solution.

b. Find y_j for $N = 10$ using a direct solution method.

c. Find the solution y_j using the Jacobi and the Gauss-Siedel methods.

d. Compare the rate of convergence and accuracy of the methods.

2. Consider the lowpass-filter

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where $\omega_c (rad/s)$ is the cutoff frequency. Using the bilinear transformation

$$s = \frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}}$$

where sampling frequency is equal $1/T$ Hz.

a. Determine $H(z)$.

b. Determine the difference equation governing $y(n)$

c. For the sampling frequency of 10kHz and cutoff frequency of 1kHz determine and display using gnuplot the impulse response for 512 time sample points and its spectrum.

Note given cutoff ω_c and the actual cutoff frequency ω_d are related as

$$\omega_c = \frac{2}{T}\, tan(\omega_d T/2)$$