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Comparing Cross-Section and Time-Series Factor Models

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We use the cross-section regression approach of Fama and MacBeth (1973) to construct cross-section factors corresponding to the time-series factors of Fama and French (2015). Time-series models that use only cross-section factors provide better descriptions of average returns than time-series models that use time-series factors. This is true when we impose constant factor loadings and when we use time-varying loadings that are natural for time-series factors and time-varying loadings that are natural for cross-section factors. (*JEL* G1, G11, G12)

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Factors in time-series asset pricing models are often motivated by evidence from Fama and MacBeth (FM 1973) cross-section regressions that average returns are related to asset characteristics. For example, the three-factor model of Fama and French (FF 1993) follows evidence (Banz 1981; Rosenberg, Reid, and Lanstein 1985; Fama and French 1992) that size (market capitalization, *MC*) and the book-to-market equity ratio (*BM*) capture differences in average stock returns missed by the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). Similarly, the five-factor model of Fama and French (FF 2015) follows evidence from FM cross-section regressions that profitability and investment capture differences in average returns missed by the three-factor model (Titman, Wei, and Xie 2004; Fama and French 2006; Novy-Marx 2013).

FM cross-section regressions are, however, a type of factor model. For example, leaving definitions of variables for later, consider the cross-section regression of stock returns for month $t, R_{it}, i=1,\ldots,n,$ on previously

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observed values of size (MC_{it-1}), the book-to-market ratio (BM_{it-1}), operating profitability (OP_{it-1}), and the rate of growth of assets (INV_{it-1}),

$$R_{it} = R_{zt} + R_{MCt} MC_{it-1} + R_{BMt} BM_{it-1} + R_{OPt} OP_{it-1} + R_{INVt} INV_{it-1} + e_{it}. \quad (1)$$

The slope estimates in Equation (1) are portfolio returns that, as indicated by the notation, can be interpreted as factors. Fama (1976, ch. 9) shows that the slope for each variable in an FM cross-section regression is the return on a portfolio of the left-hand-side (LHS) assets with weights for the assets that set the month $t-1$ portfolio value of that variable to one and zero out other explanatory variables. Fama (1976) also shows that each FM slope portfolio requires no net investment; long positions in LHS assets are financed with short positions in other LHS assets. R_{BMt}, for example, is the month t return on a zero-investment portfolio whose weights for LHS assets set the portfolio value of BM_{it-1} to one and set the portfolio values of MC_{it-1}, OP_{it-1}, and INV_{it-1} to zero. The intercept in an FM cross-section regression (R_{zt} in (1)) is the month t return on a standard portfolio of the LHS assets with weights that sum to one and zero out each explanatory variable. The intercept, which we call the level return, is the month t return common to all assets and not captured by the regression explanatory variables.

Our insight is that when the cross-section regression in Equation (1) is stacked across t, it becomes an asset pricing model that can be used in time-series applications. In this perspective, it is natural to move R_{zt} to the left side of the equation so LHS returns are in excess of R_{zt}. Since we interpret the slope estimates in (1) as factor returns, it is also natural to interchange characteristics and factors,

$$R_{it} - R_{zt} = MC_{it-1} R_{MCt} + BM_{it-1} R_{BMt} + OP_{it-1} R_{OPt} + INV_{it-1} R_{INVt} + e_{it}. \quad (2)$$

Equation (2) is a four-factor model in which the factors used to explain asset returns in excess of R_{zt} are R_{MCt}, R_{BMt}, R_{OPt}, and R_{INVt}, and the factor loadings are the time-varying MC_{it-1}, BM_{it-1}, OP_{it-1}, and INV_{it-1} characteristics.

FM regressions are a common tool in asset pricing research. Fama and French (2006), Lewellen (2015), and Bessembinder, Cooper, and Zhang (2019) are recent examples. Most applications treat FM regressions as a way to allow for variation in average returns related to characteristics. It is perhaps a short step from there to our insight that when rearranged as in Equation (2) and stacked across t, an FM cross-section regression like (1) becomes a time-series factor model with prespecified time-varying loadings. We contend that this perspective on FM regressions can enhance the way they are viewed and applied.

We use Equation (2) as a time-series model (model, not regression) to describe average returns for a wide range of left-hand-side assets, and we compare the performance of (2) in this task to that of the model of FF (2015) that uses time-series factors,

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}. \quad (3)$$

In the five-factor model in regression (3), $R_{ft}$is the risk-free rate (one-month U.S. Treasury bill rate observed at the beginning of month t), and R_{mt} is the value-weight (VW) stock market return for month t. The remaining four factors are differences between returns on diversified portfolios of small and big stocks (SMB_t), high and low BM stocks (HML_t), stocks with robust and weak profitability (RMW_t), and stocks of low and high investment firms (CMA_t, conservative minus aggressive). The intercept a_i is the pricing error for LHS asset i in the time-series regression (3). The average across t of the residual e_{it} in model (2) is the pricing error for asset i.

Though they target return variation related to the same variables, there are important differences between Equations (2) and (3). In the time-series regression (3) the factors are prespecified. As detailed later, the time-series size, value, profitability, and investment factors (SMB_t, HML_t, RMW_t, and CMA_t) of (3) are from sorts of stocks on market cap and book-to-market equity, profitability, or investment, with no attempt to optimize. Instead, a least squares time-series regression optimizes an asset's factor loadings on the prespecified factors, subject to the constraint that the factor loadings are constant and assuming the disturbances in (3) are independent and identically distributed (iid) across time. In short, the time-series regression (3) optimizes loadings on factors that are not themselves optimized.

In contrast, the factor loadings in model (2) are prespecified: they are the MC_{it-1}, BM_{it-1}, OP_{it-1}, and INV_{it-1} characteristics. The level return R_{zt} and the factor returns $R_{MCt}, R_{BMt}, R_{OPt}$, and R_{INVt}, of (2) are chosen to minimize the sum of squared residuals in the cross-section regression (1), given the values of the characteristics, MC_{it-1}, BM_{it-1}, OP_{it-1}, and INV_{it-1}, and the month t returns of the LHS assets. The factors are thus optimized month by month to the prespecified factor loadings and LHS returns but under the unrealistic assumption that the disturbances in (1) are cross-sectionally iid. In the concluding section of the paper, we argue that this optimization of factors in large part explains why, for a wide range of LHS test assets, model (2) provides better descriptions of average returns than model (3) and variants of (3).

Going back at least to Rosenberg (1974), many papers argue that factor loadings are likely to vary through time (for example, Shanken 1990; Ferson and Harvey 1991; Ferson, and Schadt 1996; Jagannathan and Wang 1996; Fama and French 1997; Avramov and Chordia 2006). Procedures to capture the variation are often somewhat arbitrary (for example, periodically reestimate the loadings or allow them to change as functions of arbitrary variables, like interest rates). An advantage of model (2) is that its time-varying (TV) factor loadings are specified: they are the MC_{it-1}, BM_{it-1}, OP_{it-1}, and INV_{it-1} characteristics that drive the month-by-month optimization of R_{zt}, $R_{MCt}, R_{BMt}, R_{OPt}, R_{INVt}$ in the cross-section regression (1). They change when the characteristics change and there is no need to estimate them.

A different view of the cross-section (CS) factors $R_{MCt,}$ R_{BMt}, R_{OPt}, and R_{INVt} is that they are just a way to construct the size, value, profitability, and investment factors of the five-factor model (3). In this perspective, the five-factor time-series regression that uses the CS factors, R_{MCt}, R_{BMt}, R_{OPt}, and R_{INVt},

$$R_{it} - R_{ft} = a_i + b_{1i}(R_{mt} - R_{ft}) + b_{2i}R_{MCt} + b_{3i}R_{BMt} + b_{4i}R_{OPt} + b_{5i}R_{INVt} + e_{it}, \quad (4)$$

is a competitor for regression (3), which uses the time-series (TS) factors, SMB_t, HML_t, RMW_t, and CMA_t.

Two papers by Back, Kapadia, and Ostdiek (BKO 2013, 2015) use FM cross-section regression slopes as factors in time-series regressions like (4). We view (4) as a time-series regression in the same family as (3) in that LHS returns are in excess of the risk-free rate, right-hand-side (RHS) factors include $R_{mt} - R_{ft}$, and CS factors simply replace the remaining TS factors of (3). In contrast, (2) is a rearrangement of the cross-section regression (1), with LHS returns in excess of R_{zt} and only CS factors on the right. Our insight, missing in BKO (2013, 2015), is that when stacked for use in time-series asset pricing applications, the characteristics that generate CS factors in cross-section regressions like (1) are time-varying factor loadings that can enhance the description of average returns by models like (2) that use only CS factors.

The time-varying loadings for the CS factors of model (2) may give it an advantage relative to the FF (2015) five-factor regression (3), which imposes constant loadings for TS factors. Leaning on the evidence that average returns vary with size and the book-to-market ratio (for example, Fama and French 1992), Fama and French (FF 1997) argue that a portfolio's loadings on the size and value factors, SMB_t and HML_t, of the FF (1993) three-factor model are likely to vary with the portfolio's size and value characteristics, MC_{it-1} (the log of market cap) and BM_{it-1} (the log of the book-to-market ratio). To level the competition with model (2), we extend the FF (1997) argument to the FF five-factor model and augment (3) with interaction variables that allow loadings for SMB_t, HML_t, RMW_t, and CMA_t to vary with the corresponding MC_{it-1}, BM_{it-1}, OP_{it-1}, and INV_{it-1} characteristics,

$$\begin{aligned} R_{it} - R_{ft} = {} & a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t \\ & + s_{i2} MC_{it-1} SMB_t + h_{i2} BM_{it-1} HML_t + r_{i2} OP_{it-1} RMW_t \\ & + c_{i2} INV_{it-1} CMA_t + e_{it}. \end{aligned} \quad (5)$$

We show results for models (3) and (4), which use constant loadings, but we are more interested in the competition between models (2) and (5), which use TV loadings.

The factors of models (1) to (5) can be motivated by rational pricing versions of the dividend discount model (FF 2015). Return momentum is a hard sell for a world of rational pricing, and one might treat momentum as an anomaly unexplained by the models outlined above. Our experience, however, is that readers are curious about how model performance changes when momentum factors are included. We examine variants of models (1) to (5) that add momentum factors. We find that momentum factors are important for explaining returns on portfolios formed on momentum, but they do not otherwise contribute much to asset pricing models.

We compare the descriptions of average returns provided by versions of models (2), (3), (4), and (5). When we pit model (3) against model (4), we find that CS factors compete well with TS factors in standard constant-slope time-series regressions that measure LHS returns in excess of the risk-free rate, R_f, and include the excess market returns, $R_m - R_f$, among the RHS factors. Model (5), which allows TV loadings for the TS factors of (3), at best only slightly improves the explanation of average returns provided by (3).

Our main result is that model (2)—which explains LHS returns in excess of the CS-level return, R_z, and uses only CS RHS factors with prespecified time-varying loadings—outperforms all other models considered. At least for the LHS portfolios we examine, the dominance of (2) is due much more to its CS factors than its TV loadings. When we limit the model by replacing the characteristics that are its TV loadings with their time-series averages, the performance of the model deteriorates only slightly, and it still produces smaller pricing errors than models other than (2).

1. The Factors

1.1 Definitions

We form the time-series (TS) size, value, profitability, and investment factors of the FF (2015) five-factor model (3) at the end of June each year. To construct the value factor *HML*, New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and (after 1972) NASDAQ stocks are first sorted into two size groups, small and big, using the end of June median market cap of NYSE stocks as breakpoint. Stocks are sorted independently into three groups on *BM* using the 30th and 70th percentiles of *BM* for NYSE stocks as breakpoints. For the sorts of June of year T, *BM* is the natural log of the ratio of book equity at the fiscal year end of $T-1$ to market cap at the end of December of $T-1$, with market cap adjusted for changes in shares outstanding between fiscal year end and December. The intersections of the 2×3 size and *BM* sorts produce six value-weight (VW) portfolios. *HML* is the average of the difference between the returns on the high and low *BM* portfolios of big stocks and the return difference for high and low *BM* portfolios of small stocks.

We construct the profitability and investment factors, *RMW* and *CMA*, in the same way as *HML* except the second sort is on either operating profitability or investment. Operating profitability, *OP*, in the sort for June of T uses accounting data for the fiscal year ending in $T-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, *INV*, is the rate of growth of total assets, $\ln(A_{T-1}/A_{T-2})$, from fiscal year end in year $T-2$ to fiscal year end in $T-1$.

The size factor *SMB* is the average of the returns on the nine small stock portfolios of the three 2×3 sorts minus the average of the returns on the nine big stock portfolios.

The book-to-market ratio *BM* used to construct the value factor *HML* is updated yearly at the end of June using an old end-of-December price. Asness and Frazzini (2013) argue that if *BM* is updated monthly using current prices, it is likely more informative about expected returns. Previous versions of this paper compare the performance of models that use value factors based on monthly and annually updated *BM*, with no clear winner. To focus better on our main issue—TS versus CS factors—we show results only for models that use annually updated *BM*.

The time-series momentum factor, *UMD* (up minus down), is constructed from 2×3 sorts in the same way as *HML* except the second sort is on *MOM*, the cumulative return for months $t-12$ to $t-2$ divided by 11 to put it in monthly units, and *UMD* is reformed monthly. Like other factors, *UMD* is the average of spreads for small and big stocks.

To have a level playing field for the cross-section (CS) and time-series (TS) factors, the LHS assets in the monthly cross-section regression (1) that produces the CS factors are the 18 VW portfolios of the 2×3 sorts that produce the TS factors *SMB*, *HML*, *RMW*, and *CMA*. We add the six VW portfolios of the 2×3 sorts that produce *UMD* to the LHS assets for models that include a CS momentum factor. Likewise, the RHS characteristics used as explanatory variables for the month t LHS returns in the simple and momentum-augmented versions of (1) are the predetermined *MC*, *BM*, *OP*, *INV*, and *MOM* characteristics for the portfolios of the 2×3 sorts. For individual stocks, *BM*, *OP*, and *INV* change once a year in June, but *MC* and *MOM* change monthly. All characteristics for portfolios are VW averages for the stocks in the portfolios, and the value (market cap) weights change monthly.

Using the same portfolios (those from the 2×3 sorts) to produce TS and CS factors is important. The cross-section regressions that generate the CS factors optimize the month-by-month description of returns on these portfolios. In contrast, the TS factor definitions are arbitrary. Since the same portfolios produce the TS and CS factors, one central issue in our tests is whether the optimization of the CS factors enhances the description of average returns for test assets beyond those that produce the factors.

In the estimates of the cross-section regression (1), the RHS characteristics are z-scores: each characteristic is rescaled so that each month the average of the 18 or (with momentum) 24 standardized values is zero and the standard deviation is one. Standardization affects the interpretation of the portfolio returns produced by (1). The CS level return R_{zt} is the month t return on a standard portfolio of the 18 or 24 LHS assets that sets the average of the regression's predicted asset returns for the month equal to the average of the realized asset returns. Since each standardized characteristic's average value is zero every month, the average predicted return for month t is the intercept, R_{zt}.

Table 1
Summary statistics for time-series (panel A) and cross-section (panel B) factors, July 1963–August 2018 (662 monthly returns)

Panel A: Summary statistics for time-series factor returns

	$R_m - R_f$	*SMB*	*HML*	*RMW*	*CMA*	*UMD*
Mean	0.54	0.26	0.33	0.25	0.28	0.67
Std. dev.	4.37	3.02	2.80	2.16	2.00	4.18
t(Mean)	3.17	2.18	2.98	2.97	3.55	4.12

Panel B: Summary statistics for cross-section factor returns

	$R_i = R_z + R_{MC}MC_i + R_{BM}BM_i + R_{OP}OP_i + R_{INV}INV_i + e_i$					$R_i = R_z + R_{MC}MC_i + R_{BM}BM_i + R_{OP}OP_i + R_{INV}INV_i + R_{MOM}MOM_i + e_i$					
	R_z	R_{MC}	R_{BM}	R_{OP}	R_{INV}	R_z	R_{MC}	R_{BM}	R_{OP}	R_{INV}	R_{MOM}
Mean	1.08	−0.15	0.07	0.09	−0.07	1.07	−0.13	0.08	0.09	−0.07	0.15
Std. dev.	4.81	1.56	1.04	0.72	0.47	4.85	1.45	0.90	0.54	0.42	0.95
t(Mean)	5.76	−2.50	1.64	3.34	−4.04	5.67	−2.33	2.26	4.22	−4.54	3.94

$R_m - R_f$ is the difference between the value-weight market return and the one-month Treasury bill rate. *SMB*, the time-series (TS) size factor, is the difference between small stock and big stock portfolio returns. *HML*, the TS value factor, is the average of small and big stock differences between value and growth portfolio returns. The portfolios that produce *HML* are formed annually, at the end of June, from independent sorts on *MC* (market cap) and *BM*, the annually updated ratio of book equity and market equity. The TS profitability and investment factors *RMW* and *CMA* are constructed like *HML*, except the component portfolios are formed on *MC* and *OP* (operating profitability) or *MC* and *INV* (investment). The TS momentum factor *UMD* is the average of small and big stock differences between high and low prior return portfolios. The portfolios are formed monthly with independent sorts on *MC* and *MOM*. R_z is the intercept, and R_{MC}, R_{BM}, R_{OP}, R_{INV}, and R_{MOM} are the slopes on *MC*, *BM*, *OP*, *INV*, and *MOM* in the Fama and MacBeth (1973) cross-section regression (1) or the with-momentum version of (1). *Mean* and *Std. dev.* are the average and standard deviation of monthly factor returns, and *t(Mean)* is the t-statistic for the average.

To set the average predicted equal to the average realized return, R_{zt} must be the average of the month t LHS returns. Thus, R_{zt} is the return on a standard portfolio with equal weights on the 18 or 24 LHS assets. Likewise, each CS factor for month t is the return on a zero-investment portfolio that sets the portfolio's value of the factor's standardized characteristic to one, and sets the values of other standardized characteristics to zero. Equivalently, each CS factor sets the value of its unstandardized characteristic one standard deviation above its cross-section mean for the month and sets other unstandardized characteristics equal to their cross-section means.

Each time-series factor is from a 2×3 sort on size and one other characteristic. Thus, when we add *UMD* to the five-factor model (3), other TS factors do not change. In contrast, CS factors are from monthly cross-section multiple regressions, and adding *MOM* changes the estimates of all factors.

1.2 Summary statistics

Table 1 shows summary statistics for the factors. Results for the time-series (TS) factors are in panel A. All TS factors have strong average returns, with t-statistics from 2.18 for the monthly size factor, *SMB*, to 4.12 for the momentum factor, *UMD*.

The units for stock characteristics in the cross-section regressions that produce CS factors are arbitrary. The momentum characteristic at the end of month $t-1$, for example, is the cumulative return from $t-12$ to $t-2$ divided by eleven. If we do not divide by eleven, each stock's *MOM* is eleven times larger, and each month's new momentum factor is the old R_{MOM} divided by eleven. Given this flexibility in the scale of CS factors, t-statistics for average factor returns, which are proportional to the ratio of mean to standard deviation and unaffected by scale, are the logical way to compare CS and TS factor returns.

Two sets of cross-section regressions (with or without *MOM*) produce two sets of CS factors, with no overlap between them. Summary statistics for these factors are in panel B of Table 1. Average CS factor returns are typically more than two standard errors from zero. The exception is the value factor of the model that does not include a momentum factor. Adding *MOM* to the cross-section regressions that include *BM* increases the t-statistic for the CS value factor average return from 1.64 to 2.26.

The ordering of size in *SMB* (small minus big) and investment in *CMA* (conservative minus aggressive) is opposite their ordering in *MC* and *INV*, and the signs of the average values of the CS factor returns R_{MC} and R_{INV} are opposite those of average *SMB* and *CMA*. Ignoring the sign differences, the two t-statistics for the average values of R_{MC}, −2.50 and −2.33, are similar to the t-statistic for the average *SMB* return, 2.18, and the t-statistics for the average values of R_{INV}, −4.04 and −4.54, are roughly similar to that of the *CMA* average, 3.55. Average returns for the TS and CS profitability factors, *RMW* and R_{OP}, also have strong t-statistics, 2.97 for the TS factor and 3.34 and 4.22 for the two versions of the CS factor.

The averages of the monthly level returns R_{zt} from the without- and with-momentum versions of cross-section regression (1), 1.07% and 1.08%, are larger than the average value-weight market return R_{mt}, 0.92% (which is not reported in Table 1). The standard deviations of R_{zt} are also larger, 4.81% and 4.85%, versus 4.36% forR_{mt}. Because we use standardized characteristics in regression (1), R_{zt} is the month t return on a standard portfolio with equal weights on the 18 or 24 LHS assets. Since half the LHS assets in (1) are small-stock portfolios, small stocks get half the weight in R_{zt}. They have much less weight in R_{mt}, so it is not surprising that the higher average returns and standard deviations of small stocks push the mean and standard deviation of R_{zt} above those of R_{mt}.

2. Perspective on the Cross-Section Factors

Table 2 shows average monthly weights from regression (1) for the portfolios of 18 LHS assets that produce the level return R_z and the CS factor returns. The 18 weights in the R_z portfolio are all 0.06 (=1/18). The average weights for CS factors in Table 2 are muted versions of the weights that produce the matching time-series factors with the same assets. *SMB*, for example, is defined as the return on a portfolio that invests 1/9th of a dollar in each of the nine small portfolios and shorts the same amount in each of the nine big portfolios. The average weights for R_{MC} repeat this general pattern with the signs reversed,

Table 2
Averages of monthly portfolio weights and standard deviations of changes in the weights for CS factors, July 1963–August 2018

	MC-BM						*MC-OP*						*MC-INV*						*Ave*
	SL	*SM*	*SH*	*BL*	*BM*	*BH*	*SW*	*SM*	*SR*	*BW*	*BM*	*BR*	*SC*	*SM*	*SA*	*BC*	*BM*	*BA*	*Mag*
Average weights																			
R_z	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
R_{MC}	−0.07	−0.06	−0.04	0.03	0.08	0.10	−0.04	−0.06	−0.08	0.07	0.07	0.04	−0.09	−0.07	−0.02	0.03	0.06	0.06	0.06
R_{BM}	−0.16	0.02	0.15	−0.13	0.07	0.20	−0.10	0.01	0.07	0.03	−0.00	−0.05	−0.11	0.01	0.04	−0.03	−0.02	−0.02	0.16
R_{OP}	−0.07	0.02	0.06	−0.02	0.01	0.05	−0.18	0.01	0.21	−0.08	−0.04	0.05	−0.05	0.04	−0.01	0.01	−0.00	−0.01	0.13
R_{INV}	0.00	−0.00	0.02	−0.04	0.02	0.07	−0.00	0.01	0.01	0.05	0.01	−0.04	−0.18	−0.05	0.17	−0.10	−0.03	0.09	0.14
Standard deviations of monthly changes																			
R_z	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
R_{MC}	0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.00	0.02	0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	
R_{BM}	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.02	
R_{OP}	0.02	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.02	0.02	0.01	0.01	0.02	0.02	0.01	0.02	0.01	0.02	
R_{INV}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	

The monthly weights for the 2×3 *MC-BM*, *MC-OP*, and *MC-INV* portfolios are from the cross-section regressions that produce the CS factors of the five-factor model (1). Using standard regression notation, the weights for a month's factors are $(X'X)^{-1}X'$. The 18 portfolios in the regressions for July of year t to June of t+1 are formed on market cap at the end of June of t and the annually updated book-to-market ratio *BM*, operating profitability *OP*, or investment *INV* measured in year $t-1$. Small and big portfolios in each set of six are denoted by S and B. Low, medium, and high *BM* are denoted by L, M, and H. Weak, medium, and robust profitability are denoted by W, M, and R, and conservative, medium, and aggressive investment are denoted by C, M, and A. The LHS variables in a month's regression are the returns on the 18 portfolios, and the RHS variables are a dummy variable (for the intercept) and each portfolio's value-weight average *MC*, *BM*, *OP*, and *INV* at the beginning of the month. The sum of a month's weights for R_z is one, and the sum of the weights for each of the other factors is zero. The average magnitude (*Ave Mag*) is the average absolute value of the average weights for the portfolios used to construct the matching TS factors: all 18 portfolios for R_z and R_{MC}; *SL*, *SH*, *BL*, and *BH* for R_{BM}; *SW*, *SR*, *BW*, and *BR* for R_{OP}; and *SC*, *SA*, *BC*, and *BA* for R_{INV}.

negative for each of the nine small portfolios and positive for each of the nine big portfolios. But the average absolute value of the average R_{MC} weights, in the last column of Table 2, is only about half the magnitude of the *SMB* weights, 0.06 versus 0.11 (=1/9). As a result, ignoring the difference in signs, the monthly mean and standard deviation of the without-momentum R_{MC} returns, −0.15 and 1.56, are about half the values for *SMB*, 0.26 and 3.02.

The 18 portfolio weights for the CS factors R_{BM}, R_{OP}, and R_{INV} in Table 2 also have intuitive patterns. Each factor's average weights for its two "high" portfolios, with high values of the factor's characteristic, are always positive, the average weights for its two "low" portfolios are always negative, and their magnitudes are often 0.10 or greater. But the average magnitudes across the two high and two low portfolios for each of the three CS factors, 0.16, 0.13, and 0.14, are less than one-third the weights of 0.50 and −0.50 that define *HML*, *RMW*, and *CMA*. As a result, the means and standard deviations of R_{BM}, R_{OP}, and R_{INV} in Table 1 are roughly one-quarter to one-third the values of their TS counterparts. The lower volatility of R_{MC}, R_{BM}, R_{OP}, and R_{INV} is a consequence of the scaling (standardization) of the characteristics that produce the CS factors via (1). The volatility of the return predictions from CS factors depends on the standardized characteristics as well as on the CS factors they multiply.

Table 2 also shows standard deviations of month-to-month changes in the portfolio weights that produce the CS factors. The levels of the weights are highly autocorrelated, and changes provide better perspective on month-to-month volatility. The standard deviations of the changes seem modest. This is not surprising since the characteristics that produce the weights (via $(X'X)^{-1}X'$) change slowly through time.

3. LHS Portfolios

Table 3 examines how well times-series and cross-section factors explain returns on a large set of LHS portfolios. The LHS portfolios include those from independent 5×5 sorts, 75 formed at the end of June on *MC* and *BM*, *OP*, or *INV*, and 25 formed monthly on *MC* and *MOM*. All use NYSE quintile breakpoints for both size (*MC*) and the second variable of a 5×5 set. Except for their quintile breakpoints, the 5×5 sorts mimic the 2×3 sorts that produce the time-series factors. The LHS portfolios from these 5×5 sorts allow us to examine how well different models capture the patterns in average returns targeted by model factors.

For a more challenging test, competing models are asked to explain the anomaly portfolio returns of Fama and French (2016). The anomalies include (i) the flat relation between univariate market beta and average return that has long plagued the CAPM (Black, Jensen, and Scholes 1972; Fama and MacBeth 1973), (ii) high average returns after share repurchases and low returns after share issues (Ikenberry, Lakonishok, and Vermaelen 1995;

Table 3
Explaining excess returns of anomaly portfolios and 5 × 5 characteristic portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: LHS assets are 185 without-momentum portfolios; factor loadings are constant regression slopes										
(3) R_m-R_f, *SMB*, *HML*, *RMW*, *CMA*	0.103	1.45	0.50	0.37	0.90	0.069	1.69	1.151	2.69	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.097	1.31	0.46	0.32	0.89	0.071	1.74	1.134	2.64	0.000
(3) R_m-R_f, *SMB*, *HML*, *RMW*, *CMA*, *UMD*	0.096	1.37	0.41	0.28	0.90	0.069	1.67	1.148	2.60	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.092	1.20	0.39	0.23	0.89	0.075	1.80	1.140	2.53	0.000
Panel A2: LHS assets are 210 without- and with-momentum portfolios; factor loadings are constant regression slopes										
(3) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.098	1.40	0.34	0.23	0.90	0.069	1.67	1.420	2.69	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.096	1.25	0.34	0.21	0.89	0.075	1.79	1.409	2.60	0.000
Panel A3: LHS assets are 185 without-momentum portfolios; loadings on *SMB*, *HML*, *RMW*, *CMA*, and *UMD* vary with characteristics										
(5) R_m-R_f, *SMB, HML, RMW, CMA*	0.095	1.38	0.43	0.31	0.91	0.067	1.64	1.137		
(5) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.100	1.51	0.44	0.32	0.91	0.066	1.58	1.250		
Panel A4: LHS assets are 210 without- and with-momentum portfolios; loadings on *SMB*, *HML*, *RMW*, *CMA*, and *UMD* vary with characteristics										
(5) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.102	1.53	0.37	0.27	0.91	0.066	1.59	1.508		

Table 3
(Continued)

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: LHS assets are 185 without-momentum portfolios; factor loadings are constant regression slopes										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.099	1.35	0.46	0.33	0.46	0.069	1.73	1.094	2.66	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.111	1.44	0.55	0.40	0.44	0.072	1.77	1.131	2.64	0.000
Panel B2: LHS assets are 210 without- and with-momentum portfolios; factor loadings are constant regression slopes										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.113	1.48	0.46	0.35	0.45	0.072	1.75	1.378	2.69	0.000
Panel B3: LHS assets are 185 without-momentum portfolios; factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.081	1.07	0.34	0.19	0.35	0.074	1.90	0.918	2.37	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.083	1.09	0.35	0.19	0.33	0.075	1.93	0.930	2.40	0.000
Panel B4: LHS assets are 210 without- and with-momentum portfolios; factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.081	1.08	0.27	0.14	0.35	0.074	1.92	1.065	2.30	0.000
Panel B5: LHS assets are 185 without-momentum portfolios; factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.074	1.01	0.30	0.16	0.39	0.071	1.82	0.957	2.47	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.076	1.11	0.31	0.19	0.46	0.066	1.71	1.023	2.64	0.000
Panel B6: LHS assets are 210 without- and with-momentum portfolios; factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.073	1.08	0.23	0.13	0.47	0.066	1.71	1.153	2.49	0.000

The tests use 210 left-hand-side (LHS) portfolios from independent 5×5 sorts on *MC* and *BM*, *OP*, *INV*, *MOM*, market beta, or accruals; conditional 5×5 sorts on *MC* and the variance of daily returns; and independent 5×7 sorts on *MC* and net share issues (repurchases, zero net share issues, and quintiles of positive net share issues). (See Appendix for definitions of variable.) The *MC-MOM* and *MC-Var* portfolios are formed monthly. The other LHS portfolios are formed annually at the end of June. Panels A1, A3, B1, B3, and B5 exclude the 25 *MC-MOM* portfolios. The others include all 210. The regressions in panel A explain returns in excess of the risk-free rate and include the excess market return $R_m - R_f$. They differ on whether they also use TS factors or CS factors and whether they include a momentum factor. Panels A1 and A2 summarize results from time-series regressions that are variants of models (3) and (4). Panels A3 and A4 summarize estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the average absolute intercept and average absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average residual standard deviation; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from monthly cross-section regressions (variants of model (1)). The time-varying factor loadings in panels B5 and B6 are predetermined size, value, profitability, investment, and momentum characteristics. Panels B3 and B4 substitute time-series average values for the time-varying characteristics of (2). Panels B1 and B2 add an intercept to (2) and use OLS regression slopes as factor loadings. The metrics in panels B1 and B2 are those in panel A. The metrics in panels B3 to B6 are analogous to those in panel A, except that a is an average model error, called the pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Model (2) $R_{it} - R_{zt} = MC_{it-1} R_{MCt} + BM_{it-1} R_{BMt} + OP_{it-1} R_{OPt} + INV_{it-1} R_{INVt} + e_{it}$.
Model (3) $R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$.
Model (4) $R_{it} - R_{ft} = a_i + b_{1i}(R_{mt} - R_{ft}) + b_{2i} R_{MCt} + b_{3i} R_{BMt} + b_{4i} R_{OPt} + b_{5i} R_{INVt} + e_{it}$,
Model (5) $R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + s_{i2} MC_{it-1} SMB_t + h_{i2} BM_{it-1} HML_t + r_{i2} OP_{it-1} RMW_t + c_{i2} INV_{it-1}\ CMA_t + e_{it}$

Loughran and Ritter 1995), (iii) low average returns of stocks of firms with large accounting accruals (Sloan 1996), and (iv) low average returns of stocks with high return variances, measured using daily returns (Ang et al. 2006). The anomaly variables and other variables are defined in the Appendix.

As in the 5×5 sorts on model characteristics described above, the first sort for the anomaly portfolios assigns stocks to NYSE quintiles on *MC*. The second sort, on an anomaly variable, also assigns stocks to NYSE quintiles, except there are seven groups for net share issues (*NI*), including net repurchases, zero net issues, and quintiles of positive net issues. The first-pass *MC* sorts and second-pass anomaly sorts are independent, with one exception. Large stocks with highly volatile returns are rare, so to avoid thin or empty portfolios, the sorts on daily variance (*VAR*) are conditional on *MC* quintile. The *MC-VAR* portfolios are reformed monthly, but portfolio formation for the other anomaly sorts is annual, at the end of June.

The patterns in average returns on the LHS portfolios of Table 3 are discussed in Fama and French (2016), and here we summarize them briefly. Lower market cap is associated with higher average returns, but the relation is noisy. Average returns increase with *BM* and *OP* (value and profitability effects), and the patterns are stronger for small stocks. The prime feature of the second pass investment (*INV*), accruals (*AC*), and volatility sorts is a large drop in average returns in the highest quintile of the variables: extreme investment, accruals, and return volatility are associated with low average returns, especially for smaller stocks. Firms that repurchase stock (negative net issues, *NI*) have higher subsequent average stock returns, but the striking feature of the *NI* sort is that stocks in the highest quintile of stock issues have the lowest average return in each size quintile. Finally, the relation between average return and univariate market beta is rather flat: stocks in the highest and lowest quintiles of beta have similar average returns.

The LHS assets cover a wide range of known patterns in average returns, but we caution that the asset pricing results that follow may be somewhat specific to these assets. Barillas and Shanken (2017) suggest that this LHS problem can be avoided by comparing models on the maximum squared Sharpe ratio that can be constructed with each model's factors. This approach does not work for models like (2) that have time-varying factor loadings since the same factors with constant loadings produce the same max squared Sharpe ratio but do not provide the same explanations of LHS returns.

4. Asset Pricing Results

Panel A of Table 3 examines how well variants of models (3), (4), and (5) explain average returns on the 5×5 and anomaly portfolios described above. The performance metrics include the *GRS* statistic of Gibbons, Ross, and Shanken (1989), which jointly tests the intercepts (pricing errors) of a model against zero. We also show the max squared Sharpe ratio for the intercepts, which is the core of *GRS*. Define a as the vector of intercepts produced by a model and Σ as the covariance matrix for the regression residuals. The max squared Sharpe ratio for the intercepts is

$$Sh^2(a) = a'\Sigma^{-1}a. \tag{6}$$

We complement *GRS* and $Sh^2(a)$ with what we call equal-weight (EW) metrics. Using A and V to indicate a cross-section average and variance, the simplest of the EW metrics are $A|a|$, the average of the absolute values of the intercepts for the LHS assets, and $A|t(a)|$, the average of the absolute values of the t-statistics for the intercepts. We estimate the proportion of the cross-section dispersion in average returns missed by a model in two ways. The first is $Aa^2/V\bar{r}$, the average of the squared intercepts for the LHS assets divided by the cross-section variance of LHS average returns. The second subtracts the squared standard error of each intercept, $s^2(a)$, to adjust $Aa^2/V\bar{r}$ for noise in the estimated intercepts. Denoting the noise-adjusted squared intercept as $\lambda^2 \equiv a^2 - s^2(a)$, the adjusted estimate of the proportion of return dispersion missed by a model is $A\lambda^2/V\bar{r}$. Low values of $Aa^2/V\bar{r}$ and $A\lambda^2/V\bar{r}$ are good news for a model; they say intercept dispersion is low relative to the dispersion of LHS average returns.

We call $A|a|$, $A|t(a)|$, $Aa^2/V\bar{r}$, and $A\lambda^2/V$ EW metrics because the averages in the statistics weight each regression equally. Unlike the EW metrics, $Sh^2(a)$ and GRS are summary measures of the magnitude of regression intercepts that account for covariances. For perspective, we also show three measures of regression fit: AR^2, the average of the regression R^2; $As(a)$, the average of the standard errors of the intercepts; and $As(e)$, the average of the standard deviations of the regression residuals.

In models (3), (4), and (5), LHS returns are in excess of the risk-free rate, R_f, and the excess market return, $R_m - R_f$, is among the RHS factors. Panels A1 and A2 of Table 3 provide evidence on models (3) and (4), which estimate constant factor loadings with ordinary least squares (OLS) regression slopes. The specific issue is whether the TS size, value, profitability, investment, and momentum factors of model (3) produce smaller pricing error metrics than the corresponding CS factors of model (4). On all metrics, the CS factors win the competition, but by small margins. There is thus no compelling evidence we should drop the TS factors from (3) and replace them with the CS factors of (4).

Panels A3 and A4 of Table 3 show results for model (5), which adds time-varying (TV) loadings for the TS factors of model (3). Despite lost degrees of freedom, R^2 increases slightly, from 0.90 to 0.91, when we add the interaction variables that produce the TV loadings. More important, the results without LHS momentum portfolios (panels A1 and A3) show that TV loadings produce slightly better pricing error metrics when the models do not have an RHS momentum factor, but with LHS momentum portfolios (panels A2 and A4), TV loadings produce slightly worse pricing error metrics when the model includes a momentum factor. We conclude that what seems like a natural approach to

TV loadings does not systematically improve the pricing error performance of TS factors, at least for the LHS assets used here.

Panel B of Table 3 summarizes the performance of variants of model (2) in which LHS returns are in excess of the level return R_{zt} and only CS RHS factors are used. Panels B5 and B6 show results for the centerpiece version of (2) in which TV loadings for CS size, value, profitability, investment, and momentum factors (R_{MCt}, R_{BMt}, R_{OPt}, R_{INVt}, and R_{MOMt}) are the prespecified characteristics (MC_{it-1}, BM_{it-1}, OP_{it-1}, INV_{it-1}, and MOM_{it-1}). For completeness, panels B1 and B2 show results for models that estimate constant OLS regression slopes for the CS factors of (2). OLS slopes are not the natural choice for a constant-loadings version of (2). The logic of (2) is that loadings for CS factors are the matching TV characteristics. The natural constant-loading version of (2) replaces each TV characteristic with its time-series average value. Panels B3 and B4 of Table 3 show that this constant-loading version of model (2) performs better on all metrics than the models of panels B1 and B2, which use OLS slopes as constant factor loadings. We do nothing further with the OLS slope models of panels B1 and B2.

It is important to be clear about how model (2) is applied in the asset pricing tests of panels B5 and B6 of Table 3. Panels B5 and B6 do not test model (2) with time-series regressions. R_z and the CS factors of (2) are from monthly first-stage estimates of the cross-section regression (1) in which the LHS returns for month t are for the 18 or (with momentum) 24 portfolios from the same 2×3 sorts that produce the TS factors, and the RHS explanatory variables are the month $t-1$ standardized size, value, profitability, investment, and momentum characteristics of these 18 or 24 portfolios. These cross-section regressions produce each month's level return, R_z, and CS factors $R_{MCt}, R_{BMt}, R_{OPt}$, R_{INVt}, and R_{MOMt}. In the second-stage asset pricing applications of (2) in panels B5 and B6 of Table 3, the loadings for the month t CS factors are the MC_{it-1}, BM_{it-1}, OP_{it-1}, INV_{it-1}, and MOM_{it-1} characteristics for the 185 or (with momentum) 210 LHS 5×5 and anomaly portfolios. The loadings and factors combine to produce predictions of month t LHS returns in excess of the CS level return, R_{zt}. The monthly prediction errors are the only estimates in the second stage: the CS factors are from the first stage, and factor loadings are the prespecified characteristics of the second-stage LHS portfolios, standardized using the monthly means and standard deviations of the characteristics for the 18 (or 24) portfolios that produce the CS factors in the first-stage estimates of regression (1). Similar comments apply to the variants of model (2) in panels B3 and B4 of Table 3, which use time-series average values of characteristics as loadings for CS factors.

The factor loadings in the models of panels B3 to B6 of Table 3 are not estimates, so *GRS* is not appropriate. We replace *GRS* with the F-statistic of Hotelling's T^2 that tests whether the expected values of the pricing errors for LHS assets are jointly zero. This is an appropriate use of T^2 since the portfolios that produce the CS factors are not among the LHS portfolios of Table 3, and the factor loadings of the models of panels B3 to B6 are prespecified characteristics, not estimates. For other performance metrics, the time-series average of an LHS portfolio's monthly prediction errors, which we call its pricing error, is analogous to the intercept in a time-series regression of the portfolio's returns on the factors of models (3), (4), or (5). If we label the pricing error a, the earlier definitions of $Sh^2(a)$ and the EW metrics apply.

The values of AR^2 in panel A of Table 3 are twice or more those of panel B. This is misleading since the LHS returns in panel A are in excess of R_f, but LHS returns in panel B are in excess of the level return R_z. In effect, AR^2 in panel B measures the average fraction of leftover variance explained by other factors after controlling for the explanatory power of the level return R_z. We can report that a similar decline in AR^2 occurs if we drop $R_m - R_f$ from the RHS of the regressions of panel A and measure LHS returns in excess of R_m. The relevant comparison of the explanatory power of the models of panels A and B is in any case provided by $As(e)$, the average across LHS assets of the time-series standard deviations of unexplained LHS returns, which is central in the average precision of pricing errors, $As(a)$. The values of $As(e)$ are similar in the two panels, as are the values of $As(a)$.

A peripheral issue addressed in Table 3 is whether inclusion of a momentum factor improves the description of average returns when LHS assets do not include momentum sorts. Panel A1 of Table 3 says adding a momentum factor modestly improves pricing error metrics in models (3) and (4), which combine $R_m - R_f$ with either TS or CS size, value, profitability, and investment factors. But panel A3 says there is slight deterioration in performance when *UMD* is added to model (5), which estimates TV loading for TS factors. In the models of panels B1, B3, and B5, which measure LHS returns in excess of R_z and use only CS RHS factors, pricing error metrics deteriorate a bit when R_{MOM} is included among the RHS factors. In short, Table 3 suggests that inclusion of a momentum factor is unnecessary and can be harmful when LHS assets are not tilted away from or toward momentum.

We turn now to the main result of the paper. The estimates of model (2) in panels B5 and B6 of Table 3 use CS factors in their natural habitat—LHS returns in excess of R_z and TV characteristic loadings for CS factors. These models dominate the models of panel A on every pricing error metric, and the competition is not close. For example, the constant-loading models (3) and (4) in panel A2 of Table 3 include RHS momentum factors, and momentum portfolios are among the LHS assets, which means these models should be compared to the estimates of model (2) in panel B6. The average absolute pricing error, $A|a|$, is 0.073 for the model of panel B6, versus 0.098 and 0.096 for the models of panel A2. The measure of pricing error dispersion adjusted for sampling error, $A\lambda^2/V\bar{r}$, is 0.13 for the model of panel B6, versus 0.23 and 0.21 for the models of panel A2. $Sh^2(a)$, which accounts for covariances as well as dispersion of pricing errors, is 1.153 in panel B6 versus 1.420 and 1.409 in panel A2.

The first model in panel B5 of Table 3 is model (2) with no RHS momentum factor and no LHS momentum portfolios. This model is comparable to the variants of models (3) and (4) in the first two lines of panel A1. Again, performance comparisons are lopsided in favor of (2). This is also true if we compare the second model in panel B5 to the third and fourth models in panel A1, all of which have RHS momentum factors but no LHS momentum portfolios. Panels A3 and A4 of Table 3 show results for model (5), which estimates TV loadings for the TS factors of model (3). Since TV loadings do not much improve the average return predictions of model (3), pricing error metrics again favor model (2) over model (5).

In short, Table 3 says model (2) of panels B5 and B6, which uses LHS returns in excess of R_z and TV loadings for CS factors, provides better descriptions of average returns than models (3), (4), and (5) of panel A. The dominance of model (2) is due more to its CS factors than their TV loadings. Panels B3 and B4 replicate the models of B5 and B6, but with TV loadings replaced by their time-series average values. This substitution produces only a small deterioration in pricing error metrics, and this constant-loading version of model (2) also dominates the constant-loading models (3) and (4), as well as model (5), which includes TV loadings for TS factors.

Time-varying loadings have long been recognized as a potential problem in applications of asset pricing models that impose constant factor loadings. For most models, we are in the dark about the nature of TV loadings. An attraction of model (2) is that its TV loadings are prespecified characteristics. At least for the LHS assets used here, however, the TV loadings of (2) are not a major source of its superior description of average returns. The TV loadings for TS factors of model (5), which seem like natural additions to the constant slope model (3), also do not add much to the explanation of average returns provided by (3).

Why are constant factor loadings almost as good as time-varying loadings in model (2)? Extending Lewellen and Nagel's (2006) CAPM logic to our multifactor question, the difference between a particular LHS asset's average monthly pricing errors from model (2) with and without TV loadings is the sum of the time-series covariances between each of the asset's standardized characteristics and the matching CS factor. Consider, for example, the contribution of market cap to average predicted monthly return. With TV loadings, market cap's contribution to the average prediction is the average of the monthly products of the asset's standardized market cap MC_{it-1} and the CS size factor R_{MCt}, and market cap's contribution to the constant-loading prediction is the product of the monthly averages of MC_{it-1} and R_{MCt}. Using A to denote time-series averages, the difference between the two is the sample covariance between the asset's standardized market cap and the CS size factor,

$$Cov(MC_{it-1}, R_{MCt}) = A(MC_{it-1} R_{MCt}) - A(MC_{it-1}) A(R_{MCt}). \qquad (7)$$

Regression (1) and model (2) use standardized characteristics with constant cross-section averages and standard deviations. Compared to raw characteristics, there is little variation through time in the standardized characteristics of the LHS assets we consider and little reason to expect the standardized characteristics for month $t-1$ to have substantial covariances with month t factor returns. As a result, it is not surprising to see little deterioration in pricing error metrics when we replace the time-varying characteristics in model (2) with their time series averages. TV loadings improve the month-by-month fit of the model, but unless a portfolio's monthly standardized characteristics anticipate next month's factor returns, they cannot have much impact on the model's pricing error.

The dominance of model (2) in Table 3 is striking, and details are warranted. Section 5, which follows, analyzes results for the 100 portfolios of the 5×5 *MC-BM*, *MC-OP*, *MC-INV*, and *MC-MOM* sorts. Section 6 turns to the 110 portfolios of the anomaly sorts. The Appendix presents details for each 5×5 sort on characteristics and each anomaly sort.

5. Results for the 100 5×5 *MC-BM*, *MC-OP*, *MC-INV*, and *MC-MOM* Portfolios

Table 4 reproduces Table 3 for the 100 portfolios of the 5×5 sorts. Since these sorts are finer versions of the 2×3 sorts that produce the TS and CS size, value, profitability, investment, and momentum factors, we expect the models to perform better in Table 4 than in Table 3, where the LHS assets also include anomaly portfolios. This is what we observe. For example, $A|a|$ ranges from 0.092 to 0.103 in panel A of Table 3, versus 0.068 to 0.085 in Table 4. Otherwise, Table 4 confirms the results in Table 3.

The models in panel A of Tables 3 and 4 combine $R_m - R_f$ with either TS or CS size, value, profitability, and investment factors. As in Table 3, the constant-slope regressions in Table 4 give CS factors a slight edge over TS factors. Allowing time-varying loadings for TS factors via (5) slightly improves explanatory power as measured by AR^2, but again has mixed effects on pricing error metrics.

The important confirming result in Table 4 is the evidence in panels B3 and B4 that model (2), which explains LHS returns in excess of the CS level return R_z and uses characteristics as time-varying loadings for CS factors, dominates the explanations of average returns for the 5×5 portfolios provided by models (3), (4), and (5) in panel A. For example, $A|a|$ ranges from 0.068 to 0.085 in panel A, versus 0.053 to 0.054 for the models of panels B3 and B4. More impressive, $A\lambda^2/V\bar{r}$, the measure of pricing error dispersion adjusted for sampling error, ranges from 0.10 to 0.22 in panel A of Table 4, but in panels B3 and B4, $A\lambda^2/V\bar{r}$ is closer to zero, 0.04 to 0.07. Model (2) with time-varying loadings for CS factors provides the best description of average returns on the portfolios of the 5×5 sorts, but it is again only slightly better on all pricing error metrics than

Table 4
Explaining excess returns of the 5 × 5 portfolios formed on model characteristics, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	*GRS*	$p(GRS)$
Panel A1: LHS assets are 75 without-momentum portfolios; factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.081	1.34	0.33	0.22	0.92	0.060	1.48	0.356	2.53	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.070	1.09	0.24	0.12	0.92	0.063	1.54	0.347	2.45	0.000
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.072	1.17	0.26	0.15	0.92	0.061	1.48	0.340	2.35	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.068	0.98	0.24	0.10	0.91	0.068	1.63	0.317	2.14	0.000
Panel A2: LHS assets are 100 without- and with-momentum portfolios; factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.082	1.29	0.21	0.14	0.92	0.063	1.52	0.594	2.95	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.083	1.15	0.22	0.13	0.91	0.069	1.65	0.596	2.89	0.000
Panel A3: LHS assets are 75 without-momentum portfolios; loadings on *SMB*, *HML*, *RMW*, *CMA*, and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.075	1.26	0.29	0.19	0.93	0.060	1.44	0.372		
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.076	1.28	0.29	0.18	0.93	0.060	1.42	0.382		
Panel A4: LHS assets are 100 without- and with-momentum portfolios; loadings on *SMB*, *HML*, *RMW*, *CMA*, and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.085	1.38	0.23	0.16	0.93	0.061	1.46	0.626		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	Sh^2(a)	T^2	$p(T^2)$
Panel B1: LHS assets are 75 without-momentum portfolios; factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.058	0.90	0.19	0.07	0.42	0.063	1.61	0.199	1.56	0.003
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.058	0.88	0.19	0.06	0.40	0.065	1.67	0.215	1.68	0.001
Panel B2: LHS assets are 100 without- and with-momentum portfolios; factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.061	0.91	0.12	0.04	0.42	0.067	1.71	0.362	2.04	0.000
Panel B3: LHS assets are 75 without-momentum portfolios; factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.054	0.86	0.18	0.06	0.46	0.060	1.55	0.188	1.47	0.008
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.053	0.86	0.18	0.07	0.50	0.058	1.50	0.185	1.45	0.011
Panel B4: LHS assets are 100 without- and with-momentum portfolios; factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.053	0.86	0.11	0.04	0.51	0.060	1.55	0.332	1.87	0.000

The tests use 100 left-hand-side (LHS) portfolios from independent 5×5 sorts on *MC* and *BM*, *OP*, *INV*, or *MOM*. (See Appendix for definitions of variables.) The *MC-MOM* portfolios are formed monthly. The other LHS portfolios are formed annually at the end of June. Panels A1, A3, B1, and B3 exclude the 25 *MC-MOM* portfolios. The others include all 100. The regressions in panel A explain returns in excess of the risk-free rate and include the excess market return $R_m - R_f$. They differ on whether they also use TS factors or CS factors and whether they include a momentum factor. Panels A1 and A2 summarize results from time-series regressions that are variants of models (3) and (4). Panels A3 and A4 summarize estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the average absolute intercept and average absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average residual standard deviation; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from monthly cross-section regressions (variants of model (1)). The time-varying factor loadings in panels B3 and B4 are predetermined size, value, profitability, investment, and momentum characteristics. Panels B1 and B2 substitute time-series average values for the time-varying characteristics of (2). The metrics in panels B1 to B4 are analogous to those in panel A, except that a is an average model error, called the pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

the models of panels B1 and B2, which replace the TV loadings of (2) with their time-series average values.

Appendix Tables A1 to A4 show separate results for each of the four 5×5 sorts on model characteristics. In every 5×5 sort and on all metrics, model (2) provides the best description of average returns. In the individual 5x5 sorts, $A\lambda^2/V\bar{r}$ (pricing error dispersion adjusted for sampling error) is close to zero for model (2), which suggests near complete descriptions of average returns. Moreover, model (2) sometimes passes or is only marginally rejected on T^2, a result that in our experience is rarely observed in GRS tests of asset pricing models.

Other details in Tables A1 to A4 are worth noting.

1. The performance metrics for model (2) are typically much better than those for models (3), (4), and (5). The exception is the 5×5 *MC-OP* sorts (Table A2) where all models perform relatively well. For example, all produce values of $A\lambda^2/V\bar{r}$, the measure of pricing error dispersion adjusted for sampling error, close to zero. We infer that profitability sorts are not a big challenge for our models.
2. Replacing the time-varying characteristics that are the factor loadings in (2) with their time-series average values typically produces slight deterioration in pricing error metrics. But TV loadings absorb more LHS return variance and so produce more precise estimates of pricing errors—higher AR^2 and thus lower $As(a)$. As a result, the constant-loadings version of (2) sometimes does better on T^2 than (2) with TV loadings—for example, in the 5×5 *MC-BM* and *MC-INV* sorts (Tables A1 and A3).
3. Even with the inclusion of the TS momentum factor *UMD*, the *MC-MOM* sorts are a long-standing problem for model (3), and the new models (4) and (5) do no better (Table A4). Based on these results, one might conclude that momentum factors miss lots of momentum average returns. Model (2) does better, producing strong performance on most pricing error metrics, including $A\lambda^2/V\bar{r}$, which rounds to zero to two decimal places. Though less impressive, model (2) also produces strong improvements in pricing error metrics in the *MC-BM* and *MC-INV* sorts (Tables A1 and A3).

6. Results for the 110 Anomaly Portfolios

Table 5 summarizes model performance for the 110 *MC-Beta*, *MC-AC*, *MC-NI*, and *MC-VAR* anomaly portfolios. The comparison of interest is the performance of model (2) of panel B2 versus models (3), (4), and (5) of panel A. Again, the tests in panel B2 measure LHS returns in excess of the level return R_z and apply prespecified time-varying loadings to CS factors. Models (3), (4), and (5) in panel A explain LHS returns in excess of R_f with time-series regressions that combine $R_m - R_f$ and either TS or CS size, value, profitability, investment, and

Table 5
Explaining excess returns of 110 anomaly portfolios, July 1963–August 2018

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)										
Panel A1: Factor loadings are constant regression slopes										
(3) $R_- R_f$, *SMB, HML, RMW, CMA*	0.118	1.53	0.59	0.45	0.88	0.075	1.84	0.688	3.14	0.000
(4) $R_m - R_f$, $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.115	1.45	0.57	0.43	0.88	0.077	1.88	0.664	3.01	0.000
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.113	1.51	0.48	0.34	0.88	0.075	1.81	0.690	3.05	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.108	1.35	0.46	0.30	0.87	0.080	1.92	0.677	2.93	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.108	1.46	0.50	0.37	0.89	0.073	1.77	0.668		
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.117	1.67	0.52	0.40	0.90	0.071	1.69	0.743		
Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)										
Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.097	1.18	0.43	0.26	0.29	0.081	2.09	0.530	2.66	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.100	1.24	0.43	0.26	0.29	0.082	2.10	0.517	2.60	0.000
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.089	1.12	0.36	0.20	0.34	0.078	2.01	0.507	2.55	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.092	1.28	0.38	0.25	0.43	0.072	1.85	0.533	2.68	0.000

The 110 left-hand-side (LHS) portfolios are from independent 5×5 sorts on *MC* and market beta or accruals, conditional 5×5 sorts on *MC* and the variance of daily returns, and independent 5×7 sorts on *MC* and net share issues (repurchases, zero net share issues, and quintiles of positive net share issues). (See Appendix for definitions of variables.) The *MC-Var* portfolios are formed monthly. The other LHS portfolios are formed annually at the end of June. All models in panel A include the excess market return$R_m - R_f$. They differ on whether they also use TS factors or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the average absolute intercept and average absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average residual standard deviation; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from monthly cross-section regressions (variants of model (1)). The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panel B are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

momentum factors. As in Tables 3 and 4, model (2) outperforms its competitors. For example, $A|a|$ ranges from 0.108 to 0.118 in panel A, versus 0.089 and 0.092 in panel B2. $A|t(a)|$ is 1.28 and 1.12 in panel B2, versus 1.45 or greater in panel A. Sh^2(a), which accounts for covariance as well as magnitude of pricing errors, is 0.507 and 0.533 in panel B, versus 0.664 to 0.743 in panel A. The models of panel B2 do not, however, provide complete descriptions of expected returns on the anomaly portfolios since all are rejected on T^2.

As in Tables 3 and 4, the strong performance of model (2) in panel B2 of Table 5 is due more to its CS factors than to its TV loadings. Again, replacing the time-varying characteristics that are the TV loadings in (2) with their time-series average values produces only a minor deterioration in pricing error metrics, and the resulting model (panel B1 of Table 5) dominates all the models in panel A.

Comparison of Tables 4 and 5 shows that for every model and metric, performance is better for the portfolios from the 5×5 sorts on model characteristics than for the anomaly portfolios. Thus, average returns on anomaly portfolios are generally a bigger challenge for our models than average returns on portfolios from 5×5 sorts on model characteristics. There are two exceptions.

1. Appendix Tables A5 to A8 report separate tests for the four anomaly sorts combined in Table 5. Results for the *MC-AC* portfolios (Table A5) are the big surprise. When models (3), (4), and (5) are asked to explain *MC-AC* portfolio returns, they are strongly rejected on *GRS*, but the version of model (2) that uses TV loadings for CS factors passes the T^2 test with p-values of 0.262 (no momentum factor) and 0.551 (with momentum factor). The returns from accruals sorts thus lose their anomaly status when explained with the TV loadings and CS factors of (2).
2. All models are cleanly rejected in the separate results for the *MC-NI* and *MC-VAR* sorts in Tables A6 and A8, but model (2) again outperforms its competitors. The *MC-Beta* sort (Table A7) is the only exception to the dominance of model (2). Model (2) performs well in the *MC-Beta* sorts, but other models, including some that estimate constant loadings for TS factors, perform equally well. We conclude that the flat relation between univariate market beta and average return apparently is not a big challenge for the multifactor models used here and perhaps should lose its anomaly status.

In the aggregate results in Tables 3, 4, and 5, the dominance of model (2) on pricing error metrics traces almost entirely to its CS factors rather than its TV loadings. The results for separate sorts in Tables A1 to A8 can, however, potentially identify sorts in which TV loadings play a bigger role. The potential is not realized. Higher AR^2 and lower $As(e)$ say TV loadings absorb LHS return variance, but judged by the pricing error metrics in Tables A1 to A8, they never substantially improve the explanation of average returns. We emphasize, however, that using model (2) in applications requires that the characteristics that are its TV loadings are available. If they are, one would use the TV loadings in (2) rather than muting their variance reducing benefits by substituting time-series average values.

7. Conclusions

We examine the performance of size, value, profitability, investment, and momentum factors from monthly FM (1973) cross-section regressions. We use these CS factors in two ways in asset pricing models. The first is as a substitute for the time-series factors of the FF (2015) five-factor model (3), extended to incorporate a TS momentum factor. The six-factor model with TS factors is the time-series regression,

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + u_i UMD_t + e_{it}. \tag{8}$$

Substituting CS for TS factors, we get

$$R_{it} - R_{ft} = a_i + b_{1i}(R_{mt} - R_{ft}) + b_{2i} R_{MCt} + b_{3i} R_{BMt} + b_{4i} R_{OPt} + b_{5i} R_{INVt} + b_{6i} R_{MOMt} + e_{it}, \tag{9}$$

which is model (4) with an added momentum factor.

Regression (9) is not the natural environment for CS factors. The CS factors are the progeny of first-stage cross-section regressions, and it is natural to use them in second-stage cross-section models (models, not regressions) that take advantage of their prespecified time-varying factor loadings,

$$R_{it} - R_{zt} = MC_{it-1} R_{MCt} + BM_{it-1} R_{BMt} + OP_{it-1} R_{OPt} + INV_{it-1} R_{INVt} + MOM_{it-1} R_{MOMt} + e_{it}, \tag{10}$$

which is model (2) augmented with a momentum factor.

Models (8) and (9) are times-series regressions, estimated separately for each LHS asset i, with constant loadings for their prespecified factors. In contrast, (10) is a monthly cross-section model in which the factors are from first-stage cross-section regressions, and factor loadings are the prespecified size, value, profitability, investment, and momentum characteristics of the LHS assets. When (10) is stacked across months, it becomes a model with time-varying factor loadings that can be used in asset pricing applications if the relevant time-series of characteristics are available for the LHS assets of interest.

We find that with or without momentum factors, model (9) explains average returns on the LHS assets we consider a bit better than model (8). Thus, in constant-slope time-series regressions that measure LHS returns in excess of the risk-free rate and include the excess market return among the RHS factors, CS size, value, profitability, investment, and momentum factors perform a bit better than their TS counterparts. Moreover, allowing natural time-varying loadings for TS factors in the manner of (5) does not noticeably improve pricing error metrics. The inference is that in applications that use constant-slope time-series regressions, there is no strong reason to drop the TS factors of (3) and (8) in favor of the CS factors of (4) and (9). But switching to CS factors is costless since the factor loadings of (4) and (9) are OLS regression slopes.

Our main result is that for the LHS assets used here, models (2) and (10), which use time-varying characteristics as loadings for CS factors, provide better descriptions of average returns than the constant-slope time-series models (4) and (9) or (3) and (8), or the models that add TV loadings to the TS factors of (3) and (8). Thus, when size, value, profitability, investment, and momentum characteristics are available, models (2) and (10) are preferred choices.

An attractive feature of our results is robustness. The conclusions outlined above are supported (i) in Table 3, which shows combined results for all our LHS assets, (ii) in Tables 4 and 5, which show separate results for the 5×5 sorts on model characteristics (Table 4) and the anomaly sorts (Table 5), and (iii) in Appendix Tables A1 to A8, which show separate results for each 5×5 sort and each anomaly sort. A ritual warning is nevertheless in order. Though the LHS assets in our tests cover a wide range of well-known patterns in average returns, the inferences they support may be somewhat special to them.

Variation through time in the overall level of the characteristics can cause variation through time in the level of the CS factors that would affect some of the paper's results, including the volatility of the monthly portfolio weights in Table 2 and the differences in Tables 3, 4, and 5 between the average pricing errors when we use monthly and average characteristics as factor loadings. To eliminate this distracting variation in the factor returns, we use standardized characteristics in the cross-section regressions that produce them. It is important to note that standardization has no effect on the results from cross-section models (2) and (10). Standardized characteristics in regression (1) rescale each month's CS factors, but the rescaling is reversed in (2) and (10) because the standardized characteristics are also used as loadings for the rescaled factors. The models' monthly predictions and errors are the same whether the inputs are raw characteristics and the factors they produce or standardized characteristics and the factors they produce. Both sets of factors are available on French's data website, along with the monthly means and standard deviations to standardize characteristics, but we expect raw characteristics and raw factors will be used in most applications.

We close by addressing a central question: What explains the dominance of models (2) and (10)? All the models we consider target cross-section variation in average returns related to the same size, value, profitability, investment, and momentum characteristics. And the CS factors of models (2) and (10) are constructed from returns on the same 2×3 *MC-BM*, *MC-OP*, *MC-INV*, and *MC-MOM* portfolios that produce the TS factors of models (3), (5), and (8). The way these returns are used to produce the TS factors is, however, arbitrary. In contrast, monthly cross-section FM OLS regressions combine returns and characteristics (via $(X'X)^{-1}X'$) on the 2×3 portfolios to produce CS factors that optimize the month-by-month description of returns on these portfolios.

The optimization that produces the CS factors uses characteristics as factor loadings. To get the full benefits of optimization when describing returns on other assets, we must also use their characteristics as factor loadings, as in (2) and (10). Simply substituting CS factors for TS factors in regression models like (4) and (9) doesn't improve much on the descriptions of average returns by (3) and (8), which use TS factors. Moreover, the optimization of the CS factors via OLS regressions is approximate. If we knew the covariance matrix of the cross-section regression disturbances in (1), optimization could be improved with generalized least squares, but optimization via OLS with characteristics as factor loadings is apparently better than the arbitrary approach that produces the TS factors, at least for describing average returns on the LHS test assets examined here.

Appendix

Variable Definitions

The data are from the Center for Research in Security Prices (CRSP) and Compustat (supplemented with hand-collected book equity data, as in Davis, Fama, and French, 2000). We form most portfolios at the end of June in each year t, but we sort firms on *MOM* and the volatility measure, *Var*, every month. We include only stocks with CRSP share codes of 10 or 11. Stocks without *MC*, *OP*, *INV*, *MOM*, or positive *BM* are not assigned when portfolios are formed and, for annual sorts, stocks missing any of these characteristics in one of the next eleven months are not included when we cumulate portfolio returns and characteristics for that month. A stock must also have *NI*, *AC*, *Beta*, or *Var* to be included when the portfolios for that anomaly are formed. The variables are:

MC: The natural log of market cap, price times shares outstanding.

BM: The annually updated ratio of book value of equity to market value of equity. Book equity in ratios for June of year t to May of $t+1$is total assets for the last fiscal yearend in calendar year $t-1$, minus liabilities, plus balance sheet deferred taxes and investment tax credit if available, minus preferred stock liquidating value if available, or redemption value if available, or carrying value, adjusted for net share issues from the fiscal yearend to the end of December of $t-1$. Market equity (market cap) is price times shares outstanding at the end of December of $t-1$, from CRSP.

OP: Operating profitability. *OP* in the sort for June of year t is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Research and development expenses reduce operating profitability.

INV: Investment, the rate of growth of total assets, $\ln(A_{t-1}/A_{t-2})$, from the last fiscal yearend in year $t-2$ to the last fiscal yearend in $t-1$.

MOM: Momentum, a stock's cumulative return from month $t-12$ to $t-2$ divided by 11. We use the fact that CRSP reports a stock's cumulative return over months with missing prices. We replace returns missing at the beginning of the $t-12$ to $t-2$ period (either because a stock was not listed yet or because a multi-month return begins before the period) or at the end (because a multi-month return extends beyond $t-2$) with the VW market return. *MOM* is not computed for month t unless a stock has at least five months of good returns in $t-12$ to $t-2$.

NI: Net stock issues, the implied growth in split-adjusted shares outstanding from the end of June in year $t-1$ to the end of June in t. *NI* is zero if CRSP's shares outstanding does not change over the twelve months. Otherwise, we compute *NI* by comparing the total growth in market cap from June $t-1$ to June t, $MC(t)/ME(t-1)$, with the growth implied by compounding the monthly without-dividend stock returns over the same period, $\Pi(1+RetX_i)$,

$$NI = \frac{MC(t)/MC(t-1)}{\prod(1+RetX_i)} - 1.$$

AC: Accruals, the change in operating working capital per split-adjusted share from $t-2$ to $t-1$ divided by book equity per split-adjusted share at $t-1$. Operating working capital is current assets minus cash and short-term investments minus current liabilities plus debt in current liabilities. We use operating working capital per split-adjusted share to adjust for the effect of changes in the scale of the firm caused by share issues and repurchases.

Beta: Market beta is measured at the end of June of year t. It is the sum of the current and previous months' slopes and is estimated using the preceding 60 months (24 minimum) of returns.

Var: Variance of daily total returns. Each stock's *Var* is estimated monthly using 60 days (20 minimum) of lagged returns.

Table A1
Explaining excess returns of 5×5 *MC-BM* portfolios, July 1963–August 2018

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)										
Panel A1: Factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.092	1.44	0.35	0.25	0.92	0.063	1.55	0.144	3.35	0.000
(4) $R_m - R_f$, $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.075	1.14	0.24	0.13	0.91	0.065	1.60	0.123	2.83	0.000
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.083	1.31	0.28	0.17	0.92	0.064	1.54	0.129	2.90	0.000
(4) $R_m - R_f$, $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.072	1.00	0.23	0.11	0.90	0.071	1.69	0.104	2.29	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.081	1.30	0.31	0.21	0.92	0.062	1.50	0.148		
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.080	1.31	0.28	0.18	0.92	0.062	1.48	0.151		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.063	0.97	0.18	0.07	0.48	0.064	1.65	0.047	1.19	0.242
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.054	0.81	0.13	0.02	0.46	0.067	1.72	0.041	1.05	0.392
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.061	0.94	0.16	0.06	0.52	0.063	1.61	0.048	1.23	0.202
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.061	0.99	0.15	0.06	0.55	0.060	1.55	0.063	1.61	0.031

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *BM*. All models in panel A include the excess market return$R_m - R_f$. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A2
Explaining excess returns of 5x5 *MC-OP* portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: Factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.064	1.06	0.18	0.06	0.93	0.060	1.47	0.085	1.98	0.003
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.057	0.89	0.15	0.01	0.92	0.063	1.55	0.075	1.72	0.016
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.057	0.92	0.15	0.03	0.93	0.060	1.46	0.075	1.69	0.019
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.061	0.85	0.19	0.03	0.91	0.068	1.62	0.079	1.74	0.015
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.052	0.88	0.14	0.02	0.93	0.059	1.43	0.086		
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.056	0.94	0.15	0.03	0.93	0.059	1.40	0.075		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.052	0.77	0.21	0.06	0.35	0.064	1.65	0.069	1.76	0.013
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.056	0.80	0.24	0.09	0.32	0.066	1.70	0.076	1.93	0.005
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.042	0.66	0.20	0.07	0.42	0.060	1.55	0.068	1.75	0.014
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.044	0.69	0.23	0.11	0.45	0.058	1.50	0.070	1.79	0.011

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *OP*. All models in panel A include the excess market return$R_m - R_f$. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A3
Explaining excess returns of 5x5 *MC-INV* portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: Factor loadings are constant regression slopes										
(3) R_m-R_f, *SMB, HML, RMW, CMA*	0.089	1.50	0.40	0.30	0.93	0.058	1.43	0.161	3.74	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.078	1.25	0.32	0.20	0.92	0.061	1.48	0.158	3.64	0.000
(3) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.075	1.27	0.33	0.22	0.93	0.059	1.42	0.148	3.33	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.072	1.08	0.28	0.14	0.91	0.065	1.56	0.131	2.88	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) R_m-R_f, *SMB, HML, RMW, CMA*	0.093	1.60	0.40	0.29	0.93	0.058	1.40	0.174		
(5) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.091	1.58	0.39	0.29	0.93	0.058	1.38	0.181		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.059	0.97	0.17	0.06	0.44	0.059	1.53	0.057	1.44	0.076
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.064	1.02	0.19	0.07	0.42	0.062	1.58	0.063	1.62	0.030
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.058	0.97	0.16	0.06	0.45	0.058	1.50	0.071	1.81	0.010
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.054	0.92	0.15	0.05	0.49	0.057	1.45	0.074	1.89	0.006

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *INV*. All models in panel A include the excess market return R_m-R_f. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A4
Explaining excess returns of 5x5 *MC-MOM* portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)										
Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: Factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.268	2.72	0.93	0.85	0.85	0.091	2.23	0.195	4.52	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.271	2.65	1.00	0.91	0.85	0.092	2.26	0.181	4.17	0.000
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.115	1.65	0.17	0.12	0.92	0.069	1.66	0.161	3.63	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.125	1.66	0.20	0.15	0.91	0.073	1.74	0.145	3.19	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.114	1.69	0.17	0.13	0.92	0.066	1.59	0.178		
Panel B: Summary of average errors, explaining returns in excess of R_z with five-factor version of model (2)										
Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.067	1.00	0.06	0.02	0.47	0.072	1.85	0.072	1.84	0.008
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.052	0.86	0.04	0.00	0.55	0.065	1.68	0.068	1.74	0.015

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *MOM*. All models in panel A include the excess market return$R_m - R_f$. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A5
Explaining excess returns of 5×5 *MC-AC* portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: Factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.124	1.85	0.87	0.70	0.91	0.066	1.63	0.173	4.01	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.124	1.79	0.84	0.66	0.91	0.068	1.67	0.148	3.42	0.000
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.123	1.83	0.79	0.61	0.91	0.067	1.62	0.158	3.55	0.000
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.112	1.54	0.66	0.45	0.90	0.073	1.74	0.121	2.66	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.114	1.74	0.69	0.52	0.92	0.065	1.59	0.151		
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.109	1.67	0.59	0.42	0.92	0.065	1.56	0.134		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.073	1.06	0.36	0.19	0.38	0.066	1.71	0.066	1.68	0.021
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.063	0.90	0.29	0.11	0.36	0.068	1.76	0.055	1.40	0.096
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.061	0.91	0.21	0.03	0.36	0.067	1.73	0.046	1.17	0.262
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.060	0.93	0.18	0.01	0.40	0.065	1.68	0.037	0.94	0.551

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *AC*. All models in panel A include the excess market return$R_m - R_f$. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A6
Explaining excess returns of 5×5 *MC-NI* portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: Factor loadings are constant regression slopes										
(3) R_m-R_f, *SMB, HML, RMW, CMA*	0.144	1.77	0.63	0.49	0.86	0.080	1.96	0.247	4.04	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.135	1.65	0.56	0.41	0.86	0.082	2.01	0.247	3.99	0.000
(3) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.133	1.63	0.54	0.39	0.86	0.081	1.96	0.233	3.68	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.120	1.39	0.50	0.33	0.85	0.087	2.08	0.241	3.72	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) R_m-R_f, *SMB, HML, RMW, CMA*	0.124	1.60	0.51	0.37	0.87	0.078	1.91	0.243		
(5) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.124	1.63	0.49	0.35	0.87	0.078	1.87	0.249		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.112	1.39	0.40	0.25	0.32	0.081	2.09	0.155	2.78	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.110	1.34	0.39	0.23	0.30	0.083	2.13	0.144	2.58	0.000
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.104	1.30	0.34	0.20	0.35	0.079	2.04	0.123	2.20	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.097	1.28	0.32	0.19	0.39	0.077	1.97	0.123	2.21	0.000

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *NI*. All models in panel A include the excess market returnR_m-R_f. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A7
Explaining excess returns of 5×5 *MC-Beta* portfolios, July 1963–August 2018

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)										
Panel A1: Factor loadings are constant regression slopes										
(3) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.063	0.86	0.28	0.02	0.88	0.076	1.86	0.078	1.81	0.010
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.068	0.90	0.27	0.01	0.88	0.078	1.90	0.056	1.30	0.154
(3) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.075	1.01	0.36	0.12	0.88	0.075	1.80	0.097	2.19	0.001
(4) $R_m - R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.076	0.98	0.32	0.04	0.87	0.079	1.89	0.068	1.50	0.056
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) $R_m - R_f$, *SMB, HML, RMW, CMA*	0.059	0.83	0.21	−0.02	0.89	0.074	1.80	0.074		
(5) $R_m - R_f$, *SMB, HML, RMW, CMA, UMD*	0.085	1.28	0.38	0.17	0.90	0.070	1.67	0.091		

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	T^2	$p(T^2)$
Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)										
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.067	0.75	0.30	−0.05	0.21	0.089	2.28	0.055	1.40	0.093
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.076	0.95	0.34	−0.00	0.23	0.087	2.25	0.064	1.63	0.028
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.070	0.79	0.40	0.09	0.28	0.084	2.16	0.067	1.72	0.017
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.068	0.93	0.28	0.05	0.43	0.074	1.89	0.069	1.76	0.013

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *Beta*. All models in panel A include the excess market return$R_m - R_f$. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

Table A8
Explaining excess returns of 5×5 *MC-Var* portfolios, July 1963–August 2018

Panel A: Summary of intercepts from regressions explaining returns in excess of R_f with five- and six-factor versions of models (3), (4), and (5)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	$Sh^2(a)$	GRS	$p(GRS)$
Panel A1: Factor loadings are constant regression slopes										
(3) R_m-R_f, *SMB, HML, RMW, CMA*	0.132	1.56	0.54	0.46	0.88	0.076	1.86	0.160	3.72	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.125	1.39	0.55	0.47	0.88	0.078	1.90	0.146	3.37	0.000
(3) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.114	1.50	0.33	0.26	0.89	0.074	1.79	0.150	3.38	0.000
(4) $R_m-R_f, R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.121	1.48	0.39	0.30	0.88	0.080	1.91	0.129	2.83	0.000
Panel A2: Loadings for *SMB, HML, RMW, CMA,* and *UMD* vary with characteristics										
(5) R_m-R_f, *SMB, HML, RMW, CMA*	0.127	1.61	0.49	0.42	0.90	0.072	1.75	0.156		
(5) R_m-R_f, *SMB, HML, RMW, CMA, UMD*	0.146	2.13	0.54	0.48	0.91	0.067	1.61	0.188		

Panel B: Summary of average errors, explaining returns in excess of R_z with four- and five-factor versions of model (2)

Model	$A\|a\|$	$A\|t(a)\|$	$Aa^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	AR^2	$As(a)$	$As(e)$	Sh^2(a)	T^2	$p(T^2)$
Panel B1: Factor loadings are average characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.131	1.44	0.49	0.37	0.24	0.089	2.29	0.148	3.77	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.148	1.71	0.52	0.41	0.26	0.088	2.26	0.142	3.62	0.000
Panel B2: Factor loadings are time-varying characteristics										
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}$	0.113	1.38	0.39	0.29	0.37	0.081	2.09	0.157	4.00	0.000
(2) $R_{MC}, R_{BM}, R_{OP}, R_{INV}, R_{MOM}$	0.140	1.97	0.50	0.43	0.52	0.070	1.80	0.166	4.25	0.000

The excess returns are for 25 portfolios formed at the end of each June using independent 5×5 sorts on *MC* and *Var*. All models in panel A include the excess market returnR_m-R_f. They differ on whether they also use TS or CS factors and whether they include a momentum factor. Panel A1 summarizes results from time-series regressions that are variants of models (3) and (4), which explain LHS returns in excess of the risk-free rate. Panel A2 summarizes estimates of model (5), which includes time-varying loadings for the TS factors of model (3). The table shows $A|a|$ and $A|t(a)|$, the averages across LHS assets of the absolute intercept and the absolute t-statistic for the intercepts; $Aa^2/V\bar{r}^2$, the average squared intercept over the cross-section variance of $\bar{r}$, the average returns on the LHS portfolios; $A\lambda^2/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $s^2(a)$, divided by the variance of $\bar{r}$; AR^2, the average regression R^2; $As(a)$, the average standard error of the intercepts; $As(e)$, the average of residual standard deviations; $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios; the *GRS* statistic of Gibbons, Ross, and Shanken (1989); and the p-value of *GRS*. Panel B summarizes results for variants of model (2) in which LHS returns are in excess of R_z. Like R_z, the RHS factors are from the monthly cross-section regressions of model (1) and its variants. The time-varying factor loadings in panel B2 are predetermined size, value, profitability, investment, and momentum characteristics. Panel B1 substitutes time-series average values for the time-varying characteristics of (2). The metrics in panels B1 and B2 are analogous to those in panel A, except that a is an average model error, called its pricing error, rather than a regression intercept, and *GRS* is replaced by the F-statistic for the Hotelling T^2 test that all expected pricing errors for the LHS portfolios are zero.

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