A1 - HMM

Artificial Intelligence

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1 Grade E-D

1.1 Question 1

$$A = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 0.9 \\ 0.5 & 0.5 \end{bmatrix}, \quad \pi = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \tag{1}$$

1.2 Question 2

$$P(X_6)\ A = P(X_7) \tag{2}$$

1.3 Question 3

$$P(X_7)\ B = P(O_7) \tag{3}$$

1.4 Question 4

$$\begin{aligned} P(O_{1:t} = o_{1:t}, X_t = x_i) &= P(O_t = o_t, O_{1:t-1} = o_{1:t-1}, X_t = x_i) \\ &= \{product\ rule\} \\ &= P(O_t = o_t | X_t = x_i, O_{1:t-1} = o_{1:t-1})\ P(X_t = x_i, O_{1:t-1} = o_{1:t-1}) \\ &= \{conditional\ independence\} \\ &= P(O_t = o_t | X_t = x_i)\ P(X_t = x_i, O_{1:t-1} = o_{1:t-1}) \end{aligned} \tag{4}$$

1.5 Question 5

- δ has TxN elements
- $\delta^i dx$ has $(T-1)xN$ elements (no predecessor for $t = 0$)

1.6 Question 6

$$\begin{aligned} P(X_t = x_i, X_{t+1} = x_j | O_{1:T} = o_{1:T}) &= \{definition\ of\ conditional\ probability\} \\ &= \frac{P(X_t = x_i, X_{t+1} = x_j, O_{1:T} = o_{1:T})}{P(O_{1:T} = o_{1:T})} \end{aligned} \tag{5}$$

The denominator of (5) can be computed using the forward algorithm as $\sum_{k=1}^{N} \alpha_T(k)$. This term represents a normalization factor.

2 Grade C

2.1 Question 7