ecalj CMD course menu

You need

Quantum mechanism, Solid state physics, and Quantum chemistry.
 Schroedinger equation, Hydrogen atom, moleculer orbitals, Brillowin zone and Band theory, Crystal symmetry, Independent-particle picture,
 Perturbation theory, Optical response, transport,...

DFT, LDA and so on

Theory. How it works (pros and cons)? How to calculate physical quantities from the results?

Computer skills

fortran, python, bash, linux, git, gnuplot, latex,...
With better skill, we can do things faster and less mistakes. Try to introduce better technology (but new technology (e.g, c++) is not necessarily good).

For research,

- Topics (knowledge of a hot topic which is now going on).
- Advanced method and theories such as GW (many-body theory).

Monday 15:00~17:00,17:20~19:20 Theory, How it works?, and Practice.

- LECTURE1: PMT method
 How to solve one-body problem? →PMT= LAPW+LMTO
- LECTURE2: Quasiparticle self-consistent GW method Theory and results until now.
 Some kinds of numerical techniques.
- Get Started!

Install ecalj, and go through "ecalj tutorial". Observe how it works.

Tuesday: 9:00~17:20, Wednesday 9:00~10:30 (or to 12:10).

- ctrl file and output.
- Numerical technique and GWinput.
- Read output (console output, data).
- Check points (How to get reliable results?)
- (For expert course) "Wannier function", Fat band analysis...

The PMT method: a new linearized method

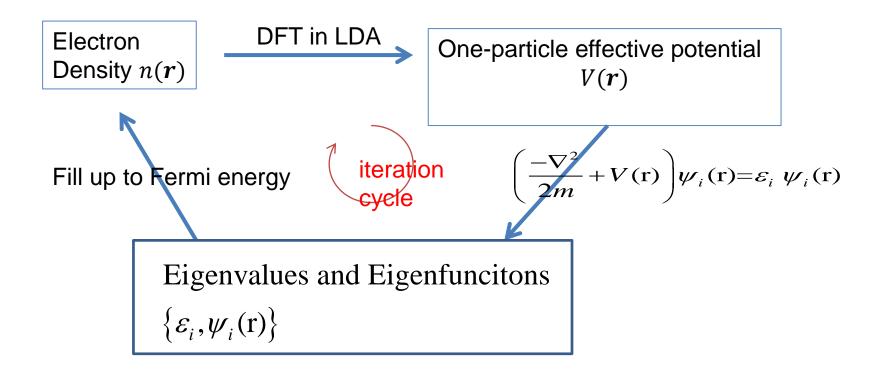
Takao Kotani (tottori-u)

Key point:

- 1. Iteration cycle.
- eigenfunctions are expanded with APWs(augmented plane wave) and MTOs(muffin-tin orbital)

Independent particle picture and total energy

These can be obtained by the density functional theory (DFT) in LDA.



This iteration cycle until converged = total energy minimization

How to represent density and so on in computer? How to solve it numerically?

Lists of the Full-potential methods

- •KKR
- Pseudopotential method
- •PAW



(why do we call them "linearized" method?

→from the view of "exact "APW method)

Finite basis set

Basis set (finite number of basis)

→ We assume eigenfunctions are given as:

$$\psi_p(\mathbf{r}) = \sum_j \alpha_p^j F_j(\mathbf{r}) \quad \left\{ F_j \right\} : j = 1, 2, ...N$$

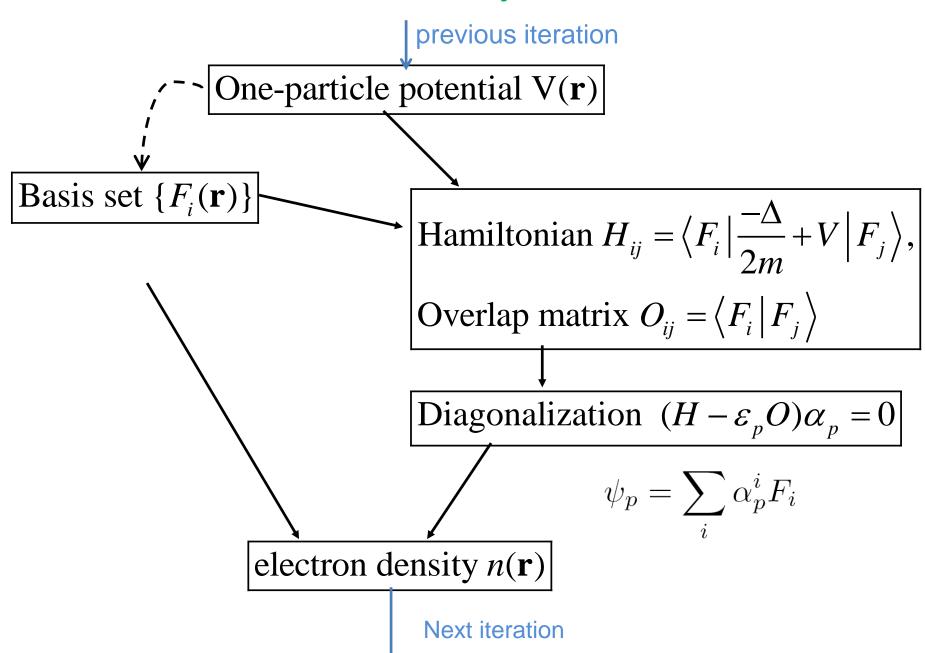
Hamiltonian
$$H_{ij} = \langle F_i | \frac{-\Delta}{2m} + V | F_j \rangle$$
,

Overlap matrix $O_{ij} = \langle F_i | F_j \rangle$

$$(H_{ij} - \varepsilon O_{ij})\alpha^{j} = 0$$

Finite dimension problem(as the same as LCAO/Gaussian)

iteration cycle



How to choose the good basis?

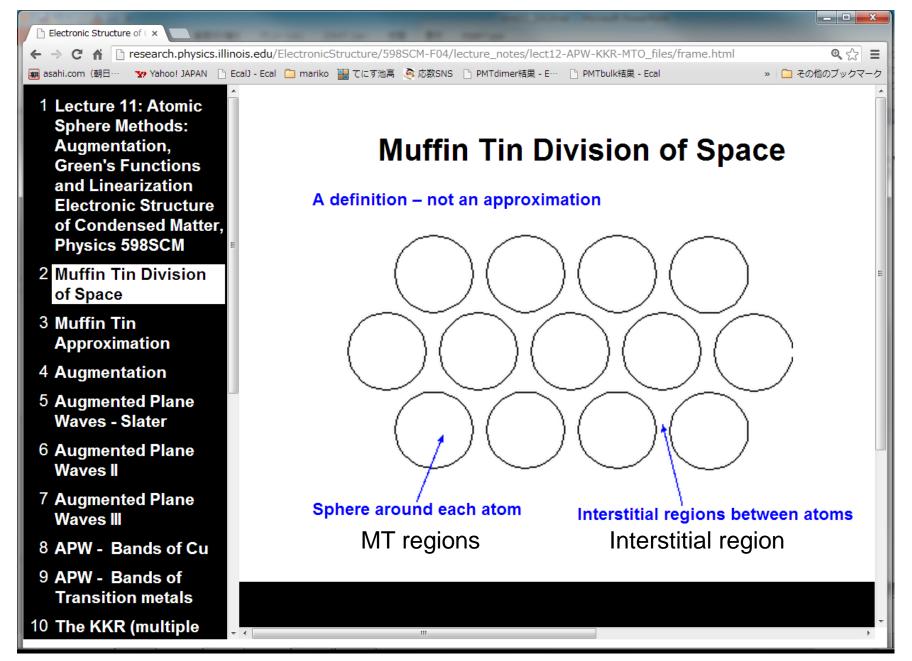
- APW (augmented plane wave)
- MTO (muffin-tin orbital)

are the names of the basis functions. Both of them are made by "augmentation".

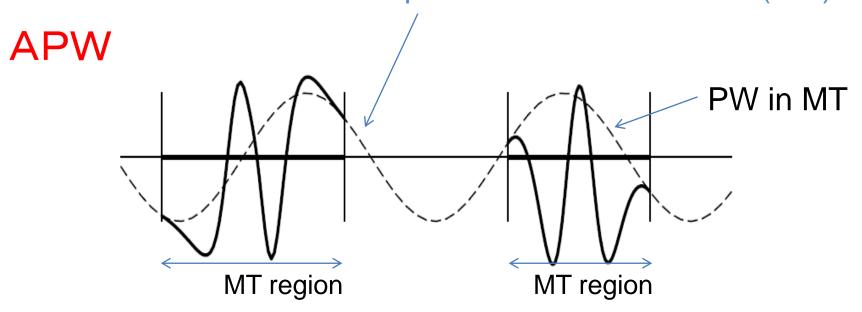
My conclusion:

To overcome shortcomings in APW basis and MTO basis, we should use both of <u>APW and MTO together</u>. →this means the PMT method

But wait... What is the APW and MTO?→ next page.



Envelope function = PlaneWave(PW)



3-components

PW + Atomic-like part — Counter part:

PW within MT

Oth 1st 2nd

- •Good for Na(3s), high energy bands.
- •Not so good for Cu(3d), O(2p)
- Systematic

Linearized APW

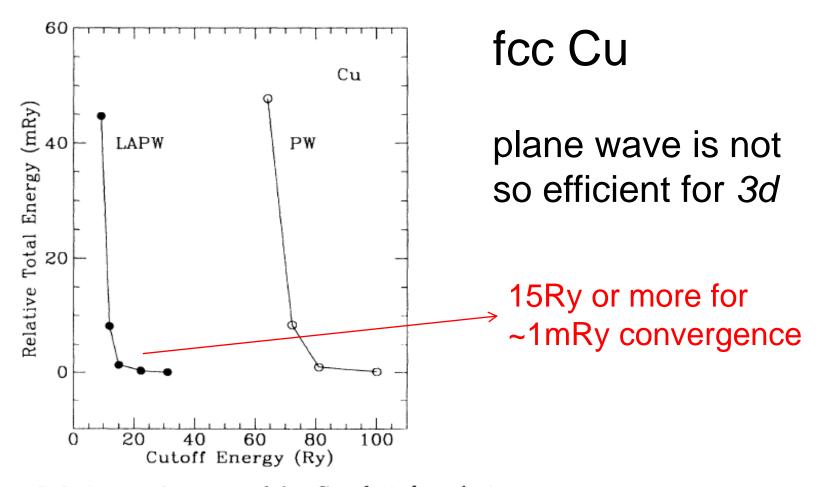
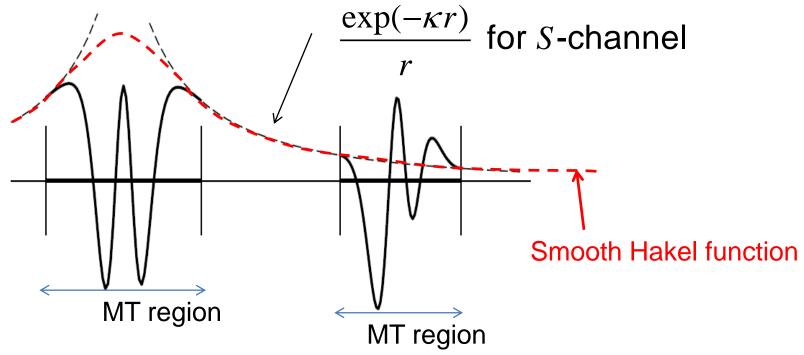


FIG. 1. Relative total energy of fcc Cu plotted against plane-wave cutoff energy.

D.Singh et al PRB49,17424

MTO

Atom-centered Hankel function, e.g,



3-components

- •Good for localized orbitals such as Cu(3d), O(2p)
- Not so good for extended states, surface.
- Not systematic

PMT = APW + MTO

T.K and M.van Schilfgaarde

Phys. Rev. B 81, 125117 (2010)

T.K, H. Kino, and H.Akai

Supercell calculations from H2 through Kr2.

Almost automatic setting of MTOs with APWs

(Energy cutoff 3~4Ry).

- J. Phys. Soc. Jpn. 82, 124714, (2013)
- J. Phys. Soc. Jpn. 84, 034702 (2015)

Basis function

MT center at **R**. Radis *R*.

$$F_{0j}(\mathbf{r}), \qquad F_{1j}(\mathbf{r}), \qquad F_{2j}(\mathbf{r})$$
 Envelope function PW or smHankel for $|\mathbf{r}| < R$ Counter part for $|\mathbf{r}| < R$ Augmentation parts Cutoff: $l \le l_{\max} \sim 4$, Radial-part expansion

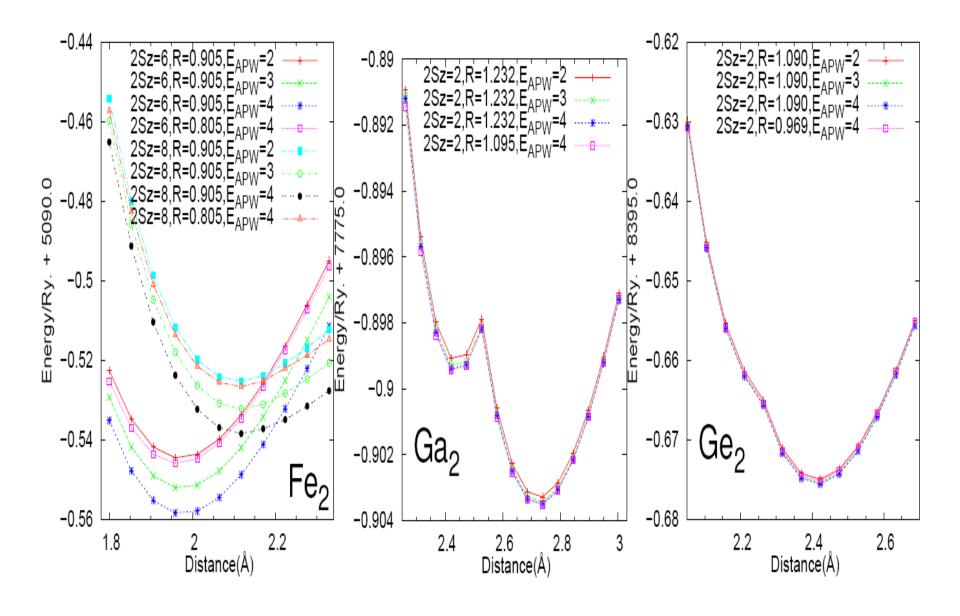
$$F_{j}(\mathbf{r}) = F_{0j}(\mathbf{r}) + F_{1j}(\mathbf{r} - \mathbf{R}) - F_{2j}(\mathbf{r} - \mathbf{R})$$

Electron density n(r) and potential V(r) are expanded in a similar manner.

Local orbitals(Io): basis functions which are Non-zero only wihtin MTs.

e.g, see http://www.wien2k.at/lapw/index.html

This is also used together.



 $N_{\text{basis}} = 1081 + 36 \times 2$, $1973 + 36 \times 2$, $3025 + 36 \times 2$

Comparison with Gaussian

		$r_{\rm e}~({\rm \AA})$	$D_e(\mathrm{Kcal/mol})$	$\omega_e \; (\mathrm{cm}^{-1})$
$H_2, 2S_z = 0$	PMT	0.749	104.678	4317.959
	PMT(NR)	0.750	104.764	4311.202
	GTO	0.752	104.552	4311.816
$O_2, 2S_z = 2$	PMT	1.218	143.741	1564.787
	PMT(NR)	1.218	144.984	1568.867
	GTO	1.220	139.815	1554.249
	VASP		143.3	
$Cr_2,2S_z=0$	PMT	1.591	32.833	813.296
	PMT(NR)	1.589	30.191	818.483
	GTO	1.595	26.192	808.148
$\text{Fe}_2, 2S_z = 6$	PMT	1.977	57.596	397.673
	PMT(NR)	1.991	58.770	386.597
	GTO	2.012	56.902	397.228
$Cu_2, 2S_z = 0$	PMT	2.218	51.169	269.326
$\mathcal{O}_{02}, 2\mathcal{O}_z = 0$	PMT(NR)	$\frac{2.210}{2.251}$	48.503	254.321
	GTO	2.251 2.251	48.645	255.768

NR: non relativistic GTO: 6-311+G(d,p)

Quasiparticle self-consistent GW Takao kotani (tottori university)

- Mean field theory.
 What the eigenvalue means?
- GW, and QSGW method
 Minimum and previous results.

Mean field theory gives an independent-particle picture

- Hartree-Fock theory
- Density functional theory

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Eigenvalues and Eigenfuncitons \{\varepsilon_i, \psi_i(\mathbf{r})\} are determined by minimization of \mathbf{E}[\{\psi_i(\mathbf{r})\}, n_i]
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The minimization determines

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n_i = 1 \text{ for } \varepsilon_i < \varepsilon_{\text{FERMI}} \text{ (occupied states)}

n_i = 0 \text{ for } \varepsilon_i > \varepsilon_{\text{FERMI}} \text{ (unoccupied states)}.
```

NOTE: we can consider

A. configuration of excited states. → QUIZ (picturize this)

B. fractional occupancy (this is very theoretical)

We can describe "ground state" and "excited state" easily by filling electron or removing electron from orbitals → Independent particle picture

But need to know its limitation.

True excited state energy is not so easy;

From a view, you can say "mean-field changes by filling electrons".

Confusing? But there is a key theorem for mean-field theories.

Janak(Koopman's) theorem

$$\frac{\partial E}{\partial n_i} = \varepsilon_i$$

when we fill/remove electrons from orbital ψ_i

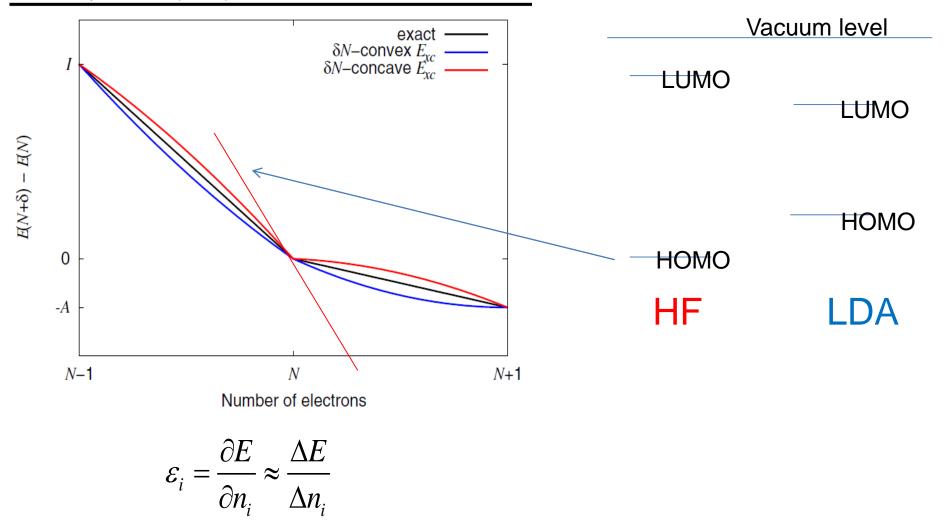
Left hand side is "True" excitation energy. (caution: this is "derivative", not one-electron filled/removed.)

Ionization energy of an atom

A.J.Cohen, P. Mori-Sanchez, W.Yang, Science321,792(2008)

PRL **100**, 146401 (2008)

PHYSICAL RE



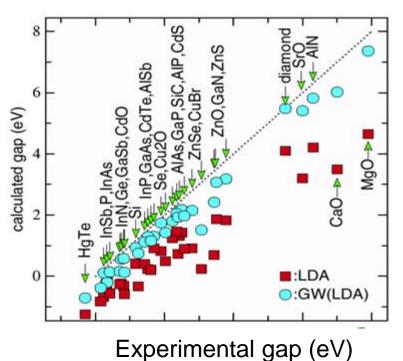
$$\varepsilon_i = \frac{\partial E}{\partial n_i} = \frac{\Delta E}{\Delta n_i}$$
 is satisfied.

If
$$\varepsilon_i = \frac{\partial E}{\partial n_i} = \frac{\Delta E}{\Delta n_i}$$
 (linear line),

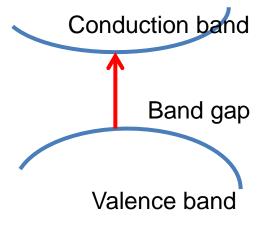
we have better excitation energies $\varepsilon_i - \varepsilon_j$

How accurate they are?

band gap
$$(\varepsilon_{\text{CBM}} - \varepsilon_{\text{VBM}})$$



LDA gives too small HF(not shown) gives too large



Key point : meanings of "eigenvalues and eigenfuncions" in the mean-field theory.

GW method and QSGW method

(preparation) What is Green's function?

*Example1: Poisson eq. $\Delta \phi = \rho(\mathbf{r})$ How to convert this to <u>integral equation</u>?

$$\Delta \frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|} = \delta(\mathbf{r} - \mathbf{r}') \qquad G_0 = \frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|} \text{ is Green function}$$

$$\phi(\mathbf{r}) = \int G_0(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}'$$

- * Example2: $\left(i\frac{\partial}{\partial t} E\right) f(t) = \rho(t)$ \Rightarrow $\left(i\frac{\partial}{\partial t} E\right) G_0(t t') = \delta(t t')$ $G_0 = \theta(t t') e^{-iE(t t')}$ or $G_0 = \theta(t' t) e^{-iE(t' t)}$ $(\delta(t t')$ means "impact force".)
- * Fourier transformation: $\theta(t-t')e^{-iE(t-t')} \leftrightarrow \frac{1}{\omega-E+i\delta}$ or ...quiz...

For given,
$$H_0 = -\frac{\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + V_{\text{H}}(\mathbf{r}) + V_{\text{XC}}(\mathbf{r}, \mathbf{r}')$$
,

we like to calculate $\varphi(\mathbf{r}, t)$ sasisfying $\left(i\frac{\partial}{\partial t} - H_0\right)\varphi(\mathbf{r}, t) = F(\mathbf{r}, t)$

$$\left(i\frac{\partial}{\partial t} - H_0\right)G_0 = \delta(t - t')\delta(\mathbf{r} - \mathbf{r}')$$

$$\Rightarrow G_0 = \frac{1}{i\frac{\partial}{\partial t} - H_0} = \frac{1}{\omega - H_0} \qquad \text{(Note boundary condition)}$$

Boundary condition along time-axis

$$H_{0} = H_{0}^{occ.} + H_{0}^{unocc.} = \sum_{occ.} \epsilon_{i} \ \varphi_{i}^{*}(\mathbf{r}) \varphi_{i} \ (\mathbf{r}') + \sum_{unocc.} \epsilon_{i} \ \varphi_{i}^{*}(\mathbf{r}) \varphi_{i} \ (\mathbf{r}')$$

$$\Rightarrow G_{0} = \frac{1}{\omega - H_{0}} = \sum_{occ.} \frac{\varphi_{i}^{*}(\mathbf{r}) \varphi_{i} \ (\mathbf{r}')}{\omega - \epsilon_{i} + i\delta} + \sum_{unocc.} \frac{\varphi_{i}^{*}(\mathbf{r}) \varphi_{i} \ (\mathbf{r}')}{\omega - \epsilon_{i} - i\delta}$$

$$= -\sum_{occ.} \varphi_{i} \ (\mathbf{r}') \varphi_{i}^{*}(\mathbf{r}) \ \theta(t' - t) e^{-i\epsilon_{i}(t' - t)}$$

$$+ \sum_{unocc.} \varphi_{i} \ (\mathbf{r}) \varphi_{i}^{*}(\mathbf{r}') \ \theta(t - t') e^{-i\epsilon_{i}(t - t')}$$

(we measure ε_i from μ , or $\varepsilon_i \rightarrow$ all excitation is positive energy)

Quiz: check this G_0 satisfy differential eq at page.24

Why? → this is convenient due to Fermi statistics (adding electron or "adding hole=removing electron").

Many body theory (rough explanation)

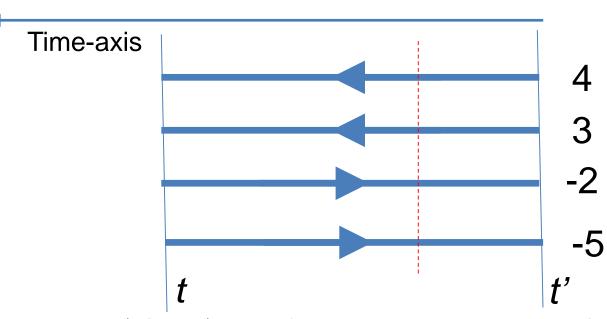
 H_0 is interpreted in the second quantization. "Fock space." An excited state can be written as

In other words, it can be written as

We need to treat a set $\Omega = \{all possible excited+ground states\}.$

Quiz: Then what the operation H_0 mean? What is $1/(\omega - H_0)$?

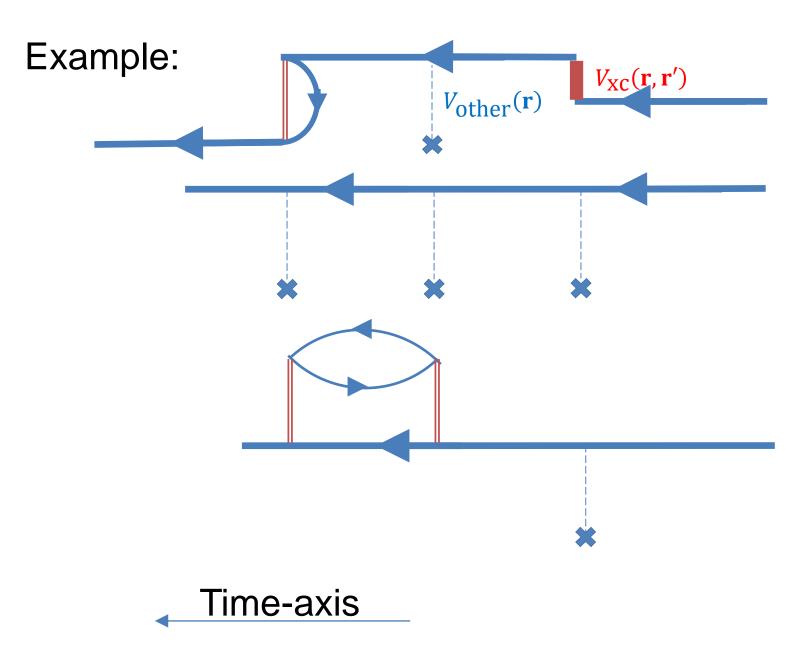
$1/(\omega - H_0)$ in MBPT (not so accurate)



これは(4,3,-2,-5)が時間がたってもそのまま、(4,3, -2, -5)であることを意味する. エネルギーは和になっている. 多体理論ではどうなるか?

 $H + V_{\text{other}}(\mathbf{r}) = H_0 + \text{ (electron-electron interaction)} - (V_H(\mathbf{r}) + V_{XC}(\mathbf{r}, \mathbf{r}')) + V_{\text{other}}(\mathbf{r})$

Illustration of Many-body perturbation theory(MBPT)



GW approximation

We concentrate on how G_0 is modified by $H_0 + (H - H_0)$ We assume H_0 is not so bad as starting point...

$$H_0 = -\frac{\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + V_{\text{H}}(\mathbf{r}) + V_{\text{XC}}(\mathbf{r}, \mathbf{r}')$$

1.Exchane effect.

 $(H - H_0)$ makes jump to another electron. Effect of Fock term. (and remove self-interaction)

2. Correlation effect.

When a test charge is moving in electron sea.

The test charge is moving

- → cause polarization of electrons
- → affect force to the test charge.

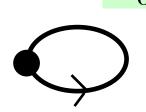
Particle of $H_0 \rightarrow$ Quasiparticle picture.

GW approximation: 1電子波動関数の時間発展

Start from any H_{0}

$$H_0 = -\frac{\nabla^2}{2m} + V_{\text{eff}}(\mathbf{r}, \mathbf{r}') \iff G_0 = \frac{1}{\omega - H_0}$$
$$n(\mathbf{r}) = G_0(\mathbf{r}, \mathbf{r}, 0-)$$

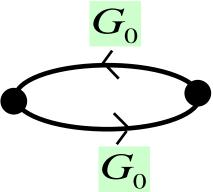
e,q. $=H^{LDA}$



2. $\Pi = -iG_0 \times G_0$ Polarization function

 Π is density response for one-particle potential.

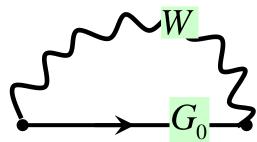
$$\Pi(\mathbf{r},\mathbf{r}',t-t') = \frac{\delta n(\mathbf{r},t)}{\delta V(\mathbf{r}',t')}$$



 $W = \varepsilon^{-1}v = (1 - \Pi v)^{-1}v$ W in the RPA $v(\mathbf{r}, \mathbf{r}') = \frac{e^2}{|\mathbf{r} - \mathbf{r}'|}$ (time-dependent Hartree approx.)

Self-energy (Many-body effect is downfolded into one-body propagetor)

$$\Sigma = i G_0 W$$
 Self-energy



が「交換相関」を与える。

interpretation 1: exchange hopping + electron moving is other electrons

2: screened Coulomb interaction + Coulomb hole

$$G_0 \rightarrow n \rightarrow V^H$$
 also

$$H(\mathbf{r},\mathbf{r}',\omega) = -\frac{\nabla^2}{2} + V^{\text{ext}}(\mathbf{r}) + V^{\text{H}}(\mathbf{r}) + \Sigma(\mathbf{r},\mathbf{r}',\omega)$$

$$\Rightarrow G = \frac{1}{\omega - H}$$

$$NOTE$$
: In $\int d\mathbf{r}' dt' H(\mathbf{r}, \mathbf{r}', t-t') \varphi(\mathbf{r}', t')$, $V^{\text{ext}}(\mathbf{r})$ is takes as $V^{\text{ext}}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$

 ω -dependence of $\Sigma(\mathbf{r},\mathbf{r}',\omega)$ is very problematic

QSGW: How to determine better H₀?

$$H(\varepsilon_{i})|\varphi_{i}(\mathbf{r})\rangle = \varepsilon_{i}|\varphi_{i}(\mathbf{r})\rangle$$

$$\left(\frac{-\Delta}{2m} + V_{\text{ext}} + V_{\text{H}} + \Sigma(\varepsilon_{i})\right)|\varphi_{i}(\mathbf{r})\rangle = \varepsilon_{i}|\varphi_{i}(\mathbf{r})\rangle$$

It is not so easy to treat energy-dependent potential!

$$\Sigma(\mathbf{r},\mathbf{r}',\omega) \rightarrow \omega$$
-independent $V_{xc}(\mathbf{r},\mathbf{r}')$

$$H_0 = \frac{-\Delta}{2m} + V_{\text{ext}} + V_{\text{H}} + V_{\text{xc}}(\mathbf{r}, \mathbf{r}') \rightarrow \Sigma(\mathbf{r}, \mathbf{r}', \omega) \quad \underline{\qquad}$$

An average procedure (not shown here)

This idea is supported by Quasiparticle picture (Landau-Silin's).

GW and the "RPA total energy"

We start from $H_{LDA} \rightarrow \{\varepsilon_i, \psi_i(\mathbf{r})\}$

$$E = E_{0k} + E_{0ext} + E_{H} + E_{X} + E_{C}$$

$$= \sum_{i}^{\text{OCC.}} \left\langle \varphi_{i} \left| \frac{-\Delta}{2m} \right| \varphi_{i} \right\rangle + \sum_{i}^{\text{OCC.}} \left\langle \varphi_{i} \left| V_{\text{ext}}(\mathbf{r}) \right| \varphi_{i} \right\rangle + E_{\text{H}} + E_{\text{X}} + E_{\text{C}}$$

$$E_{i} = \frac{\partial E}{\partial n_{i}} = \left\langle \varphi_{i}(\mathbf{r}) \middle| \frac{-\Delta}{2m} + V_{\text{ext}} + V_{\text{H}} + \Sigma(\varepsilon_{i}) \middle| \varphi_{i}(\mathbf{r}) \right\rangle$$

$$= \varepsilon_{i} + \left\langle \varphi_{i}(\mathbf{r}) \middle| \left(\Sigma(\varepsilon_{i}) - V_{\text{XC}}^{\text{LDA}} \right) \middle| \varphi_{i}(\mathbf{r}) \right\rangle$$

This is one-shot GW

Historically,
$$E_i = \varepsilon_i + \left\langle \varphi_i(\mathbf{r}) \middle| \left(\Sigma(E_i) - V_{\mathrm{XC}}^{\mathrm{LDA}} \right) \middle| \varphi_i(\mathbf{r}) \right\rangle$$
, (but $E_i = \varepsilon_i + \left\langle \varphi_i(\mathbf{r}) \middle| \left(\Sigma(\varepsilon_i) - V_{\mathrm{XC}}^{\mathrm{LDA}} \right) \middle| \varphi_i(\mathbf{r}) \right\rangle$ is better)³³

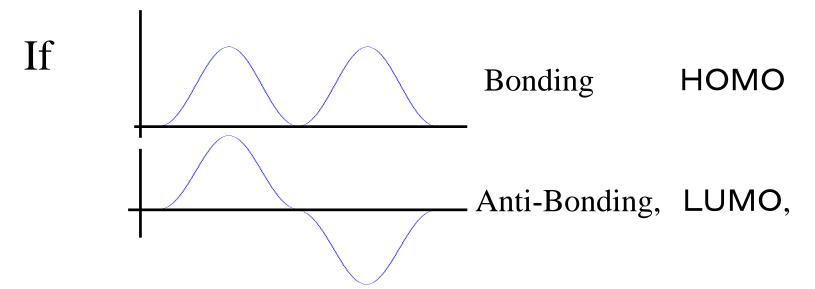
Non-local potential term (as Fock exchange term) is important.

I. Localized electrons ← LDA+U type effect
 (Onsite non-locality. self-interaction included).
 It can break time-reversal symmetry → No orbital moment.

II. Extended electrons ← GW type effect for semiconductor.

Important to describe band gap. Off-site non-locality. (required to distinguish "bonding orbital" and "anti-bonding" orbitals.) → next page

Schematic explanation on off-site exchange

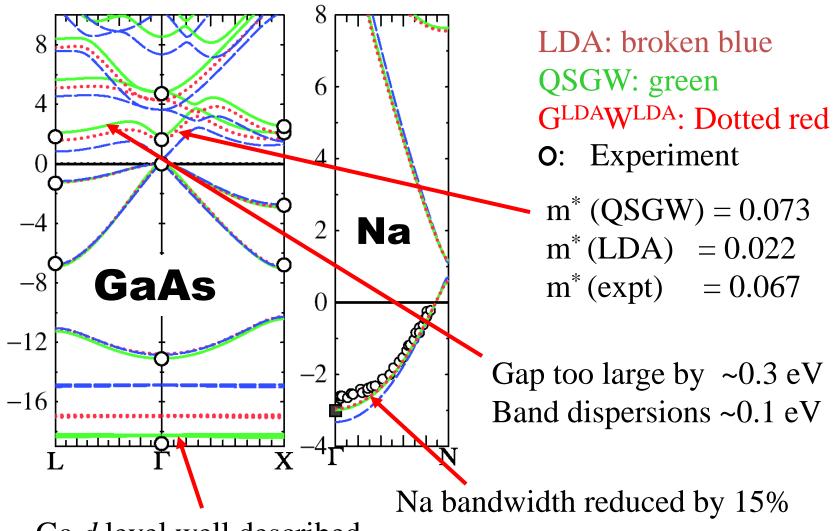


square of them are the same.

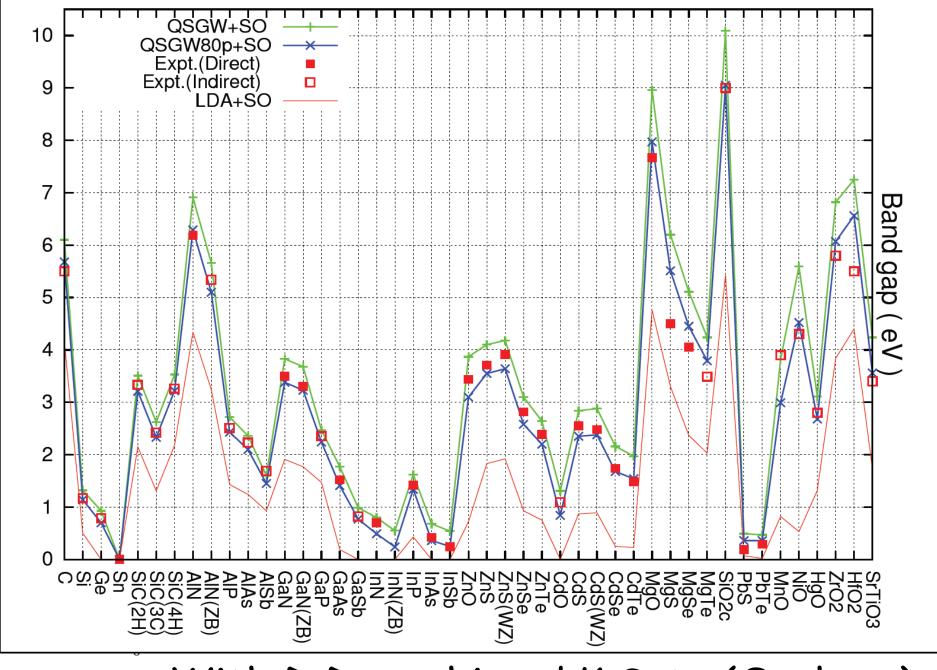
$$\langle \psi_{\text{bonding}} | V(r) | \psi_{\text{bonding}} \rangle = \langle \psi_{\text{anti-bonding}} | V(r) | \psi_{\text{anti-bonding}} \rangle.$$

→ Local potential can not distinguish LUMO and HOMO.

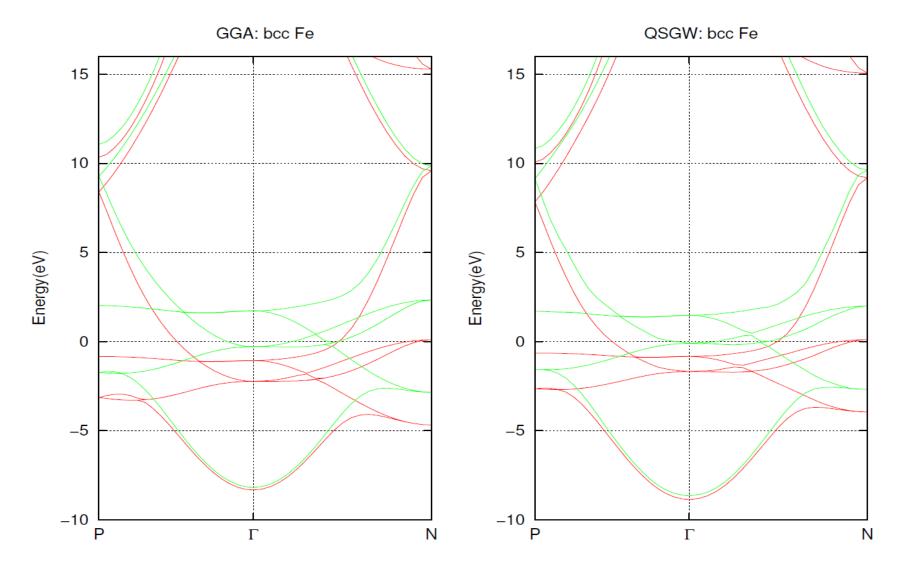
sp bonded systems



Ga d level well described



With D.Deguchi and K.Sato (Osaka-u)



金属的なもの→QSGWでOK. ハイブリッド法では場合によりうまくいかない。

Physics in QSGW

LDA(GGA) → homogeneous gas OK! (Physics)

Hartree-Fock → H-atom OK! (chemistry)

"True results" may be between its middle;

- \rightarrow Hybrid methd (B3LYP, HSE...) $\frac{1}{4} \times HF + \frac{3}{4} \times LDA$
- •Problem: the mixing ratio may be dependent on materials.

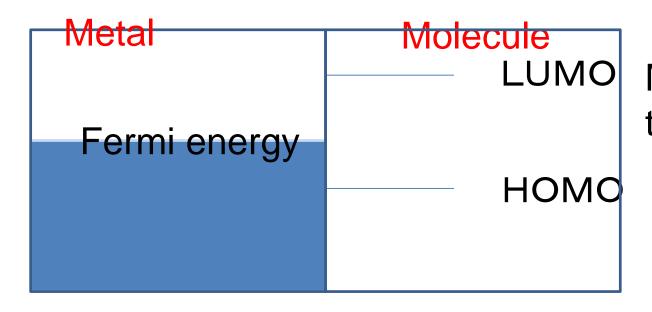
- —QSGW: Instead of bare Coulmb v,
- we use "Dynamically Screened Coulom interaction W".
- W is determined self-consistently.

Independent-particle theory where we take into account the charge fluctuation at the RPA self-consistently.

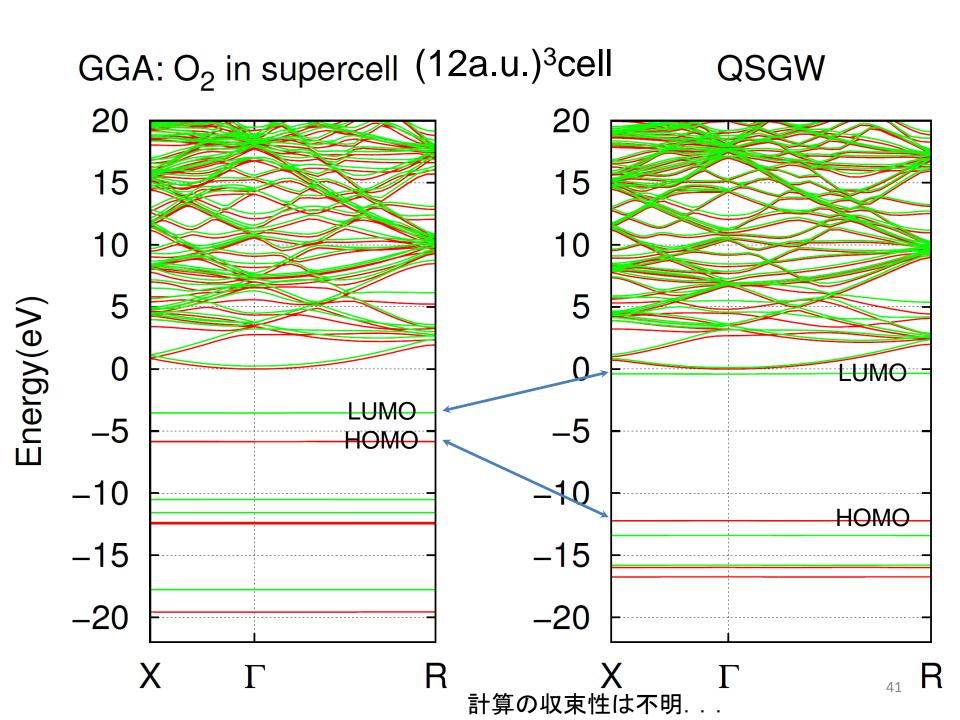
Why we need QSGW?

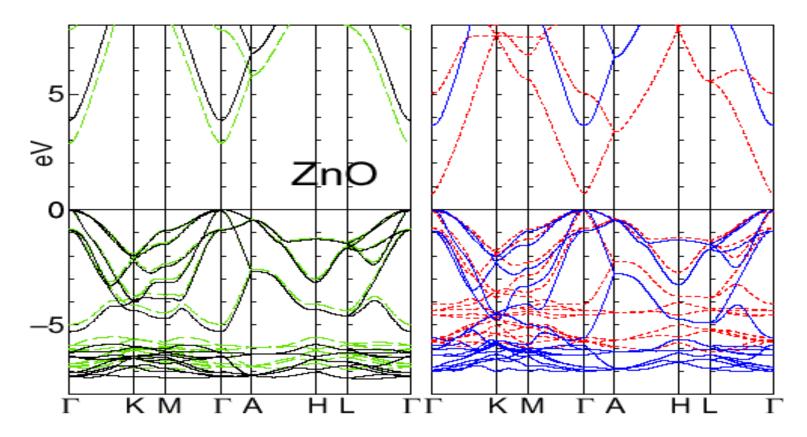
We need good independent-particle picture in order to calculate linear responses (magnetic optical, transport...). H0 (eigenvalue and eigenfunctions) are required.

- * Band gap, Effective mass
- * Relative position of levels. LUMO—HOMO, Fermi energy...



Molecule on top of metal.





Black:QSGW

Red:LDA

3.87eV

0.71eV

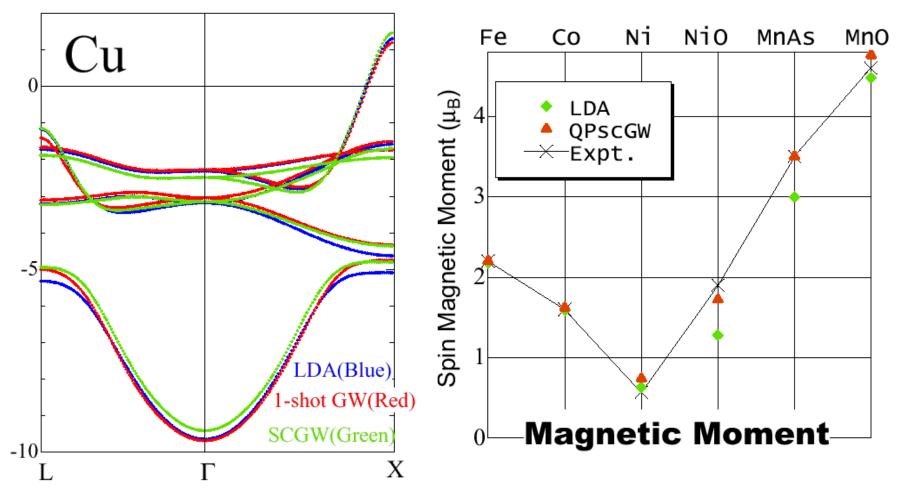
Green: GLDAWLDA (Z=1,Offdiagonal included) 3.00eV

Blue: e-only self-consistency 3.64eV

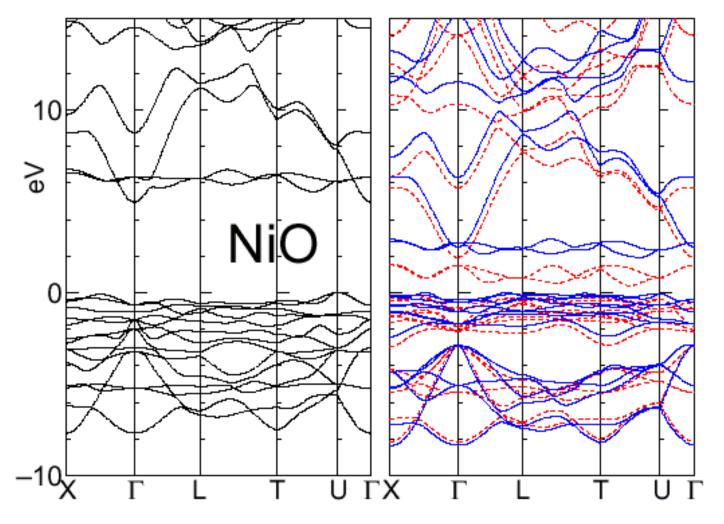
Experiment(+correction) 3.60eV

Kotani et al PRB76,165106(2007)

d systems

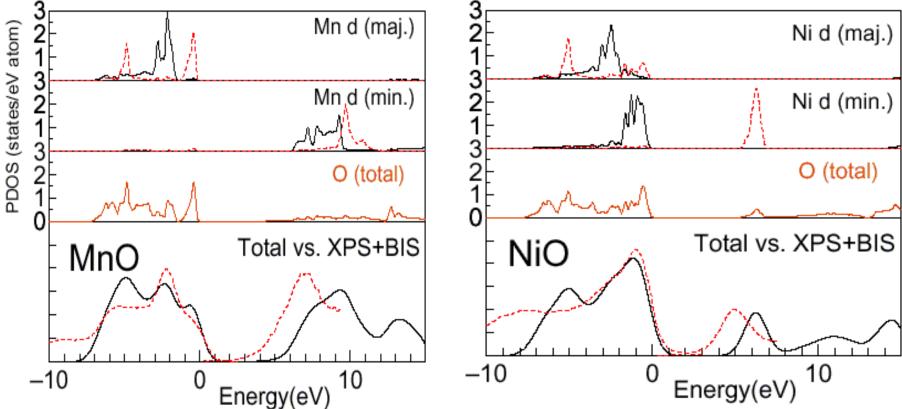


"No Pushing down 3d-band" in GW.

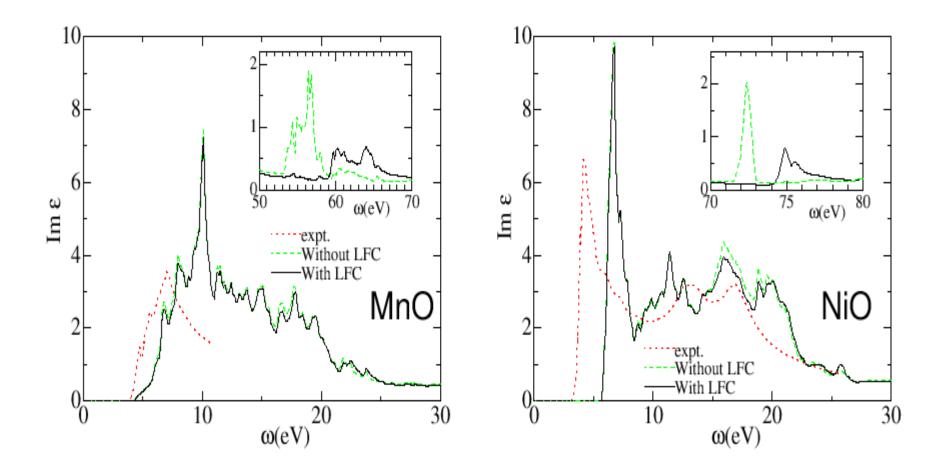


Black:QSGW Red:LDA Blue: e-only

Black:t2g Red:eg

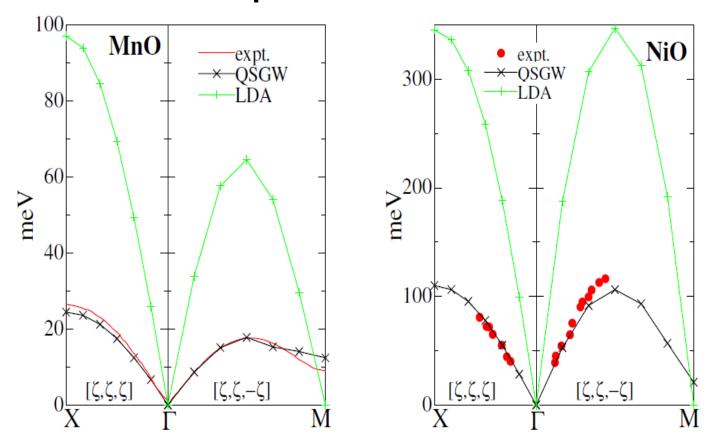


Red(bottom): expt



Black: Im eps with LFC Red: expt

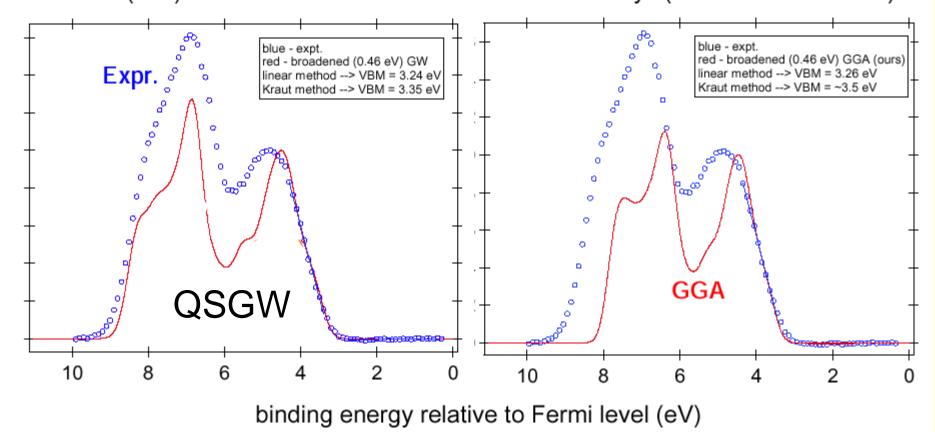
Spin wave dispersion based on QSGW



J.Phys.C20 (2008) 295214, PRB83, 060404(R) (2011) for CaFe2As2 for spectrum of χ^{+-} .

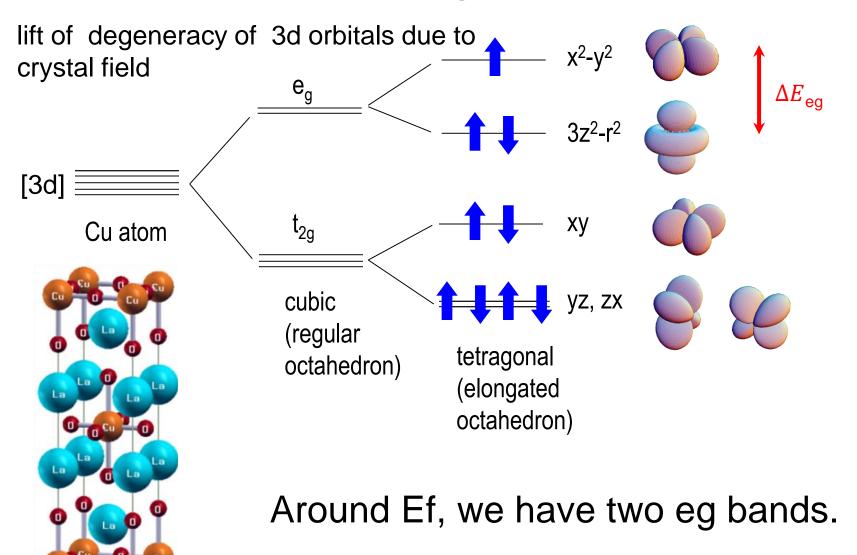
SrTiO3 Valence DOS

fit of properly broadened theoretical DOS with experiment n-STO(001) VB excited with monochromatic AlK α x-rays (resolution = 0.46 eV)

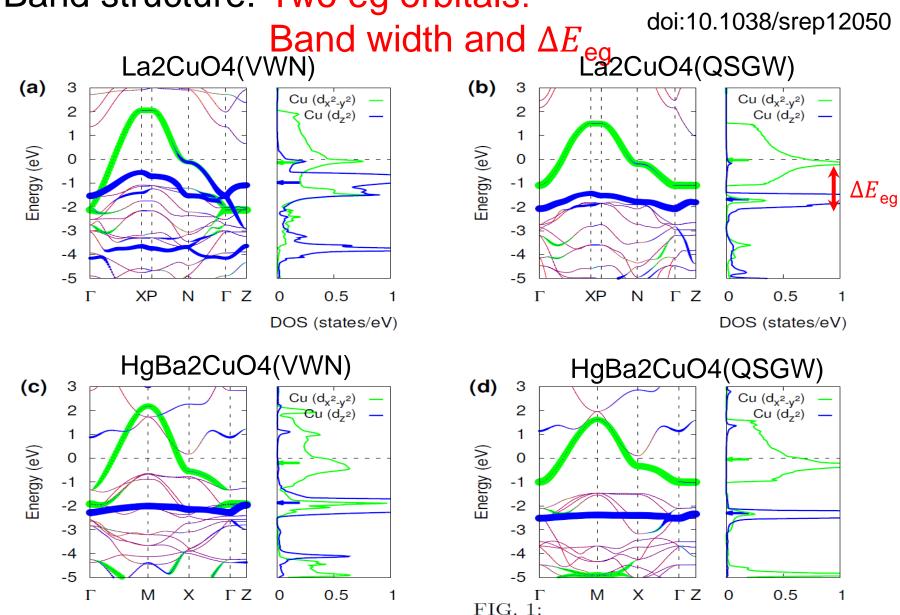


S. A. Chambers et al, Surface Sci 554,81-89 (2004)

Electron configuration



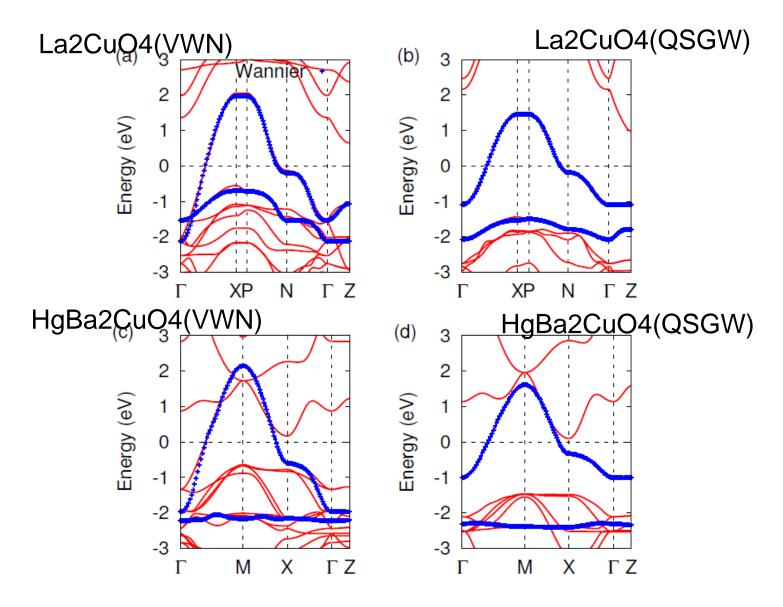
Band structure: Two eg orbitals.



DOS (states/eV)

DOS (states/eV₂)

Wannier function method. Extract tight-binding parameters

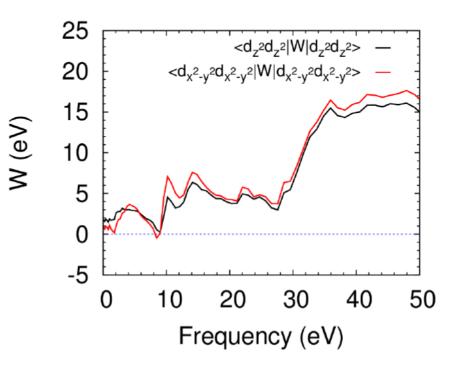


Miyake's Maxloc Wannier code is re-implemented(TK and H.Kino).

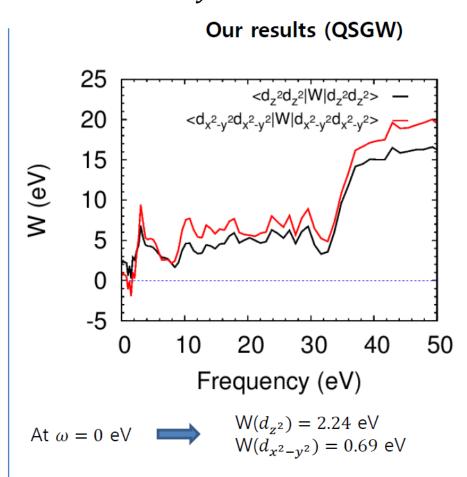
La₂CuO₄

Effective interaction $U(\omega)$ for d_{z^2} and $d_{x^2-v^2}$



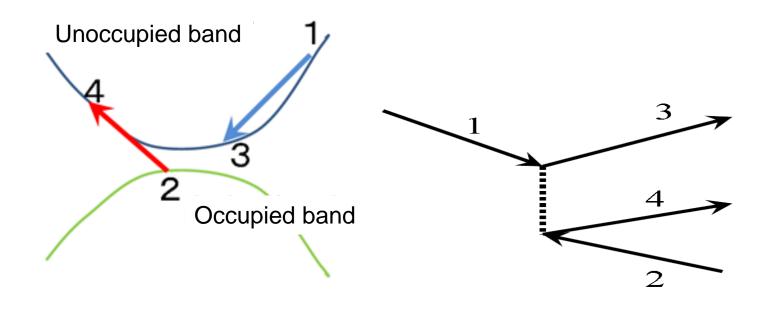


At
$$\omega=0$$
 eV \longrightarrow $W(d_{z^2})=1.71$ eV $W(d_{x^2-y^2})=0.77$ eV



^{*} There is an anisotropy in W between $d_{x^2-y^2}$ and d_{z^2}

Impact ionization rate(auger process)



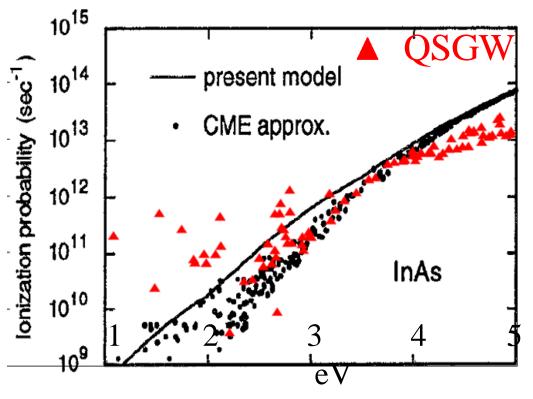
Fermi's Golden rult → transition rate. Sum up for all final states. Matrix element of transition is <1,3 | electron-electron interaction | 4,2>

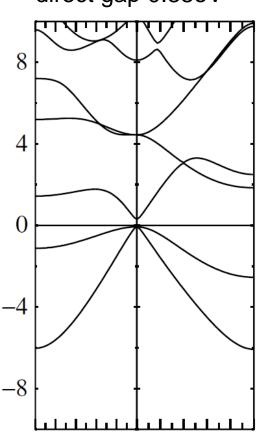
(Energy conservation and Momentum conservation)

InAs impact ionization rate

(=auger process

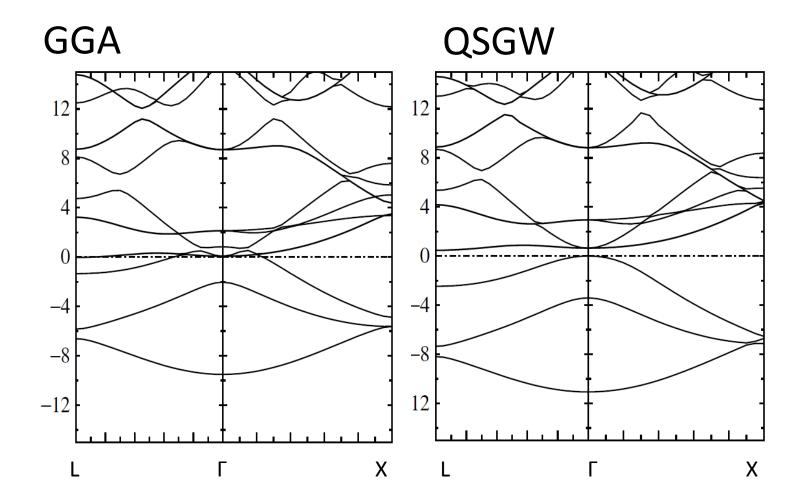
= high energy electron lose energy with e-h pair)
direct gap 0.38eV



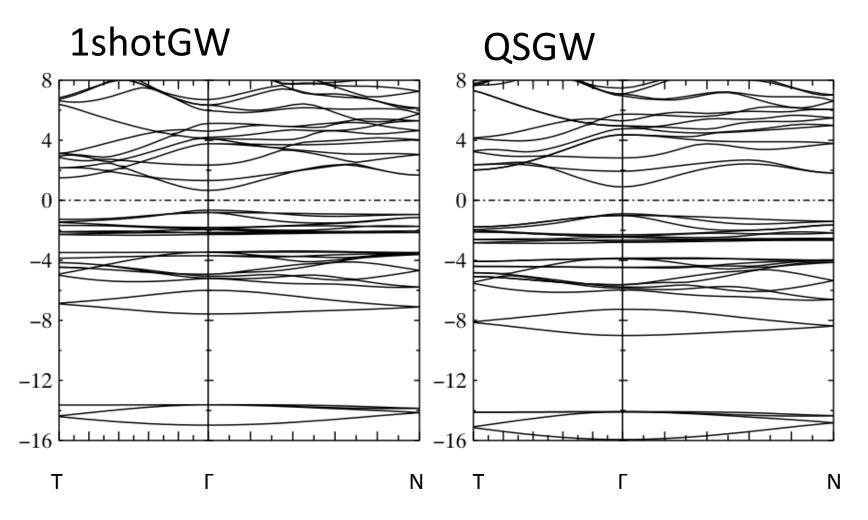


Present model: Sano and Yoshii JAP77 2020 (1995)

YH3(fcc structure 6x6x6)



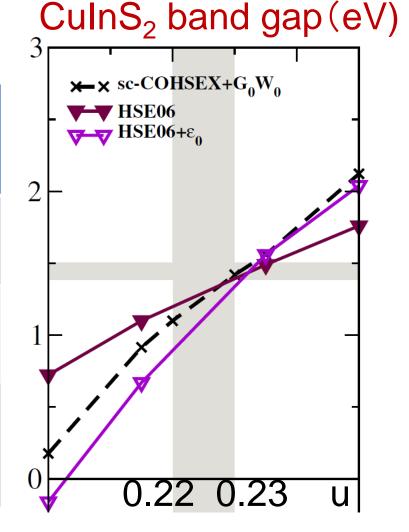
CuGaSe2 (eight atoms in cell, 2x2x2. ~3hours per iteration by single core)



Band gap GGA:0.4eV, GW(1shot noZ) 1.37 eV, QSGW(1.79eV) Exper. 1.63, 1.67, 1.73eV

Band gap for CuAB₂ (eV)

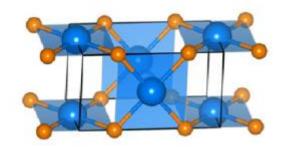
A B	S	Se
Al	3.62 (3.49)	2.91 (2.67)
Ga	2.83 (2.43)	1.69 (1.68)
In	1.22 _(u=0.22) 1.48 _(u=0.23) (1.53)	(1.04)

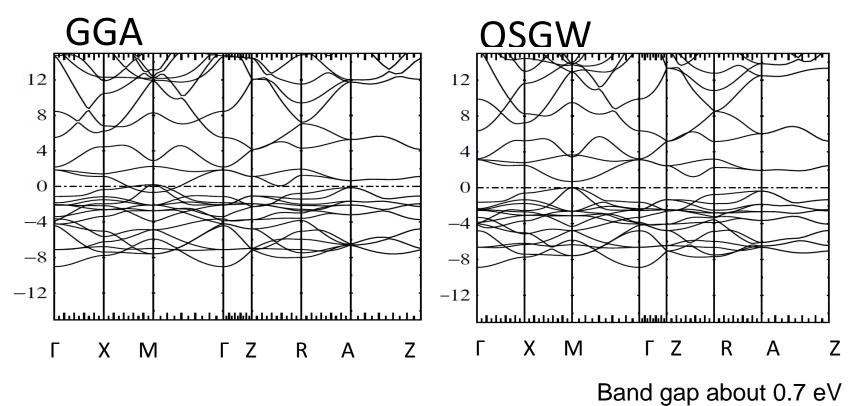


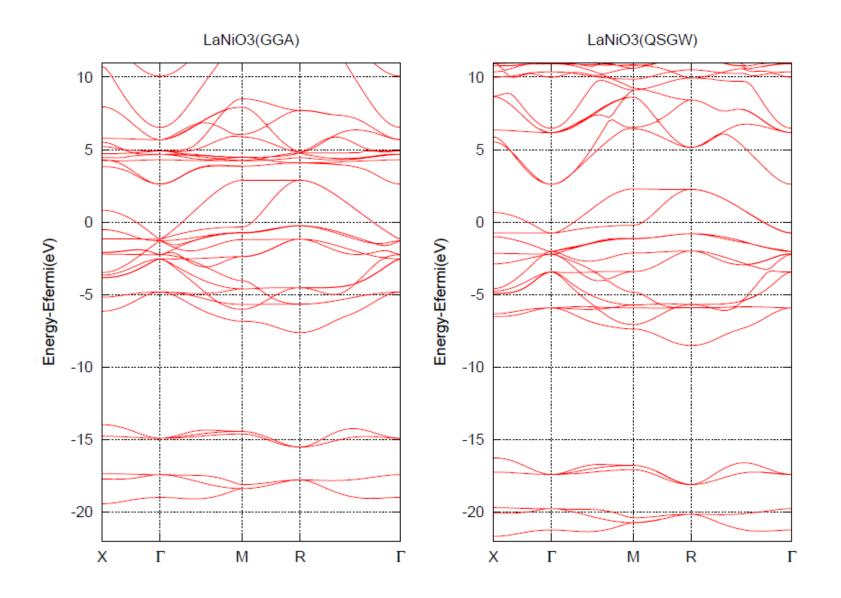
Experiments in (...)

J. Vidal, PRL 104, 056401 (2010)

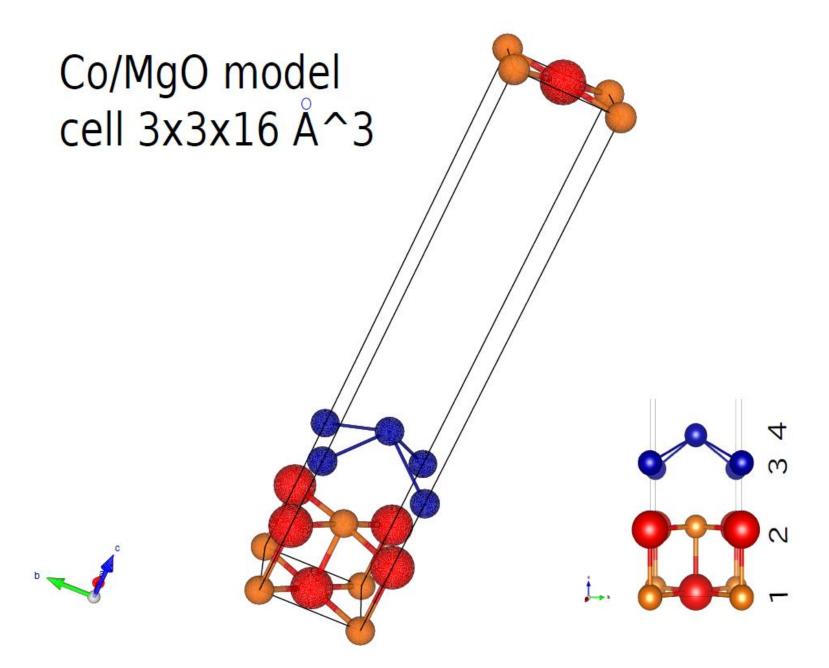
PdO(tetragonal, four atoms per cell 3x3x2 calculation, 1hour per iteration)

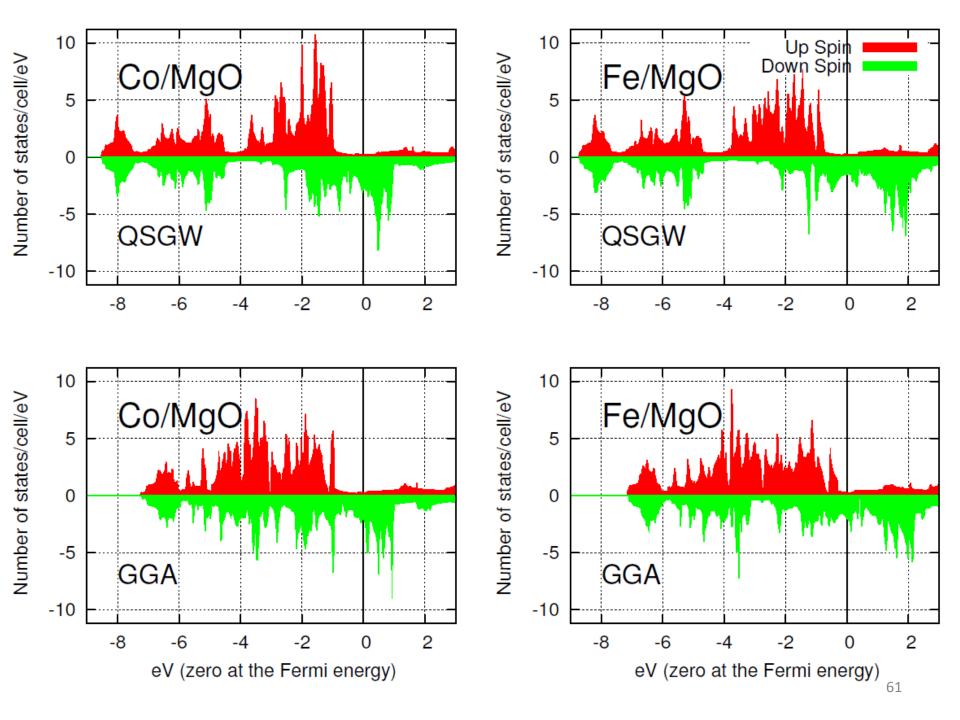






LaNiO3





summary

- Mean field theory and excitation energy LDA, Hartree-Fock and QSGW Janak theorem, finite vs.infinite system
- RPA total energy QSGW method
- Some Results
 Band gap, GaAs and Na, Cu
 NiO, Spin Wave, ZnO, LaMnO3,
 InAs (impact ionization), YH3, CuGaSe2,PdO
 - How QSGW works for atoms and molecules?
 See F.Bruneval J.Chem.Phys 136,194107(2012)