

## Excerpt of the proof of Theorem 5.8

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We recall the definition of  $\mathcal{G}(\mathcal{D}, \mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3)$  as given in Theorem 5.8 of [1] by cases: first on the last applied rule in the compliance derivation  $\mathcal{D}$  and then on the shapes of the global types  $\mathbf{G}_1$ ,  $\mathbf{G}_2$  and  $\mathbf{G}_3$ .

Note that the  $\mathbf{h}$ ,  $\mathbf{v}$  and  $\mathbf{w}$  in the definition below are the names of the participants acting as interfaces in the composition dealt with in the proof of Theorem 5.8. Such names have to be explicitly provided as extra arguments in any actual implementation of the function  $\mathcal{G}$ .

$$\text{Case } \mathcal{D} = \begin{array}{c} \dots \\ [\text{COMP-0}] \\ \dots \end{array} \quad \mathcal{G}(\mathcal{D}, \mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3) = \mathbf{G}_1$$

$$\text{Case } \mathcal{D} = \begin{array}{c} \mathcal{D}_i \quad \forall i \in I \\ [\text{COMP-O/I-L}] \\ \dots \end{array}$$

$$\mathcal{G}(\mathcal{D}, \mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3) = \begin{cases} \mathbf{q} \rightarrow \mathbf{v} : \{\lambda_i. \widehat{\mathbf{G}}_i\}_{i \in I'} & \begin{array}{l} \text{if } \mathbf{G}_1 = \mathbf{h} \rightarrow \mathbf{p} : \{\lambda_j. \mathbf{G}'_j\}_{j \in J} \quad I \subseteq J \\ \mathbf{G}_2 = \mathbf{q} \rightarrow \mathbf{v} : \{\lambda_i. \mathbf{G}''_i\}_{i \in I'} \quad I' \subseteq I \\ \text{where } \widehat{\mathbf{G}}_i = \mathbf{v} \rightarrow \mathbf{h} : \lambda_i. \mathbf{h} \rightarrow \mathbf{p} : \lambda_i. \mathcal{G}(\mathcal{D}_i, \mathbf{G}'_i, \mathbf{G}''_i, \mathbf{G}_3) \end{array} \\ \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathcal{G}(\mathcal{D}, \mathbf{G}'_l, \mathbf{G}_2, \mathbf{G}_3)\}_{l \in L} & \text{if } \mathbf{G}_1 = \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathbf{G}'_l\}_{l \in L} \quad \mathbf{h} \notin \{\mathbf{t}, \mathbf{u}\} \\ \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathcal{G}(\mathcal{D}, \mathbf{G}_1, \mathbf{G}''_l, \mathbf{G}_3)\}_{l \in L} & \begin{array}{l} \text{if } \mathbf{G}_1 = \mathbf{h} \rightarrow \mathbf{p} : \{\lambda_j. \mathbf{G}'_j\}_{j \in J} \\ \mathbf{G}_2 = \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathbf{G}''_l\}_{l \in L} \quad \mathbf{v} \notin \{\mathbf{t}, \mathbf{u}\} \end{array} \end{cases}$$

$$\text{Case } \mathcal{D} = \begin{array}{c} \mathcal{D}_i \quad \forall i \in I \\ [\text{COMP-I/O-L}] \\ \dots \end{array}$$

$$\mathcal{G}(\mathcal{D}, \mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3) = \begin{cases} \mathbf{p} \rightarrow \mathbf{h} : \{\lambda_i. \widehat{\mathbf{G}}_i\}_{i \in I'} & \begin{array}{l} \text{if } \mathbf{G}_1 = \mathbf{p} \rightarrow \mathbf{h} : \{\lambda_j. \mathbf{G}'_j\}_{j \in J} \quad I \subseteq J \\ \mathbf{G}_2 = \mathbf{v} \rightarrow \mathbf{q} : \{\lambda_i. \mathbf{G}''_i\}_{i \in I'} \quad I' \subseteq I \\ \text{where } \widehat{\mathbf{G}}_i = \mathbf{h} \rightarrow \mathbf{v} : \lambda_i. \mathbf{v} \rightarrow \mathbf{q} : \lambda_i. \mathcal{G}(\mathcal{D}_i, \mathbf{G}'_i, \mathbf{G}''_i, \mathbf{G}_3) \end{array} \\ \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathcal{G}(\mathcal{D}, \mathbf{G}'_l, \mathbf{G}_2, \mathbf{G}_3)\}_{l \in L} & \text{if } \mathbf{G}_1 = \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathbf{G}'_l\}_{l \in L} \quad \mathbf{h} \notin \{\mathbf{t}, \mathbf{u}\} \\ \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathcal{G}(\mathcal{D}, \mathbf{G}_1, \mathbf{G}''_l, \mathbf{G}_3)\}_{l \in L} & \begin{array}{l} \text{if } \mathbf{G}_1 = \mathbf{h} \rightarrow \mathbf{p} : \{\lambda_j. \mathbf{G}'_j\}_{j \in J} \\ \mathbf{G}_2 = \mathbf{t} \rightarrow \mathbf{u} : \{\lambda_l. \mathbf{G}''_l\}_{l \in L} \quad \mathbf{v} \notin \{\mathbf{t}, \mathbf{u}\} \end{array} \end{cases}$$

$$\text{Case } \mathcal{D} = \begin{array}{c} \mathcal{D}_i \quad \forall i \in I \\ [\text{COMP-O/I-R}] \\ \dots \end{array}$$

$$\mathcal{G}(\mathcal{D}, G_1, G_2, G_3) = \begin{cases} q \rightarrow w : \{\lambda_i.\widehat{G}_i\}_{i \in I'} & \text{if } G_1 = h \rightarrow p : \{\lambda_j.G'_j\}_{j \in J} \quad I \subseteq J \\ & G_3 = q \rightarrow v : \{\lambda_i.G'''_i\}_{i \in I'} \quad I' \subseteq I \\ & \text{where } \widehat{G}_i = w \rightarrow h : \lambda_i.h \rightarrow p : \lambda_i.\mathcal{G}(\mathcal{D}_i, G'_i, G''_i, G_3) \\ t \rightarrow u : \{\lambda_l.\mathcal{G}(\mathcal{D}, G'_l, G_2, G_3)\}_{l \in L} & \text{if } G_1 = t \rightarrow u : \{\lambda_l.G'_l\}_{l \in L} \quad h \notin \{t, u\} \\ t \rightarrow u : \{\lambda_l.\mathcal{G}(\mathcal{D}, G_1, G''_l, G_3)\}_{l \in L} & \text{if } G_1 = h \rightarrow p : \{\lambda_j.G'_j\}_{j \in J} \\ & G_2 = t \rightarrow u : \{\lambda_l.G''_l\}_{l \in L} \quad v \notin \{t, u\} \end{cases}$$

$$\text{Case } \mathcal{D} = \begin{array}{c} \mathcal{D}_i \quad \forall i \in I \\ \text{---} \\ \text{---} \\ \dots \end{array} \quad [\text{COMP-I/O-R}]$$

$$\mathcal{G}(\mathcal{D}, G_1, G_2, G_3) = \begin{cases} p \rightarrow h : \{\lambda_i.\widehat{G}_i\}_{i \in I'} & \text{if } G_1 = p \rightarrow h : \{\lambda_j.G'_j\}_{j \in J} \quad I \subseteq J \\ & G_3 = v \rightarrow q : \{\lambda_i.G'''_i\}_{i \in I'} \quad I' \subseteq I \\ & \text{where } \widehat{G}_i = h \rightarrow v : \lambda_i.v \rightarrow q : \lambda_i.\mathcal{G}(\mathcal{D}_i, G'_i, G_2, G''') \\ t \rightarrow u : \{\lambda_l.\mathcal{G}(\mathcal{D}, G'_l, G_2, G_3)\}_{l \in L} & \text{if } G_1 = t \rightarrow u : \{\lambda_l.G'_l\}_{l \in L} \quad h \notin \{t, u\} \\ t \rightarrow u : \{\lambda_l.\mathcal{G}(\mathcal{D}, G_1, G_2, G''')\}_{l \in L} & \text{if } G_1 = h \rightarrow p : \{\lambda_j.G'_j\}_{j \in J} \\ & G_3 = t \rightarrow u : \{\lambda_l.G'''_l\}_{l \in L} \quad w \notin \{t, u\} \end{cases}$$

$$\text{Case } \mathcal{D} = \begin{array}{c} \mathcal{D}_i \quad \forall i \in I \\ \text{---} \\ \text{---} \\ \dots \end{array} \quad [\text{COMP-O-A}]$$

$$\mathcal{G}(\mathcal{D}, G_1, G_2, G_3) = \begin{cases} h \rightarrow p : \{\lambda_i.\mathcal{G}(\mathcal{D}_i, G'_i, G_2, G_3)\}_{i \in I} & \text{if } G_1 = h \rightarrow p : \{\lambda_i.G'_i\}_{i \in I} \\ t \rightarrow u : \{\lambda_j.\mathcal{G}(\mathcal{D}, G'_j, G_2, G_3)\}_{j \in J} & \text{if } G_1 = t \rightarrow u : \{\lambda_j.G'_j\}_{j \in J} \quad h \notin \{t, u\} \end{cases}$$

$$\text{Case } \mathcal{D} = \begin{array}{c} \mathcal{D}_i \quad \forall i \in I \\ \text{---} \\ \text{---} \\ \dots \end{array} \quad [\text{COMP-O-A}]$$

$$\mathcal{G}(\mathcal{D}, G_1, G_2, G_3) = \begin{cases} p \rightarrow h : \{\lambda_i.\mathcal{G}(\mathcal{D}_i, G'_i, G_2, G_3)\}_{i \in I} & \text{if } G_1 = p \rightarrow h : \{\lambda_i.G'_i\}_{i \in I} \\ t \rightarrow u : \{\lambda_j.\mathcal{G}(\mathcal{D}, G'_j, G_2, G_3)\}_{j \in J} & \text{if } G_1 = t \rightarrow u : \{\lambda_j.G'_j\}_{j \in J} \quad h \notin \{t, u\} \end{cases}$$

## References

- [1] Franco Barbanera and Mariangiola Dezani-Ciancaglini. Partial typing for open compliance in multiparty sessions. submitted, 2024.