FFT y sus Aplicaciones

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UBA
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Traning Camp 2024

TC ARG

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Diamond

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accenture

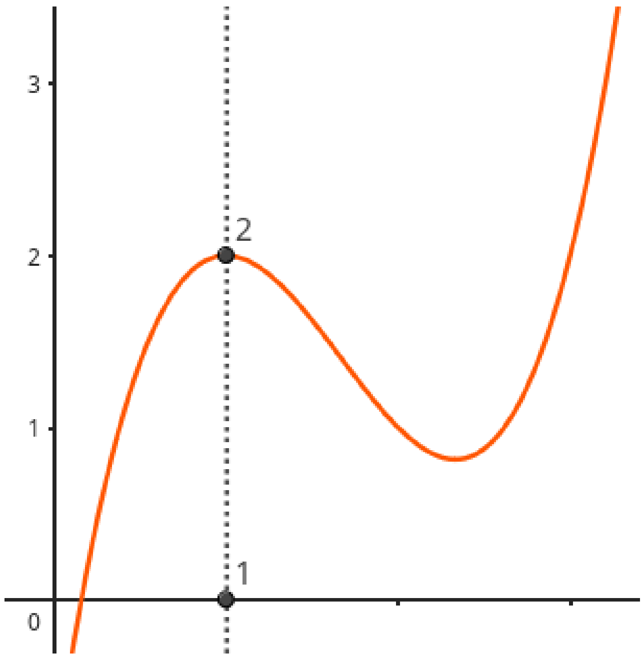
Polinomios

$$\mathbb{R}[X] \quad \mathbb{C}[X] \quad \mathbb{Z}_p[X]$$

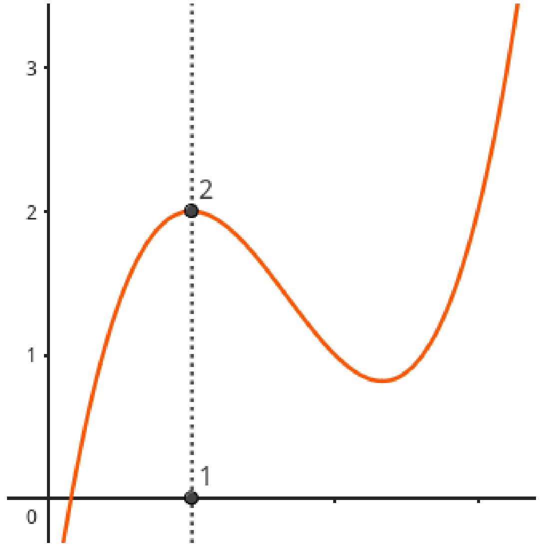
$$P(X) = a_0 + a_1 X + a_2 X^2 + ... + a_n X^n$$

$$\mathrm{ev}_1(P) = P(1)$$

$$\mathrm{ev}_1 : \mathbb{R}[X] \to \mathbb{R}$$

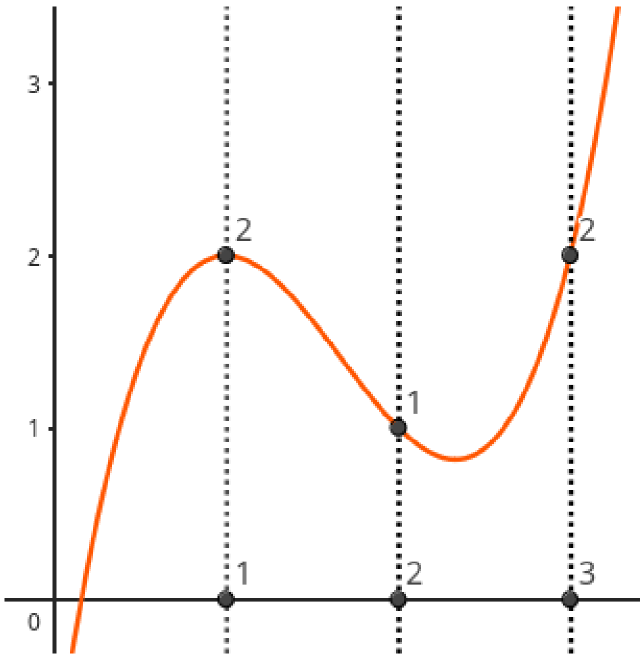


$$\mathrm{ev}_1\left(x^3 - 5x^2 + 7x - 1\right) = 2$$

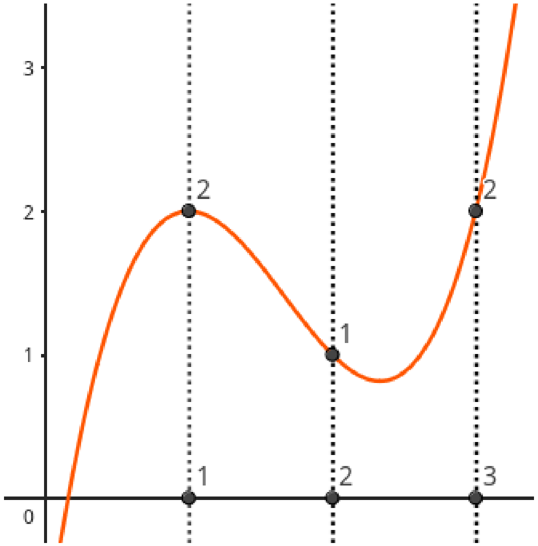


$$\mathrm{ev}_{1,2,3}(P) = \begin{pmatrix} P(1) \\ P(2) \\ P(3) \end{pmatrix}$$

$$\mathrm{ev}_{1,2,3} : \mathbb{R}[X] \to \mathbb{R}^3$$



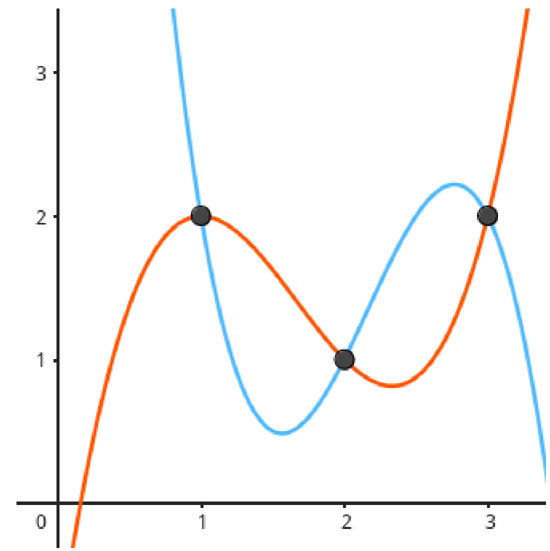
$$\mathrm{ev}_{1,2,3}(x^3 - 5x^2 + 7x - 1) = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$



$$\mathrm{ev}_{1,2,3}({\color{orangered}\bullet}) = \mathrm{ev}_{1,2,3}({\color{cornflowerblue}\bullet}) = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$${\color{orangered}\bullet} = {\color{orangered} x^3 - 5x^2 + 7x - 1}$$

$${\color{cornflowerblue}\bullet} = {\color{cornflowerblue} -2x^3 + 13x^2 - 26x + 17}$$



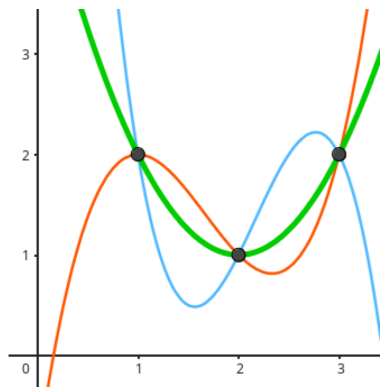
Polinomios | Evaluación Múltiple

$$\mathrm{ev}_{1,2,3}(\textcolor{orangered}{\bullet}) = \mathrm{ev}_{1,2,3}(\textcolor{skyblue}{\bullet}) = \mathrm{ev}_{1,2,3}(\textcolor{green}{\bullet}) = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\textcolor{orangered}{\bullet} = x^3 - 5x^2 + 7x - 1$$

$$\textcolor{skyblue}{\bullet} = -2x^3 + 13x^2 - 26x + 17$$

$$\textcolor{green}{\bullet} = x^2 - 4x + 5$$

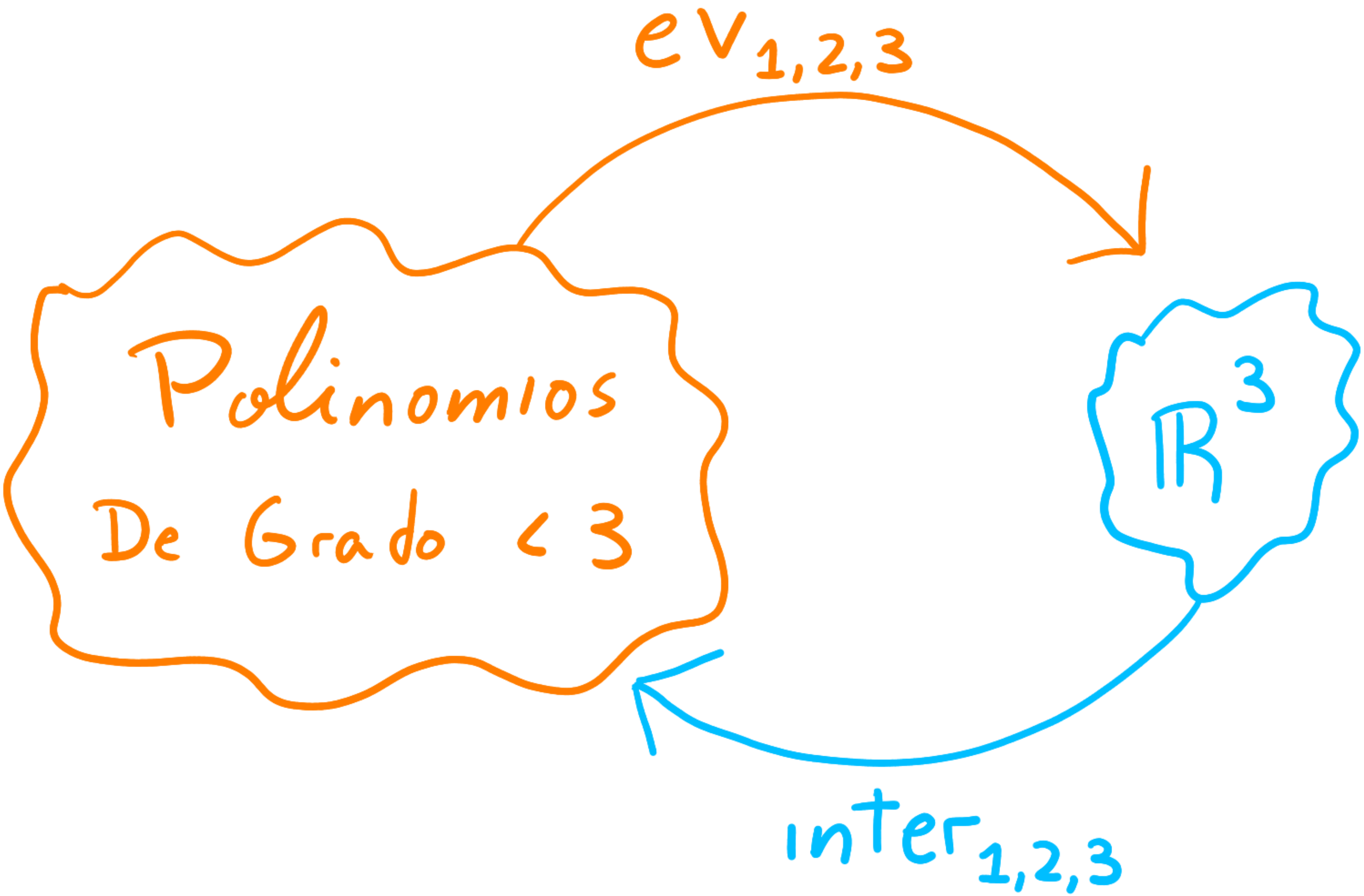


$$\mathrm{ev}_{1,2,3}(x^2 - 4x + 5) = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\mathrm{inter}_{1,2,3}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = x^2 - 4x + 5$$

$$\mathrm{ev}_{1,2,3} : \mathbb{R}[X]_{\leq 2} \to \mathbb{R}^3$$

$$\mathrm{inter}_{1,2,3} : \mathbb{R}^3 \to \mathbb{R}[X]_{\leq 2}$$


$ev_{1,2,3}$
Polinomios
De Grado < 3
$\mathbb{R}^3$
$inter_{1,2,3}$

$$\mathrm{ev}_{r,r^2,r^3,\ldots,r^k} \qquad \mathrm{inter}_{r,r^2,r^3,\ldots,r^k}$$

k **potencia de dos**

r **raíz primitiva** k**-ésima de la unidad**

$$r^k = 1 \quad r^{\frac{k}{2}} \neq 1$$

FFT

$\mathbb{C}$ $\mathbb{Z}_p$

$$p = 998244353$$

$$p - 1 = 2^{23} \cdot 119$$

$$(a_0 + a_1 x + a_2 x^2)(b_0 + b_1 x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

$$(a_0 + a_1 x + a_2 x^2)(b_0 + b_1 x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

$$0 \leq c_i < p$$

Teorema Chino del Resto

$$(a_0 + a_1 x + a_2 x^2)(b_0 + b_1 x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

$$0 \leq c_i < p_1 p_2$$

Dados dos conjuntos A, B, encontrar todos los t que se pueden escribir como $t = a + b$

Dados dos conjuntos A, B, encontrar todos los t que se pueden escribir como $t = a + b$

$$A = \{0, 1, 3\} \quad B = \{2, 5\}$$

$$\left(x^0 + x^1 + x^3\right)\left(x^2 + x^5\right)$$

$$= x^{0+2} + x^{0+5} + x^{1+2} + x^{1+5} + x^{3+2} + x^{3+5}$$

$$= x^2 + x^3 + 2x^5 + x^6 + x^8$$

Dados dos conjuntos A, B, C, encontrar todos los t que se pueden escribir como $t = a + b + c$

$$A = \{0, 1, 3\} \quad B = \{2, 5\} \quad C = \{7, 8, 9\}$$

$$\left(x^0 + x^1 + x^3\right)\left(x^2 + x^5\right)\left(x^7 + x^8 + x^9\right)$$

Dado un conjunto S, encontrar todos los t que se pueden escribir como suma de elementos de S

$$S = \{1, 3, 5\}$$

$$(x^0 + x^1)(x^0 + x^3)(x^0 + x^5)$$

$$(1 + x^1)(1 + x^3)(1 + x^5)$$

$$
\begin{aligned}
&(1+x)(1+x)(1+x)(1+x)(1+x) \\
&= 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5 \\
&= \binom{0}{5} + \binom{1}{5}x + \binom{2}{5}x^2 + \binom{3}{5}x^3 + \binom{4}{5}x^4 + \binom{5}{5}x^5
\end{aligned}
$$

$$(1 + x)^k$$

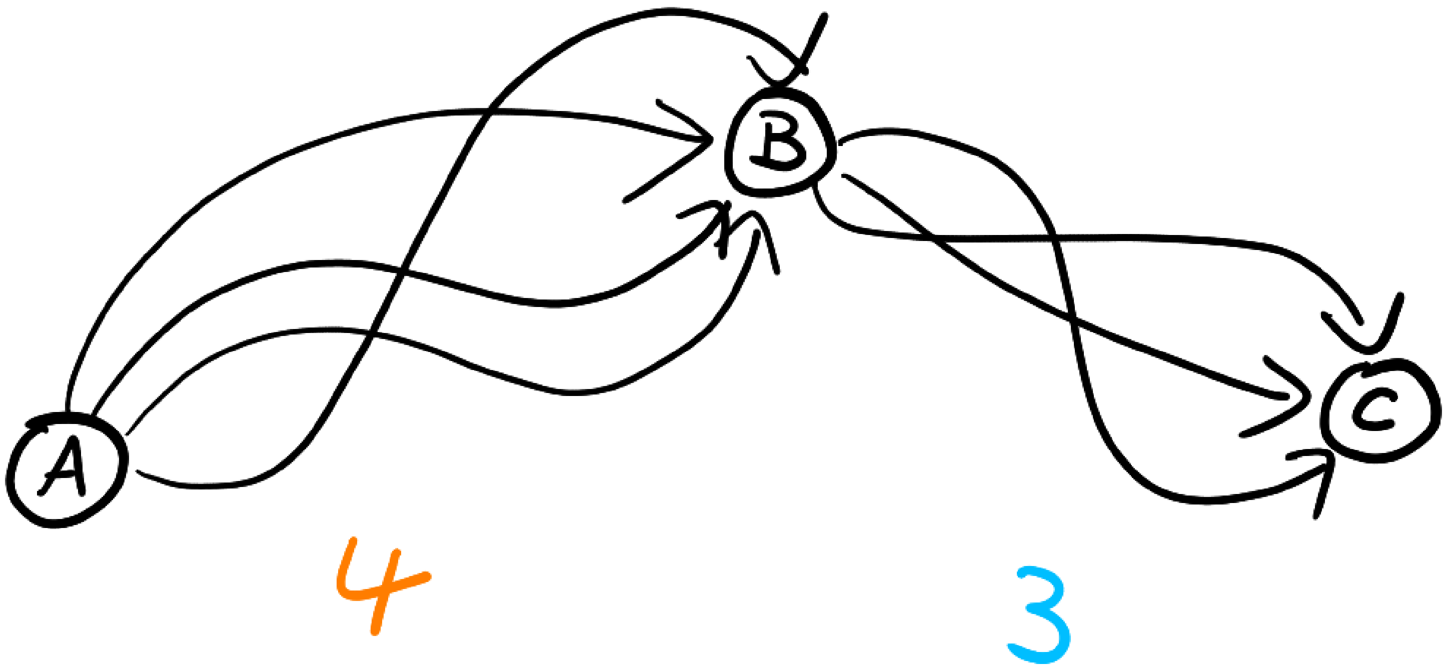
$\times$	1	$3x$	$2x^2$
2	2	$6x$	$4x^2$
$5x$	$5x$	$15x^2$	$10x^3$
$2x^2$	$2x^2$	$6x^3$	$4x^4$

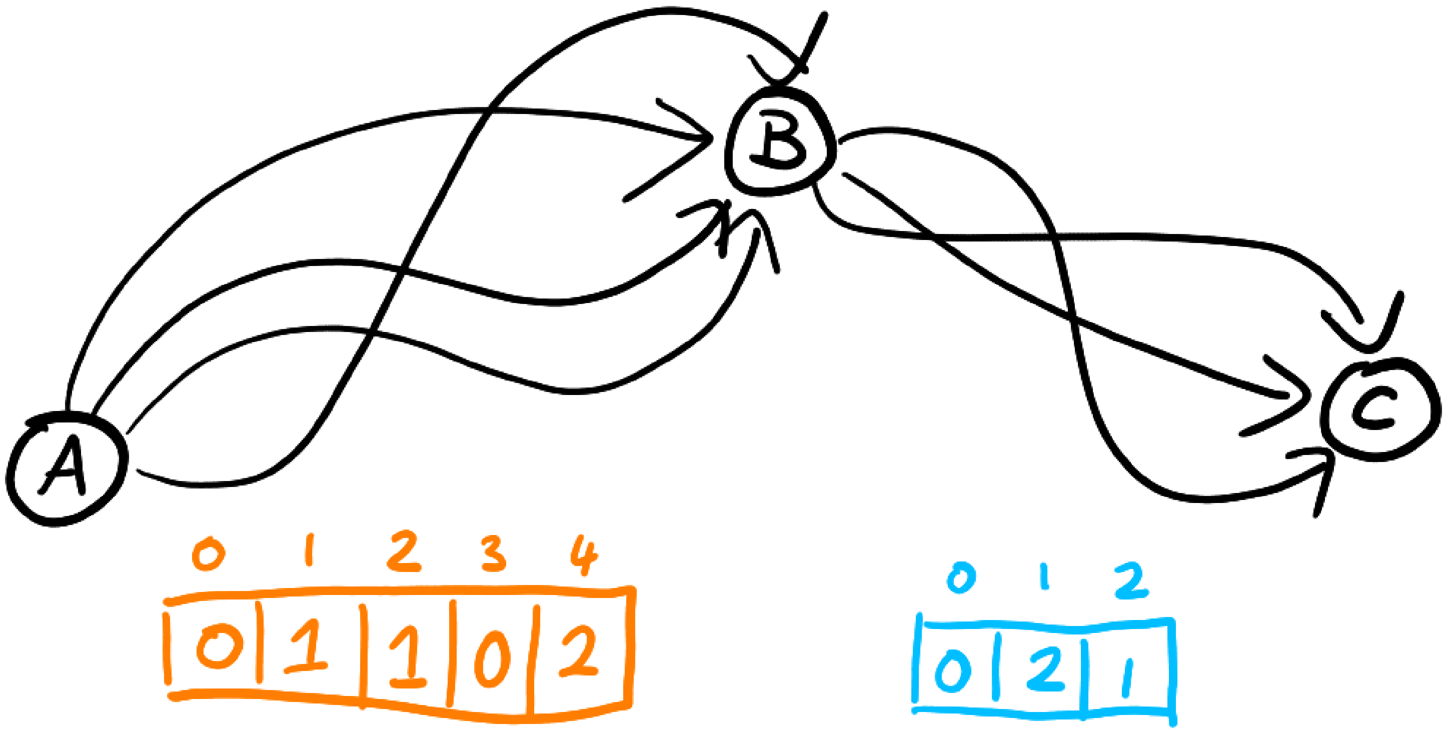
Usos | Convolución

$\times$	1	3	2
2	2	6	4
5	5	15	10
2	2	6	4

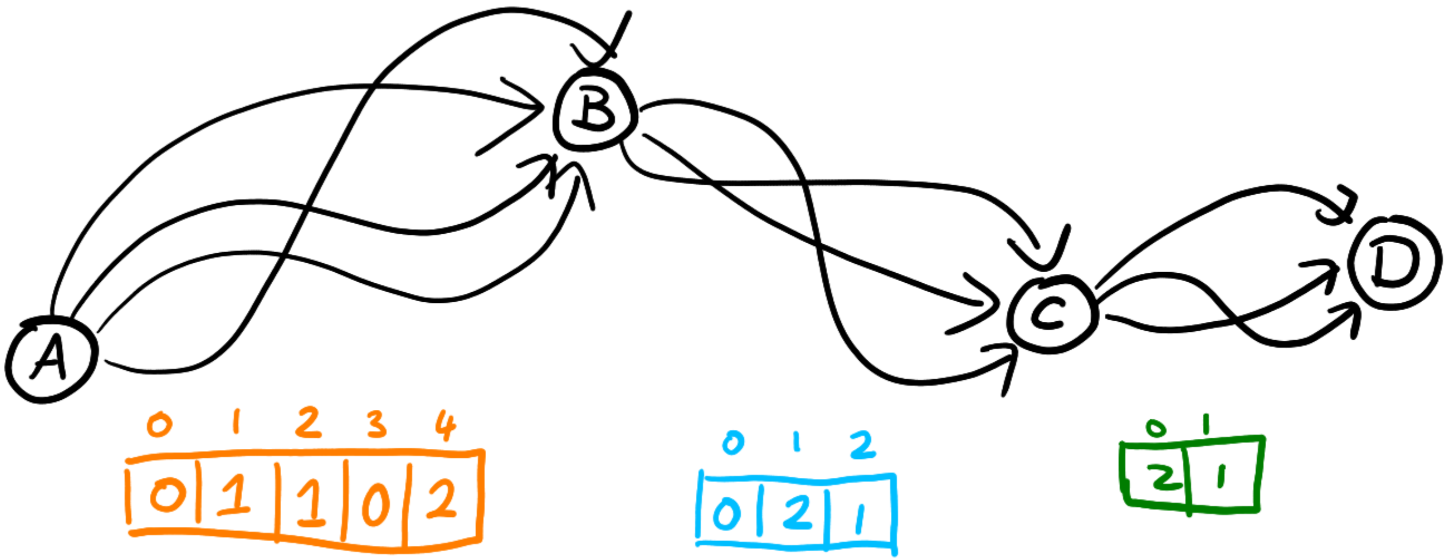
$$\text{conv} = [\bullet, \bullet, \bullet, \bullet, \bullet]$$

$$\text{conv}[t] = \sum_{i+j=t} A[i] \cdot B[j]$$


A
B
C
4
3

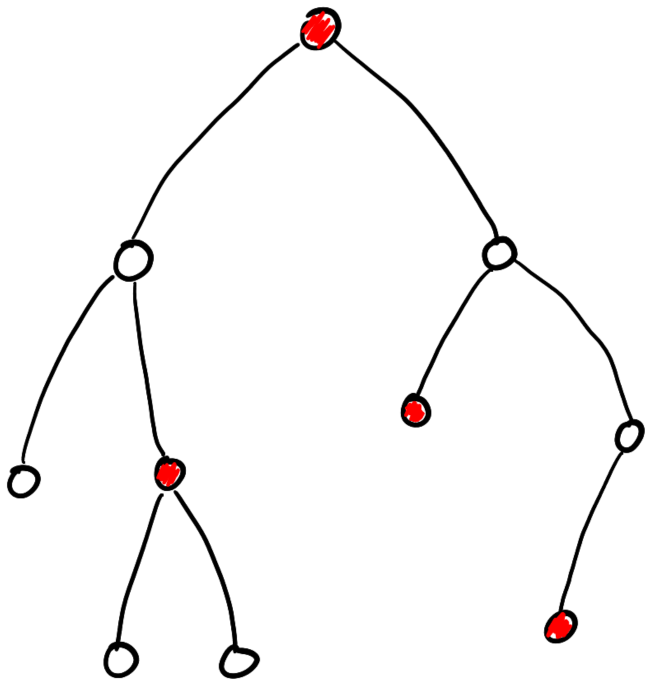


$$AC[t] = \sum_{i+j=t} AB[i] \cdot BC[j]$$


A
B
C
D
0 1 2 3 4
0 1 1 0 2
0 1 2
0 2 1
0 1
2 1

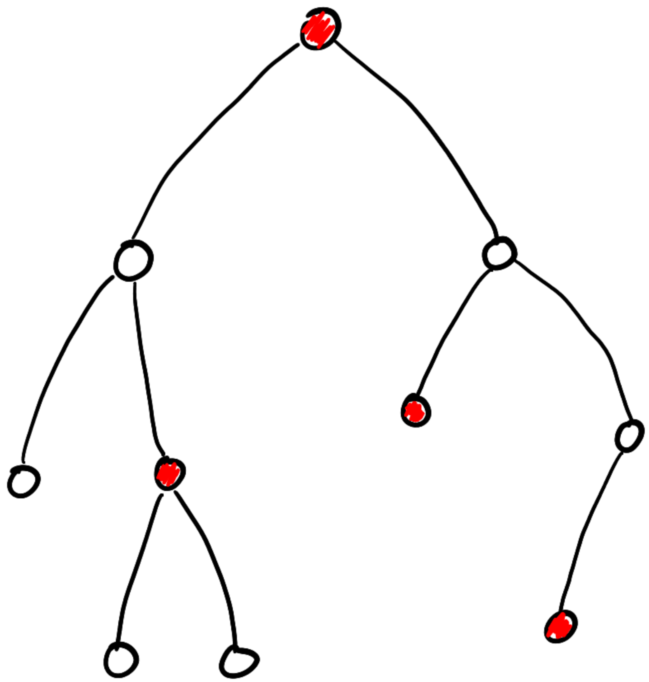
$$\text{con}[i] = \prod_{h \in \text{ hijos}(i)} \text{sin}[h]$$

$$\text{sin}[i] = \prod_{h \in \text{ hijos}(i)} \text{con}[h] + \text{sin}[h]$$



$$\mathrm{con}[i] = \mathrm{shift}\left(\underset{h \in \mathrm{\ hijos}(i)}{\mathrm{conv}} \ \mathrm{sin}[h]\right)$$

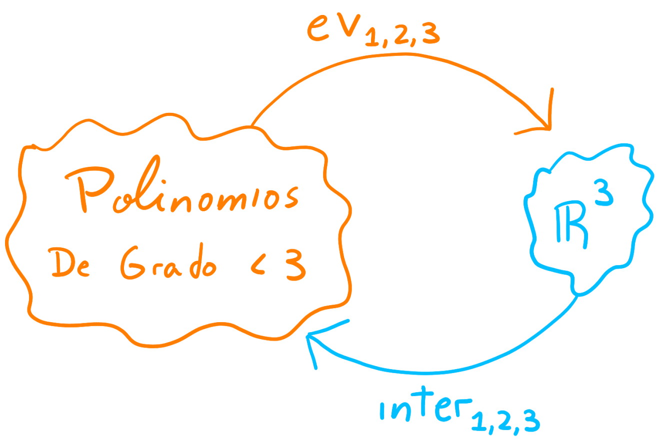
$$\mathrm{sin}[i] = \underset{h \in \mathrm{\ hijos}(j)}{\mathrm{conv}} \ \mathrm{con}[h] + \mathrm{sin}[h]$$



Ejemplo | Independent Sets

$$\mathrm{con}[i] = \mathrm{shift}\left(\underset{h \in \mathrm{hijos}(i)}{\mathrm{conv}} \ \mathrm{sin}[h]\right)$$

$$\mathrm{sin}[i] = \underset{h \in \mathrm{hijos}(j)}{\mathrm{conv}} \ \mathrm{con}[h] + \mathrm{sin}[h]$$



Truquito Mágico

Calcular $\mathrm{inter}_{0,1,2,\ldots,n}$ **en** $O(n^2)$**.**

Truquito Mágico

Dado un polinomio $P(X)$,

$$\Delta P(X) = P(X+1) - P(X)$$

$$\Delta\left(3X^2 - 1\right) = 3(X+1)^2 - 1 - \left(3X^2 - 1\right) = 6X + 3$$

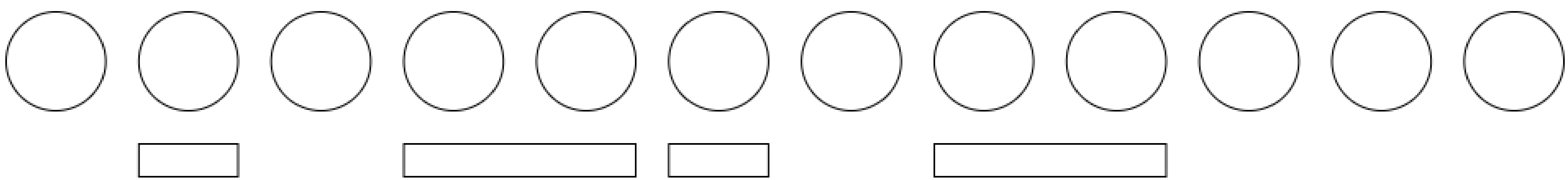
Truquito Mágico

	0	1	2	3	4
P	1	4	3	5	6
ΔP	3	-1	2	1	
ΔΔP	-4	3	-1		
ΔΔΔP	7	-4			
ΔΔΔΔP	-11				

Truquito Mágico

	0	1	2	3	4
P	1	-4	6	-4	1
ΔP	-1	3	-3	1	
ΔΔP	1	-2	1		
ΔΔΔP	-1	1			
ΔΔΔΔP	1				

Tenés n ($n \leq 10^9$) bolitas y queres armar m ($m < 2^{15}$) conjuntos disjuntos que consisten o de una sola bolita o de dos adyacentes. Cuántas formas hay?



Codificando DPs

$$\mathrm{dp}[i][k] = \mathrm{dp}[i-1][k] + \mathrm{dp}[i-1][k-1] + \mathrm{dp}[i-2][k-1]$$

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$$\mathrm{dp}[i][k] = \mathrm{dp}[i-1][k] + \mathrm{dp}[i-1][k-1] + \mathrm{dp}[i-2][k-1]$$

$$\mathrm{dp}[i] = \mathrm{dp}[i-1] + \mathrm{shift}(\mathrm{dp}[i-1]) + \mathrm{shift}(\mathrm{dp}[i-2])$$

Codificando DPs

$$\mathrm{dp}[i][k] = \mathrm{dp}[i-1][k] + \mathrm{dp}[i-1][k-1] + \mathrm{dp}[i-2][k-1]$$

$$\mathrm{dp}[i] = \mathrm{dp}[i-1] + X\ \mathrm{dp}[i-1] + X\ \mathrm{dp}[i-2]$$

Codificando DPs

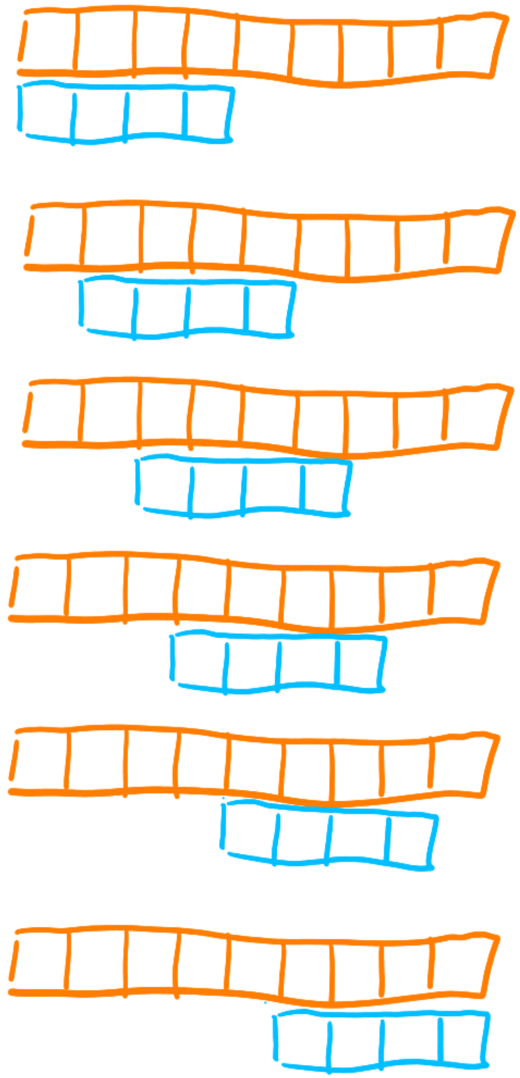
$$\mathrm{dp}[i] = (1 + X)\ \mathrm{dp}[i-1] + X\ \mathrm{dp}[i-2]$$

Codificando DPs

$$\mathrm{dp}[i] = (1 + X)\ \mathrm{dp}[i-1] + X\ \mathrm{dp}[i-2]$$

$$\begin{pmatrix} \mathrm{dp}[i] \\ \mathrm{dp}[i-1] \end{pmatrix} = \begin{pmatrix} 1+X & X \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{dp}[i-1] \\ \mathrm{dp}[i-2] \end{pmatrix}$$

Usos | Cross-correlation




0
1
1
0
1
0
1
0
0


1
0
1
1

$s = \textbf{aabaab}$

$t = \textbf{a**a}$

$$s = \textbf{aaba}$$

$$t = \textbf{a**a}$$

$$\textbf{a} \rightarrow 1 \qquad \textbf{b} \rightarrow 2 \qquad \textbf{*} \rightarrow 0$$

$$\sum_{0 \le i < 4} t[i](s[i] - t[i])^2$$

$$s = \textbf{aaba}$$

$$t = \textbf{a**a}$$

$$\textbf{a} \rightarrow 1 \qquad \textbf{b} \rightarrow 2 \qquad \textbf{*} \rightarrow 0$$

$$\sum_{0 \leq i < 4} s[i]^2 t[i] - 2s[i]t[i]^2 + t[i]^3$$

$$s = \textbf{aabaab}$$

$$t = \textbf{a**a}$$

$$\textbf{a} \rightarrow 1 \qquad \textbf{b} \rightarrow 2 \qquad \textbf{*} \rightarrow 0$$

$$\sum_{0 \le i < 4} s[i+k]^2 t[i] - 2s[i+k]t[i]^2 + t[i]^3$$

Tengo una pila de n piedras y tengo un conjunto de movidas válidas

$$V \subset \{1, 2, ..., n\}$$

Estoy en posición ganadora o perdedora?

Usos | Nim

Tengo una pila de n piedras y tengo un conjunto de movidas válidas

$$V \subset \{1, 2, ..., n\}$$

Estoy en posición ganadora o perdedora?

$$n = 8 \qquad V = \{2, 5\}$$

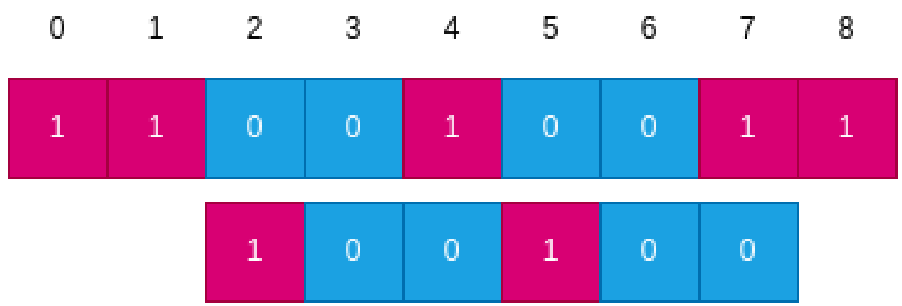
0	1	2	3	4	5	6	7	8
P	P	G	G	P	G	G	P	P

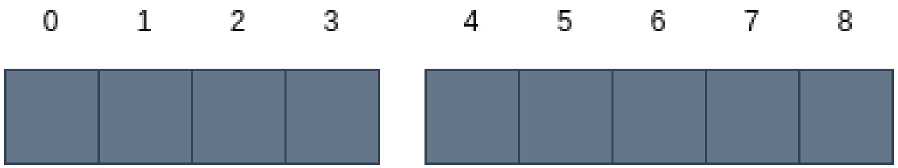
0	1	2	3	4	5	6	7	8
1	1	0	0	1	0	0	1	1

5	4	3	2	1	0
1	0	0	1	0	0

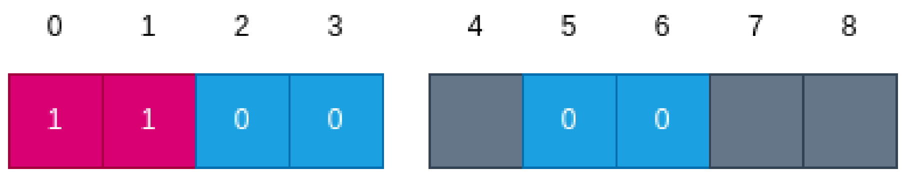
Usos | Nim

0	1	2	3	4	5	6	7	8
1	1	0	0	1	0	0	1	1
	1	0	0	1	0	0		


0 1 2 3 4 5 6 7 8
1 1 0 0 1 0 0 1 1
1 0 0 1 0 0


0
1
2
3
4
5
6
7
8

0	1	2	3	4	5	6	7	8
1	1	0	0					


0 1 2 3 4 5 6 7 8
1 1 0 0 0 0

Usos | Nim

0	1	2	3	4	5	6	7	8
1	1	0	0	1	0	0	1	1

Preguntas?

Fin

sugerencias/comentarios:

https://reedef.dev/feedback