Ingeniería Eléctrica
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS UNIVERSIDAD DE CHILE

Thesis Defense

GraphSLAM Algorithm Implementation for Solving Simultaneous Localization and Mapping

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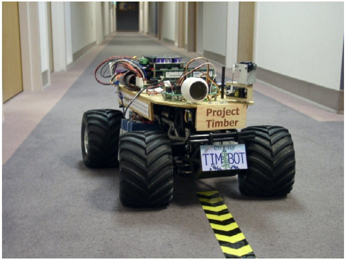
Departamento de Ingeniería Eléctrica
Facultad de Ciencias Físicas y Matemáticas
Universidad de Chile

April 18, 2016

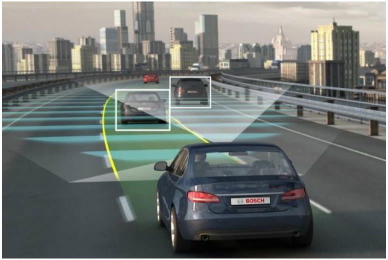
Motivation

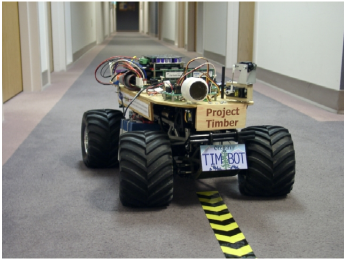
Robots Before

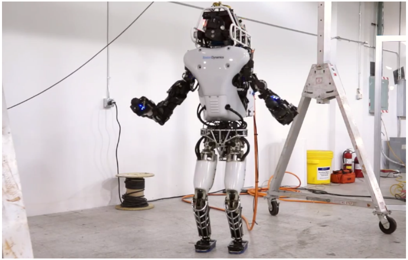


Motivation

Robots Before	Robots Today







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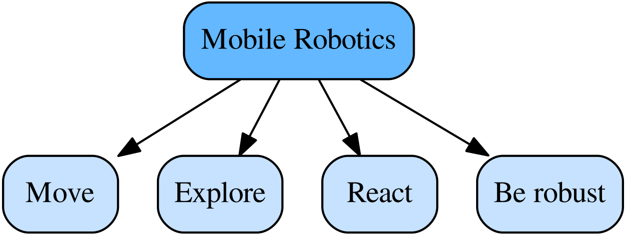

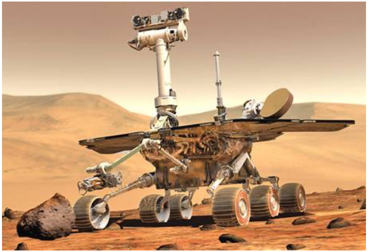

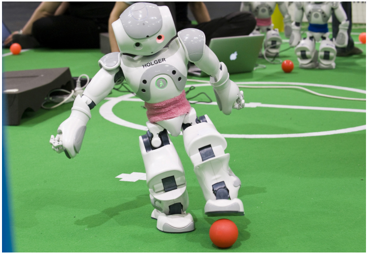

Simultaneous Localization and Mapping

SLAM

The problem were an agent must simultaneously estimate its current position (localization), and construct a map of its environment (mapping)

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The problem were an agent must simultaneously estimate its current position (localization), and construct a map of its environment (mapping)

We have (information):

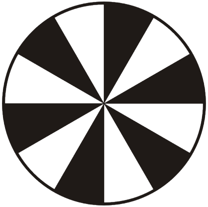
- Robot odometry
- Landmarks measurements

We need (states):

- Robot position
- Robot orientation

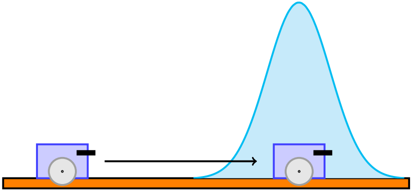
Robot pose

- Landmarks position



Motion Model:

$$\boldsymbol{x}_k = \boldsymbol{g}(\boldsymbol{u}_k, \boldsymbol{x}_{k-1}) + \boldsymbol{\delta}_k$$



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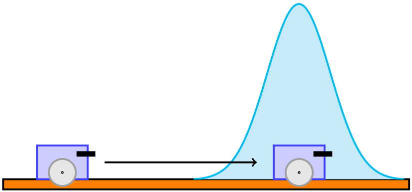
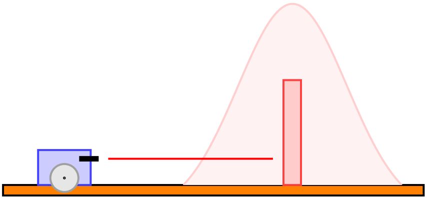


Measurement Model:

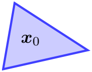
$$\boldsymbol{z}_k^i = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{m}_j, i) + \boldsymbol{\varepsilon}_k^i$$



Both contaminated with Gaussian noise.

GraphSLAM

GraphSLAM: SLAM $\Rightarrow$ graph



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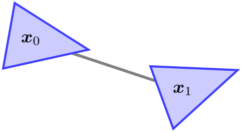

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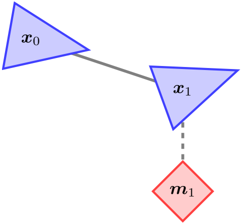

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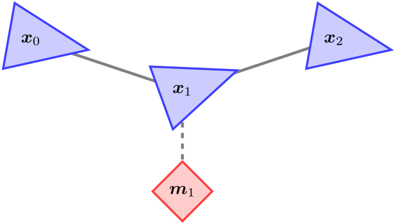

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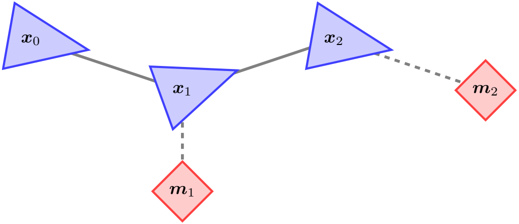

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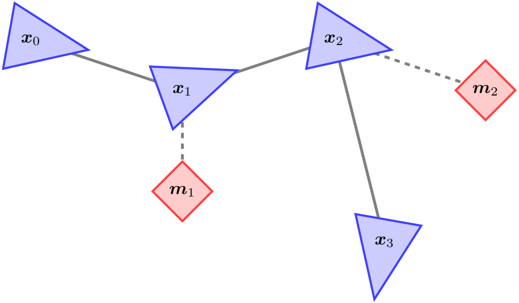

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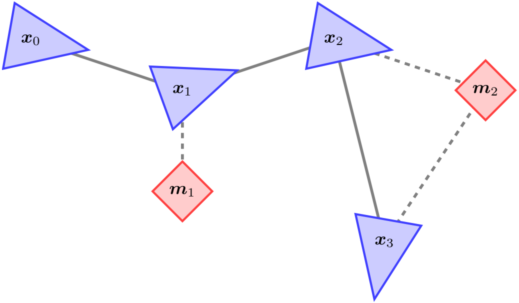

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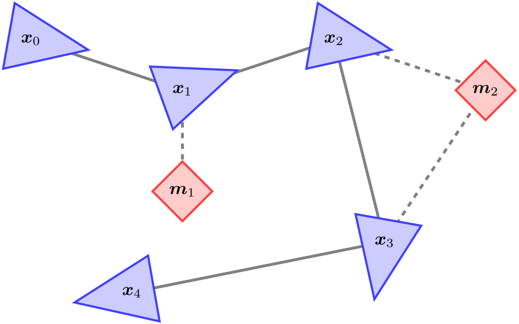

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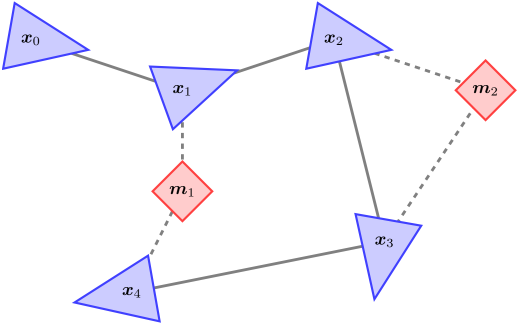

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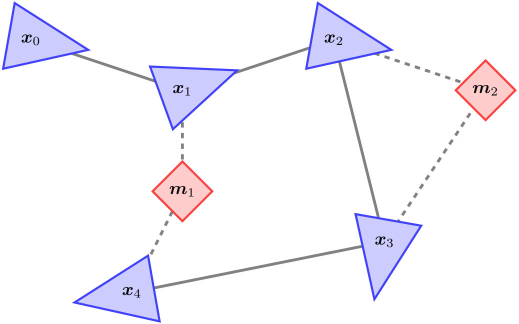

Nodes are: Robot poses, or
Landmarks positions

Edges are: Odometry, or
Measurements

Goal: Find the graph that more accurately represents the data.

Mathematical Formulation of GraphSLAM

SLAM posterior probability:

$$\boxed{p(\boldsymbol{x}_{0:k}, \boldsymbol{m} | \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k})}$$

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Mathematical Formulation of GraphSLAM

SLAM posterior probability:

pose map

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measurements odometry

Using models in $-\log(p(\boldsymbol{y}|\boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k}))$:

$$\boxed{F(\boldsymbol{y}) := \sum \boldsymbol{e}_{ij}(\boldsymbol{y})^T \boldsymbol{\Omega}_{ij} \boldsymbol{e}_{ij}(\boldsymbol{y})}$$

We need the maximum of this equation.

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measurements odometry

Using models in $-\log(p(\boldsymbol{y}|\boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k}))$:

For all edges in graph

information matrix

$$\boxed{F(\boldsymbol{y}) := \sum \boldsymbol{e}_{ij}(\boldsymbol{y})^T \boldsymbol{\Omega}_{ij} \boldsymbol{e}_{ij}(\boldsymbol{y})}$$

error function $[\boldsymbol{x}_{0:k}, \boldsymbol{m}]$

We need the maximum of this equation.

Mathematical Formulation of GraphSLAM

- $\mathrm{g}^2\mathrm{o}$: General Graph Optimization framework

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- g^{2}o: General Graph Optimization framework

- Gauss-Newton Algorithm:
  1. Taylor Approximation:

$$F(\breve{\boldsymbol{y}} + \boldsymbol{\Delta y}) = k + 2\boldsymbol{b}\boldsymbol{\Delta y} + \boldsymbol{\Delta y}^T \boldsymbol{H}\boldsymbol{\Delta y}$$

- $\boldsymbol{b}$: Total system information vector
- $\boldsymbol{H}$: Total system information matrix

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- g^{2}o: General Graph Optimization framework
- Gauss-Newton Algorithm:
  1. Taylor Approximation:

$$F(\breve{y} + \boldsymbol{\Delta y}) = k + 2\boldsymbol{b}\boldsymbol{\Delta y} + \boldsymbol{\Delta y}^T \boldsymbol{H} \boldsymbol{\Delta y}$$

  2. Minimization:

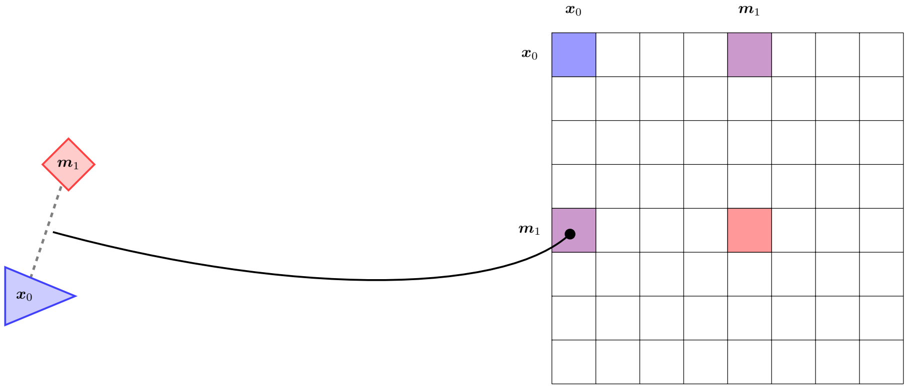
$$\boldsymbol{H}\boldsymbol{\Delta y}^* = -\boldsymbol{b}$$

  3. Update:

$$\boldsymbol{y}^* = \breve{y} + \boldsymbol{\Delta y}^*$$

- $\boldsymbol{b}$: Total system information vector
- $\boldsymbol{H}$: Total system information matrix

Information matrix $\boldsymbol{H}$ is intrinsically ***sparse***.



Structure of the Linearized System

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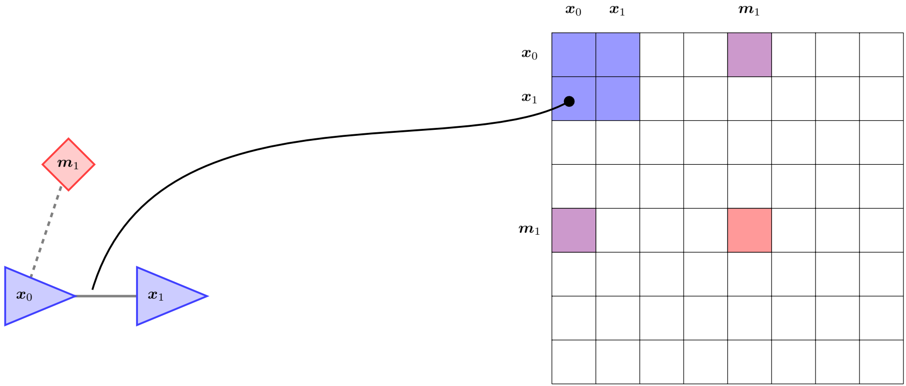

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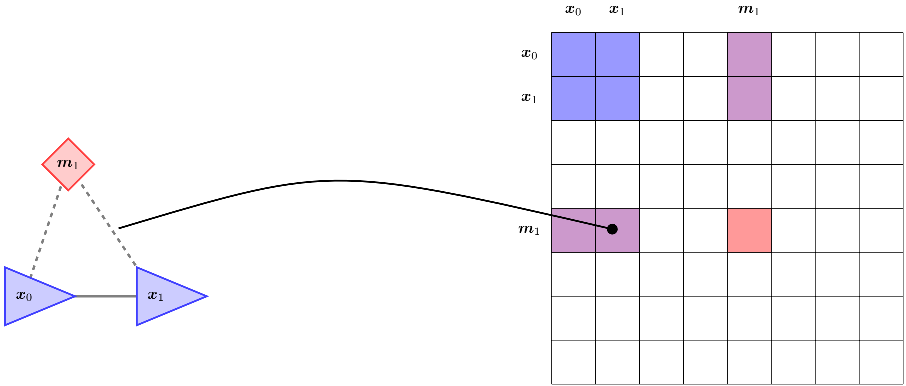

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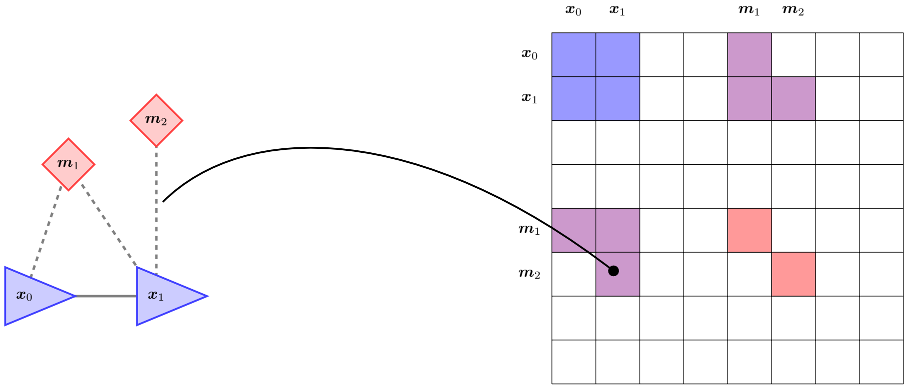

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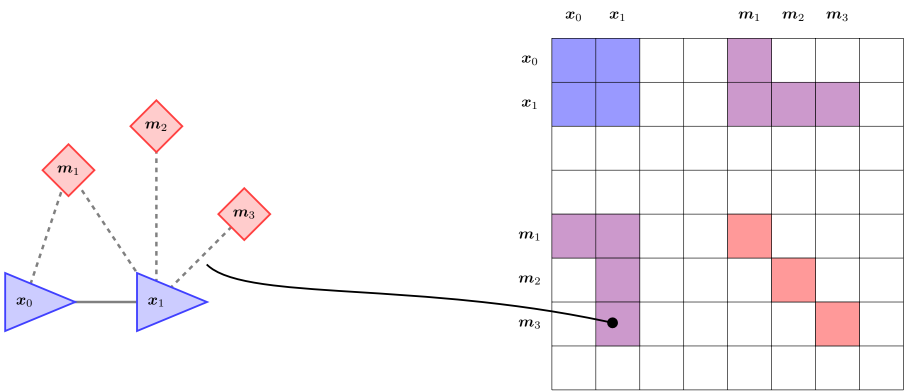

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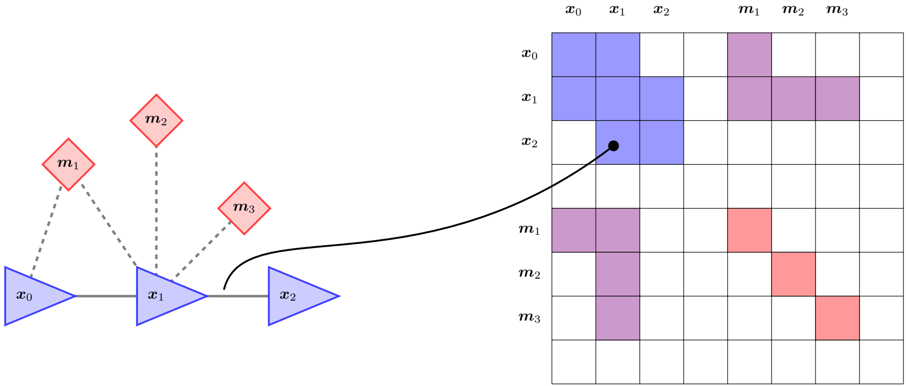

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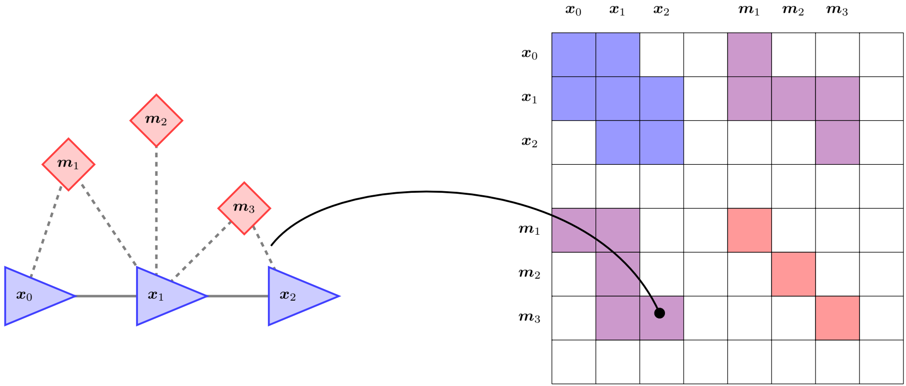

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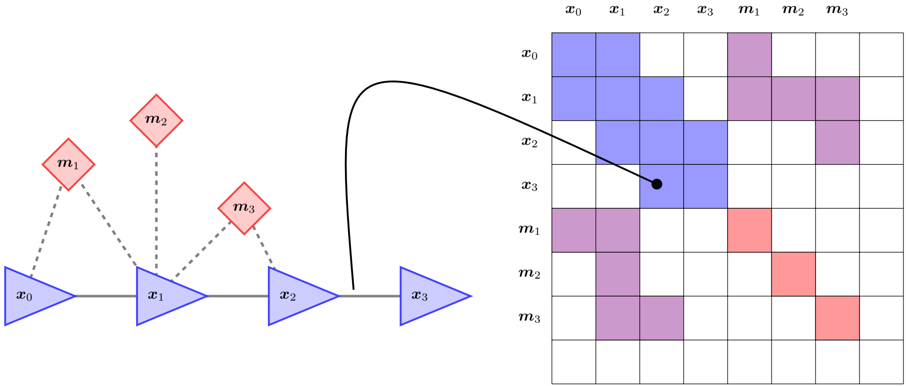

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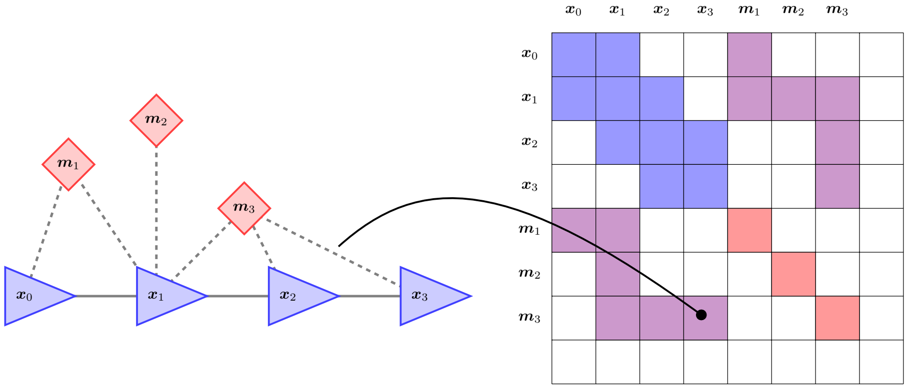

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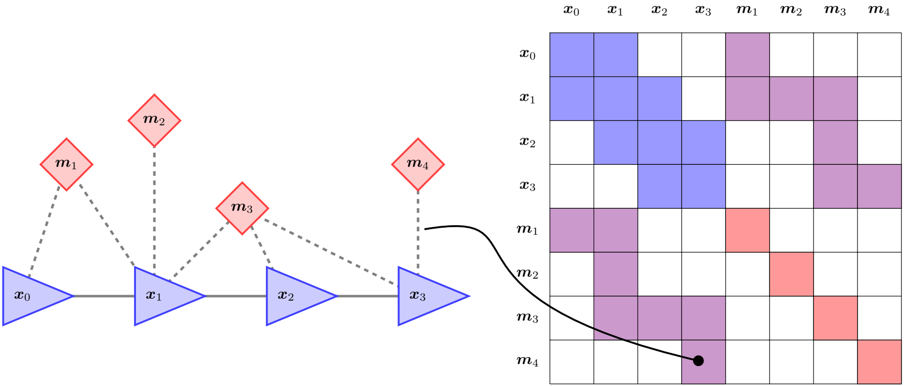
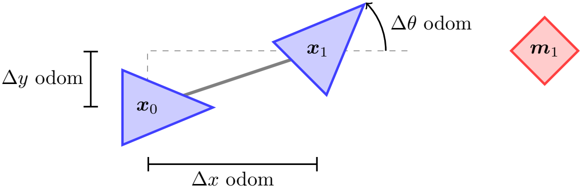


g^2o Protocol

Graph Element	Notation
Pose Node	`VERTEX_SE2 id x y a`
Landmark Node	`VERTEX_XY id x y`
Fix a Node	`FIX id`

g^2o Protocol



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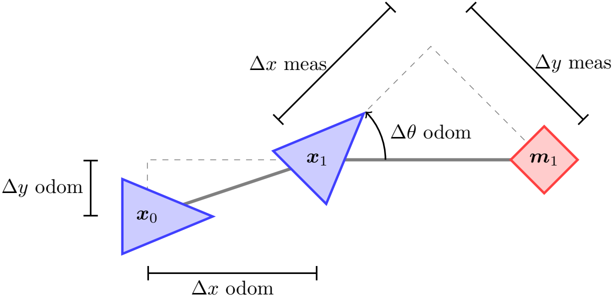
Odometry Edge	`EDGE_SE2 id1 id2 dx dy da ipxx ipxy ipxa ipyy ipya ipaa`

One must specify the information matrix of edges.

g^2o Protocol



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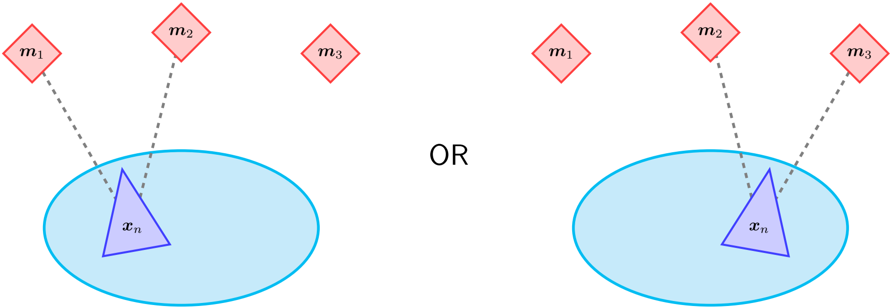
Measurement Edge	`EDGE_SE2_XY id1 id2 dx dy ilxx ilxy ilyy`

One must specify the information matrix of edges.

Correspondence Problem



- In the real world, **correspondence** between measurements and landmarks is **unknown**.
- **Data association**: assignment of measurements to landmarks.

Correspondence Test

Premise: Compute the likelihood of two landmarks being the same.

$$\pi_{i=j} := \det(2\pi\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i,j}})^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\boldsymbol{\mu}_{\boldsymbol{\Delta}_{i,j}}^{T}\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i,j}}^{-1}\boldsymbol{\mu}_{\boldsymbol{\Delta}_{i,j}}\right\}$$

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We define threshold χ.

If $\pi_{j=k} \geq \chi$:

- Merge landmarks i and j

Else:

- Leave them separated

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Problem: Matrix $\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i,j}}$ is expensive to compute.

Distant Test

- Merges landmark according to the distance.
- Avoids the computation of $\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i,j}}$ for *easy* cases.

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Maximum distance at which associations can still be made:

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Maximum distance, independent of $\sigma_{\boldsymbol{\delta}_{i,j}}$:

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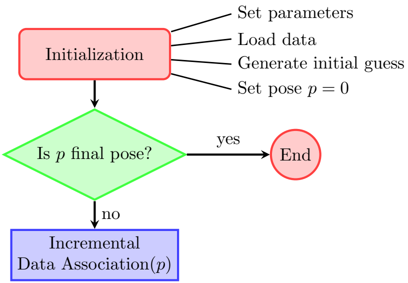
The computation saved with this method may be low.

The Final Algorithm

- Initialization
  - Set parameters
  - Load data
  - Generate initial guess
  - Set pose $p = 0$

The Final Algorithm



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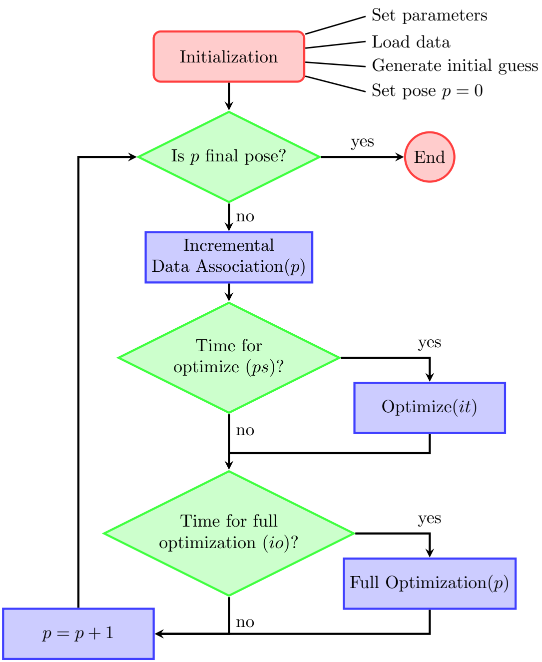


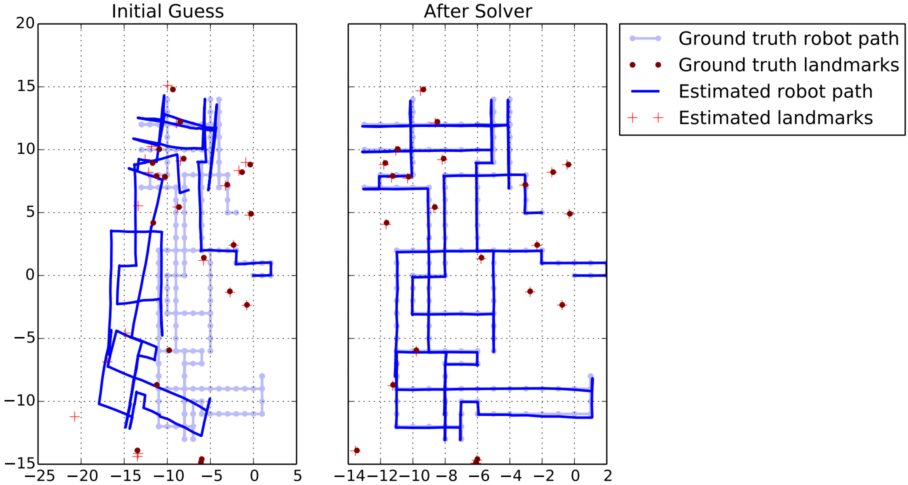
The Final Algorithm

Algorithm 1 Incremental Data Association Function

```
function IncrementalDataAssociation(p)
    for all landmarks l_p observed in p do
        for all previous poses q up to p do
            for all landmarks l_q observed in q do
                if distance(l_p, l_q) < dt then
                    if correspondenceTest(i, j) ≥ χ then
                        optimizer.merge(i, j)
                    end if
                end if
            end for
        end for
    end for
end function
```

Results - Simulation - Known Data Association

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w
300	40	1000	1000	1000	20	1



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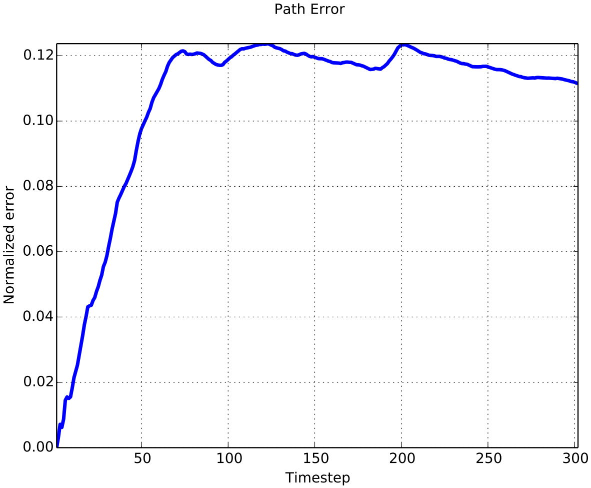

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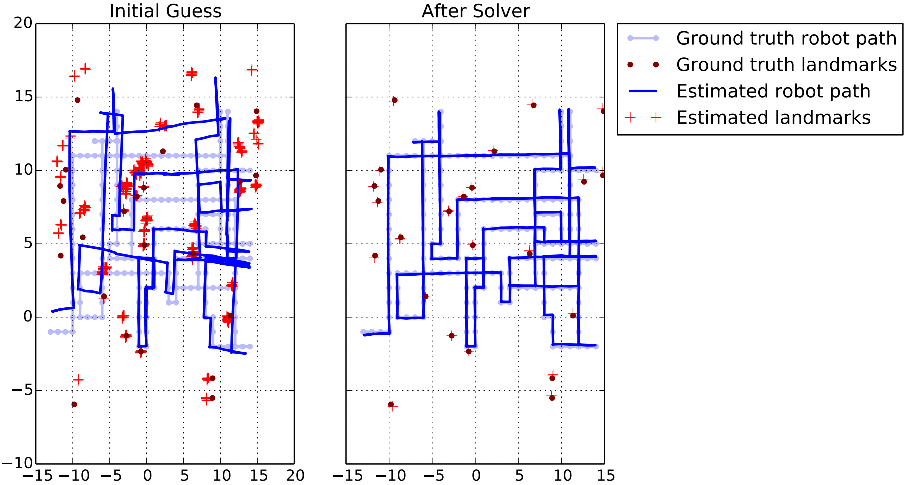
n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
400	30	1000	10000	1000	20	1	0.1	∞	400	10	$13[s]$



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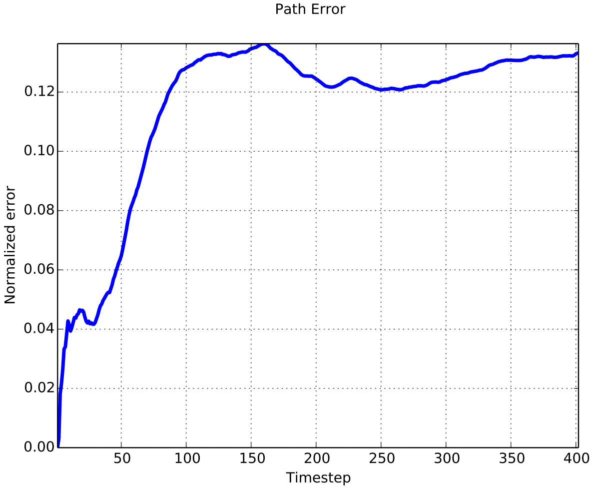
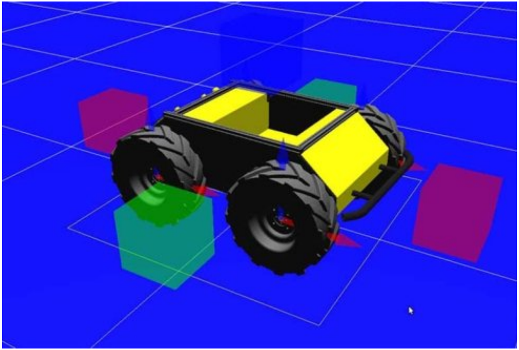


Results - Real Data - Husky a200 ROS Simulation

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
4700	18	10000	10000	1000	10	1	∞	0.5	500	10	$3[min]$



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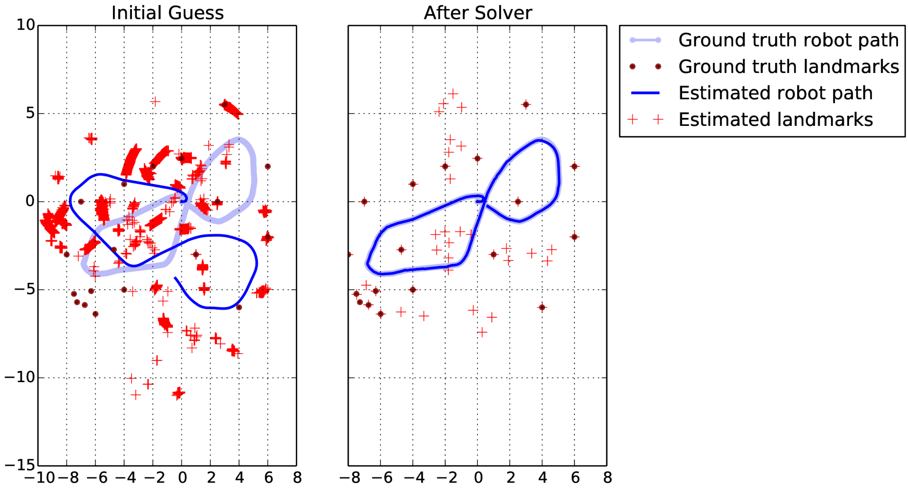

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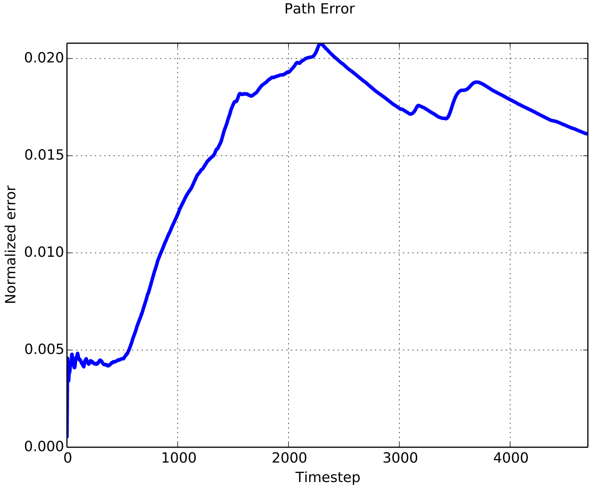
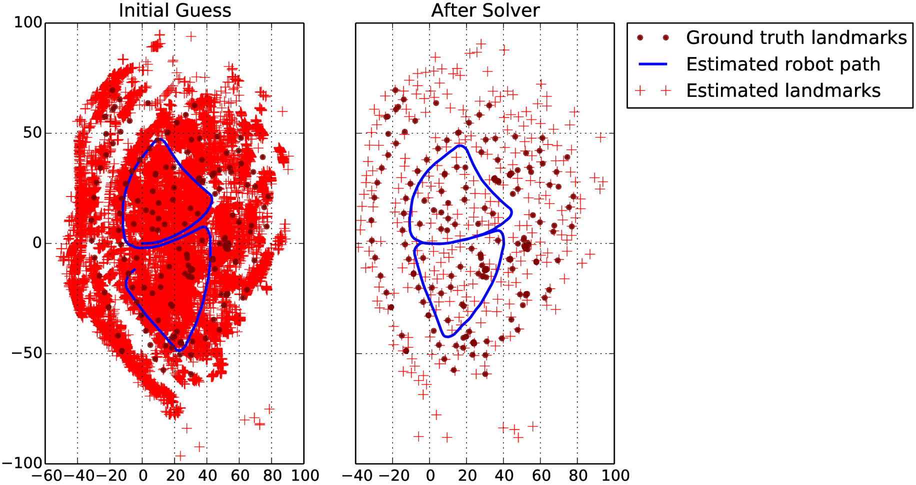


Results - Real Data - Parque O'Higgins

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
8130	174	100000	100000	100	10	1	∞	3	500	10	$4[hrs]$

Results - Real Data - Parque O'Higgins

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
8130	174	100000	100000	100	10	1	∞	3	500	10	$4[hrs]$

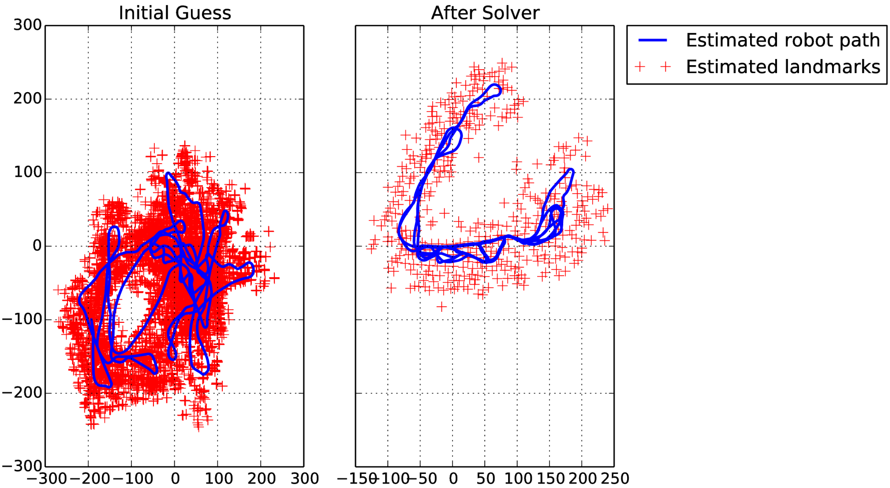


Results - Real Data - Victoria Park

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
61763	-	8000	100000	5	15	1	∞	5	500	10	$10[hrs]$

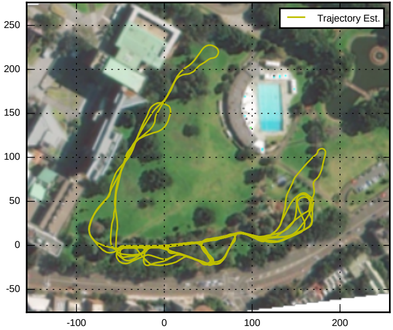
Results - Real Data - Victoria Park

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
61763	-	8000	100000	5	15	1	∞	5	500	10	$10[hrs]$



Results - Real Data - Victoria Park

n_p	n_l	i_{op}	i_{oa}	i_{lp}	it	k_w	χ	dt	io	ps	t
61763	-	8000	100000	5	15	1	∞	5	500	10	$10[hrs]$



GraphSLAM

iSAM

Conclusions

- GraphSLAM was successfully implemented for the 2D scenario using $\mathbf{g}^2\mathbf{o}$.
- A **correspondence test** was implemented to deal with the correspondence problem.
- The **distant test** is used to speed up the algorithm.
- The implementation works in the **known** and **unknown data association**, for **simulated** and **real data**.
- The biggest drawbacks are the **computational time** and the **false positives**.
- **Repository**: `https://github.com/francocurotto/GraphSLAM`.

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Thesis Defense

GraphSLAM Algorithm Implementation for Solving Simultaneous Localization and Mapping

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