Lecture 1: Anti Derivatives

Franco Vidal
Math 125

March 28, 2023

Understanding the antiderivative

$$\frac{dy}{dx}x^6 = 6x^5$$

- The anti derivative is doing the above process backwards.
- x^6 is an antiderivative of $6x^5$
- an anti derivative of a function $f(x)$ on an interval is a function $F(x)$ for $F'(x) = f(x)$
- hence, we say x^6 is an antiderivative of $6x^5$

How to find the anti derivative

Method: guess and check
ex:

- $f(x) = x^3$
  - guess: $F(x) = x^4$
  - check: $F'(x) = 4x^3$

The above example shows that the two solutions are fairly close with the exception of the constant present in the seconds one
Remember:

- $\frac{d}{dx}(C \cdot g(x)) = C\frac{d}{dx}(g(x))$

For the previous example, set $C = \frac{1}{4}$

- guess: $F(x) = \frac{1}{4}x^4$
- check: $F'(x) = \frac{1}{4}(4x^3) = x^3$

Another example:

- $f(x) = xe^{x^2}$
  - idea: work chain rule backwards
    - guess: $F(x) = e^{x^2}$
    - check: $F'(x) = e^{x^2} \cdot 2x = 2xe^{x^2}$
      - This is almost correct but not quite

            * guess: $F(x) = \frac{1}{2}e^{x^2}$
            * check: $F'(x) = \frac{1}{2}(e^{x^2} \cdot 2x) = xe^{x^2}$
                · $F(x) = \frac{1}{2}e^{x^2}$ is an antiderivative of $f(x) = xe^{x^2}$

Example:

- $f(x) = x^3$
    - try $F(x) = \frac{1}{4}x^4 + \pi$
    - $F'(x) = \frac{1}{4}(4x^3 + 0) = x^3$
    - hence, $F(x) = \frac{1}{4}x^4 + \pi$ is also an antiderivative of $f(x) = 3$
- there are more than one antiderivative for any given function

General antiderivatives

- if $f(x)$ is one antiderivative of $f(x)$ on an interval, then $F(x) + C$ is the general antiderivative of $f(x)$ on that interval
- new notation: $\int f(x)\,dx$ is called the indefinite integral of f(x), it's the general antiderivative of f(x)
    - it's a set of functions of x

Example: guess and check

- $\int (3x+1)dx = -\frac{1}{3}cos(3x+1) + C$
- guess: $F(x) = cos(3x+1)$
- check: $F'(x) = -sin(3x+1) \cdot 3 = -3sin(3x+1)$
- guess: $-\frac{1}{3}cos(3x+1)$
- check: $\frac{d}{dx}(-\frac{1}{3}cos(3x+1)) = -\frac{1}{3}(-sin(3x+1) \cdot 3) = sin(3x+1)$