Implicit Differentiation

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Math 124

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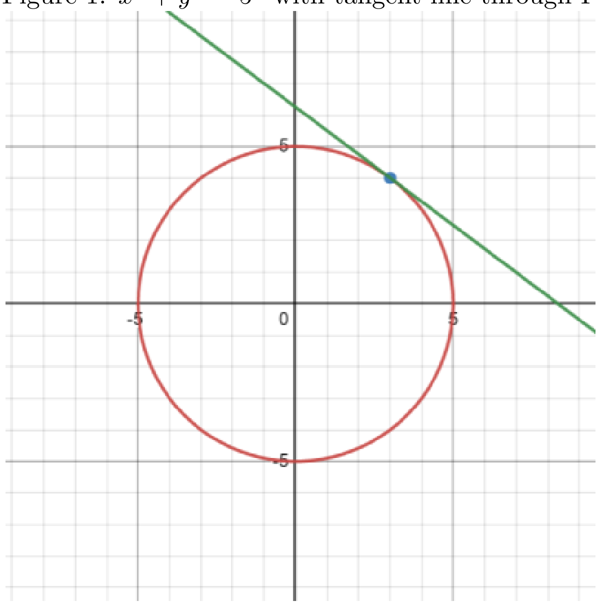
1 Circle Example

For a circle with equation $x^2 + y^2 = 5^2$ With a Point P = (3,4), find the tangent line using implicit differentiation

Figure 1: $x^2 + y^2 = 5^2$ with tangent line through P



$$x^2 + y^2 = 5^2 \tag{1}$$

$$(x^2 + y^2)\frac{dy}{dx} = (5)\frac{dy}{dx} \tag{2}$$

$$(x^2)\frac{dy}{dx} + (y^2)\frac{dy}{dx} = 0 \tag{3}$$

Because we're differentiating in terms of x, we treat y differently

$$2x + (y^2)\frac{dy}{dx} \tag{4}$$

$$2x + 2yy' = 0 \tag{5}$$

We then solve for y'

$$2yy' = 0 - 2x\frac{2yy'}{2y} = \frac{0 - 2x}{2y}y' = \frac{-x}{y} \tag{6}$$

We then plug in P (3,4)

$$y'|_{(3,4)} = \frac{-3}{4} \tag{7}$$

This gives us the slope of the tangent line at P, in order to find the equation of the line, we plug it into the slope equation:

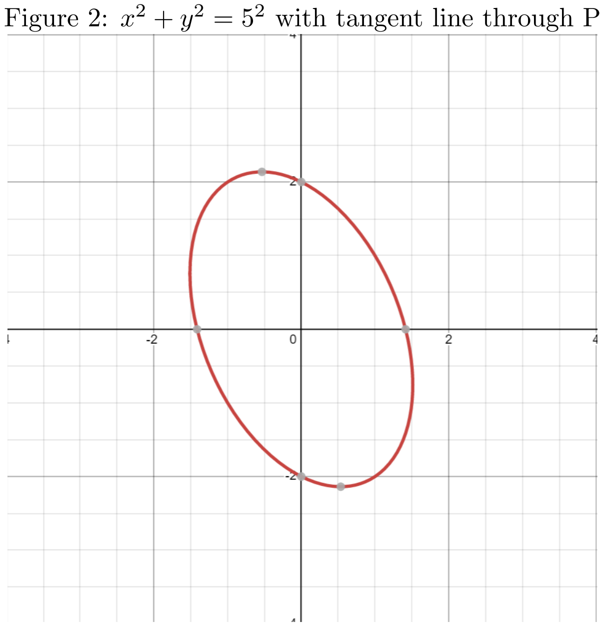
$$y = mx(x - x_1) + y_1 \tag{8}$$

$$y = \frac{-3}{4}(x - 3) + 4 \tag{9}$$

2 Ellipse Example

For a curve of equation $2x^2 + xy + y^2 = 4$, find the derivative

Figure 2: $x^2 + y^2 = 5^2$ with tangent line through P



$$2x^2 + xy + y^2 = 4 \tag{10}$$

$$\frac{dy}{dx}(2x^2 + xy + y^2) = \frac{dy}{dx}(4) \tag{11}$$

$$\frac{dy}{dx}(2x^2)\frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0 \tag{12}$$

$$4x + x\frac{dy}{dx} + \frac{dy}{dx}(x)y + 2y\frac{dy}{dx} = 0 \tag{13}$$

$$4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \tag{14}$$

Move all of the numbers without a $\frac{dy}{dx}$ to the other side of the equation, then solve.

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -4x - y \tag{15}$$

$$\frac{dy}{dx}(x + 2y) = -4x - y \tag{16}$$

Divide both sides by $(x + 2y)$

$$\frac{dy}{dx} = \frac{-4x - y}{x + 2y} \tag{17}$$