academiacesarvallejo.edu.pe

Ciclo INTENSIVO UNI

ACADEMIA CÉSAR VALLEJO

ACADEMIA CÉSAR VALLEJO

TRIGONOMETRÍA

Dirigida 6

PROBLEMA 1

Determine la variación de la siguiente expresión:

$$E=\cos\left(\frac{\pi|x|}{1+x^2}\right)+1$$

A) [2; 4] B) [1; 4] C) [1; 2]

D) [0; 1] E) [1; 3]

RESOLUCIÓN

$x^2=|x|^2$

$$E=\cos\left(\frac{\pi|x|}{1+|x|^2}\right)+1$$

• si $x=0$

$$E=\underbrace{\cos(0)}_{1}+1 \longrightarrow E=2$$

• si $x\neq 0$

$$E=\cos\left(\frac{\frac{\pi|x|}{|x|}}{\frac{1+|x|^2}{|x|}}\right)+1$$

$$E=\cos\left(\frac{\pi}{\frac{1}{|x|}+|x|}\right)+1$$

$$\boxed{|x|+\frac{1}{|x|}\geq 2\ ;\quad |x|>0}$$

$$0<\frac{1}{|x|+\frac{1}{|x|}}\leq\frac{1}{2}$$

$$0<\frac{\pi}{|x|+\frac{1}{|x|}}\leq\frac{\pi}{2}$$

Entonces

$$0\leq\cos\left(\frac{\pi}{|x|+\frac{1}{|x|}}\right)<1$$

$$1\leq\underbrace{\cos\left(\frac{\pi}{|x|+\frac{1}{|x|}}\right)+1}_{E}<2$$

Finalmente

$E\in[1;2]$

PROBLEMA 3

Si $\frac{\pi}{8} \leq x \leq \frac{\pi}{4}$, determine la suma del máximo y mínimo valor de la expresión

$$E = \frac{1 - \tan^2 x + 4\tan x}{\tan x}$$

A) 10 B) 8 C) 6

D) 4 E) 12

$\tan\theta \in \mathbb{R} \longleftrightarrow \theta \neq \frac{\pi}{2}; \frac{3\pi}{2}; \ldots$

$\theta \neq (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$

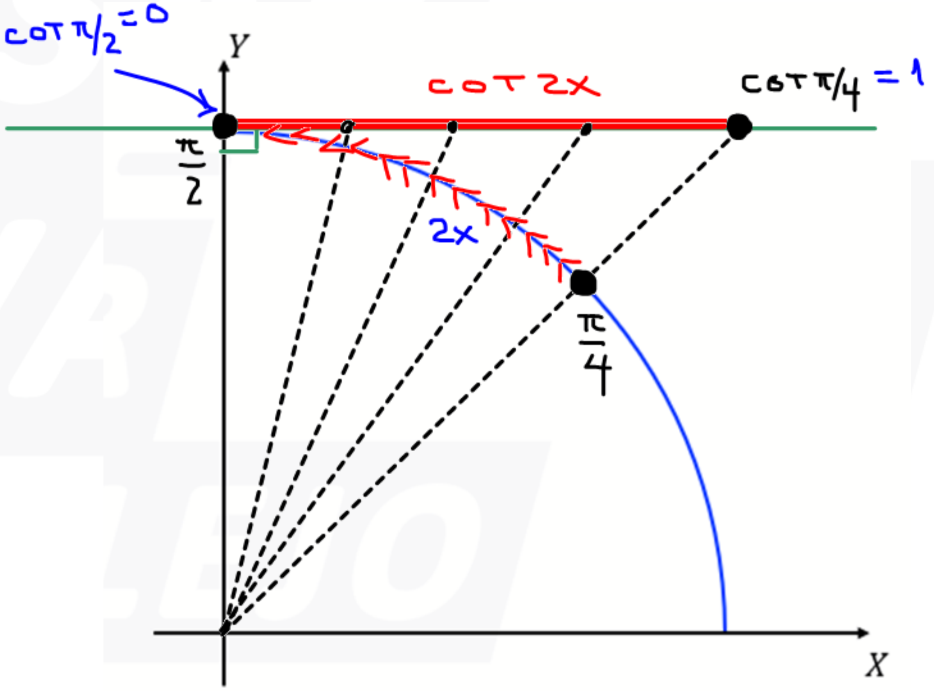
$$\boxed{\cot\theta - \tan\theta = 2\cot 2\theta}$$

RESOLUCIÓN

$$E = \frac{1}{\tan x} - \frac{\tan^{\cancel{2}} x}{\cancel{\tan x}} + \frac{4\cancel{\tan x}}{\cancel{\tan x}}$$

$$E = \cot x - \tan x + 4 \quad ; \quad \frac{\pi}{8} \leq x \leq \frac{\pi}{4}$$

$$\longrightarrow \quad E = 2\cot 2x + 4 \quad ; \quad \frac{\pi}{4} \leq 2x \leq \frac{\pi}{2}$$



En la C.T

$$0 \leq \cot 2x \leq 1$$

$$0 \leq 2\cot 2x \leq 2$$

$$4 \leq \underbrace{2\cot 2x + 4}_{E} \leq 6$$

$$\therefore \quad E_{máx} + E_{mín} = 6 + 4 = 10$$

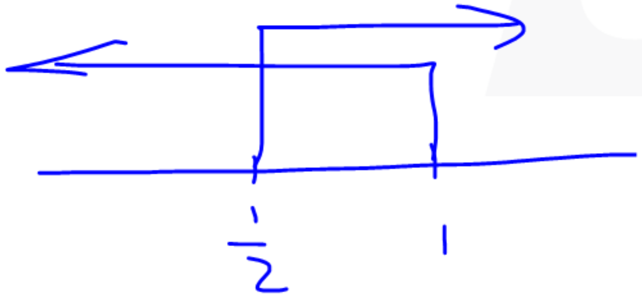
PROBLEMA 6

Determine los valores de la variable, tal que la expresión esté correctamente definida.

$E=\sqrt{1-\operatorname{sen}x}+\sqrt{2\operatorname{sen}x-1};\ 0\le x\le\pi$

A) $\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$ B) $\left[0;\frac{5\pi}{6}\right]$ C) $\left[\frac{\pi}{6};\frac{\pi}{2}\right]$

D) $\left[\frac{\pi}{6};\pi\right]$ E) $\left[0;\frac{\pi}{2}\right]$



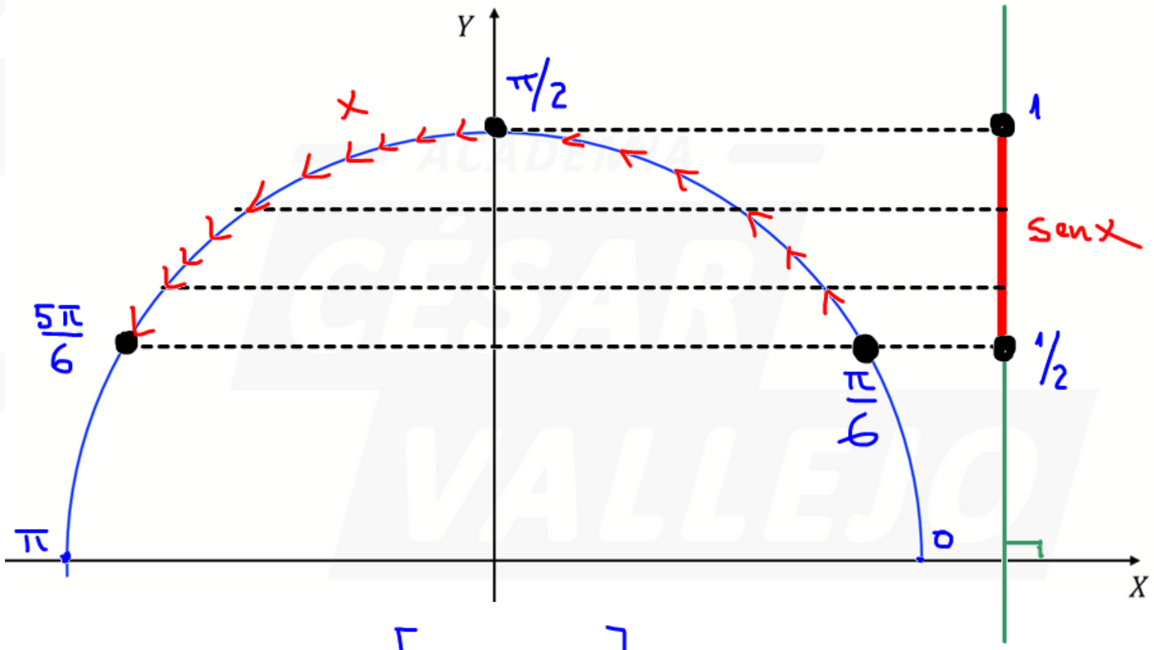
RESOLUCIÓN

$E=\sqrt{1-\operatorname{sen}x}+\sqrt{2\operatorname{sen}x-1};\ 0\le x\le\pi$

$1-\operatorname{sen}x\ge 0 \quad\wedge\quad 2\operatorname{sen}x-1\ge 0$

$\rightarrow\quad 1\ge\operatorname{sen}x \quad\wedge\quad \operatorname{sen}x\ge\frac{1}{2}$

$\rightarrow\quad \frac{1}{2}\le\operatorname{sen}x\le 1;\quad 0\le x\le\pi$



$\therefore\ x\in\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$

PROBLEMA 10

Determine el máximo valor de la siguiente expresión:

$E=\sqrt{2(1-\operatorname{sen}x)(1-\cos x)}$

A) $1+\sqrt{5}$ B) $1+\sqrt{2}$ C) $\sqrt{2}$

D) $1+\sqrt{3}$ E) $1+2\sqrt{3}$

$-3<a<2$

$\rightarrow 0\leq |a| < \text{mayor}\{|2|;|-3|\}$

$\rightarrow 0\leq |a| < 3$

RESOLUCIÓN

$$E=\sqrt{2(1-\operatorname{sen}x)(1-\cos x)}$$

$$E=\sqrt{(1-\operatorname{sen}x-\cos x)^2}$$

$$E=|1-\operatorname{sen}x-\cos x|$$

$$\boxed{-\sqrt{2}\leq \operatorname{sen}x+\cos x\leq \sqrt{2}\ ;\ \forall x\in\mathbb{R}}$$

$$\sqrt{2}\geq -\operatorname{sen}x-\cos x\geq -\sqrt{2}$$

$$\underbrace{1+\sqrt{2}}_{+}\geq 1-\operatorname{sen}x-\cos x\geq \underbrace{1-\sqrt{2}}_{-}$$

$$\rightarrow 0\leq \underbrace{|1-\operatorname{sen}x-\cos x|}_{E}\leq 1+\sqrt{2}$$

$$\therefore E_{máx}=1+\sqrt{2}$$

PROBLEMA 11

Calcule el intervalo al que pertenece θ, para que verifique $3\tan^2\theta \leq 1$; $k \in \mathbb{Z}$.

A) $\left[-\frac{\pi}{12}+k\pi; \frac{\pi}{12}+k\pi\right]$

B) $\left[-\frac{\pi}{8}+k\pi; \frac{\pi}{8}+k\pi\right]$

C) $\left[-\frac{\pi}{3}+k\pi; \frac{\pi}{3}+k\pi\right]$

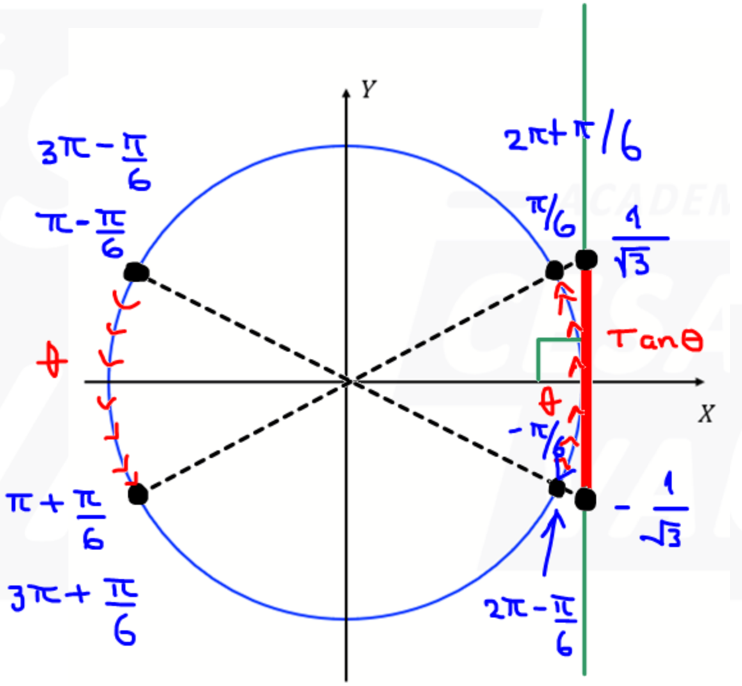
D) $\left[-\frac{\pi}{6}+\frac{k\pi}{2}; \frac{\pi}{6}+\frac{k\pi}{2}\right]$

E) $\left[-\frac{\pi}{6}+k\pi; \frac{\pi}{6}+k\pi\right]$

RESOLUCIÓN

$3\,\mathrm{Tan}^2\theta \leq 1 \longrightarrow \mathrm{Tan}^2\theta \leq \frac{1}{3}$

$|\mathrm{Tan}\theta| \leq \frac{1}{\sqrt{3}} \longrightarrow -\frac{1}{\sqrt{3}} \leq \mathrm{Tan}\theta \leq \frac{1}{\sqrt{3}}$



En la C.T

$\theta \in \left[-\frac{\pi}{6}; \frac{\pi}{6}\right] \cup \left[\pi-\frac{\pi}{6}; \pi+\frac{\pi}{6}\right] \cup \left[2\pi-\frac{\pi}{6}; 2\pi+\frac{\pi}{6}\right] \cup \left[3\pi-\frac{\pi}{6}; 3\pi+\frac{\pi}{6}\right] \cup$

......

En general

$\theta \in \left[k\pi-\frac{\pi}{6}; k\pi+\frac{\pi}{6}\right]; k \in \mathbb{Z}$

PROBLEMA 13

Si $2|\operatorname{sen} 2\theta| \leq \sqrt{3}$, calcule la variación de la expresión $2\sqrt{3}|\cos\theta|$, donde $2\theta \in \langle 0; 2\pi\rangle$.

A) $[0; \sqrt{3}]$

B) $[0; \sqrt{3}] \cup [3; 2\sqrt{3}\rangle$

C) $[0; 2\sqrt{3}]$

D) $[0; \sqrt{3}] \cup [2; 2\sqrt{3}]$

E) $[0; 2] \cup [3; 2\sqrt{3}]$

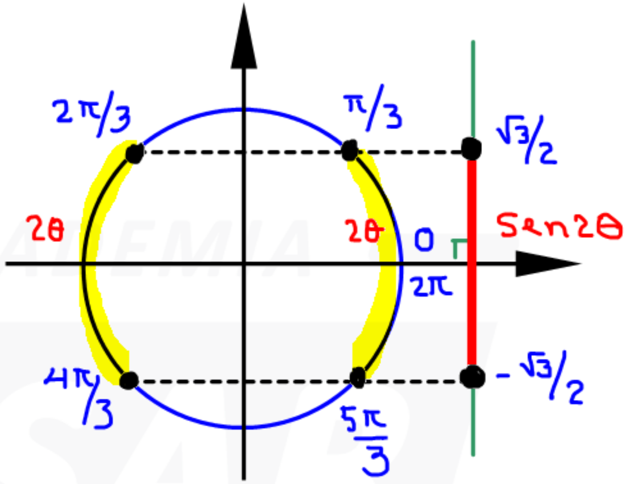
$$-\frac{\sqrt{3}}{2} \leq \operatorname{Sin} 2\theta \leq \frac{\sqrt{3}}{2} \rightarrow 0 \leq \underbrace{\operatorname{sen}^2 2\theta}_{1-\cos^2 2\theta} \leq \frac{3}{4}$$

$$\rightarrow \frac{1}{4} \leq \cos^2 2\theta \leq 1 \rightarrow \frac{1}{2} \leq |\cos 2\theta| \leq 1$$

$\cos 2\theta \rightarrow 2\cos^2\theta - 1$

RESOLUCIÓN

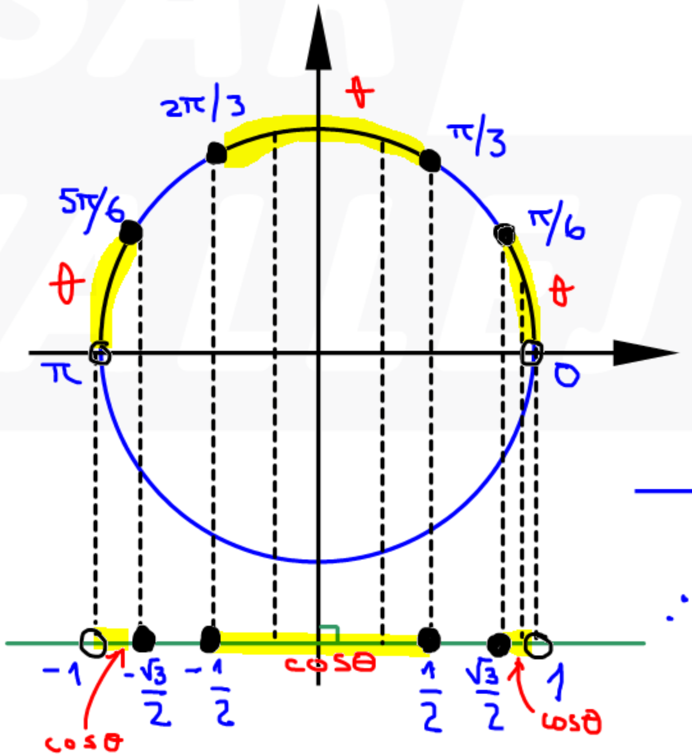
$$2|\operatorname{sen} 2\theta| \leq \sqrt{3} \longrightarrow -\frac{\sqrt{3}}{2} \leq \operatorname{Sen} 2\theta \leq \frac{\sqrt{3}}{2}; \quad 0 < 2\theta < 2\pi$$



En la c.T

$$2\theta \in \left\langle 0; \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}; \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}; 2\pi\right\rangle$$

$$\longrightarrow \theta \in \left\langle 0; \frac{\pi}{6}\right] \cup \left[\frac{\pi}{3}; \frac{2\pi}{3}\right] \cup \left[\frac{5\pi}{6}; \pi\right\rangle$$



En la c-T

$$\cos\theta \in \left\langle -1; -\frac{\sqrt{3}}{2}\right] \cup \left[-\frac{1}{2}; \frac{1}{2}\right] \cup \left[\frac{\sqrt{3}}{2}; 1\right\rangle$$

$$\longrightarrow |\cos\theta| \in \left[0; \frac{1}{2}\right] \cup \left[\frac{\sqrt{3}}{2}; 1\right\rangle$$

$$\therefore 2\sqrt{3}|\cos\theta| \in [0; \sqrt{3}] \cup [3; 2\sqrt{3}\rangle$$

PROBLEMA 15

. Si $0 \le x \le \frac{\pi}{2}$, determine la suma del máximo y mínimo valor de la expresión

$E=|\text{sen}x|+|\cos x|$

A) $1+\sqrt{2}$ B) $1+\sqrt{3}$ C) 2

D) $2+\sqrt{3}$ E) 1

$$\text{Sen}x + \cos x = \sqrt{2}\,\text{Sen}\left(x+\frac{\pi}{4}\right)$$

$$\text{Sen}x - \cos x = \sqrt{2}\,\text{Sen}\left(x-\frac{\pi}{4}\right)$$

$$\cos x - \text{Sen}\,x = \sqrt{2}\,\cos\left(x+\frac{\pi}{4}\right)$$

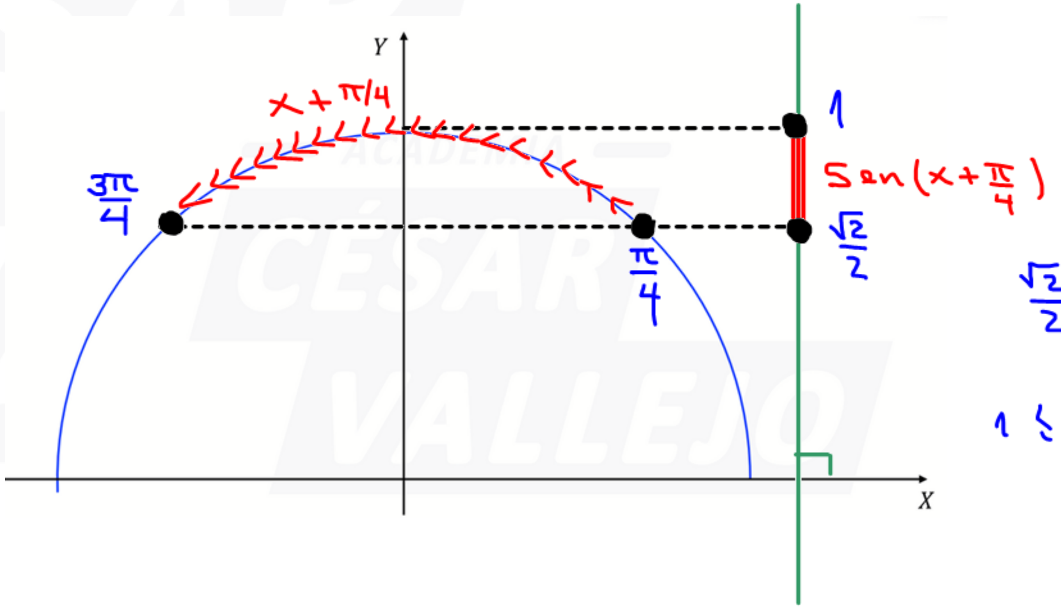
RESOLUCIÓN

$$E = \underbrace{|\text{sen}x|}_{+} + \underbrace{|\cos x|}_{+} \quad ; \quad 0 \le x \le \frac{\pi}{2}$$

$$E = \text{Sen}x + \cos x \quad ; \quad 0 \le x \le \frac{\pi}{2}$$

$$E = \sqrt{2}\,\text{Sen}\left(x+\frac{\pi}{4}\right) ; \quad \frac{\pi}{4} \le x+\frac{\pi}{4} \le \frac{3\pi}{4}$$

En la C.T



$$\frac{\sqrt{2}}{2} \le \text{Sen}\left(x+\frac{\pi}{4}\right) \le 1$$

$$1 \le \underbrace{\sqrt{2}\,\text{Sen}\left(x+\frac{\pi}{4}\right)}_{E} \le \sqrt{2}$$

$$\therefore\ E_{máx} + E_{mín} = \sqrt{2} + 1$$

PROBLEMA 20

Si $\tan\beta = \sqrt{3}\,\text{sen}\,\theta$, $\theta \in \langle -2; 2\rangle$, halle los valores que toma β en el intervalo $\left\langle \frac{\pi}{2}; \frac{3\pi}{2} \right\rangle$.

A) $\left[\frac{5\pi}{6}; \frac{4\pi}{3}\right]$

B) $\left[\frac{2\pi}{3}; \frac{4\pi}{3}\right]$

C) $\left[\frac{3\pi}{4}; \frac{4\pi}{3}\right]$

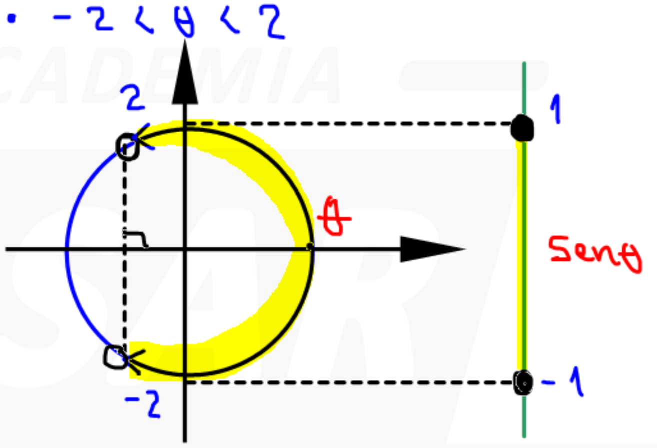
D) $\left[\frac{2\pi}{3}; \frac{5\pi}{6}\right]$

E) $\left[\frac{2\pi}{3}; \frac{3\pi}{4}\right]$

RESOLUCIÓN

$\boxed{\tan\beta = \sqrt{3}\,\text{sen}\,\theta}$

• $-2 < \theta < 2$

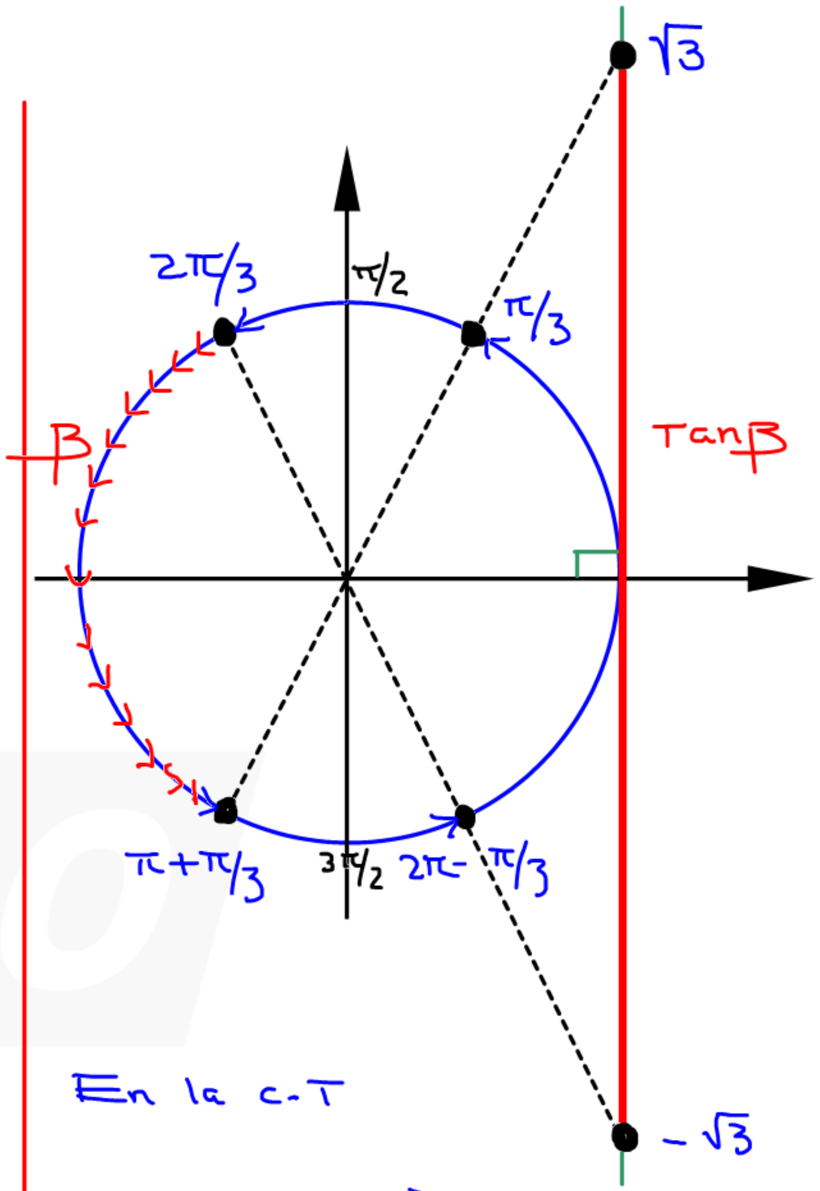


En la C.T

$-1 \le \text{sen}\,\theta \le 1$

→ $-\sqrt{3} \le \sqrt{3}\,\text{sen}\,\theta \le \sqrt{3}$

→ $-\sqrt{3} \le \tan\beta \le \sqrt{3}$; $\frac{\pi}{2} < \beta < \frac{3\pi}{2}$



En la C.T

$\beta \in \left[\frac{2\pi}{3}; \frac{4\pi}{3}\right]$

PROBLEMA 21

. Si $\dfrac{1-2\sqrt{2}}{2} \leq \dfrac{2\csc\beta+1}{2} \leq \dfrac{1}{2}$; $\beta \in \langle 0;\ 2\pi \rangle$,

calcule la variación de $\sqrt{\cot\beta+2}$.

A) $\left[1;\ \sqrt{2}\right]$

B) $\left[\sqrt{2};\ \sqrt{3}\right]$

C) $\left[1;\ \sqrt{3}\right]$

D) $\left[0;\ \sqrt{2}\right]$

E) $\left[\sqrt{2};\ 2\right]$

$0 \leq |a| \leq 1$

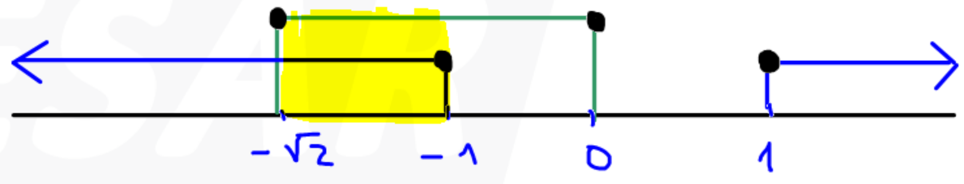
$\rightarrow -1 \leq a \leq 1$

RESOLUCIÓN

$$\frac{1}{2} - \frac{2\sqrt{2}}{2} \leq \frac{2\csc\beta}{2} + \frac{1}{2} \leq \frac{1}{2}$$

$\longrightarrow -\sqrt{2} \leq \csc\beta \leq 0 \quad ; \quad 0 < \beta < 2\pi$

Por teoría : $\csc\beta \leq -1 \ \vee \ \csc\beta \geq 1$



$\longrightarrow -\sqrt{2} \leq \csc\beta \leq -1$

$2 \geq \underbrace{\csc^2\beta}_{1+\cot^2\beta} \geq 1 \longrightarrow 1 \geq \cot^2\beta \geq 0$

$1 \geq |\cot\beta| \geq 0 \longrightarrow -1 \leq \cot\beta \leq 1$

$1 \leq \cot\beta + 2 \leq 3 \longrightarrow 1 \leq \sqrt{\cot\beta+2} \leq \sqrt{3}$

PROBLEMA 23

Si $0 \leq x \leq \pi$, determine los valores de la variable, tal que la expresión esté correctamente definida.

$E = \log(\operatorname{sen}x) + \log(-\cos x)$

A) $\left\langle 0; \frac{\pi}{2} \right\rangle$

B) $\left[0; \frac{5\pi}{6} \right]$

C) $\left\langle \frac{\pi}{2}; \pi \right\rangle$

D) $\left[\frac{\pi}{2}; \pi \right]$

E) $\left[0; \frac{\pi}{2} \right]$

RESOLUCIÓN

PROBLEMA 24

Si $\operatorname{sen}\theta \in \left[-\frac{3}{4}; -\frac{1}{2}\right] \cup \left\langle \frac{3}{4}; 1 \right\rangle$,

calcule la variación de $|\tan\theta|$.

A) $\left[\frac{3\sqrt{7}}{7}; +\infty\right\rangle$

B) $\left[\frac{\sqrt{3}}{3}; +\infty\right\rangle$

C) $\left[\frac{\sqrt{3}}{3}; +\infty\right\rangle - \left\{3; \frac{\sqrt{7}}{7}\right\}$

D) $\left\langle 3\frac{\sqrt{7}}{7}; +\infty\right\rangle$

E) $[0; +\infty\rangle - \left\{\frac{\sqrt{3}}{3}; 3\frac{\sqrt{7}}{7}\right\}$

RESOLUCIÓN

PROBLEMA 25

Determine la variación de
$\cot(\operatorname{sen}^2 x+\operatorname{sen}x)$; $x\in\langle 0;\ \pi\rangle$

A) $[0;\ +\infty\rangle$

B) $\langle 0;\ +\infty\rangle$

C) $[\cot 3;\ +\infty\rangle$

D) $\langle \cot 3;\ +\infty\rangle$

E) $[1;\ +\infty\rangle$

RESOLUCIÓN

PROBLEMA

RESOLUCIÓN

ACADEMIA
CÉSAR VALLEJO

GRACIAS

SÍGUENOS:

academiacesarvallejo.edu.pe