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TRIGONOMETRÍA

Dirigida 7

PROBLEMA 1

Determine el rango de la función f si

$f(x)=6\operatorname{sen}x-2\cos 2x-1$

A) $\left[-\frac{21}{4};7\right]$

B) $\left[-\frac{21}{4};6\right]$

C) $[-4;7]$

D) $[-7;7]$

E) $\left[-\frac{21}{4};5\right]$

RESOLUCIÓN

- $f(x)=6\operatorname{sen}x-2\cos 2x-1$

$\cos 2x \to 1-2\operatorname{sen}^2 x$

$f(x)=4\operatorname{sen}^2 x+6\operatorname{sen}x-3$

Completamos cuadrados

$f(x)=4\left(\operatorname{sen}x+\frac{3}{4}\right)^2-\frac{21}{4}$

- Asumimos que: $-1\leq \operatorname{sen}x\leq 1$

$-\frac{1}{4}\leq \operatorname{sen}x+\frac{3}{4}\leq \frac{7}{4}$

$0\leq\left(\operatorname{sen}x+\frac{3}{4}\right)^2\leq\frac{49}{16}\to 0\leq 4\left(\operatorname{sen}x+\frac{3}{4}\right)^2\leq\frac{49}{4}$

$-\frac{21}{4}\leq \underbrace{4\left(\operatorname{sen}x+\frac{3}{4}\right)^2-\frac{21}{4}}_{f(x)}\leq 7$

$\therefore \operatorname{Ran} f=\left[-\frac{21}{4};7\right]$

PROBLEMA 3

Determine el dominio de la función definida por

$$f_{(x)} = \sqrt{\frac{\text{vers}(x) - \text{cov}(x)}{\text{vers}(x) + \text{cov}(x)}}$$

A) $\left[\frac{\pi}{4}; \frac{3\pi}{4}\right]$ B) $\left[\frac{\pi}{4}; \frac{5\pi}{4}\right]$ C) $\left[\frac{\pi}{2}; \frac{3\pi}{4}\right]$

D) $\left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$ E) $\left[\frac{3\pi}{4}; \frac{5\pi}{4}\right]$

$\text{vers}(x) = 1 - \cos(x)$

$\text{cov}(x) = 1 - \text{sen}(x)$

$\text{ex} - \sec(x) = \sec(x) - 1$

RESOLUCIÓN

$$f_{(x)} = \sqrt{\frac{\text{vers}(x) - \text{cov}(x)}{\text{vers}(x) + \text{cov}(x)}}$$

$$\frac{\text{vers}(x) - \text{cov}(x)}{\text{vers}(x) + \text{cov}(x)} \geq 0$$

$$\frac{\text{Sen}x - \cos x}{2 - (\text{Sen}x + \cos x)} \geq 0$$

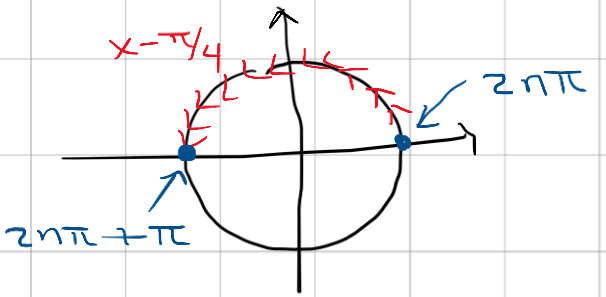
$2 + : \forall x \in \mathbb{R}$

$-\sqrt{2} \leq \text{Sen}x + \cos x \leq \sqrt{2}; x \in \mathbb{R}$

$\rightarrow \text{Sen}x - \cos x \geq 0$

$\sqrt{2}\,\text{Sen}\left(x - \frac{\pi}{4}\right) \geq 0$

$\rightarrow \text{Sen}\left(x - \frac{\pi}{4}\right) \geq 0$



$x - \frac{\pi}{4} \in [2n\pi ; 2n\pi + \pi]; n \in \mathbb{Z}$

$\rightarrow x \in \left[2n\pi + \frac{\pi}{4} ; 2n\pi + \frac{5\pi}{4}\right]$

$\therefore \text{Dom}f = \left[2n\pi + \frac{\pi}{4} ; 2n\pi + \frac{5\pi}{4}\right]$

$n \in \mathbb{Z}$

PROBLEMA 5

Determine el rango de la función f si

$$f_{(x)} = \cos\left(\frac{\pi}{4}\sqrt{4x - x^2}\right)$$

A) $\left[0; \frac{1}{2}\right]$ B) $\left[0; \sqrt{2}\right]$ C) $[0; 1]$

D) $\left[0; \frac{\sqrt{2}}{2}\right]$ E) $\left[\frac{1}{2}; \frac{\sqrt{2}}{2}\right]$

RESOLUCIÓN

$$f_{(x)} = \cos\left(\underbrace{\frac{\pi}{4}\sqrt{4x - x^2}}_{\alpha}\right)$$

- $4x - x^2 \geq 0$

$\rightarrow x^2 - 4x \leq 0$

$\rightarrow x(x-4) \leq 0$

$\rightarrow 0 \leq x \leq 4$

- $\alpha = \frac{\pi}{4}\sqrt{4x - x^2}$

$\alpha = \frac{\pi}{4}\sqrt{-(x^2 - 4x)}$

$\alpha = \frac{\pi}{4}\sqrt{-(x-2)^2 + 4}$

- $0 \leq x \leq 4 \rightarrow -2 \leq x-2 \leq 2$

$0 \leq (x-2)^2 \leq 4 \rightarrow 0 \geq -(x-2)^2 \geq -4$

$\rightarrow 4 \geq -(x-2)^2 - 4 \geq 0$

$\rightarrow 2 \geq \sqrt{-(x-2)^2 - 4} \geq 0$

$\rightarrow \frac{\pi}{2} \geq \underbrace{\frac{\pi}{4}\sqrt{-(x-2)^2 - 4}}_{\alpha} \geq 0$

- $f(x) = \cos\alpha, \quad 0 \leq \alpha \leq \frac{\pi}{2}$

$\rightarrow 0 \leq \underbrace{\cos\alpha}_{f(x)} \leq 1$

$\therefore \text{Ran}f = [0; 1]$

PROBLEMA 7

Dada la siguiente función f con regla de correspondencia:

~~$f(x) = |1 - \text{cov}(x)| 1 + \text{cov}(x)$~~

determine el rango.

$$f(x) = |1 - \text{cov}(x)| + 1 + \text{cov}(x)$$

A) $\left[-\frac{1}{2}; \frac{1}{2}\right]$

B) $[-1; 1]$

C) $[0; 1]$

D) $\left[0; \frac{3}{2}\right]$

E) $[2; 4]$

RESOLUCIÓN

$$f(x) = |1 - \underbrace{\text{cov}(x)}_{1-\text{Sen}x}| + 1 + \underbrace{\text{cov}(x)}_{1-\text{Sen}x}$$

$$\boxed{f(x) = |\text{Sen}x| + 2 - \text{Sen}x}$$

- $0 \leq \text{Sen}x \leq 1 \rightarrow |\text{Sen}x| = \text{Sen}x$

  $\rightarrow f(x) = 2$

- $-1 \leq \text{Sen}x \leq 0 \rightarrow |\text{Sen}x| = -\text{Sen}x$

  $f(x) = 2 - 2\text{Sen}x$

  $-1 \leq \text{Sen}x \leq 0 \rightarrow 2 \geq -2\text{Sen}x \geq 0$

  $4 \geq \underbrace{2 - 2\text{Sen}x}_{f(x)} \geq 2$

- Tenemos

  $f(x) \in [2; 4] \cup \{2\} = [2; 4]$

  $\therefore \text{Ran}f = [2; 4]$

PROBLEMA 9

Señale la verdad (V) o falsedad (F) de las siguientes proposiciones:

I. Si $0<x<\frac{\pi}{2}$, entonces $\operatorname{sen}2x>\cos x$.

II. Si $0<x<\frac{\pi}{2}$, entonces $\cos 2x>\operatorname{sen}x$.

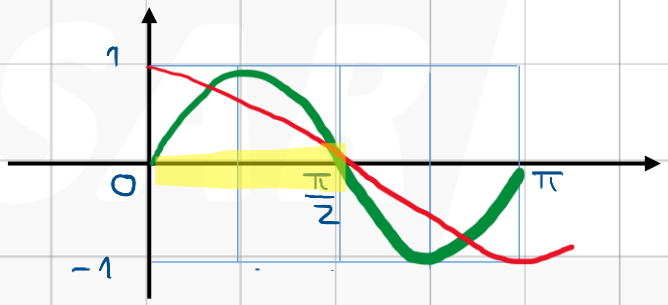
III. Si $\frac{\pi}{2}<x<\pi$, entonces $\operatorname{sen}x>\cos x$.

A) FFV B) FFF C) FVF
D) FVV E) VVF

RESOLUCIÓN

I. Si $0<x<\frac{\pi}{2}$, entonces $\operatorname{sen}2x>\cos x$.

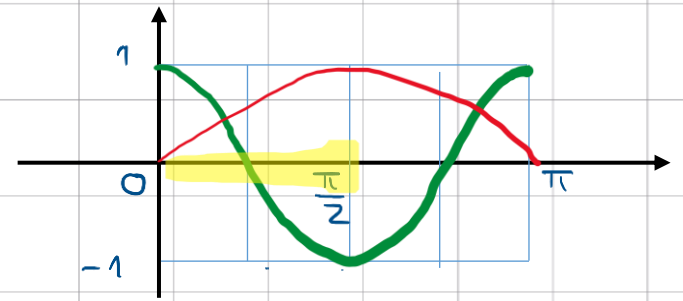
- $f(x)=\operatorname{sen}2x \rightarrow T=\frac{2\pi}{2}=\pi$
- $g(x)=\cos x \rightarrow T=2\pi$



$\rightarrow$ (F)

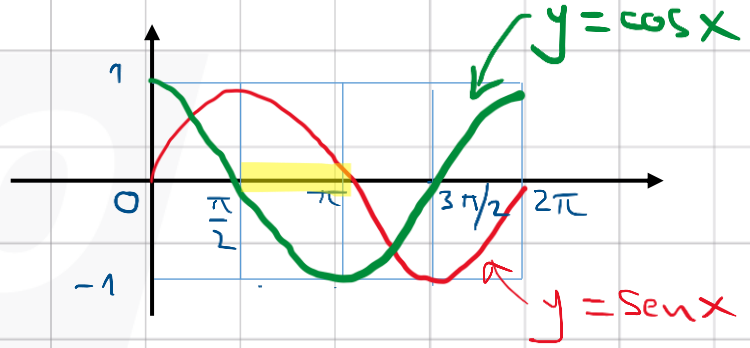
II. Si $0<x<\frac{\pi}{2}$, entonces $\cos 2x>\operatorname{sen}x$.

- $f(x)=\cos 2x \rightarrow T=\frac{2\pi}{2}=\pi$
- $g(x)=\operatorname{Sen}x \rightarrow T=2\pi$



$\rightarrow$ (F)

III. Si $\frac{\pi}{2}<x<\pi$, entonces $\operatorname{sen}x>\cos x$.



$\rightarrow$ (V)

PROBLEMA 11

Respecto a la función

$f(x)=4\text{sen}^4x+4\cos^4x$

señale la verdad (V) o falsedad (F) de las siguientes proposiciones:

I. Si $\frac{3\pi}{4}<x<\pi$, entonces f es creciente.

II. La función tiene periodo $T=\frac{\pi}{2}$.

III. Si $x=\frac{\pi}{2}$, entonces f es mínimo.

A) FFV B) VVF C) VFV
D) VFF E) VVV

RESOLUCIÓN

$$f(x)=4(\text{sen}^4x+\cos^4x)$$

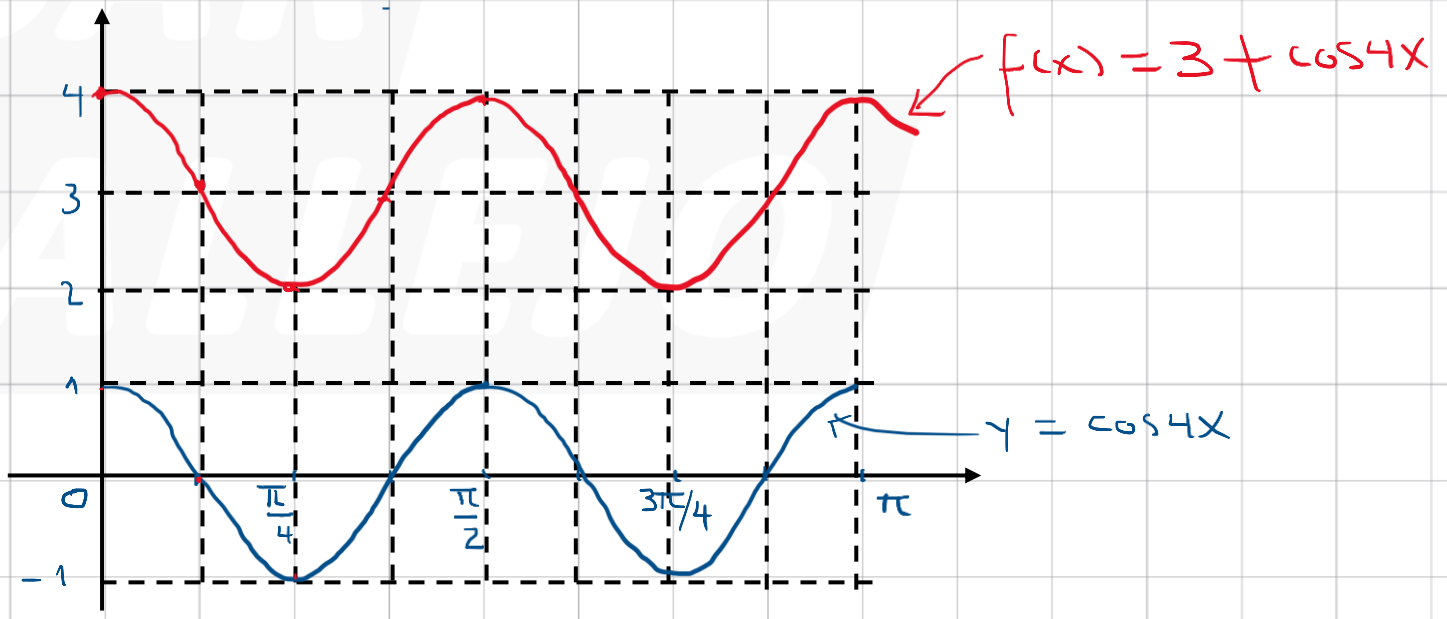
$$f(x)=4\left(\frac{3}{4}+\frac{1}{4}\cos 4x\right)$$

$$\boxed{f(x)=3+\cos 4x}$$

i) $y=\cos 4x$

$$T=\frac{2\pi}{4}=\frac{\pi}{2}$$

ii) $y=3+\cos 4x$



I) V II) V III) F

PROBLEMA 13

Sea la función f definida por
$f_{(x)}=\text{sen}(x)+\cos(x)+\text{sen}(x)\cos(x)+1$,
Calcule el máximo valor de la función f.

A) $1+\sqrt{2}$

B) 3

C) $\frac{3}{2}+\sqrt{2}$

D) $3+\sqrt{2}$

E) $\frac{3}{2}+2\sqrt{2}$

RESOLUCIÓN

$$f(x) = \text{sen}x + \cos x + \text{sen}x\cos x + 1$$

$$f(x) = \text{sen}x + 1 + \cos x(1 + \text{sen}x)$$

$$f(x) = (1+\text{sen}x)(1+\cos x)$$

$$2f(x) = 2(1+\text{sen}x)(1+\cos x)$$

$$2f(x) = (1 + \text{sen}x + \cos x)^2$$

$$f(x) = \frac{1}{2}\left(1 + \underbrace{\text{sen}x + \cos x}_{-\sqrt{2} \le \quad \le \sqrt{2}}\right)^2 \quad ; \quad x \in \mathbb{R}$$

$$\rightarrow f_{máx} = \frac{1}{2}(1+\sqrt{2})^2$$

$$\therefore f_{máx} = \frac{3}{2} + \sqrt{2}$$

PROBLEMA 15

¿Para que los valores de x en $\left[\frac{\pi}{4};\frac{7\pi}{4}\right]$ la función f, definida por $f_{(x)}=\sqrt{|\operatorname{sen}x|-|\cos x|}$ no está definida?

A) $x\in\left[\frac{3\pi}{2};\frac{5\pi}{2}\right]$

B) $x\in\left[\frac{6\pi}{4};\frac{9\pi}{4}\right]$

C) $x\in\left\langle\frac{3\pi}{4};\frac{5\pi}{4}\right\rangle$

D) $x\in\left[\frac{3\pi}{3};\frac{5\pi}{3}\right]$

E) $x\in\left[\frac{3\pi}{4};\frac{3\pi}{2}\right]$

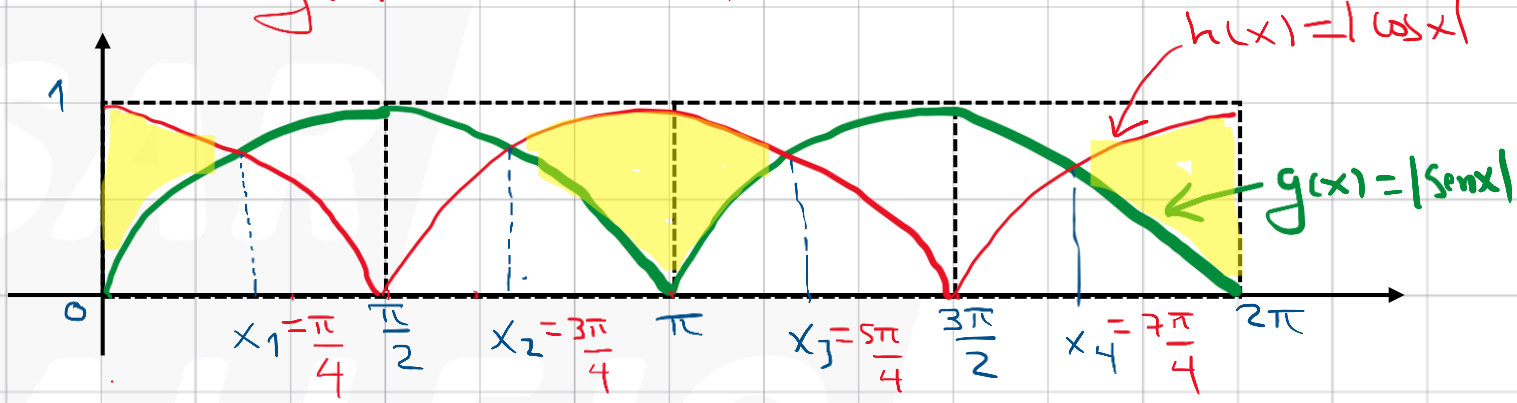
RESOLUCIÓN

$f_{(x)}=\sqrt{|\operatorname{sen}x|-|\cos x|}$ no está definida?

$\rightarrow |\operatorname{sen}x|-|\cos x|<0$

$\rightarrow |\operatorname{sen}x|<|\cos x|\ ;\ \frac{\pi}{4}\le x\le\frac{7\pi}{4}$

$g(x)$ $\quad$ $h(x)$



- $g(x)=h(x)\rightarrow |\operatorname{sen}x|=|\cos x|$

$\rightarrow |\operatorname{Tan}x|=1$

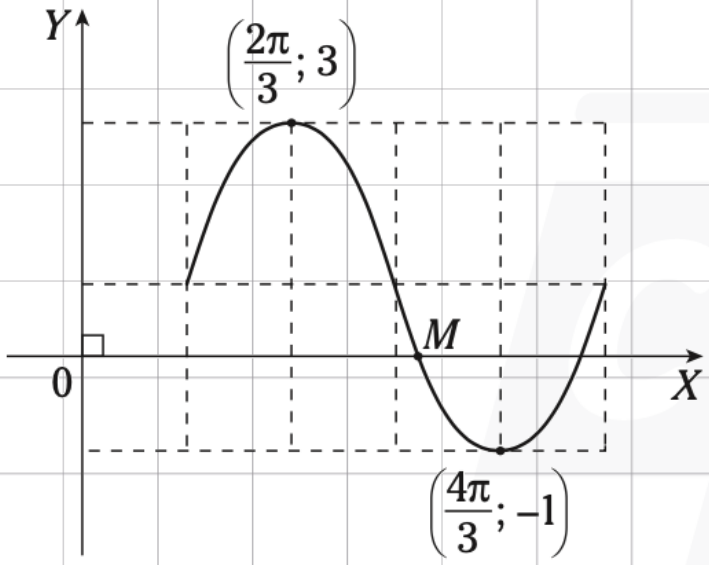
$\rightarrow \operatorname{Tan}x=\pm 1$

$\rightarrow x=\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4}$

- $g(x)<h(x)\rightarrow x\in\left[0;\frac{\pi}{4}\right\rangle\cup\left\langle\frac{3\pi}{4};\frac{5\pi}{4}\right\rangle\cup\left\langle\frac{7\pi}{4};2\pi\right]$

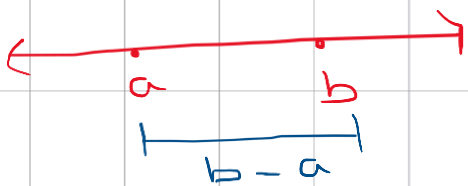
PROBLEMA 17

En el sinusoide de la función mostrada, ¿cuál es la abscisa del punto M?



A) $\frac{2\pi}{9}$ B) $\frac{5\pi}{9}$ C) $\frac{10\pi}{9}$

D) $\frac{14\pi}{9}$ E) $\frac{7\pi}{9}$



RESOLUCIÓN

$$f(x) = A\,Sen(Bx + C) + D$$

$A > 0$; $B > 0$

- $A = \frac{f_{máx} - f_{min}}{2} \rightarrow A = 2$
- $D = \frac{f_{máx} + f_{min}}{2}$

$\rightarrow D = 1$

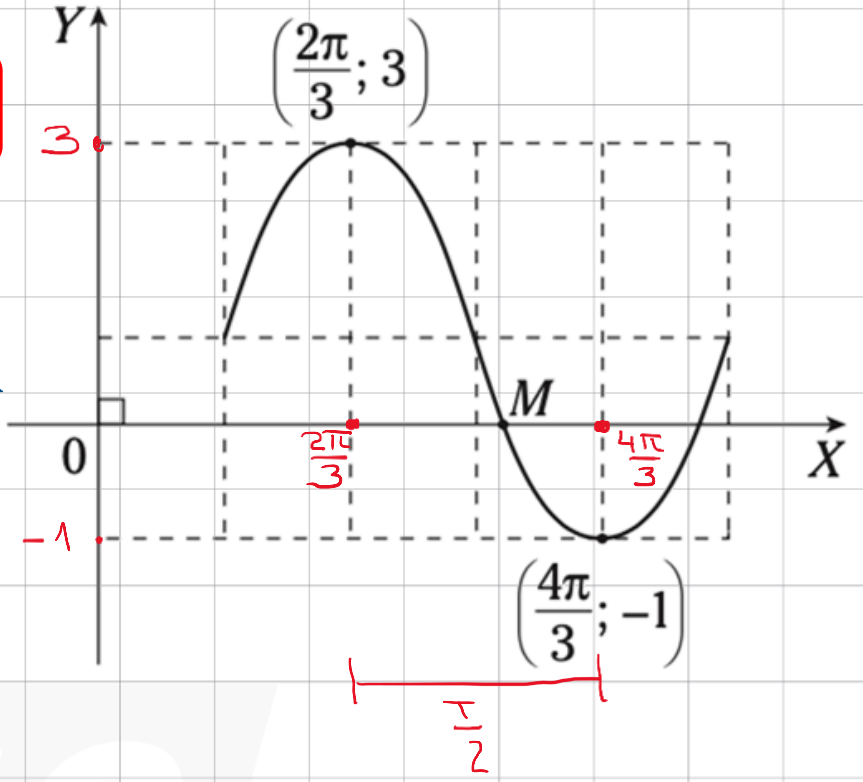
- Periodo: $T = \frac{2\pi}{B}$

$\frac{T}{2} = \frac{4\pi}{3} - \frac{2\pi}{3} \rightarrow \frac{\pi}{B} = \frac{2\pi}{3}$

$\rightarrow B = 3/2$

$$f(x) = 2\,Sen\left(\frac{3}{2}x + C\right) + 1$$

- $f\left(\frac{2\pi}{3}\right) = 3 \rightarrow C = -\pi/2$



- Si $f(x) = 0$

$2\,Sen\left(\frac{3x}{2} - \frac{\pi}{2}\right) + 1 = 0$

$\cos\frac{3x}{2} = \frac{1}{2}$

$\frac{3x}{2} = \frac{\pi}{3}; \frac{5\pi}{3}; \ldots$

$x = \frac{2\pi}{9}; \frac{10\pi}{9}; \ldots \therefore M = \left(\frac{10\pi}{9}; 0\right)$

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