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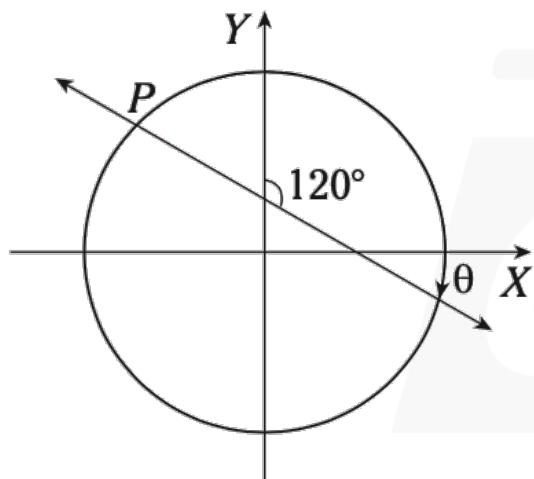
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TRIGONOMETRÍA

Dirigida 5

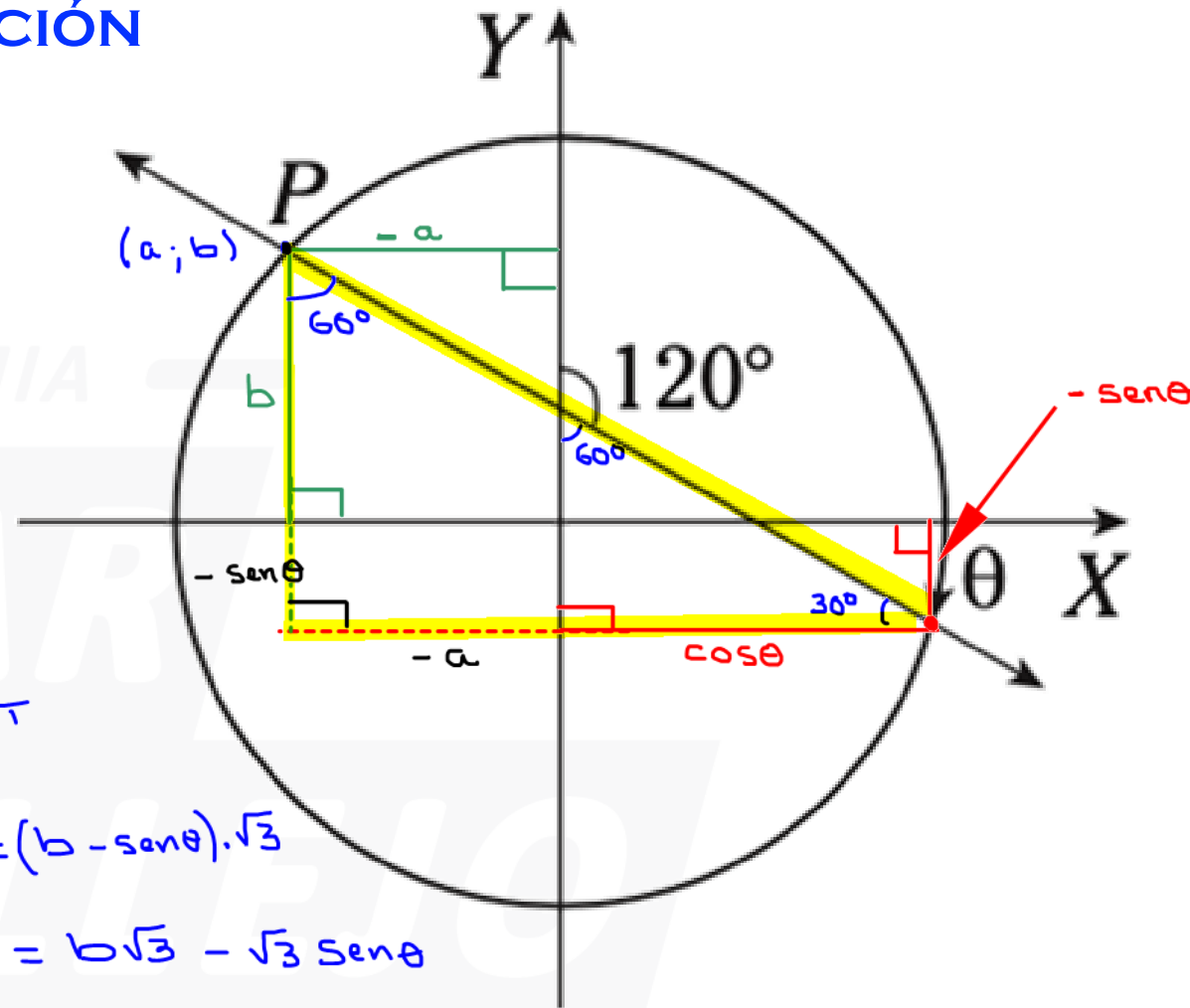
PROBLEMA 1

En la circunferencia trigonométrica, si $P(a; b)$, halle $a + \sqrt{3}b$.



- A) $\cos \theta - \sqrt{3} \sin \theta$
- B) $\cos \theta + \sqrt{3} \sin \theta$
- C) $\sin \theta - \sqrt{3} \cos \theta$
- D) $\sin \theta + \sqrt{3} \cos \theta$
- E) $\cos \theta - \sin \theta$

RESOLUCIÓN



En la c.T

$$\cos \theta - a = (b - \sin \theta) \cdot \sqrt{3}$$

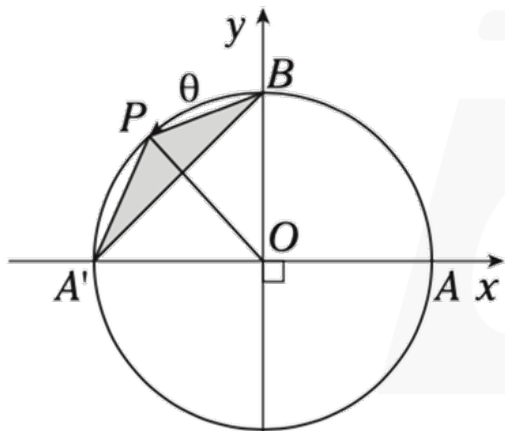
$$\cos \theta - a = b\sqrt{3} - \sqrt{3} \sin \theta$$

$$\rightarrow \cos \theta + \sqrt{3} \sin \theta = a + b\sqrt{3}$$

PROBLEMA 3

En la circunferencia trigonométrica mostrada, calcule el área de la superficie mostrada.

$$(\widehat{ABP} = \theta)$$



A) $\frac{1}{2}(\sin(\theta) + \cos(\theta) + 1)$

B) $\frac{1}{2}(\sin(\theta) + \cos(\theta) - 1)$

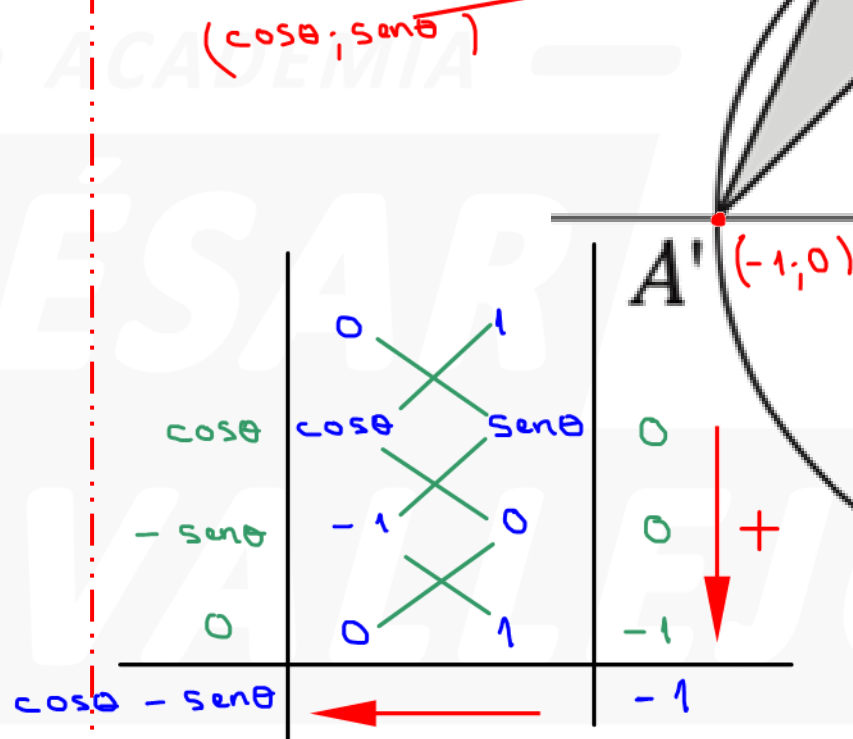
C) $\frac{1}{2}(\sin(\theta) - \cos(\theta) + 1)$

D) $\frac{1}{2}(\cos(\theta) - \sin(\theta) + 1)$

☒ E) $\frac{1}{2}(\sin(\theta) - \cos(\theta) - 1)$

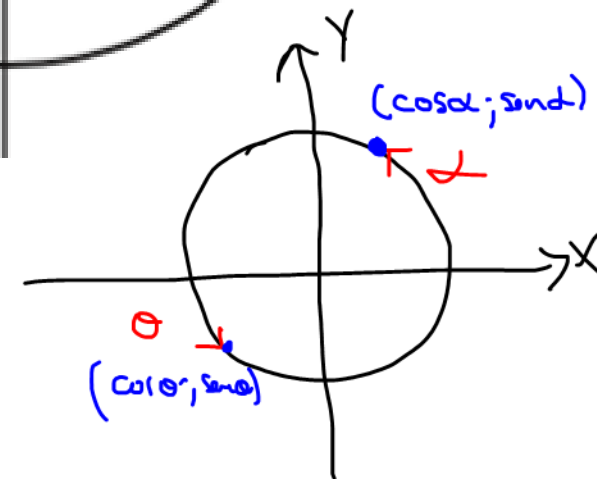
RESOLUCIÓN

¿Iden el área "S"?



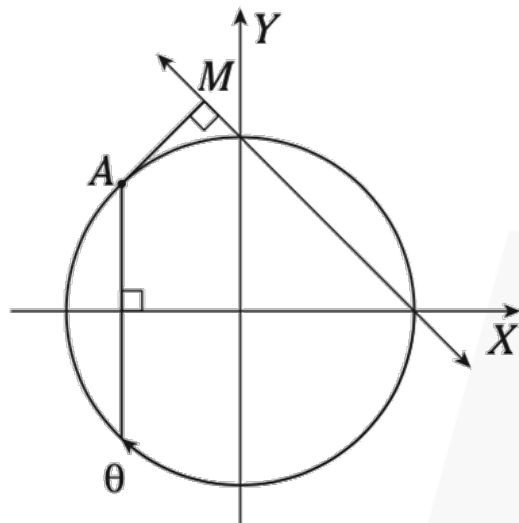
$$S = \frac{(-1) - (\cos \theta - \sin \theta)}{2}$$

$$\therefore S = \frac{\sin \theta - \cos \theta - 1}{2}$$



PROBLEMA 5

En la circunferencia trigonométrica, halle AM .



A) $\frac{1 + \operatorname{sen} \theta + \cos \theta}{\sqrt{2}}$

B) $\frac{1 - \operatorname{sen} \theta + \cos \theta}{\sqrt{2}}$

C) $\frac{1 + \operatorname{sen} \theta - \cos \theta}{\sqrt{2}}$

D) $\frac{\operatorname{sen} \theta + \cos \theta - 1}{\sqrt{2}}$

E) $\frac{1 - \operatorname{sen} \theta - \cos \theta}{\sqrt{2}}$

RESOLUCIÓN

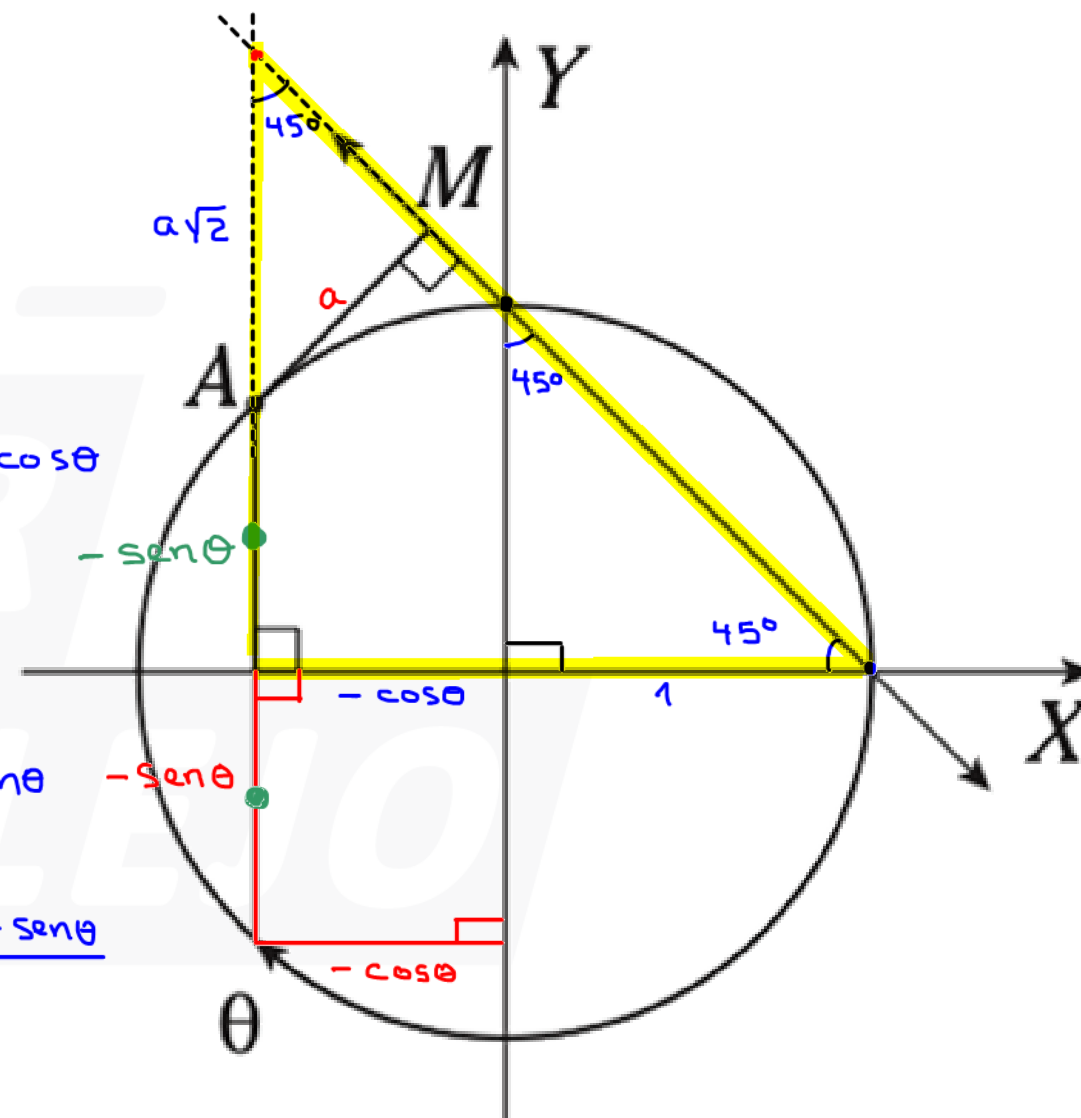
Piden "a"

En la C.T

$$a\sqrt{2} - \operatorname{sen} \theta = 1 - \cos \theta$$

$$a\sqrt{2} = 1 - \cos \theta + \operatorname{sen} \theta$$

$$\therefore a = \frac{1 - \cos \theta + \operatorname{sen} \theta}{\sqrt{2}}$$



- A) $(\sin\theta + \cos\theta)^{-1}$
 B) $(\sin\theta - \cos\theta)^{-1}$
 C) $(\cos\theta - \sin\theta)^{-1}$
 D) $-(\cos\theta - \sin\theta)^{-1}$
 E) $-(\tan\theta + \cot\theta)^{-1}$

RESOLUCIÓN

Finden "a".

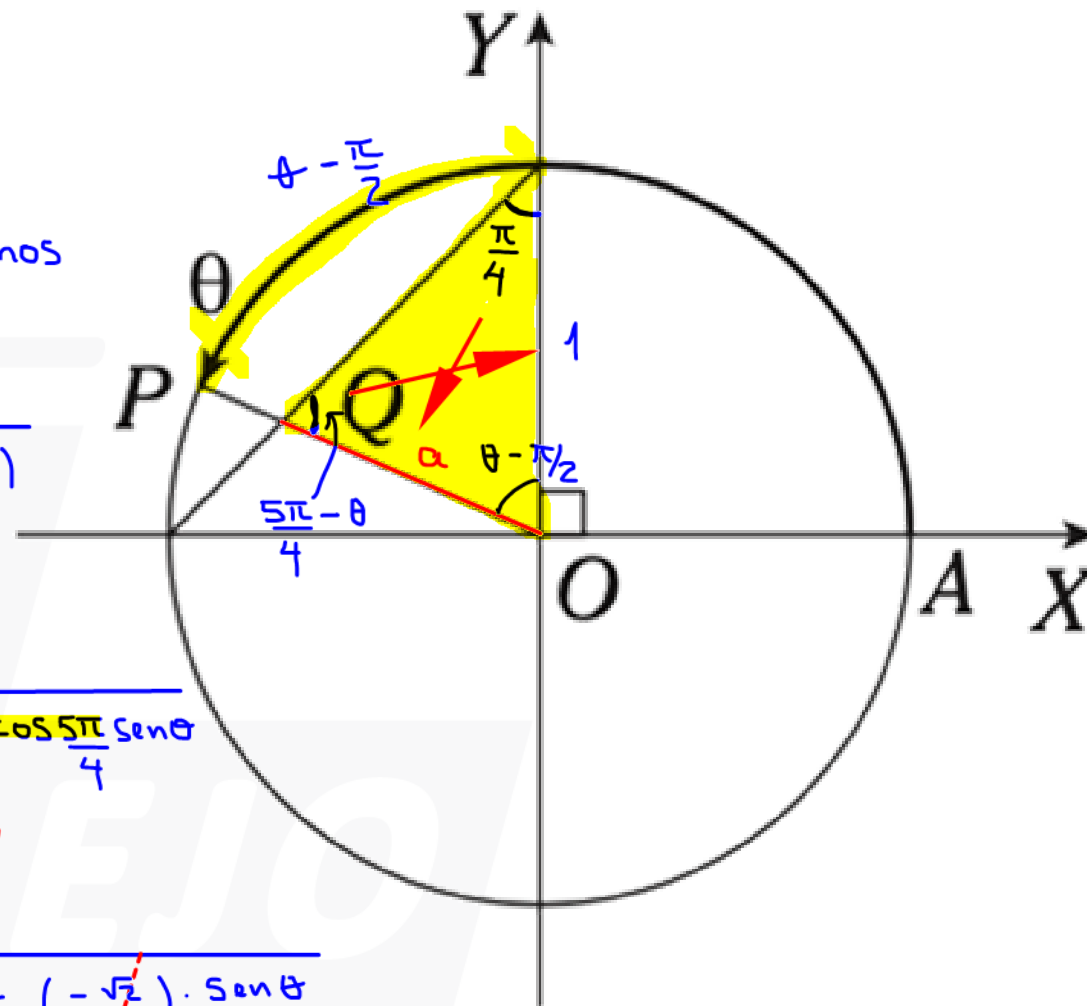
Por teorema de senos

$$\frac{a}{\sin \frac{\pi}{4}} = \frac{1}{\sin(\frac{5\pi}{4} - \theta)}$$

$$a = \frac{\sin \frac{\pi}{4}}{\sin \frac{5\pi}{4} \cos \theta - \cos \frac{5\pi}{4} \sin \theta}$$

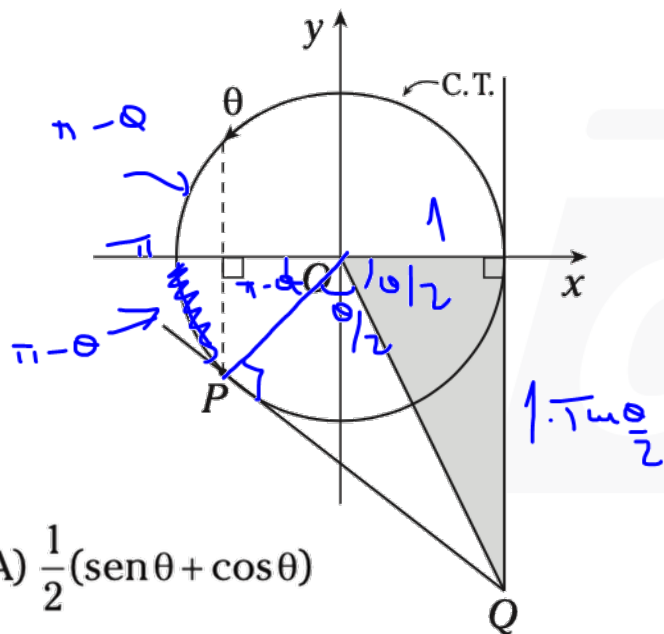
$$a = \frac{\frac{\sqrt{2}}{2}}{\left(-\frac{\sqrt{2}}{2}\right) \cos \theta - \left(-\frac{\sqrt{2}}{2}\right) \cdot \sin \theta}$$

$$a = \frac{1}{-\cos\theta + \sin\theta} \longrightarrow a = \frac{1}{\sin\theta - \cos\theta}$$



PROBLEMA 9

Calcule el área de la región sombreada en términos de θ .



A) $\frac{1}{2}(\sin \theta + \cos \theta)$

B) $2 \cos \frac{\theta}{2}$

C) $\frac{1}{2}(\cos \theta - \sin \theta)$

D) $\frac{1}{2}(\csc \theta - \cot \theta)$

E) $\frac{1}{2}(\csc \theta + \cot \theta)$

$$S = 1 \cdot \frac{\pi \theta}{2}$$

$$S = \frac{\csc \theta - \cot \theta}{2}$$

RESOLUCIÓN

Por el teorema de Pitágoras:

$$a^2 = (a - \sin \theta)^2 + (1 - \cos \theta)^2$$

$$a^2 = a^2 + \sin^2 \theta - 2a \sin \theta +$$

$$1 + \cos^2 \theta - 2 \cos \theta$$

$$2a \sin \theta = 2 - 2 \cos \theta$$

$$a = \frac{1 - \cos \theta}{\sin \theta}$$

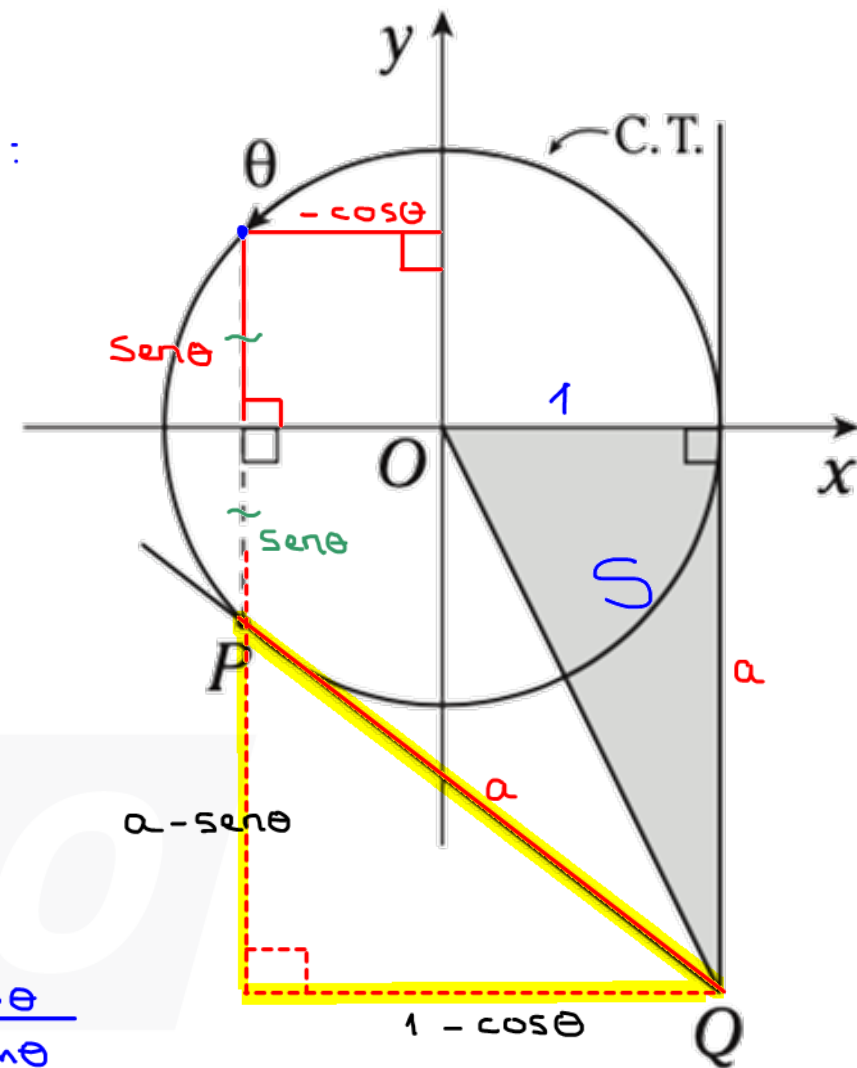
$$a = \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}$$

$$\rightarrow a = \csc \theta - \cot \theta$$

Luego

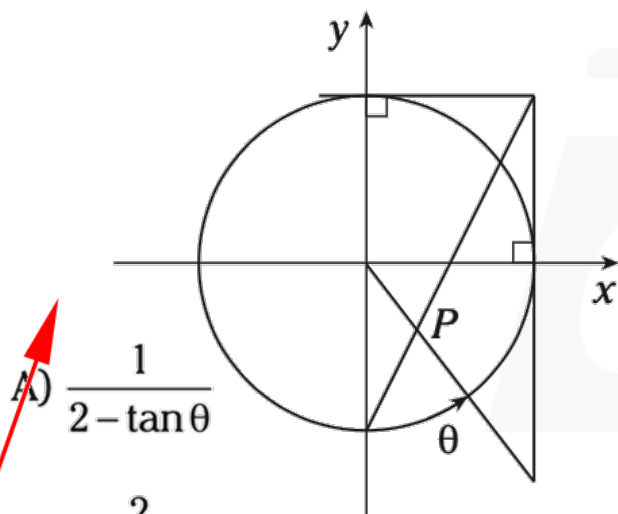
$$S = \frac{1 \cdot a}{2}$$

$$\rightarrow S = \frac{\csc \theta - \cot \theta}{2}$$



PROBLEMA 11

En la circunferencia trigonométrica, calcule la abscisa del punto P en términos de θ .



A) $\frac{1}{2 - \tan \theta}$

B) $\frac{2}{2 - \tan \theta}$

C) $\frac{1}{1 - \tan \theta}$

D) $\frac{2}{1 - \tan \theta}$

E) $\frac{2}{\tan \theta - 2}$

RESOLUCIÓN

Piden "a"

Por semejanza de triángulos

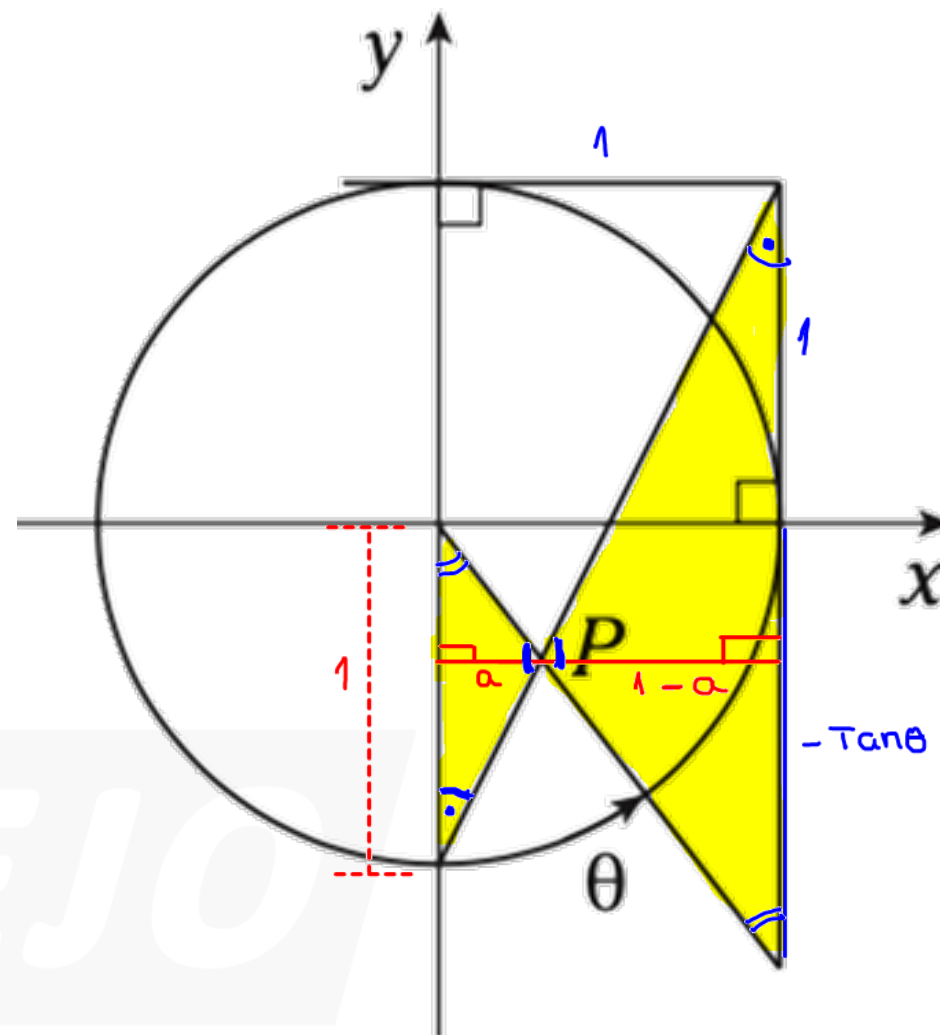
$$\frac{a}{1} = \frac{1-a}{1 - \tan \theta}$$

$$a - a \tan \theta = 1 - a$$

$$2a - a \tan \theta = 1$$

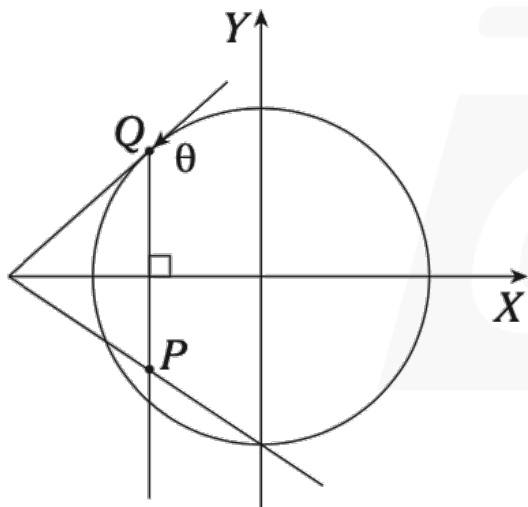
$$a(2 - \tan \theta) = 1$$

$$\therefore a = \frac{1}{2 - \tan \theta}$$

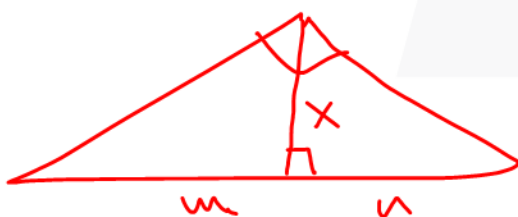


PROBLEMA 15

En el gráfico se muestra la circunferencia trigonométrica. Halle la ordenada del punto P en términos de θ .



- A) $\text{sen}^2\theta$
~~B) $-\text{sen}^2\theta$~~
 C) $\cos\theta$
 D) $-\cos^2\theta$
 E) $\cos\theta - \text{sen}\theta$



$$x^2 = mn$$

RESOLUCIÓN

Piden: $-a$

En el $\triangle AQO$

$$(\text{sen}\theta)^2 = (b)(-\cos\theta)$$

$$b = \frac{\text{sen}^2\theta}{-\cos\theta}$$

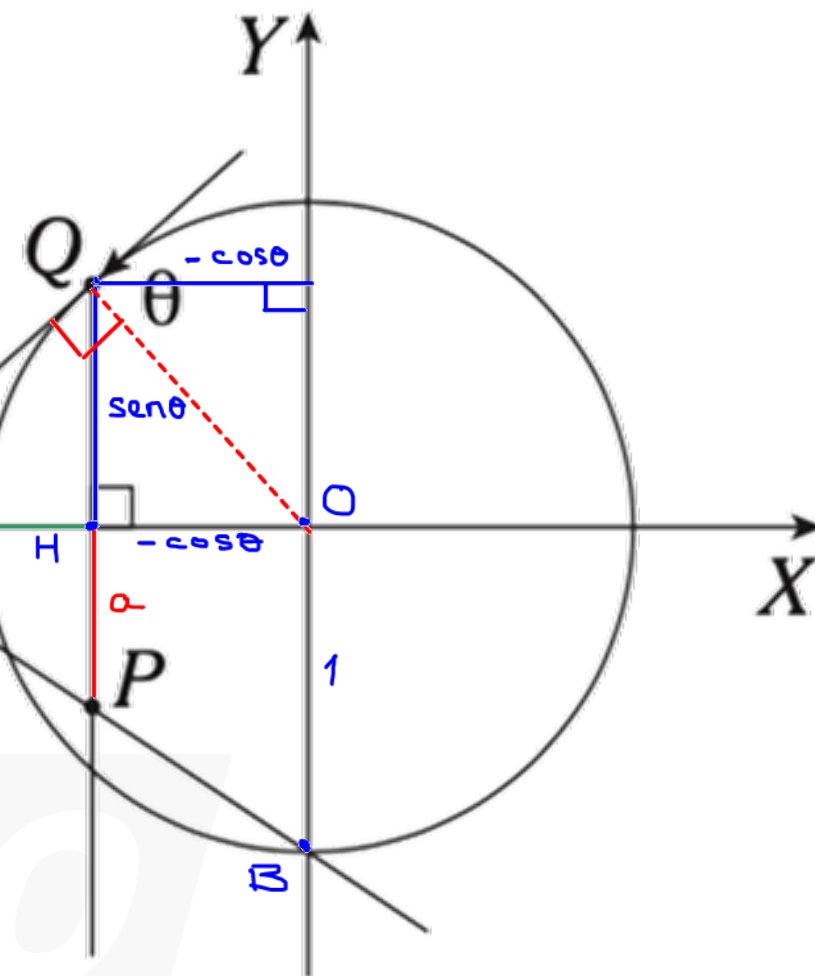
$$\triangle AOB \sim \triangle AHP$$

$$\frac{a}{b} = \frac{1}{b - \cos\theta}$$

$$a = \frac{-\frac{\text{sen}^2\theta}{\cos\theta}}{-\frac{\text{sen}^2\theta}{\cos\theta} - \cos\theta} = \frac{-\text{sen}^2\theta}{-\text{sen}^2\theta - \cos^2\theta} = \frac{-\text{sen}^2\theta}{-(\text{sen}^2\theta + \cos^2\theta)}$$

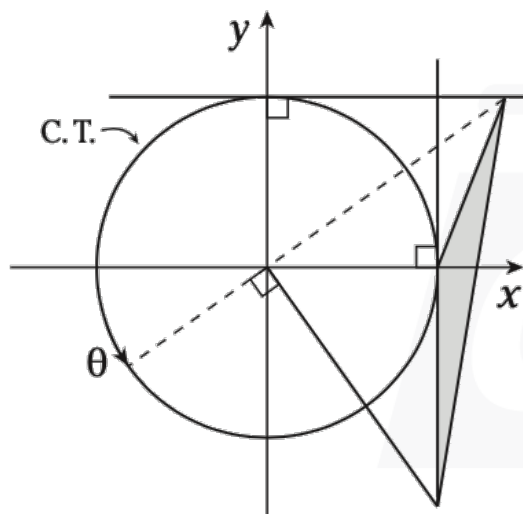
$$\rightarrow a = \text{sen}^2\theta$$

$$\therefore -a = -\text{sen}^2\theta$$



PROBLEMA 17

Del gráfico mostrado, calcule el área de la región sombreada.



A) $\frac{\tan(\theta)}{2}(1 + \tan(\theta))$

B) $\cot(\theta)(1 + \cot(\theta))$

C) $\tan(\theta)(1 + \tan(\theta))$

☒ D) $\frac{\cot(\theta)}{2}(\cot(\theta) - 1)$

E) $2\cot(\theta)(\cot(\theta) - 1)$

RESOLUCIÓN

Piden el área "S"

En la c.T.

$$S = \frac{b \cdot h}{2} \quad \text{--- (I)}$$

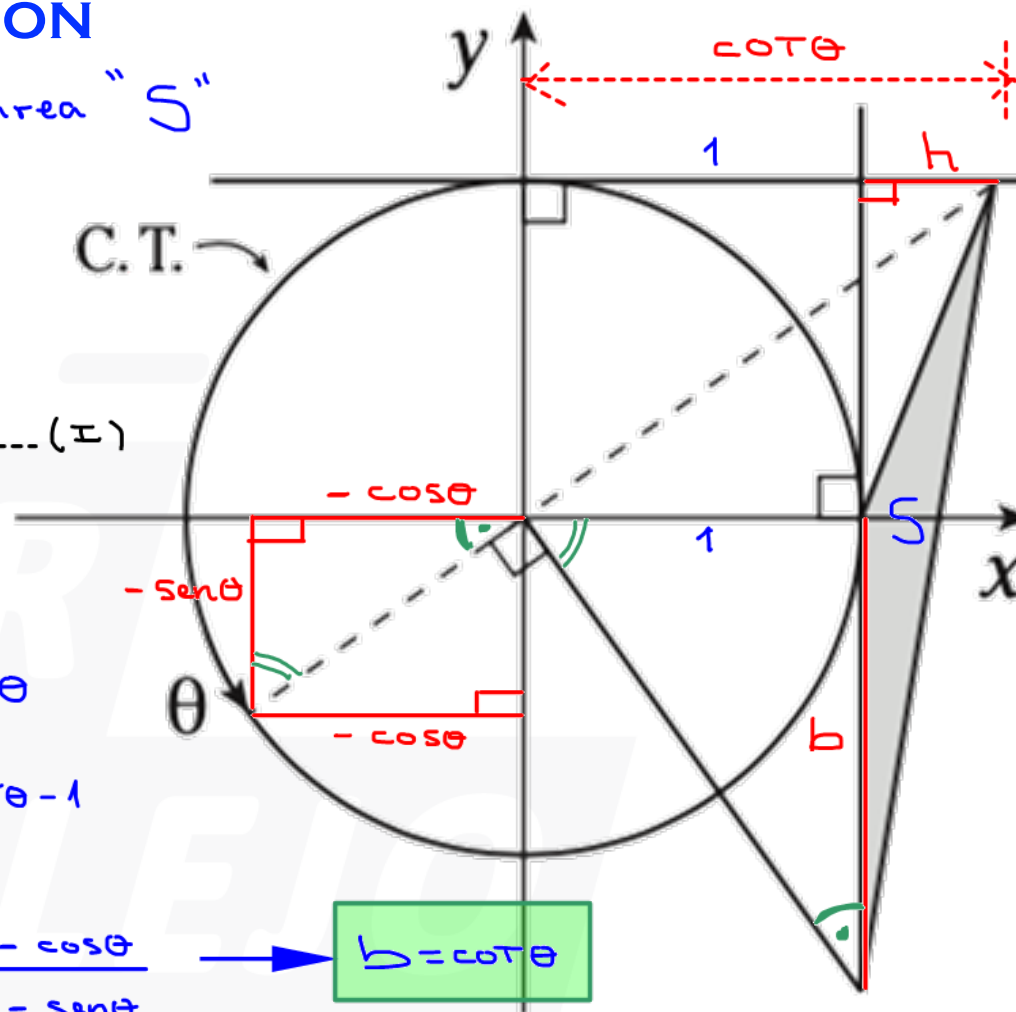
$$\bullet \quad 1 + h = \cot \theta$$

$$\rightarrow h = \cot \theta - 1$$

$$\bullet \quad \frac{b}{1} = \frac{-\cos \theta}{-\sin \theta} \rightarrow b = \cot \theta$$

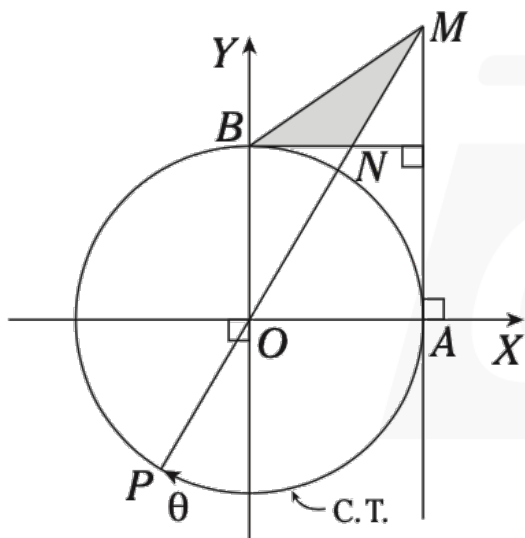
En (I)

$$S = \frac{\cot \theta (\cot \theta - 1)}{2}$$



PROBLEMA 19

Si PM pasa por el origen de coordenadas, determine el área de la región triangular BNM .



A) $\frac{1}{2}(1 + \cot \theta)$

B) $\frac{1}{2}(1 + \tan \theta)$

C) $\frac{1}{2}(1 - \cot \theta)$

D) $\frac{1}{2}(1 - \tan \theta)$

E) $\frac{1}{2}(\tan \theta - \cot \theta)$

RESOLUCIÓN

Piden el área "S"

$$S = \frac{b \cdot h}{2} \quad \text{----- (I)}$$

$$\bullet b = \cot \theta$$

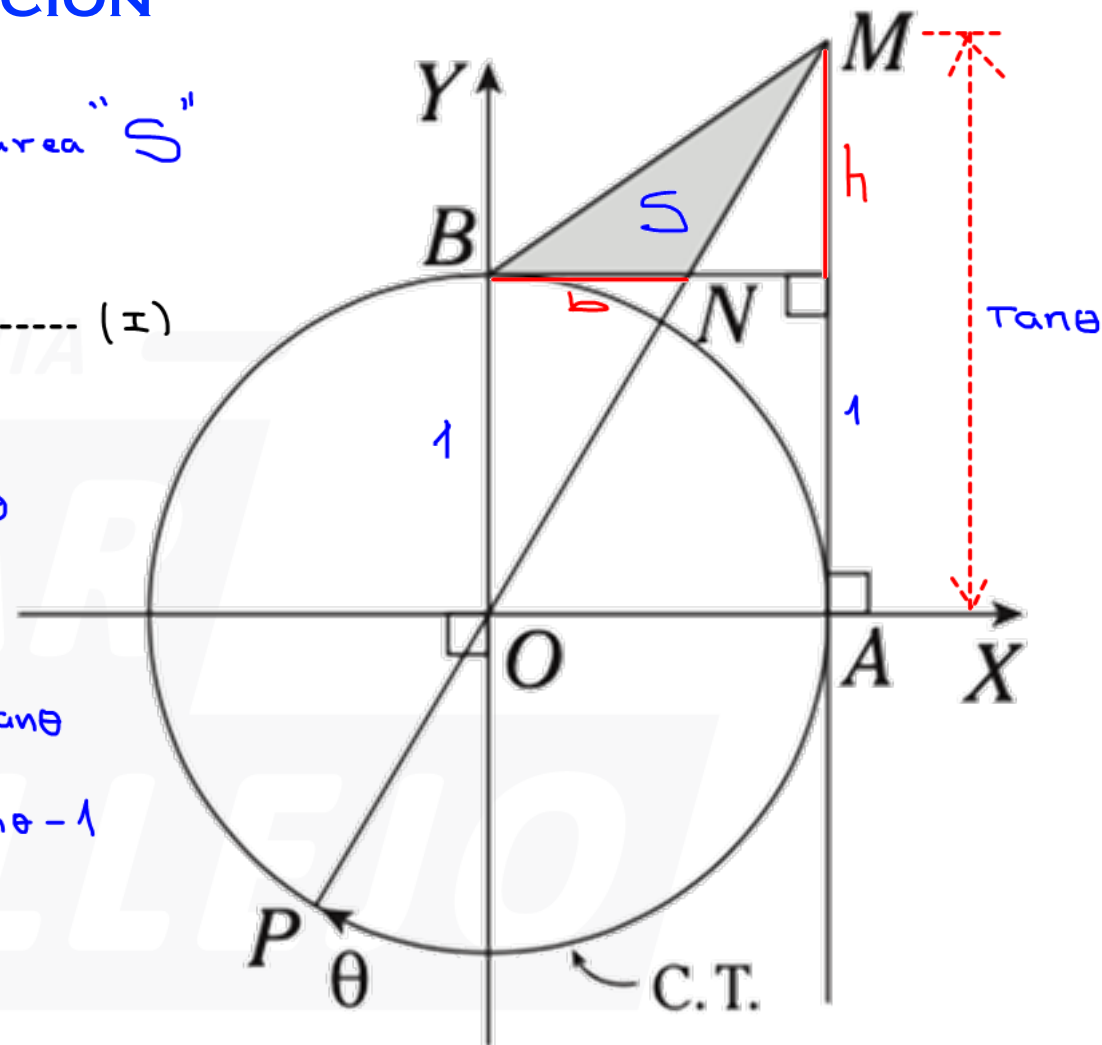
$$\bullet h + 1 = \tan \theta$$

$$h = \tan \theta - 1$$

$$\text{En (I)}$$

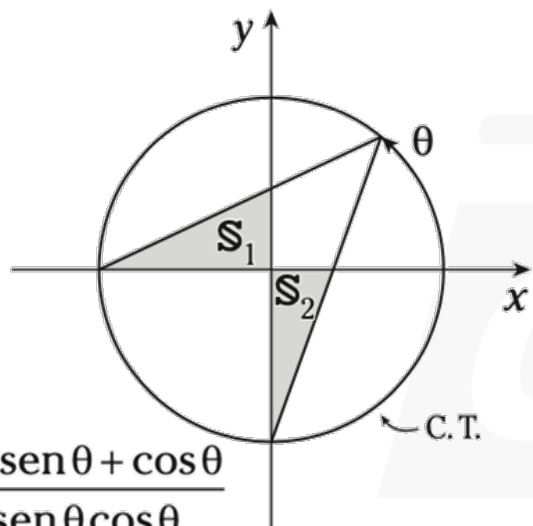
$$S = \frac{\cot \theta (\tan \theta - 1)}{2} \longrightarrow S = \frac{\cot \theta \tan \theta - \cot \theta}{2}$$

$$\therefore S = \frac{1 - \cot \theta}{2}$$



PROBLEMA 21

En el gráfico, calcule $S_1 + S_2$ en términos de θ .



RESOLUCIÓN

$$S_1 = \frac{1 \cdot \tan \frac{\theta}{2}}{2} = \frac{\tan \frac{\theta}{2}}{2}$$

$$S_1 = \frac{\csc \theta - \cot \theta}{2}$$

$$\rightarrow S_1 = \frac{1 - \cos \theta}{2 \sin \theta}$$

$$S_2 = \frac{1 \cdot \tan(\frac{\pi}{4} - \frac{\theta}{2})}{2}$$

$$S_2 = \frac{\csc(\frac{\pi}{2} - \theta) - \cot(\frac{\pi}{2} - \theta)}{2}$$

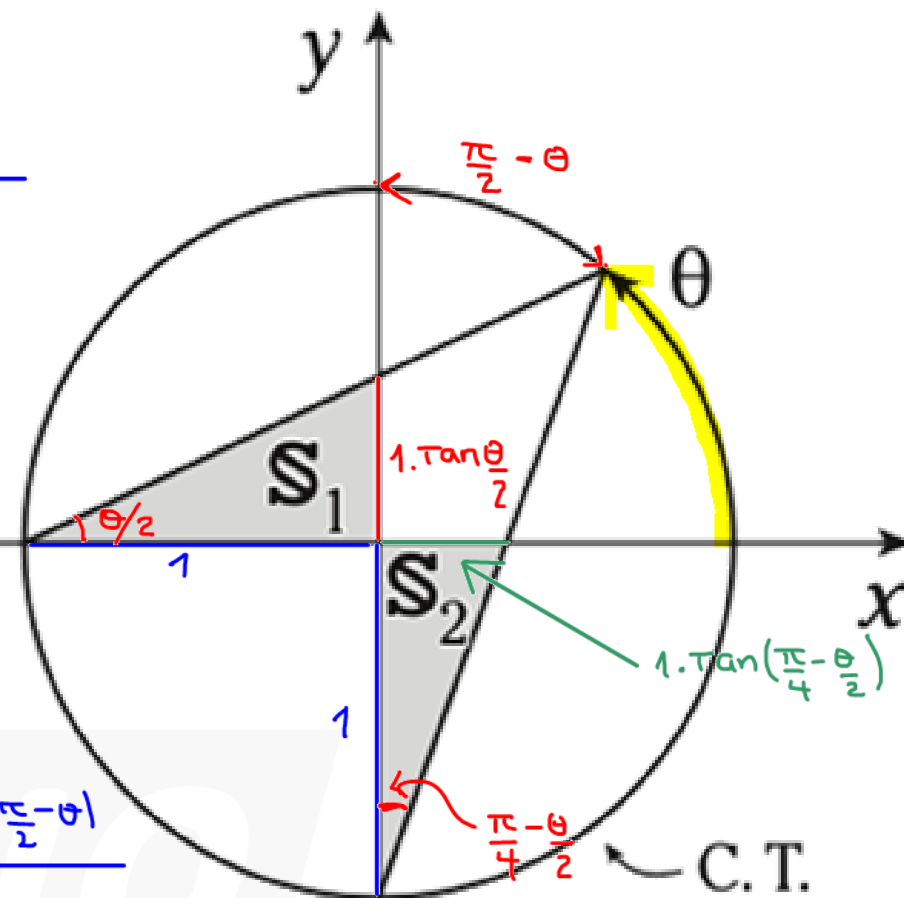
$$S_2 = \frac{\sec \theta - \tan \theta}{2} \rightarrow$$

$$S_2 = \frac{1 - \sin \theta}{2 \cos \theta}$$

Luego

$$S_1 + S_2 = \frac{1 - \cos \theta}{2 \sin \theta} + \frac{1 - \sin \theta}{2 \cos \theta} = \frac{2 \cos \theta - 2 \cos^2 \theta + 2 \sin \theta - 2 \sin^2 \theta}{4 \sin \theta \cos \theta}$$

$$\therefore S_1 + S_2 = \frac{\sin \theta + \cos \theta - 1}{2 \sin \theta \cos \theta}$$



A) $\frac{1 + \sin \theta + \cos \theta}{\sin \theta \cos \theta}$

B) $\frac{\sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta}$

C) $\frac{1 - \sin \theta + \cos \theta}{\sin \theta \cos \theta}$

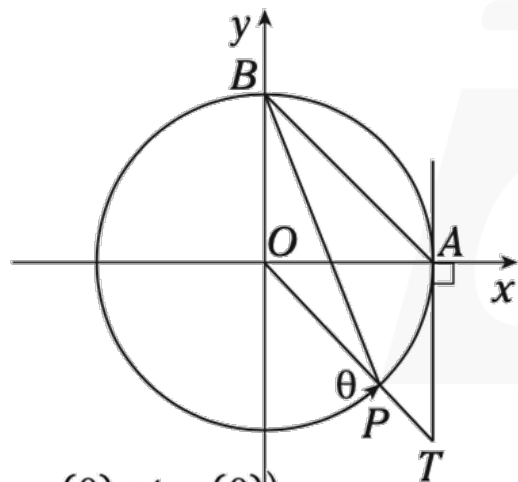
D) $\frac{1 + \sin \theta - \cos \theta}{2}$

E) $\frac{\sin \theta + \cos \theta - 1}{2 \sin \theta \cos \theta}$

$$\tan x = \csc 2x - \cot 2x$$

PROBLEMA 23

En el gráfico muestra una circunferencia trigonométrica con el arco θ cuyo extremo del arco es P . Determine el área de la región cuadrangular $ABPT$ (en u^2).



A) $\frac{1}{2}(1 + \cos(\theta) + \tan(\theta))$

B) $\frac{1}{2}(1 - \cos(\theta) - \tan(\theta))$

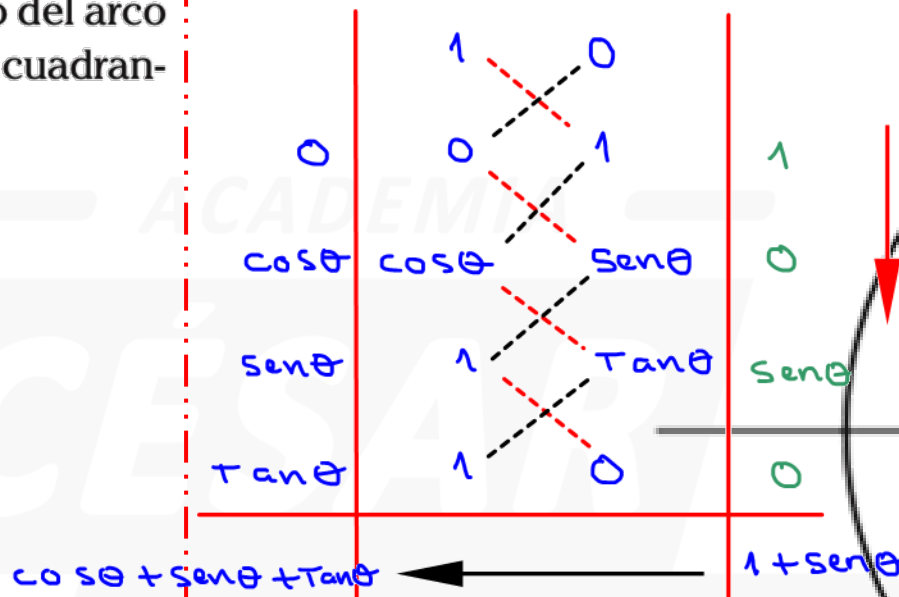
C) $\frac{1}{2}(1 + \sin(\theta) + \tan(\theta))$

D) $\frac{1}{2}(1 - \sin(\theta) - \tan(\theta))$

E) $\frac{1}{2}(\sin(\theta) + \cos(\theta))$

RESOLUCIÓN

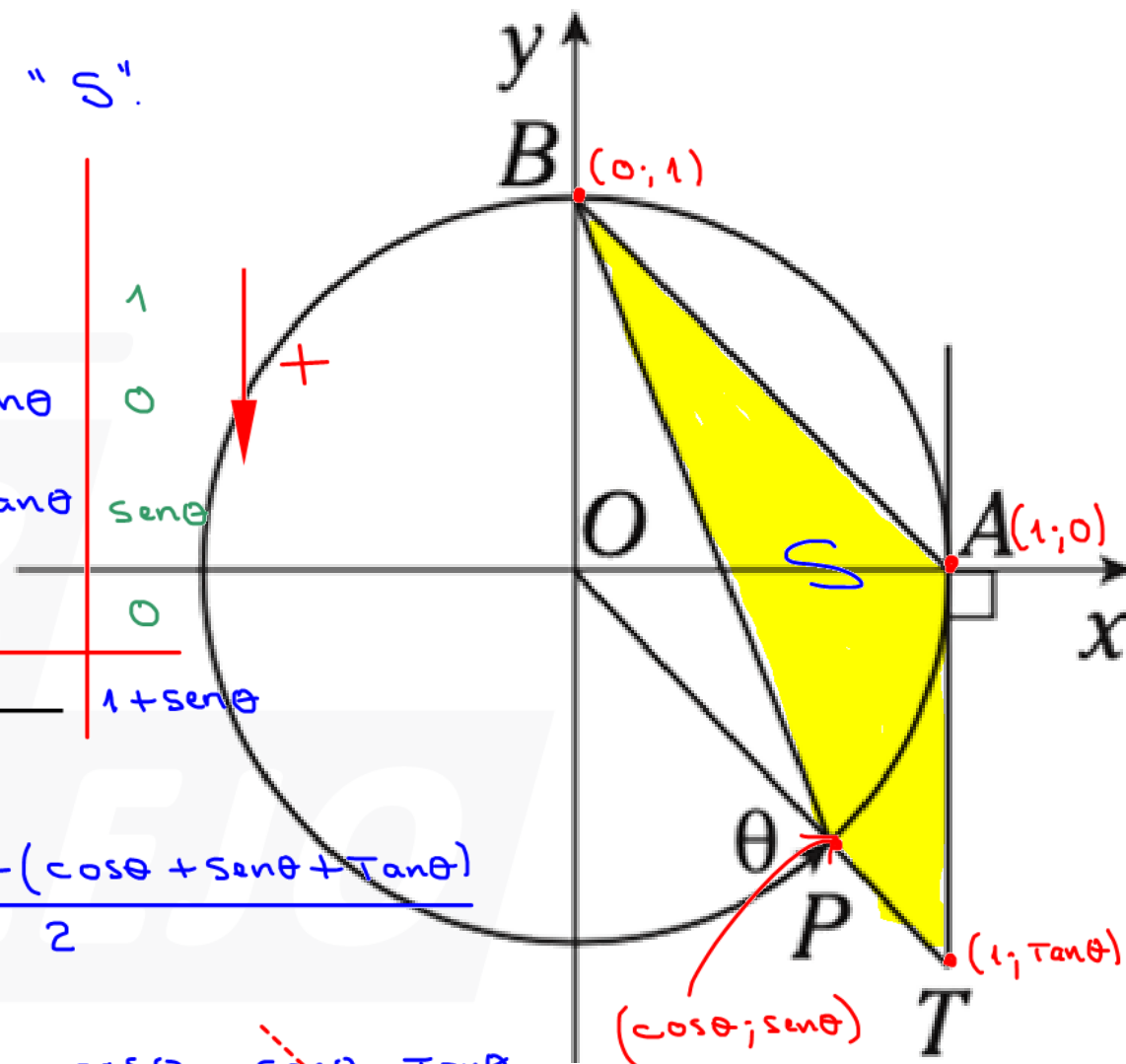
Piden el área "S".



$$S = \frac{(1 + \sin \theta) - (\cos \theta + \sin \theta + \tan \theta)}{2}$$

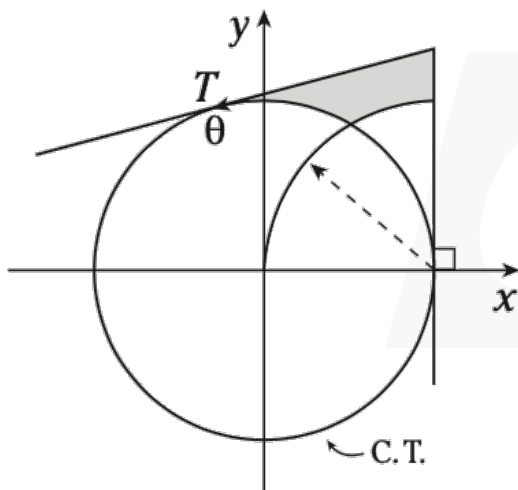
$$S = \frac{1 + \sin \theta - \cos \theta - \sin \theta - \tan \theta}{2}$$

$$\therefore S = \frac{1 - \cos \theta - \tan \theta}{2}$$



PROBLEMA 25

Si \mathbb{S} es el área de la región sombreada, calcule $\mathbb{S} + \frac{\sqrt{3}}{4} + \frac{\pi}{6}$. Considere que T es punto de tangencia.



A) $\frac{2\csc\theta + \cot\theta}{2}$

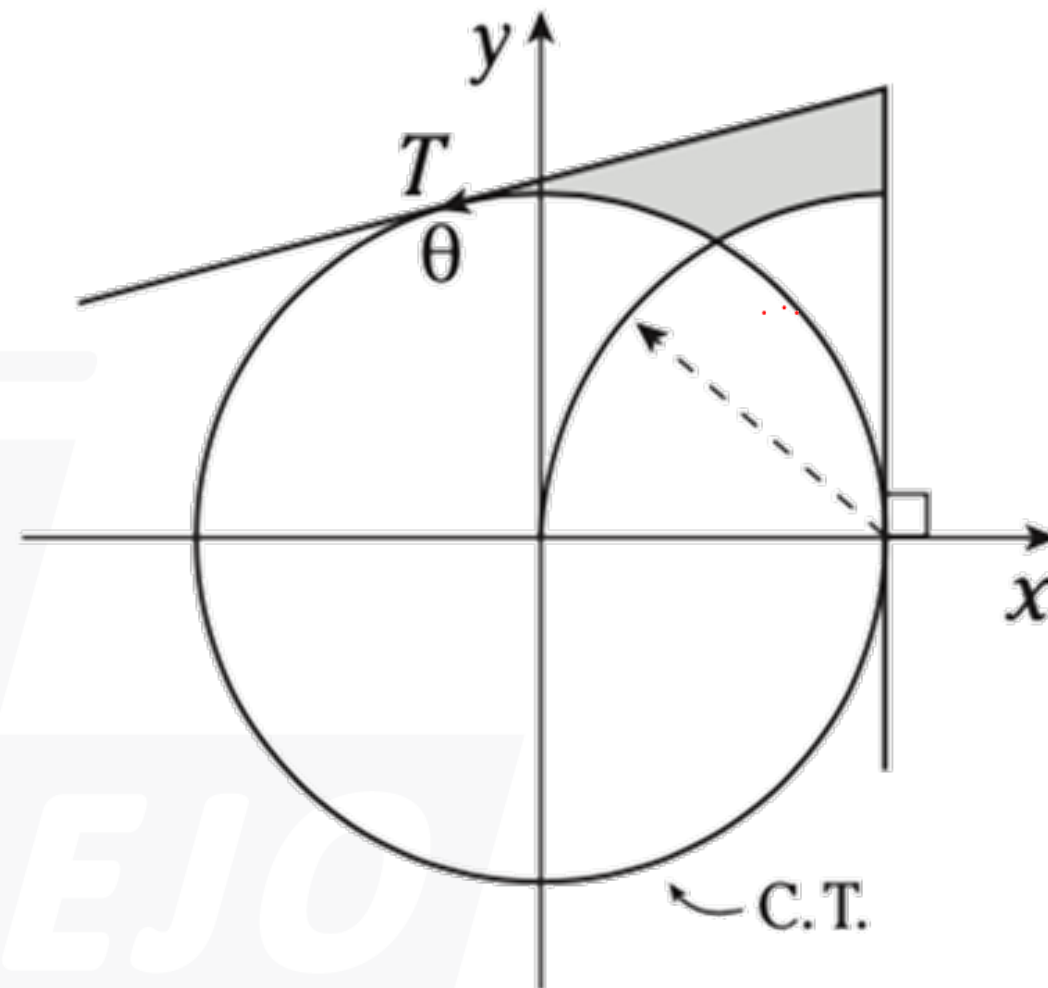
B) $\frac{2\sec\theta - \csc\theta}{2}$

C) $\frac{\csc\theta + 2\cot\theta}{2}$

D) $\frac{2\csc\theta - \cot\theta}{2}$

E) $\frac{\csc\theta - 2\cot\theta}{2}$

RESOLUCIÓN



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