

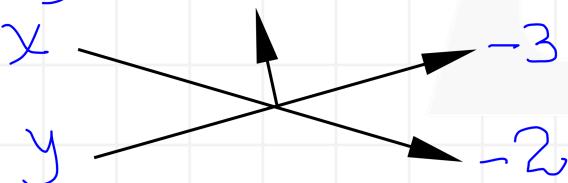
EJERCICIO

Grafique $xy - 2x - 3y + 6 = 0$.

Resolución

Se tiene:

$$xy - 2x - 3y + 6 = 0$$

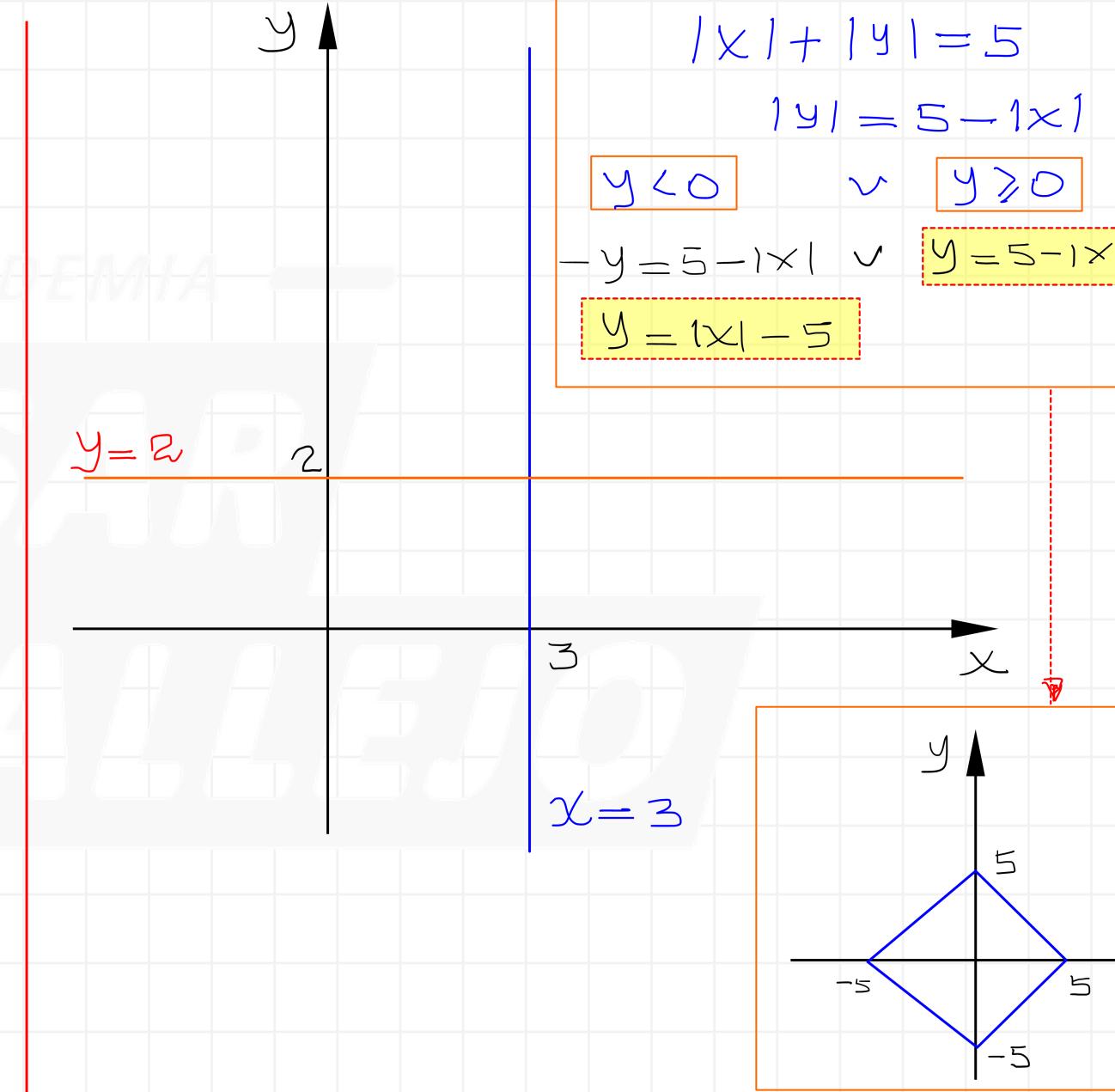


$$(x-3)(y-2) = 0$$

$$x-3 = 0 \vee y-2 = 0$$

$$x=3 \quad \vee \quad y=2$$

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ÁLGEBRA

Gráfica de relaciones

Semana 07

Docente: César Velásquez M.

1. La gráfica de la relación

$$R = \{(x; y) \in \mathbb{R} / y^2 - xy - 2x^2 = 0 \vee xy - 4x - 3y + 12 = 0\}$$

genera un cuadrilátero. Determine el área de dicho cuadrilátero.

A) $\frac{25}{2}$

B) 12

C) $\frac{23}{2}$

D) 11

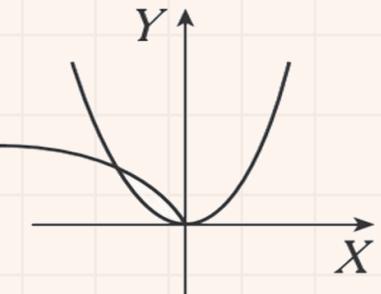
E) $\frac{21}{2}$

Resolución

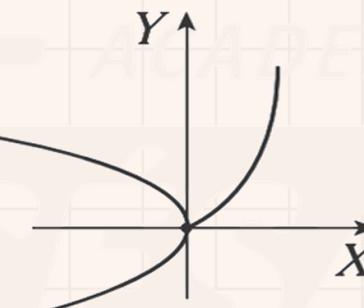
2. Determine la gráfica de la siguiente relación:

$$f = \{(x; y) \in \mathbb{R}^2 / x = y^2 \vee x = \sqrt{y}\}$$

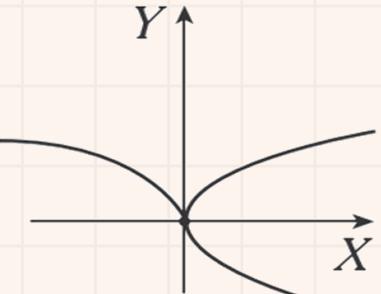
A)



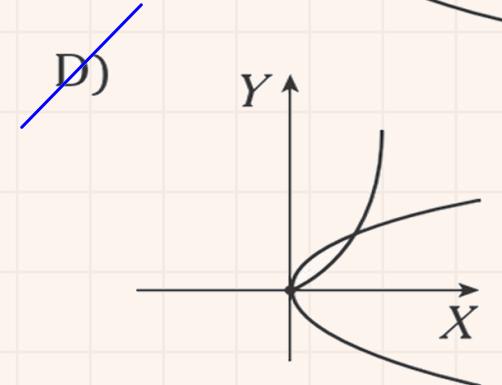
B)



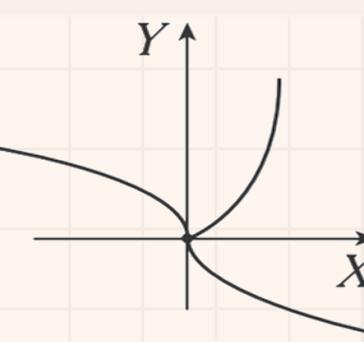
C)



D)



E)

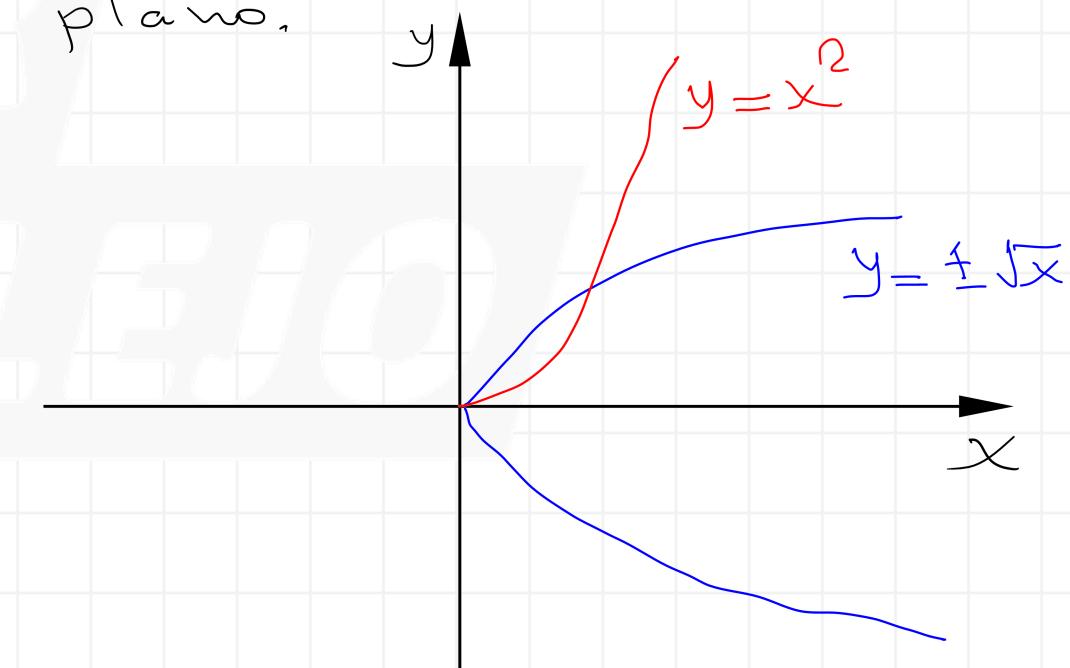


Resolución

$$\begin{cases} x = y^2 \\ y = \pm \sqrt{x} \\ x > 0 \end{cases}$$

$$\begin{array}{l} \checkmark x = \sqrt{y} \\ \checkmark y = x^2 \\ x > 0 \end{array}$$

Graficando en el mismo
plano.



3. Dados los conjuntos

$$A = \{(x; y) \in \mathbb{R}^2 / |x| \leq 3 \wedge |y| \leq 3\}$$

$$B = \{(x; y) \in \mathbb{R}^2 / y \geq |x - h|\}$$

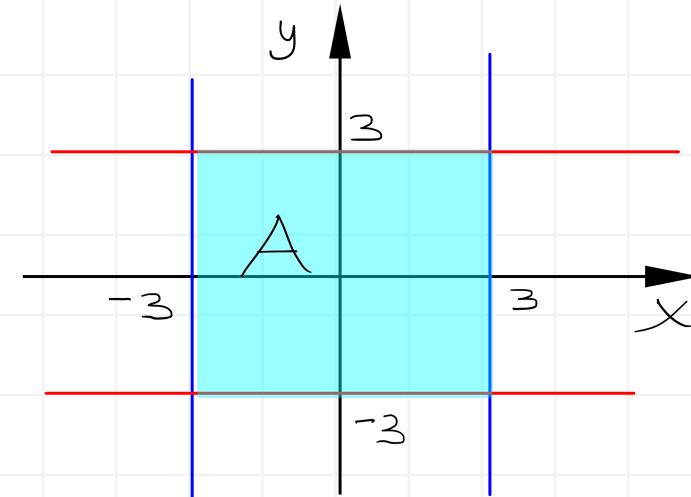
$$C = A \cap B$$

si el área de la región que representa el conjunto C es $4,5 \text{ m}^2$, halle el conjunto de valores de h .

- A) $(-3; 3)$
- B) $(0; 3]$
- C) $\{3; -3\}$
- D) $[-3; 0]$
- E) $\{4; -4\}$

Resolución

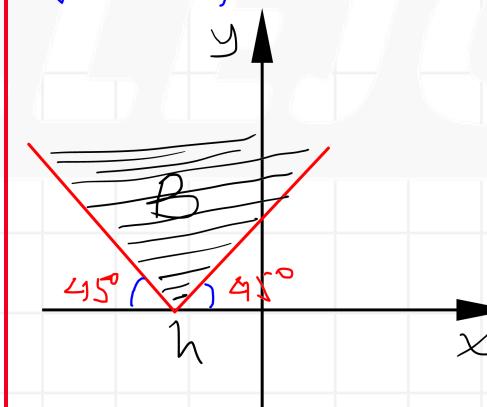
De A: $-3 \leq x \leq 3 \wedge -3 \leq y \leq 3$



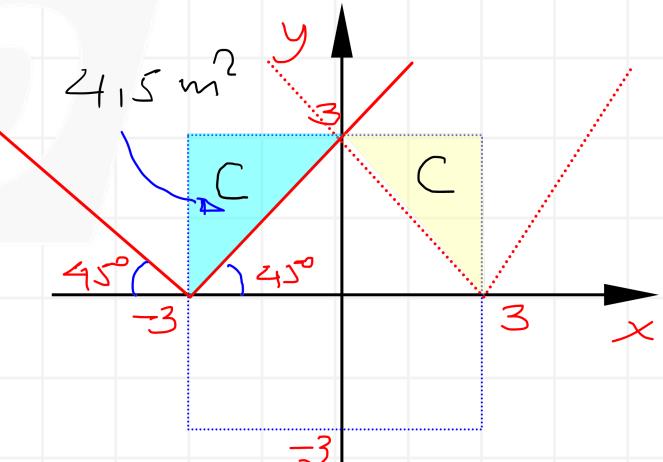
DE B:

$$y \geq |x - h|$$

$$\forall (h; 0)$$



Luego $C = A \cap B$



$\therefore h = \{-3; 3\}$

4. Determine el área generada por la relación

$$R = \{(x; y) \in \mathbb{R}^2 / x^2 + y^2 - 4x - 2y + 1 \leq 0 \wedge x + y - 3 \geq 0\}$$

- A) π
- B) 2π
- C) 4π
- D) $\frac{3\pi}{2}$
- E) $\frac{\pi}{2}$

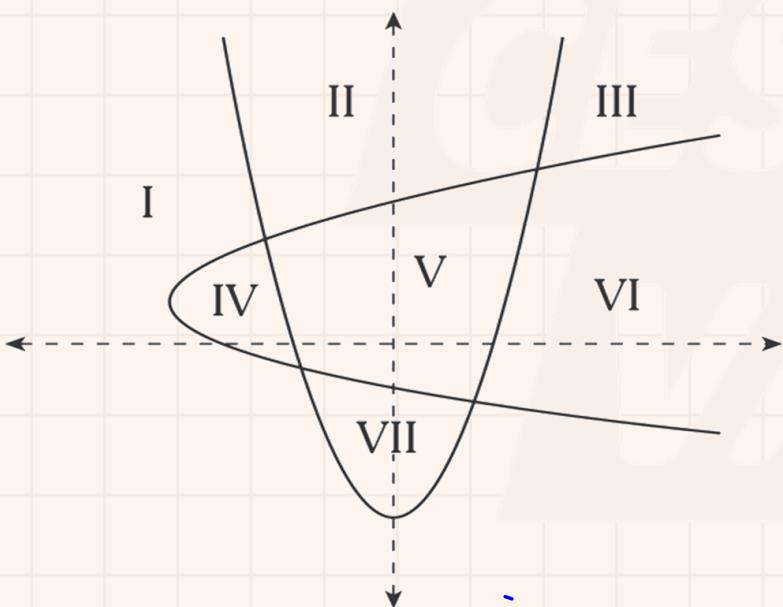
Resolución

Resolución

5. Sea el siguiente sistema de inecuaciones:

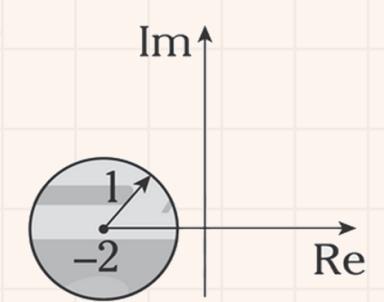
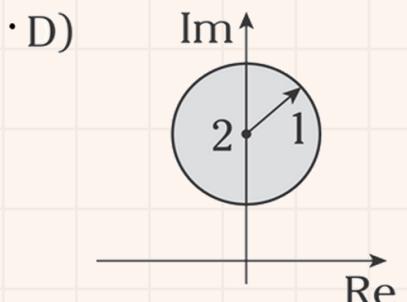
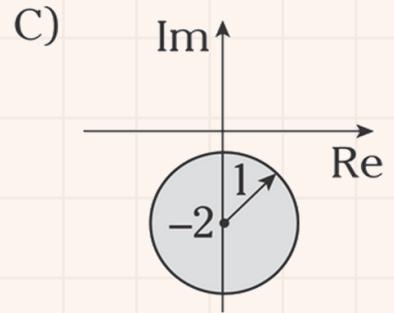
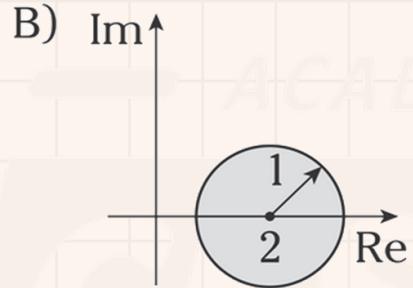
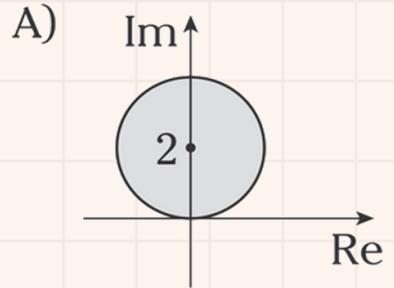
$$\begin{cases} x^2 - y - 4 \geq 0 \\ x - y^2 + 2y + 3 \geq 0 \end{cases}$$

entonces, el conjunto solución está representado por la región



- A) IV y VI
- B) V
- C) II y VII
- D) I y III
- E) II y VI

6. Grafique la región determinada por todos los números complejos z , tal que $|z-2i| \leq 1$.

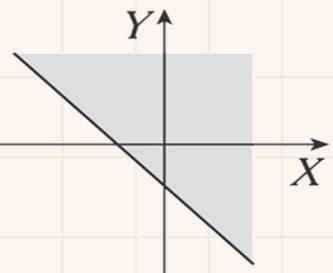
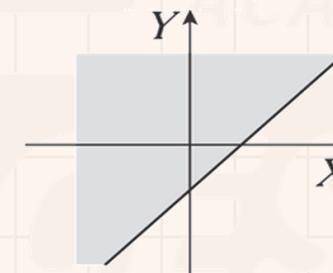
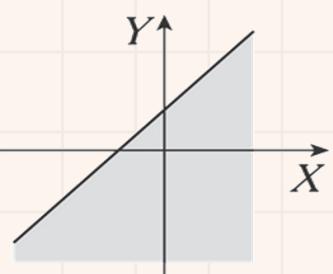
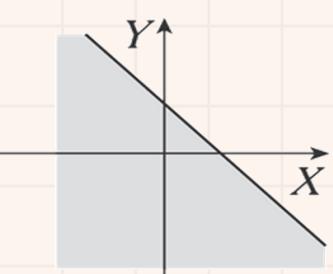
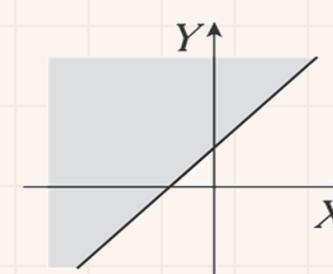


Resolución

Resolución

7. Determine la gráfica que mejor represente al conjunto

$$A = \{z - 1 + 2i \in \mathbb{C} / 2\operatorname{Im}(z) - 3\operatorname{Re}(z) \leq 3\}$$

- A) 
- B) 
- C) 
- D) 
- E) 

8. Determine el número de puntos en común entre las gráficas de las relaciones siguientes:

$$f = \{(x; y) \in \mathbb{R}^2 \mid x = \sqrt{4 - y^2}\}$$

$$g = \{(x; y) \in \mathbb{R}^2 \mid x = \sqrt{4 - y}\}$$

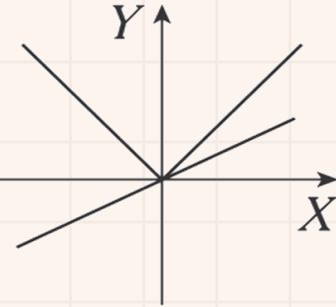
- A) 1
- B) 2
- C) 3
- D) 4
- E) 5

Resolución

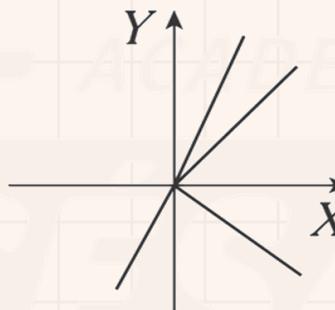
9. Determine la gráfica de

$$f = \{(x; y) \in \mathbb{R}^2 / (x - 2y)(x - |y|) = 0\}$$

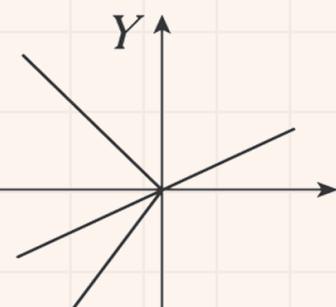
A)



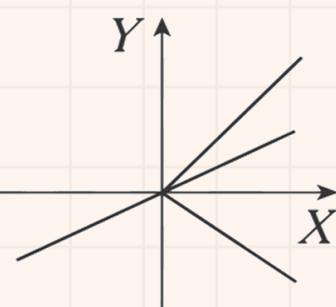
B)



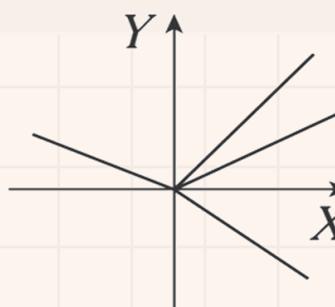
C)



D)



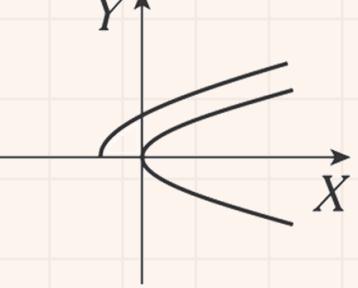
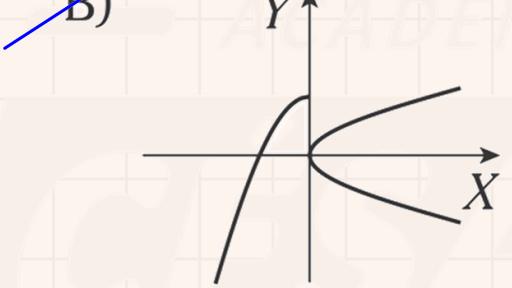
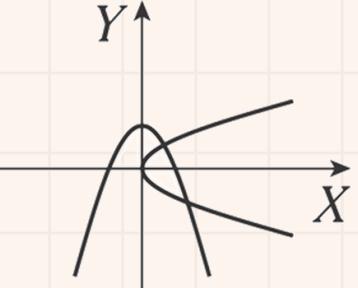
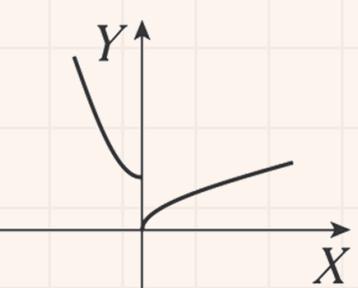
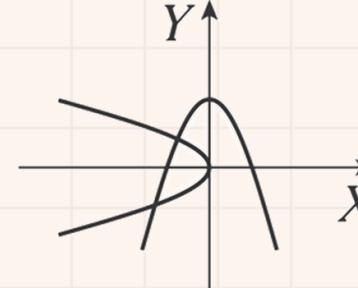
E)



Resolución

10. Determine la gráfica de la siguiente relación:

$$f = \{(x; y) \in \mathbb{R}^2 / (x - y^2)(x + \sqrt{1-y}) = 0\}$$

- A) 
- B) 
- C) 
- D) 
- E) 

Resolución

De la igualdad:

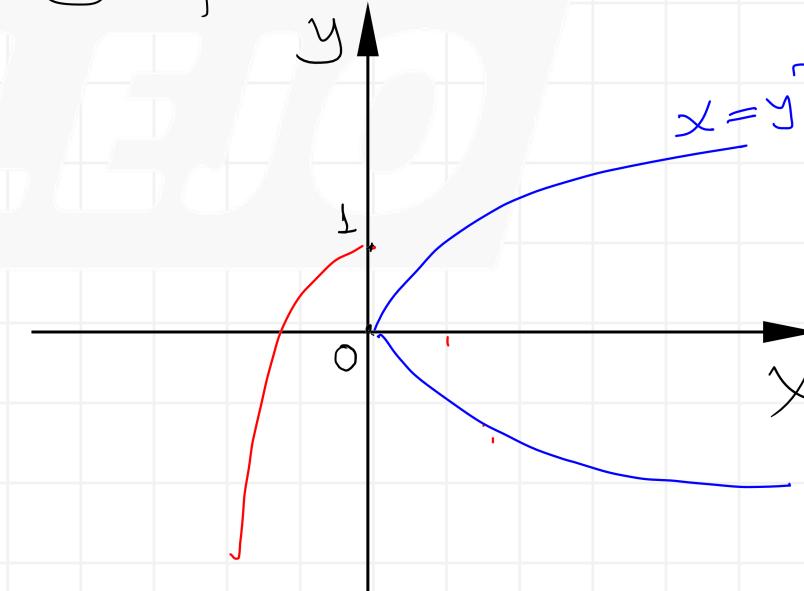
$$x - y^2 = 0 \quad \vee \quad x + \sqrt{1-y} = 0 ; 1-y \geq 0$$

$$x = y^2 \quad \vee \quad \sqrt{1-y} = -x$$

$$x \geq 0 \quad \vee \quad 1-y = x^2 ; -x \geq 0$$

$$(x = y^2 ; x \geq 0) \vee (y = 1-x^2 ; x \leq 0) \quad y \leq 1$$

Graficando.

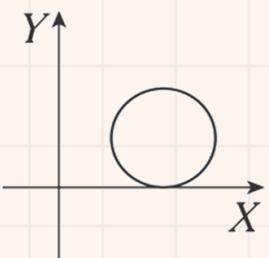


Resolución

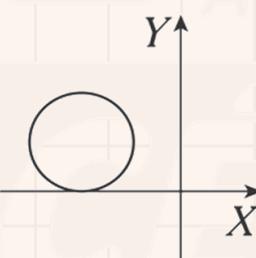
11. Determine la gráfica que mejor represente al conjunto

$$A = \{(x; y) \in \mathbb{R}^2 \mid x^2 + y^2 - 4x + 6y + 9 = 0\}$$

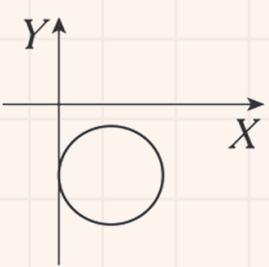
A)



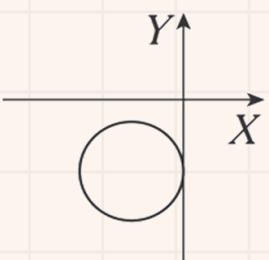
B)



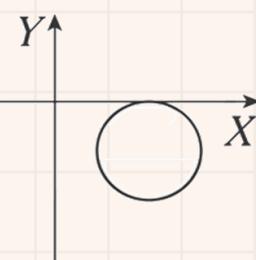
C)



D)



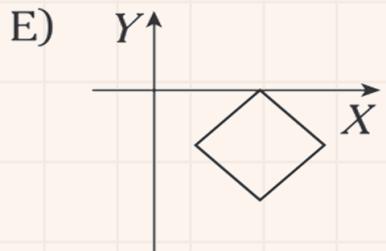
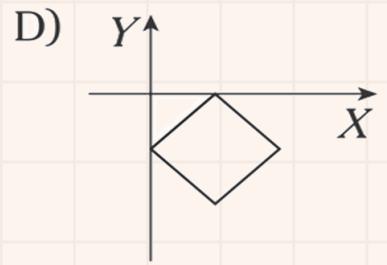
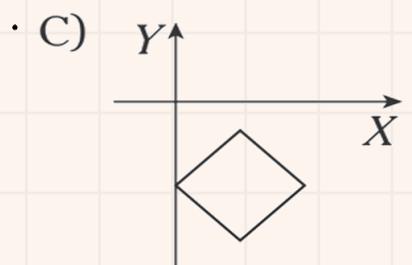
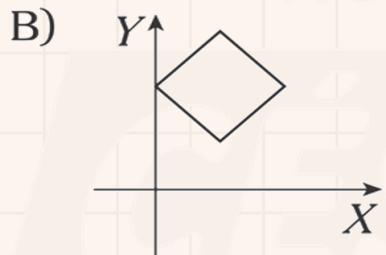
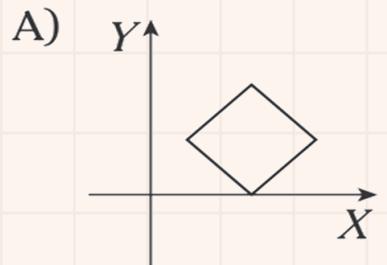
E)



Resolución

12. Determine la gráfica que mejor represente al conjunto

$$A = \{(x; y) \in \mathbb{R}^2 / |x - 2| + |y + 3| = 2\}$$

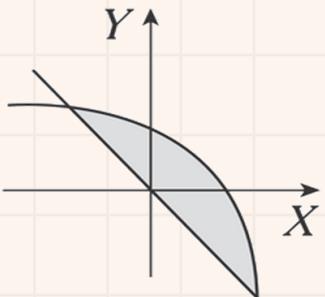


Resolución

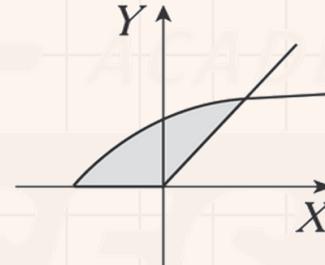
13. Determine todos los $(x; y) \in \mathbb{R}^2$, tal que

$$|x| - x \leq y \leq \sqrt{4 - x}$$

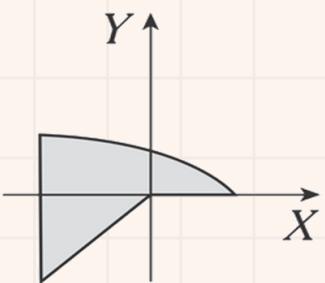
A)



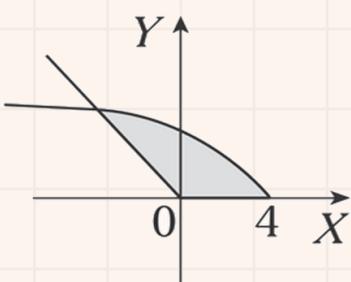
B)



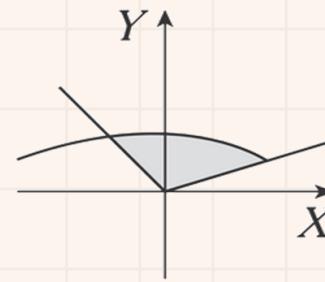
C)



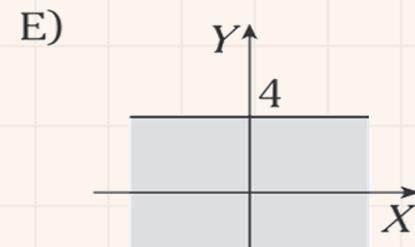
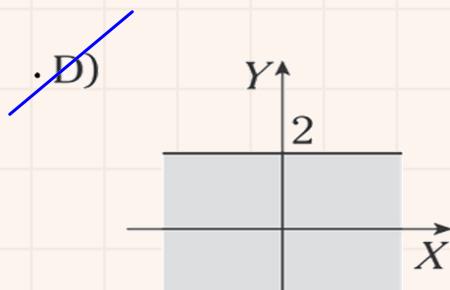
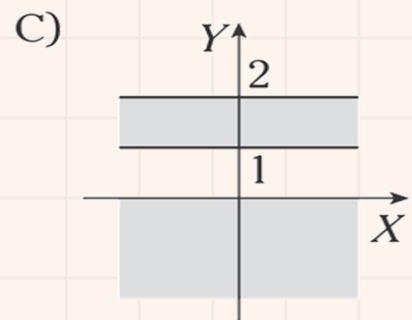
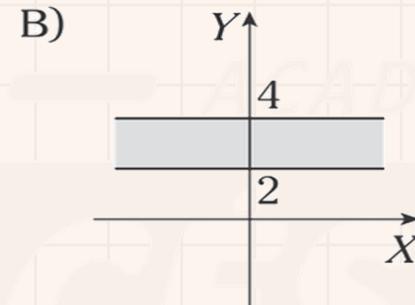
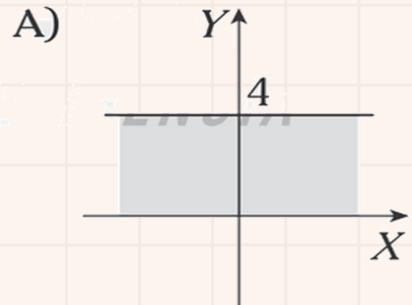
. D)



E)



14. Determine la gráfica de la relación
 $R = \{(x; y) \in \mathbb{R}^2 / |y| + y \leq 4\}$



Resolución

Tenemos: $|y| + y \leq 4$

Ahora

$y \leq 0$

$|y| + y \leq 4$

$0 \leq 4$ (Verdad)

$y \geq 0$

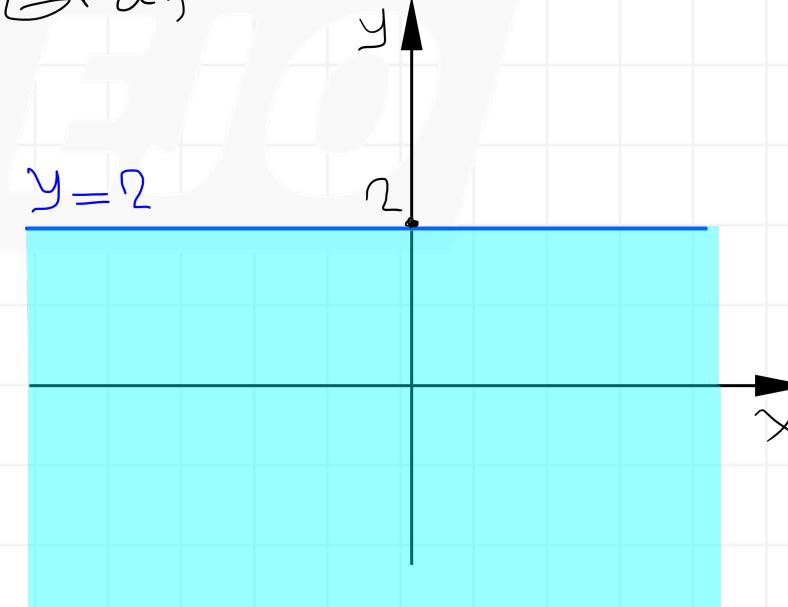
✓

$|y| + y \leq 4$

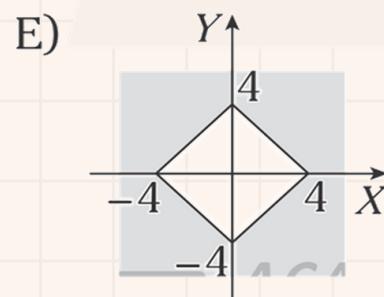
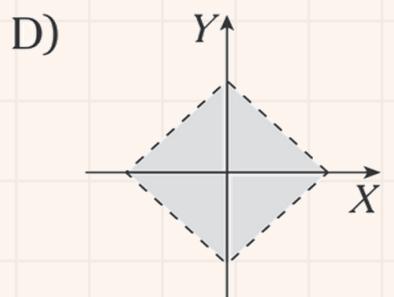
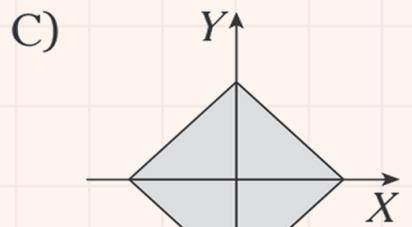
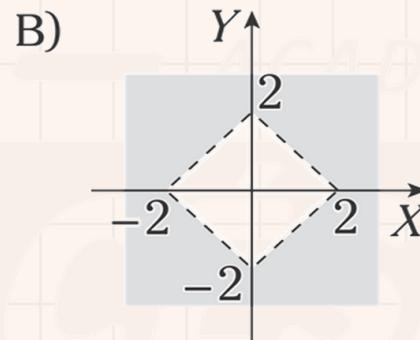
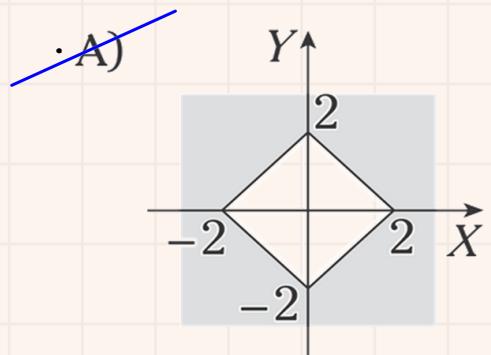
$|2y| \leq 4$

$y \leq 0$ ✓ $y \leq 2 ; y \geq 0$

Graficando:



15. Sea $\|(x; y)\| = |x| + |y|$ para $(x; y) \in \mathbb{R}^2$. Determine la región C , donde $C = \{(x; y) : \|(x; y)\| \geq 2\}$.

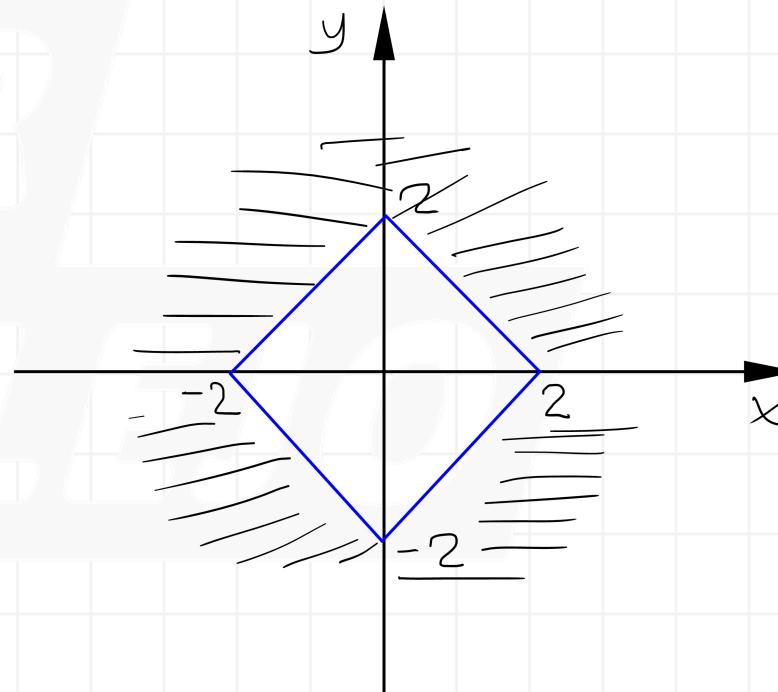


Resolución

Se tiene $\|(x; y)\| \geq 2$

Pero por dato: $\|(x; y)\| = |x| + |y|$

$$\rightarrow |x| + |y| \geq 2$$



16.

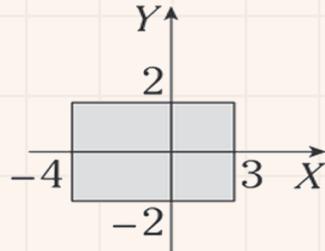
Dados los conjuntos

$$A = \{x \in \mathbb{R} / 2(x+3)(x-3) \leq (x+2)(x-3)\}$$

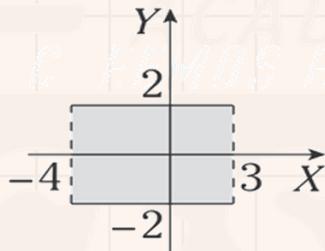
$$B = \{x \in \mathbb{R} / 4 - x^2 > 0\}$$

si $R = \{(x; y) \in \mathbb{R}^2 / x \in B \wedge y \in A\}$, halle la gráfica de R .

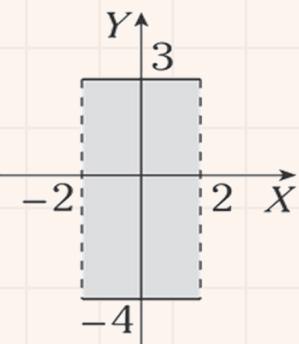
A)



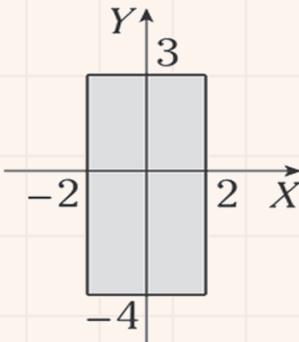
B)



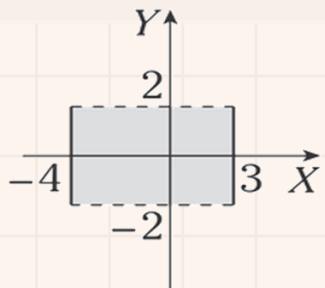
. C)



D)



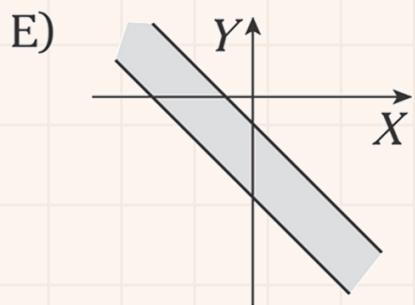
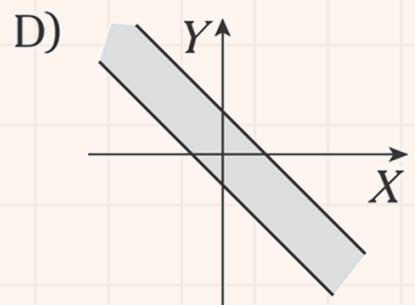
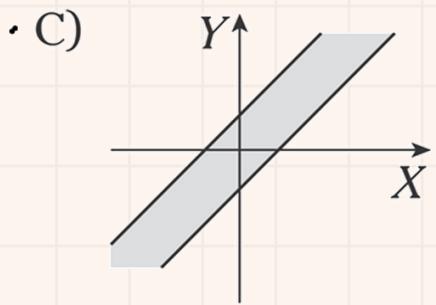
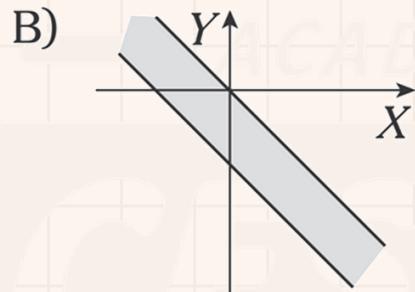
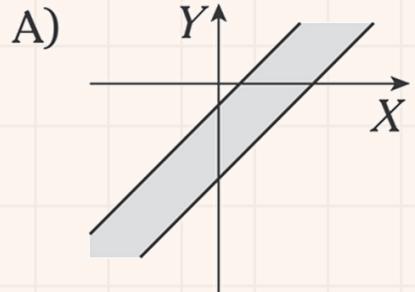
E)



Resolución

17. Determine la gráfica de la siguiente relación:

$$A = \{(x; y) \in \mathbb{R}^2 \mid |y - 2x + 3| \leq 5\}$$



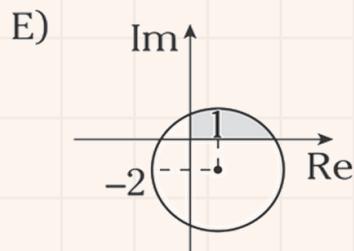
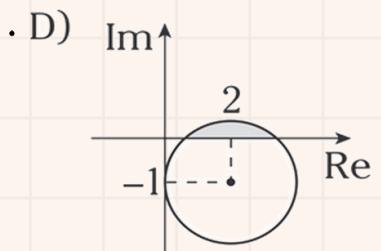
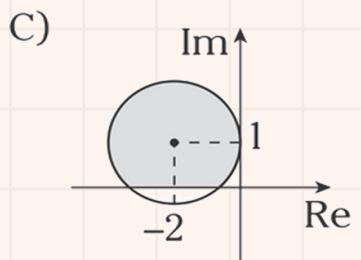
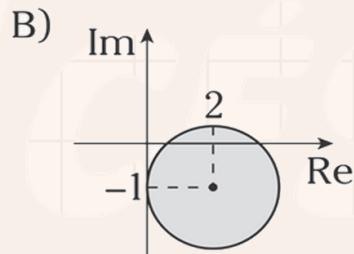
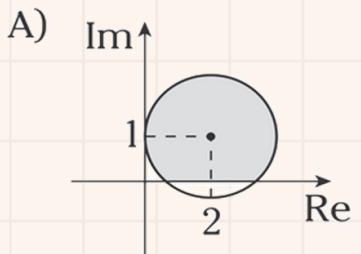
Resolución

Resolución

18.

Determine todos los $z \in \mathbb{C}$, tal que cumplan las siguientes condiciones:

- $|z - 2 + i| \leq |2e^{\frac{\pi}{5}i}|$
- $\arg(z) \in \left[0; \frac{\pi}{2}\right]$



19. Determine el área de la región formada por la intersección de los siguientes conjuntos:

$$A = \{z \in \mathbb{C} / |z - 1 + i| \leq 2\}$$

$$B = \{z \in \mathbb{C} / \arg(z) \in [0; \pi]\}$$

A) $\frac{2\pi}{3} - \sqrt{3} u^2$

B) $\frac{4\pi}{3} u^2$

C) $\frac{2\pi}{3} u^2$

D) $\frac{4\pi}{3} - \sqrt{3} u^2$

E) $\frac{5\pi}{3} - \sqrt{3} u^2$

Resolución

20. Determine el mayor argumento del complejo

$z \in N$ si

$$N = \{z \in \mathbb{C} / |z - 10\sqrt{3} - 10i| \leq 10\}$$

- A) $\frac{\pi}{4}$
- B) $\frac{\pi}{3}$
- C) $\frac{2\pi}{3}$
- D) $\frac{\pi}{2}$
- E) π

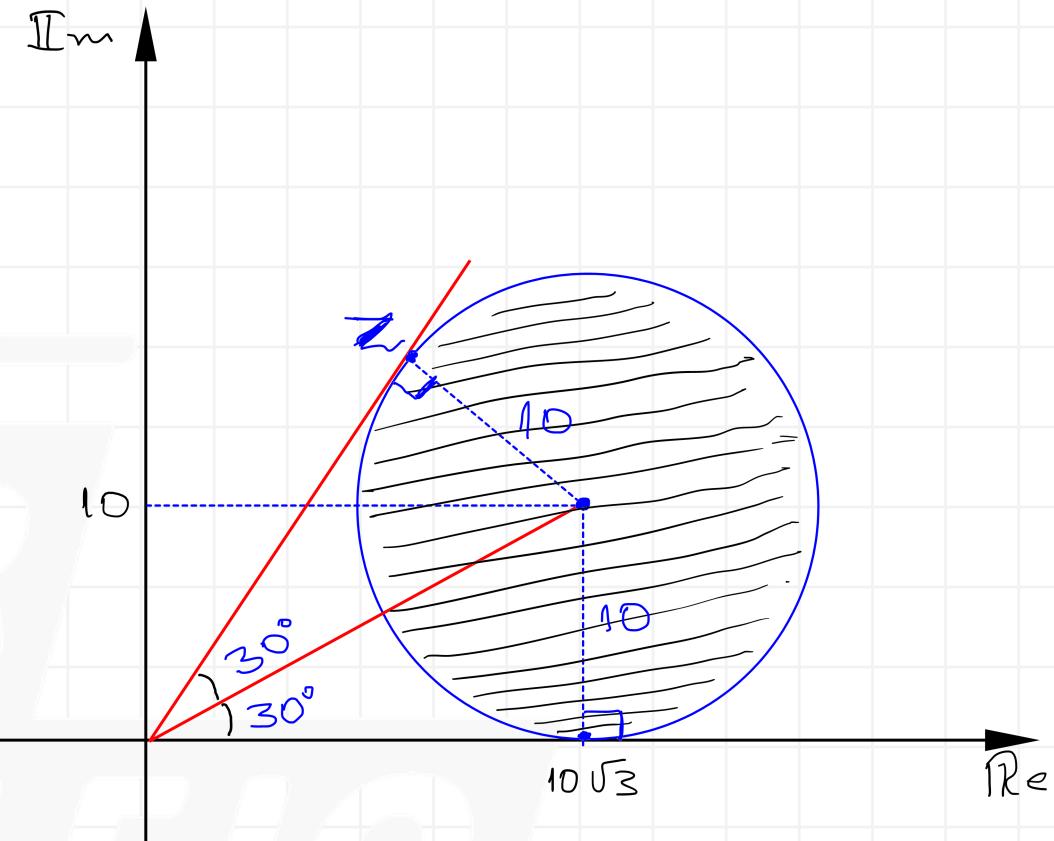
Resolución

Se tiene: $|z - 10\sqrt{3} - 10i| \leq 10$

Circunferencia

Centro: $(10\sqrt{3}, 10)$ Radio: $r=10$

Graficando.



∴ el mayor argumento de z es:

$$60^\circ = \frac{\pi}{3}$$

21. Si se tiene que

$$M = \{z \in \mathbb{C} / \operatorname{Im}(z) \geq \operatorname{Re}^2(z) - 4; \operatorname{Im}(z) \leq \operatorname{Re}^2(z) + 4\}$$

entonces determine el máximo valor de

$|z_1 - z_2|$, tal que $z_1 \wedge z_2 \in M$.

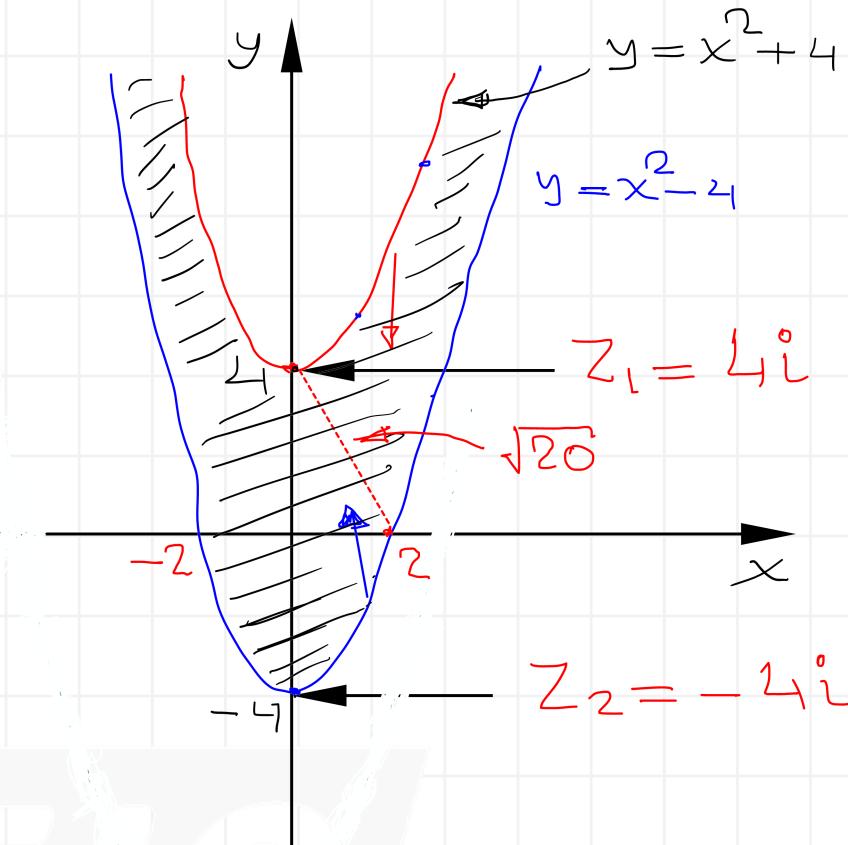
- A) 6
- B) 9
- C) 8
- D) 7
- E) 5

Resolución

Sea $Z = x + y i$; luego se tiene

$$y \geq x^2 - 4; \quad y \leq x^2 + 4$$

Graficando en el mismo plano.

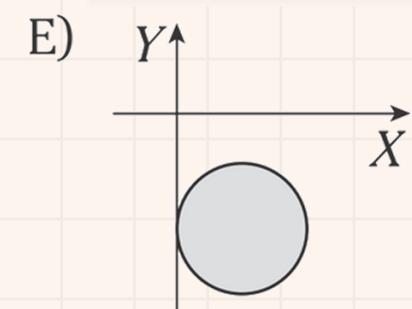
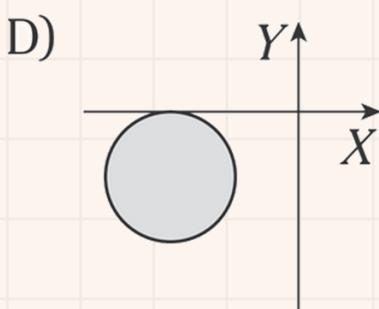
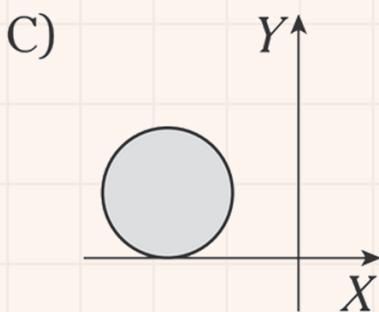
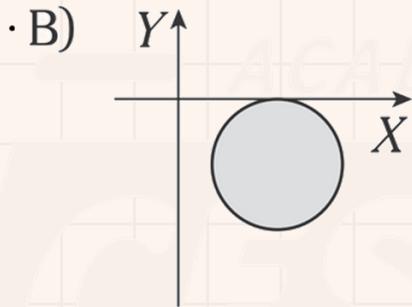
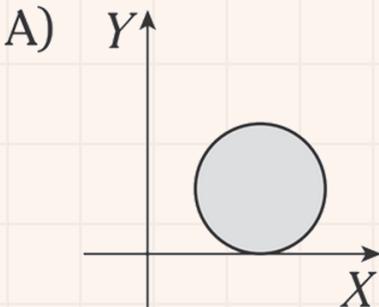


Notamos que:

$$\max |z_1 - z_2| = |4i - (-4i)| = |8i|$$

$$\therefore \max |z_1 - z_2| = 8$$

22. Si $M = \{\bar{z} + 1 - i \in \mathbb{C} / |\bar{z}^* + 2 + i| \leq 2\}$,
indique su gráfica.



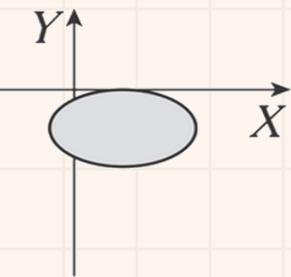
Resolución

23.

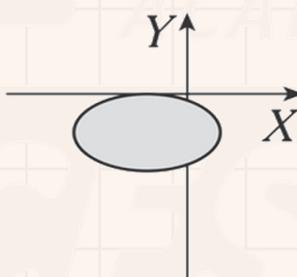
Determine la región que representa

$$A = \{ z + 1 - i / |\operatorname{Re}(z) + 2\operatorname{Im}(z)i| \leq 2 \}$$

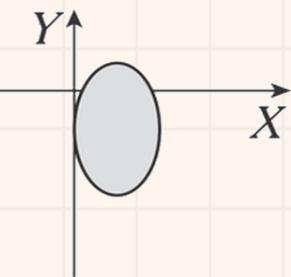
· A)



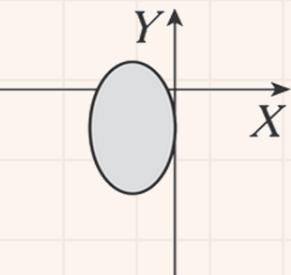
B)



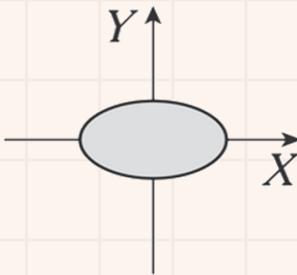
C)



D)



E)



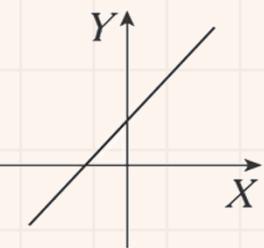
Resolución

24.

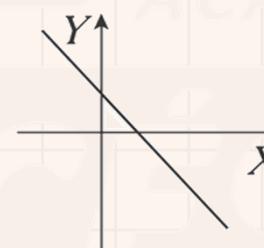
Grafique el siguiente conjunto:

$$A = \left\{ z \in \mathbb{C} \mid \operatorname{Re}\left(z - \frac{1}{z}\right) = 0 \right\}$$

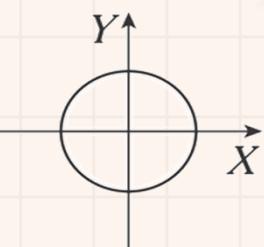
A)



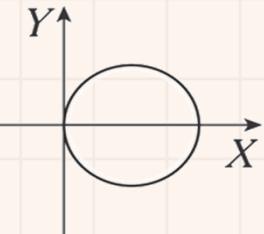
B)



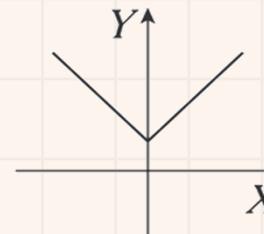
C)



D)



E)



Resolución

Sea $z = x + y i$, luego

$$z - \frac{1}{z} = x + y i - \frac{1}{x+yi} \times \frac{(x-yi)}{(x-yi)}$$

$$= x + y i - \frac{x-yi}{x^2+y^2}$$

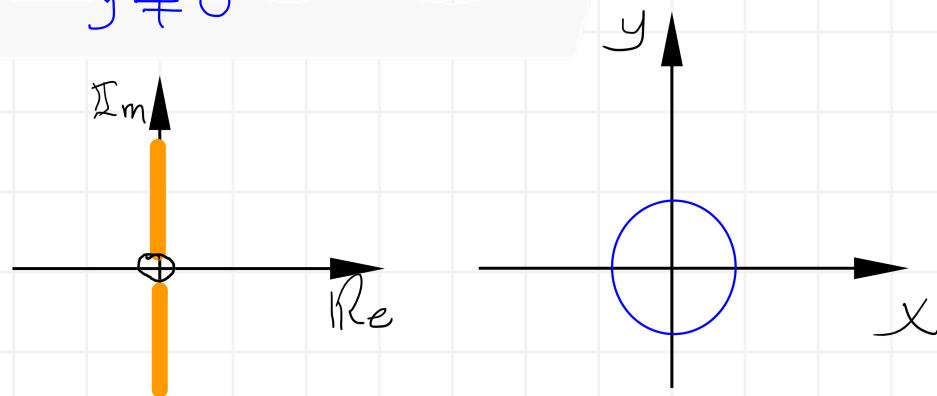
$$= x - \frac{x}{x^2+y^2} + \left(y + \frac{y}{x^2+y^2}\right)i$$

Ahora: $x - \frac{x}{x^2+y^2} = 0$

$$x(x^2+y^2-1) = 0$$

$$x=0 \vee x^2+y^2=1$$

$$y \neq 0$$



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ÁLGEBRA

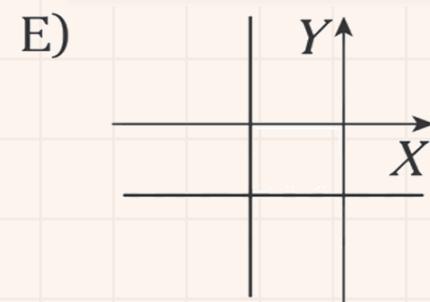
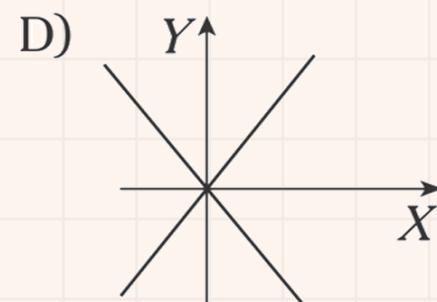
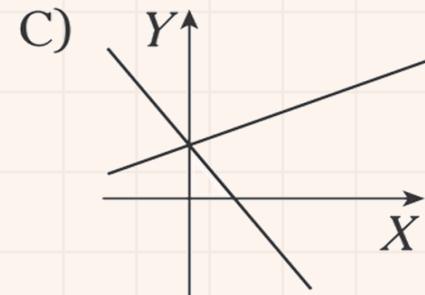
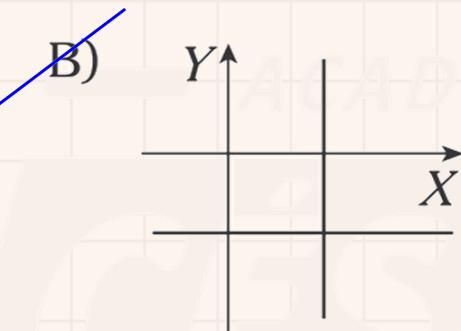
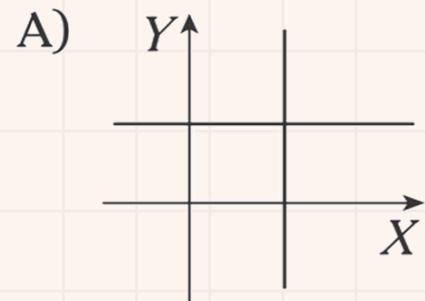
Gráfica de relaciones

Semana 07

Evaluación en línea

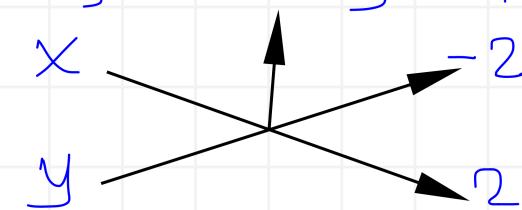
1. Grafique

$$xy = 2y - 2x + 4$$



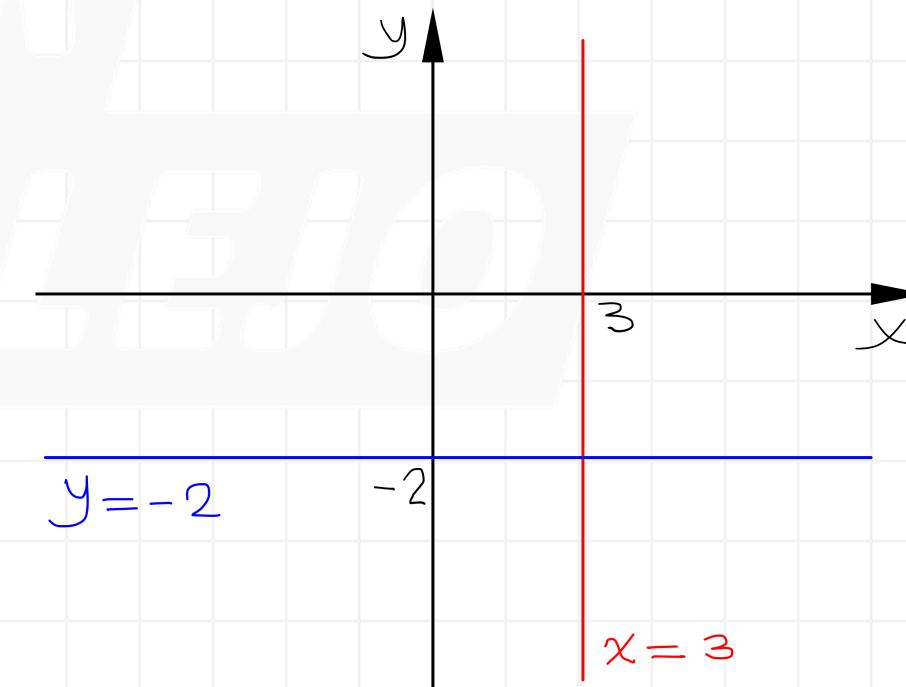
Resolución

Se tiene: $xy + 2x - 2y - 4 = 0$



$$(x-2)(y+2)=0$$

$$x=2 \quad \vee \quad y=-2$$



2. Determine el área de la región solución del sistema

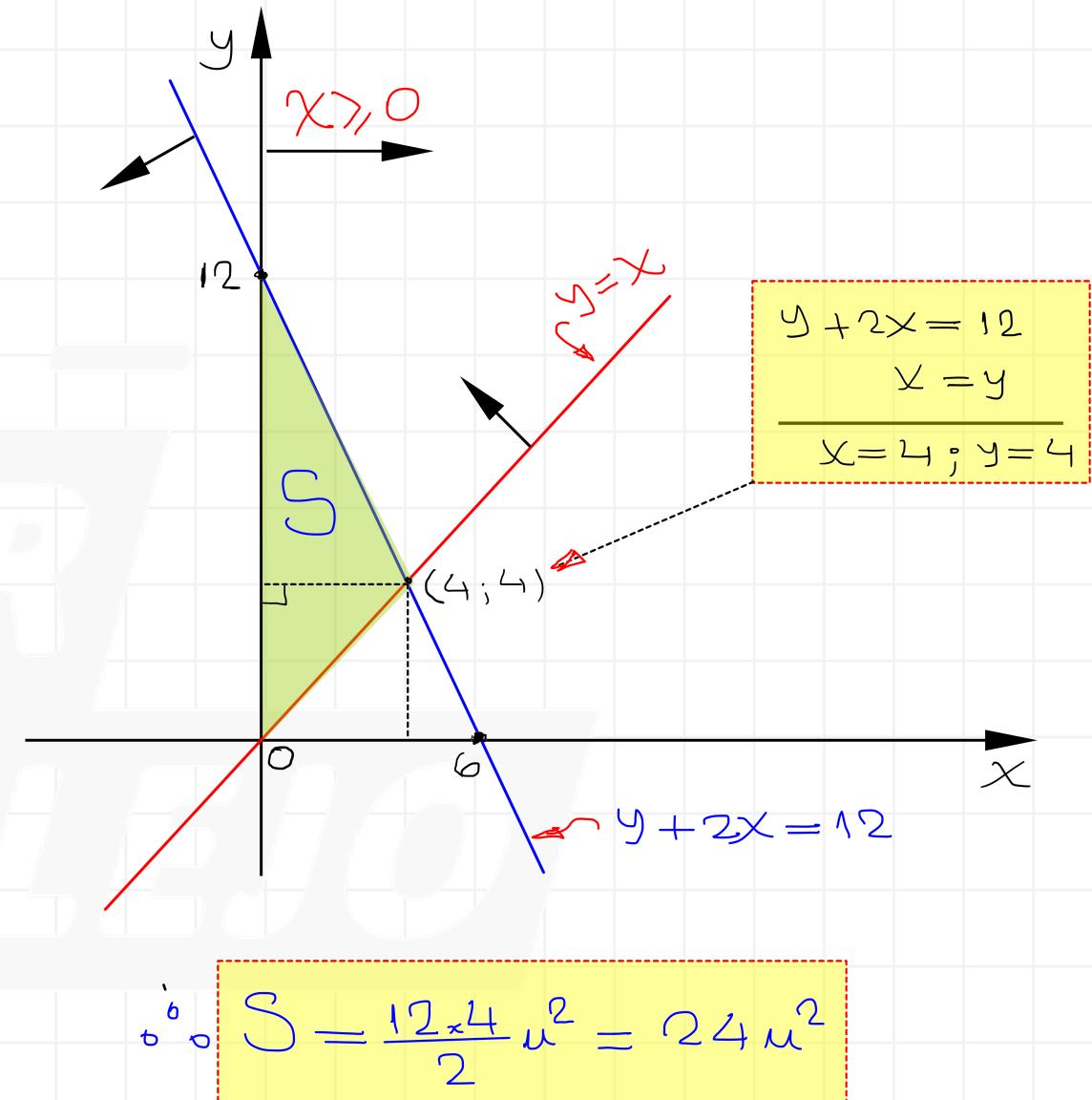
$$\begin{cases} y + 2x \leq 12 \\ y \geq x \\ x \geq 0 \end{cases}$$

- A) 32 u^2 B) 18 u^2 C) 16 u^2
 D) 24 u^2 E) 36 u^2

Resolución

$$y + 2x = 12$$

x	y
0	12
6	0



3. Dado el conjunto

$$A = \{z \in \mathbb{C} / |z - 2 + 2i| = 2\}$$

si $z_1 \in A$, tal que $|z_1|$ sea máximo. Luego, halle $\operatorname{Re}(z_1) + \operatorname{Im}(z_1)$.

- A) 4 B) $2\sqrt{2}$
 D) $4 + 2\sqrt{2}$

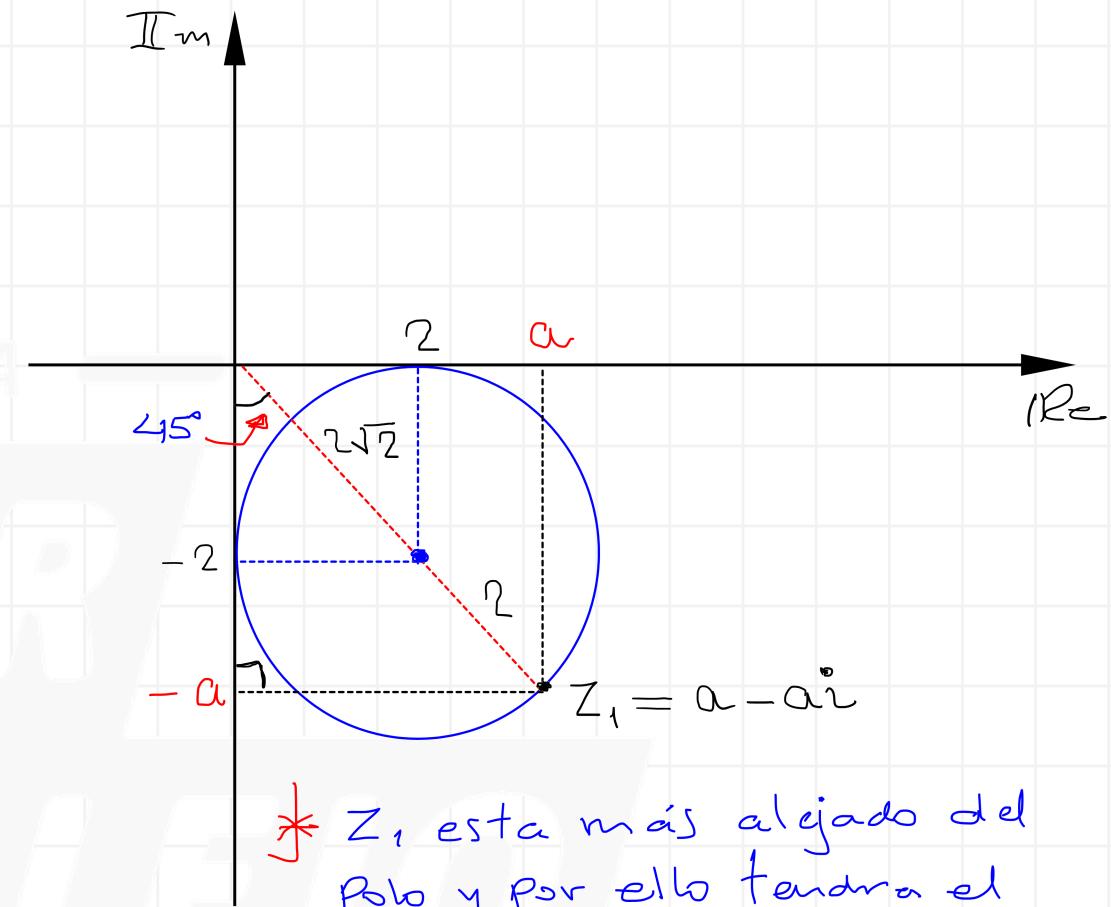
- C) 0
 E) $2 + \sqrt{2}$

Resolución

$$|z - 2 + 2i| = 2 \text{ (Circunferencia)}$$

$$\text{Centro: } C = (2, -2) \quad \text{Radio: } r = 2$$

Graficando.



* z_1 está más alejado del polo y por ello tendrá el mayor módulo.

→ $\operatorname{Re}(z_1) = a$ y $\operatorname{Im}(z_1) = -a$

$$\therefore \operatorname{Re}(z_1) + \operatorname{Im}(z_1) = 0$$



GRACIAS

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