Advanced Fixed Income and Credit Case Study
Deutsche Bank: Finding Relative Value Trades

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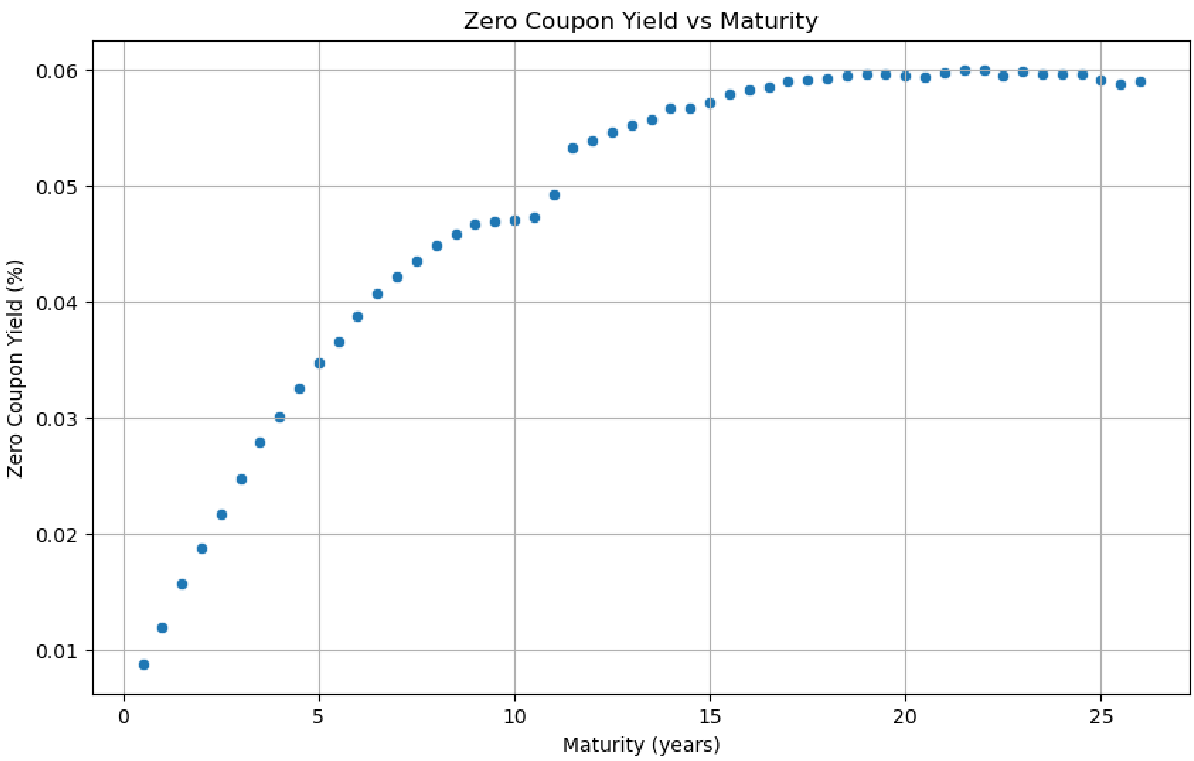
We strongly recommend readers of this report to read the Jupyter notebook that we used to answer all the questions of this assignment.

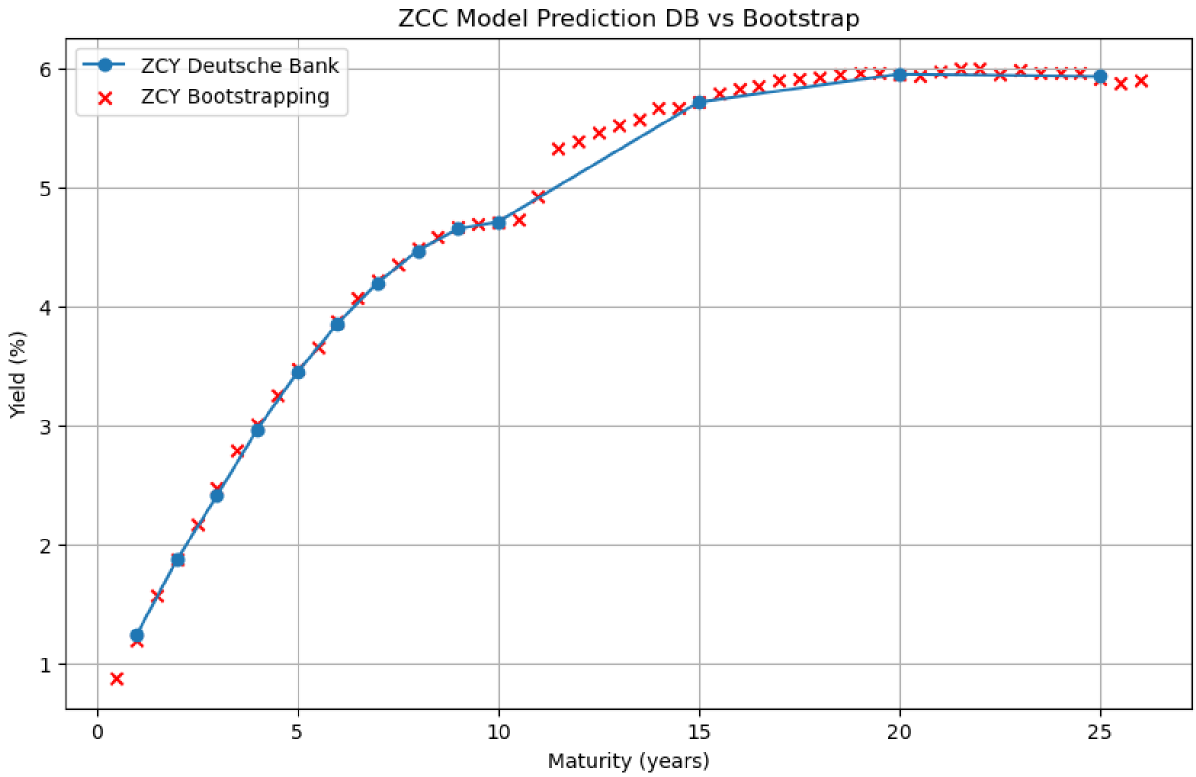
QUESTION 1: Bootstrapping

Q1.1

We computed the following yield curve based on the dataset we were asked to work on:



Q1.2



Deutsche Bank should buy bonds where the market zero-coupon yields are higher than those predicted by their model, as these bonds are likely undervalued. Conversely, they should sell bonds where the market yields are lower than the model's predictions, suggesting overvaluation. For example, bonds with maturities around 10-15 years show higher market yields, which would be buy opportunities.

Q1.3

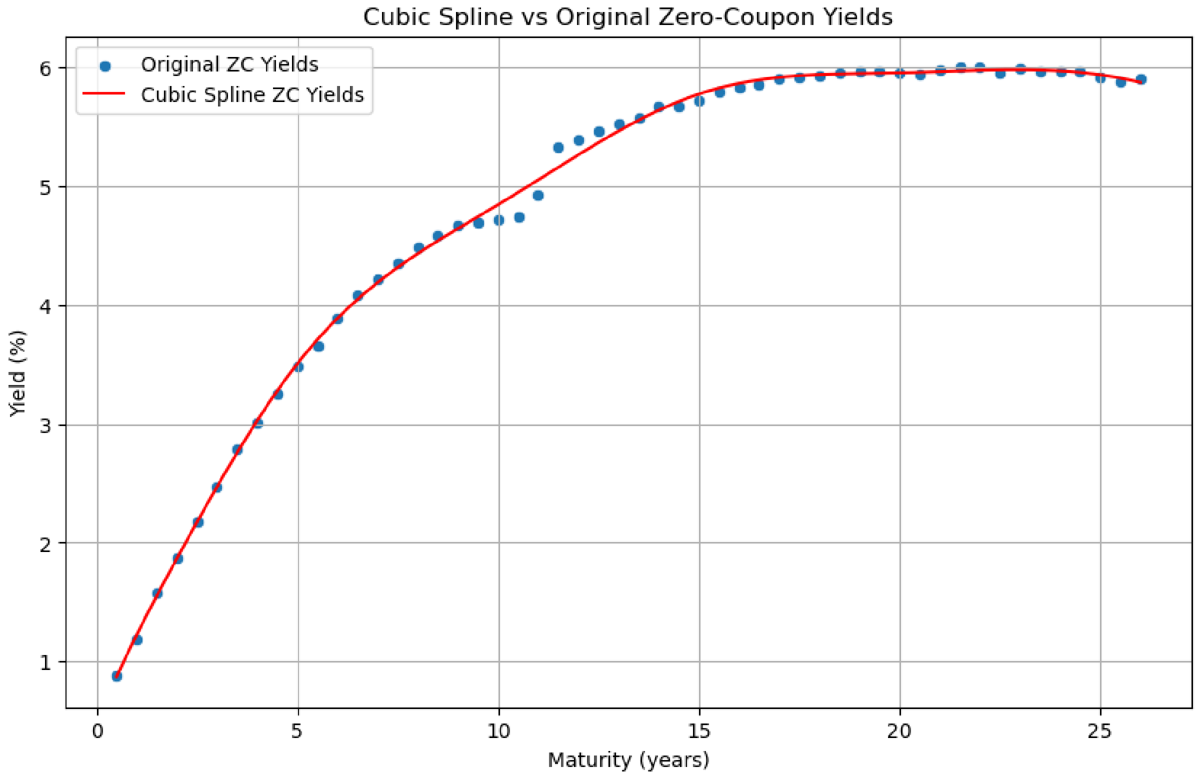
However, this strategy is not risk-free arbitrage. Beyond interest rate risk, model risk, and liquidity risk, it's crucial to recognize that older bonds, especially those that are off-the-run (i.e., no longer the most recent issuance), can be less liquid and harder to trade compared to on-the-run (benchmark) bonds. These older bonds may be rarer in the market, and their lower liquidity often commands a premium, which is reflected in their repo rates. As a result, executing buy or sell decisions for these bonds could be more challenging and expensive. Additionally, transaction costs and the potential difficulty in sourcing these bonds could significantly reduce the anticipated profits from the strategy. Managing these factors carefully, particularly the liquidity and availability of bonds in the repo market, is essential to the successful execution of this approach.

QUESTION 2: Cubic splines

Q2.1

This method fits a cubic spline (k=3) to the data, using specified internal knots at 2, 5, 10, 15, and 20-year maturities. The spline is constrained to pass precisely through these specified knot points, ensuring the curve accurately reflects these maturities. Between the knots, the spline smoothly interpolates the yield curve without being forced through any other maturities.

To create a smooth, continuous plot, we evaluate the spline over 500 equally spaced points across the maturity range, providing a fine-grained and visually continuous yield curve.



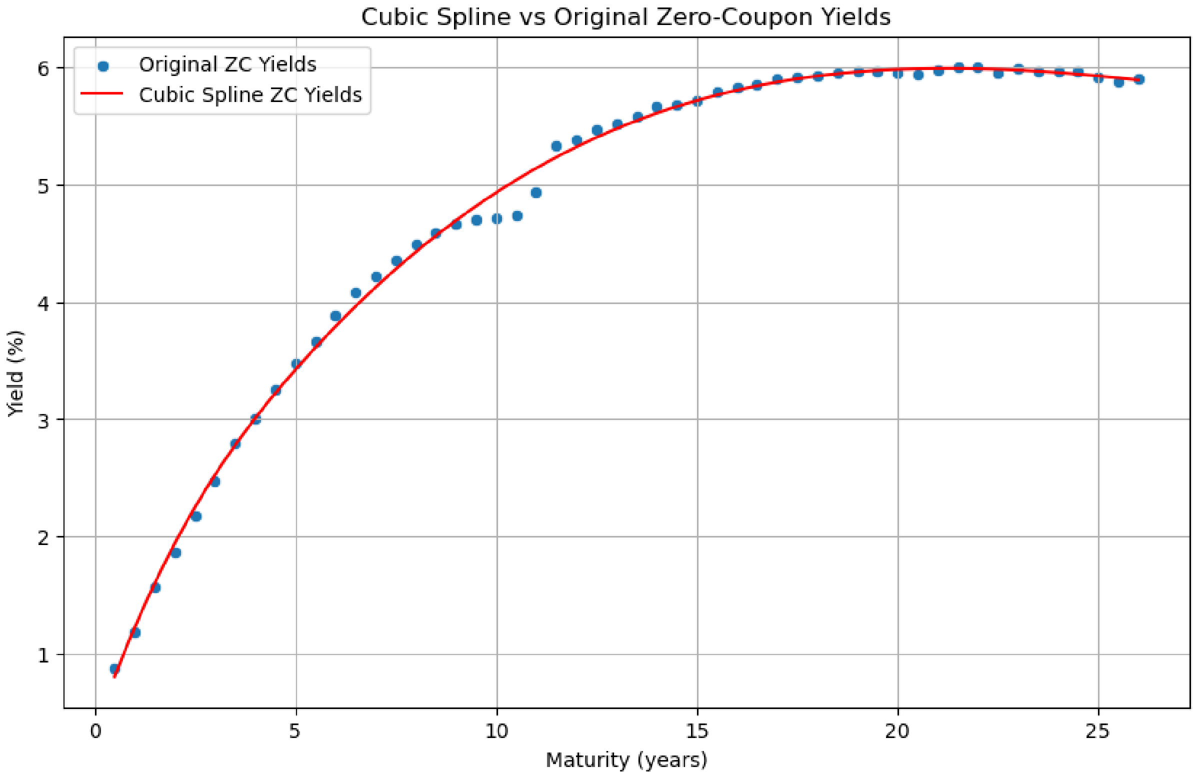
Q2.2

The cubic spline is more appropriate when we are talking about liquid maturities and where there are multiple maturities close to each other. This is due to the fact that the boostrapping method doesn't try to smooth the curve and maybe produce some local small jumps . By reading this curve, you may interpret those jumps as potential buy or sell opportunity. However, with the cubic spline, the interpolation is smoother and represents more accurately the gradual change of yield accross time. When we are looking at markets with fewer observed points or irregular yield movements, bootstrapping may better capture abrupt changes due to supply-demand factors or market anomalies, which might not be as well represented by a spline. The market may be representing some specific risk for a range of maturity, creating a local dynamic and if the spline doesn't have a knot in this area, it will fail to capture it due to it's nature.

The best example here is the 10 to 15 year maturities : when looking at the boostrap, we can clearly see the jump around 11-12 year that may indicate a buy opportunity in this area. However, when using the cubic spline, the closest know we are using in this area at 10 and 15 year. This mean we model will produce a smooth curve between those two points, regardless of the granularity of the yield in between.

In our opinion, it doesn't really change the conclusions of the question 1.2, the bank has just to be aware of both models limitation and knowing which one to use for given conditions ! Using cubic spline wouldn't not signal buy the bonds we mentioned in Q1.2, but this is because the model is less appropriate in this case.

Q2.3



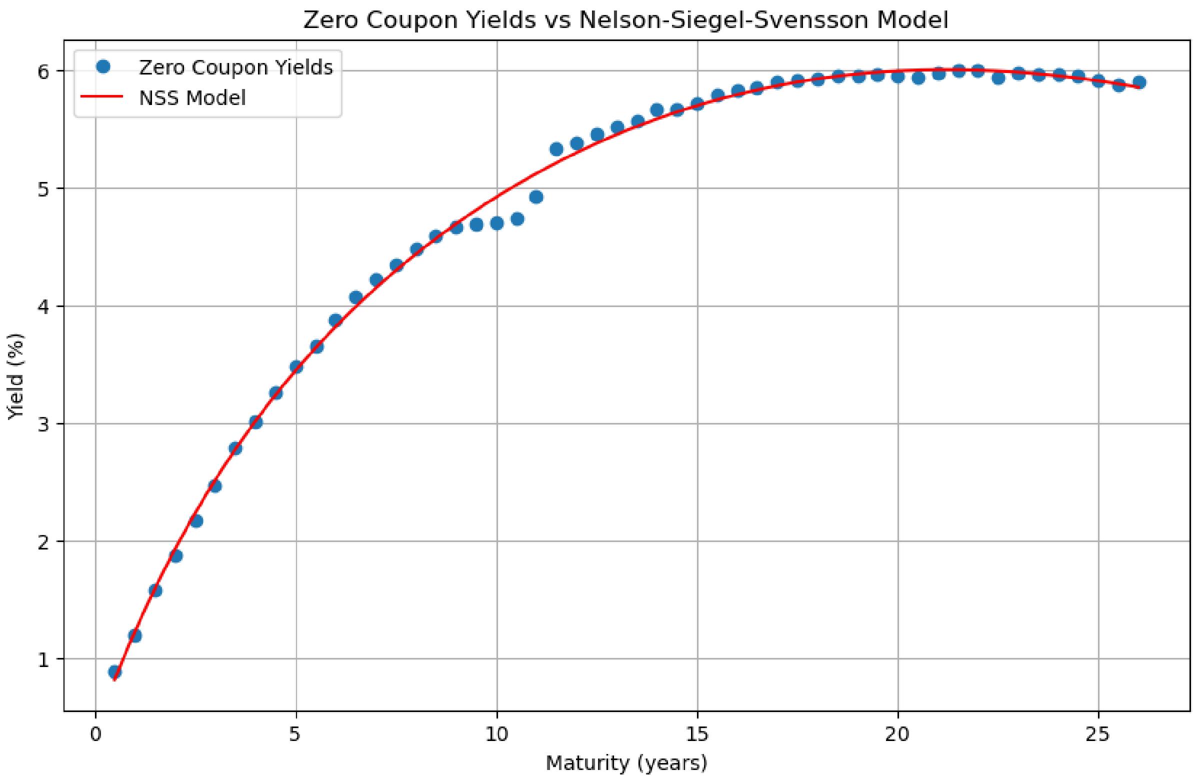
When using knots only at the 5-year and 20-year maturities, the cubic spline model provides a general approximation of the yield curve's shape but fails to capture crucial local variations, particularly around the 10-year mark. The model's limited knots result in a smoother curve that overlooks fluctuations in the intermediate term, especially between the 9-year and 13-year maturities. As a result, the model adequately fits maturities beyond 13 years but lacks precision in representing the curve's structure in the mid-range, where there is a notable dip in the yield data.

This limitation reflects how reducing the number of knots can simplify the model to the detriment of accuracy, especially in regions with significant yield curve inflections. By omitting intermediate knots, the model underestimates the curve's complexity and fails to accommodate fluctuations that are essential for modeling the yield curve's dynamic nature, particularly around the 10-year maturity.

To improve the model, adding intermediate knots at key points—such as around 10 years—would better capture the curve's true shape. This would allow for a more accurate representation of the yield curve without oversimplifying important trends. However, care must be taken to avoid excessive knots, which could lead to overfitting and reduce the model's generalizability. The objective is to balance the selection of knot points to enhance accuracy without replicating the granular fit of a bootstrap approach, thereby preserving the advantages of smooth interpolation.

QUESTION 3: Nelson Siegel model

Q3.1



This code implements and fits the Nelson-Siegel-Svensson (NSS) model to approximate the zero-coupon yield curve. The nss_model function breaks down the yield curve into four distinct components, each shaped by the parameters beta0, beta1, beta2, beta3, lambda1, and lambda2, which together capture the yield curve's level, slope, and curvature characteristics. Specifically:

- **beta0** represents the long-term level of the yield curve.
- **beta1** controls the slope, typically affecting short-term yields.
- **beta2** and beta3 adjust the curvature, influencing intermediate yields.
- **lambda1 and lambda2** are decay factors that influence how quickly the slope and curvature components transition over different maturities

Using curve_fit, the model optimizes these parameters by fitting them to the observed zero-coupon yields. Once the fitting is done, the parameters are used to compute the fitted yields over a range of maturities. The NSS model creates a smooth curve that represents the overall yield curve without forcing it to pass through each individual data point, reflecting the underlying structure of the curve's level, slope, and curvature.

While this approach smooths the yield curve, a limitation appears around the 8-11 year maturities, where the model's structure can lead to overestimations and cause a slight underestimation in the subsequent 11-13 year range. These biases may impact relative value analyses, as local variations are not fully captured in these segments. An effective improvement could involve applying the NSS model over smaller, localized maturity ranges—using subsets of maturities (e.g., six points at a time)—and averaging the results to create a more granular approximation. I applied this method during a previous internship on a bond trading desk, and it was highly appreciated for providing a more accurate approximation.

Q3.2

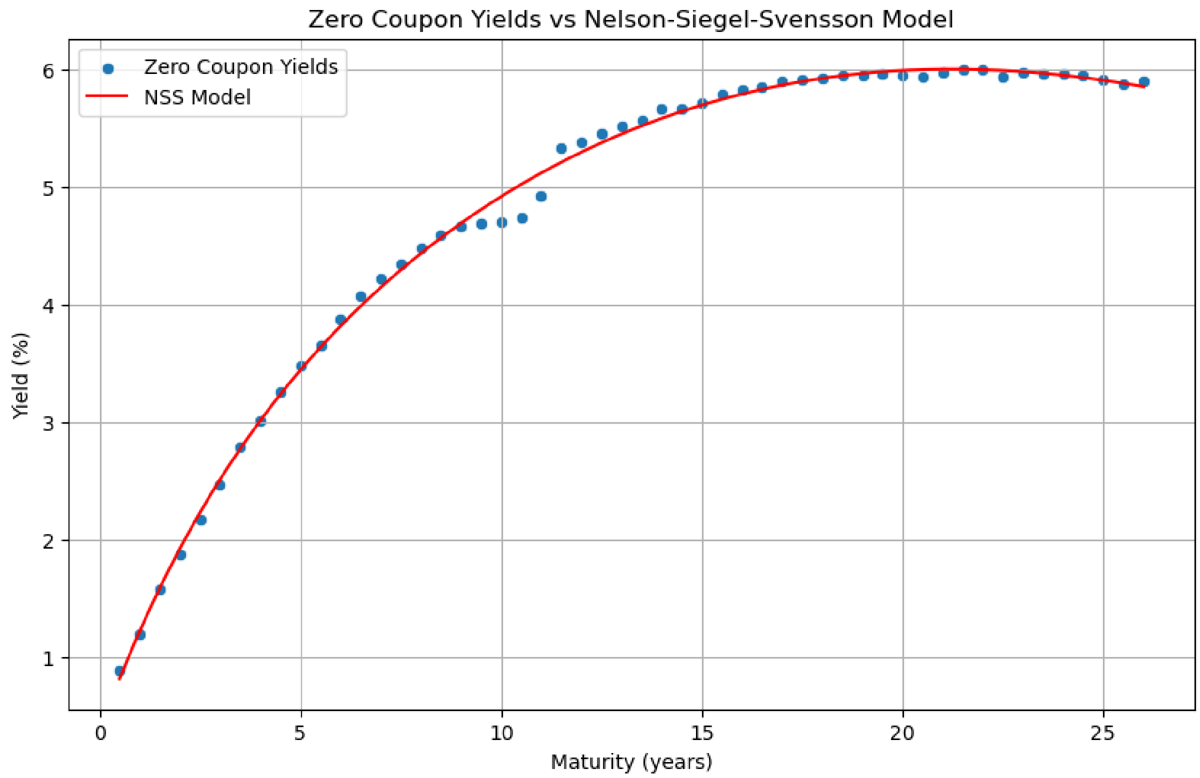
The Nelson-Siegel-Svensson (NSS) model and the cubic spline model represent two distinct approaches to yield curve fitting, each with unique strengths and limitations. The NSS model produces a smooth curve that captures the overall yield curve structure by modeling its level, slope, and curvature, without needing to pass through specific data points. This flexibility makes it well-suited for capturing broader yield curve dynamics, especially over longer maturities, where smoothness and trend continuity are prioritized.

In contrast, the cubic spline model excels at capturing localized movements between closely spaced maturities (such as the 2-6 year range) due to its knot-based structure. However, this structure can lead to overfitting at the specified knot points, which forces the curve to align at these specific maturities (e.g., the 10-year mark). This rigid alignment often prevents the model from accurately representing local variations and may distort the yield curve at the shorter and longer ends.

Overall, the NSS model offers a more adaptable and realistic yield curve representation, particularly when covering a broad maturity range. Its design allows it to better capture the general shape of the yield curve, making it ideal for applications that require a smooth, overarching view rather than a detailed alignment with specific maturities.

Q3.3

beta0: -0.13975641184820958
beta1: 0.14343919558672738
beta2: 0.08819760780763944
beta3: 0.5491774235219888
lambda1: 0.24629974342688665
lambda2: 0.05902331479003359



In the Nelson-Siegel-Svensson (NSS) model, each parameter offers insights into different aspects of the yield curve's shape. **Beta0** represents the long-term rate level, and while its value here is -0.1397, which suggests negative rates, the yield curve itself stabilizes around 5-6% for longer maturities. This implies that **Beta0 captures a positive baseline level** for the yield curve over the long term, aligning with observed stable positive rates in the plot.

Beta1 (0.1434) governs the slope, and its positive value reflects **an upward-sloping yield curve—indicating that shorter maturities start at lower rates and increase over time**. **Beta2 (0.0882)** influences the curvature, indicating a hump in the medium-term maturities (around 5-10 years), where yields rise to a peak before leveling off. **Beta3 (0.5491)** introduces additional flexibility, potentially **capturing more complex shapes in the longer-term section**, such as a second hump or further curvature.

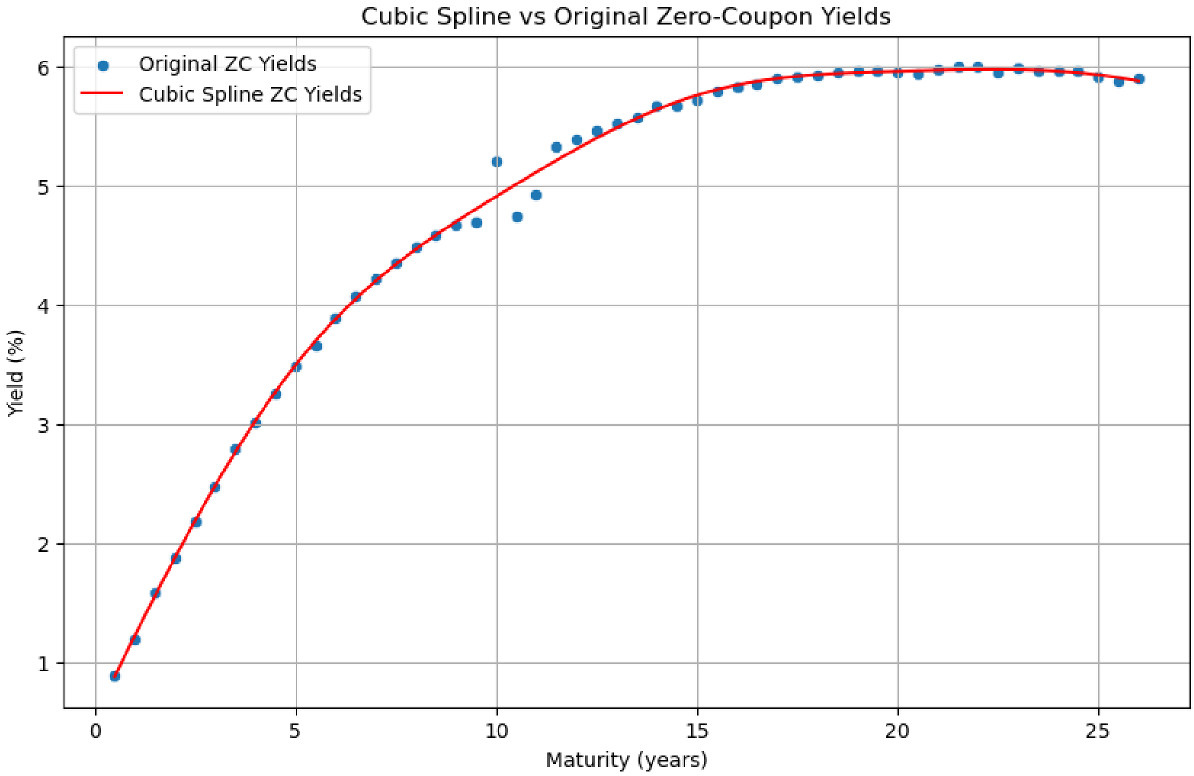
The lambda1 (0.2463) and **lambda2 (0.0590)** parameters determine the decay rates of the slope and curvature effects, respectively. **The relatively higher value of lambda1**

indicates that the upward slope extends for a more sustained period, while **the lower lambda2 suggests that the curvature introduced by Beta3 is localized and diminishes more quickly**.

In summary, this NSS model effectively represents the yield curve's essential dynamics: an upward slope, a medium-term hump, and some additional complexity in the long end, creating a smooth and realistic approximation of the actual curve.

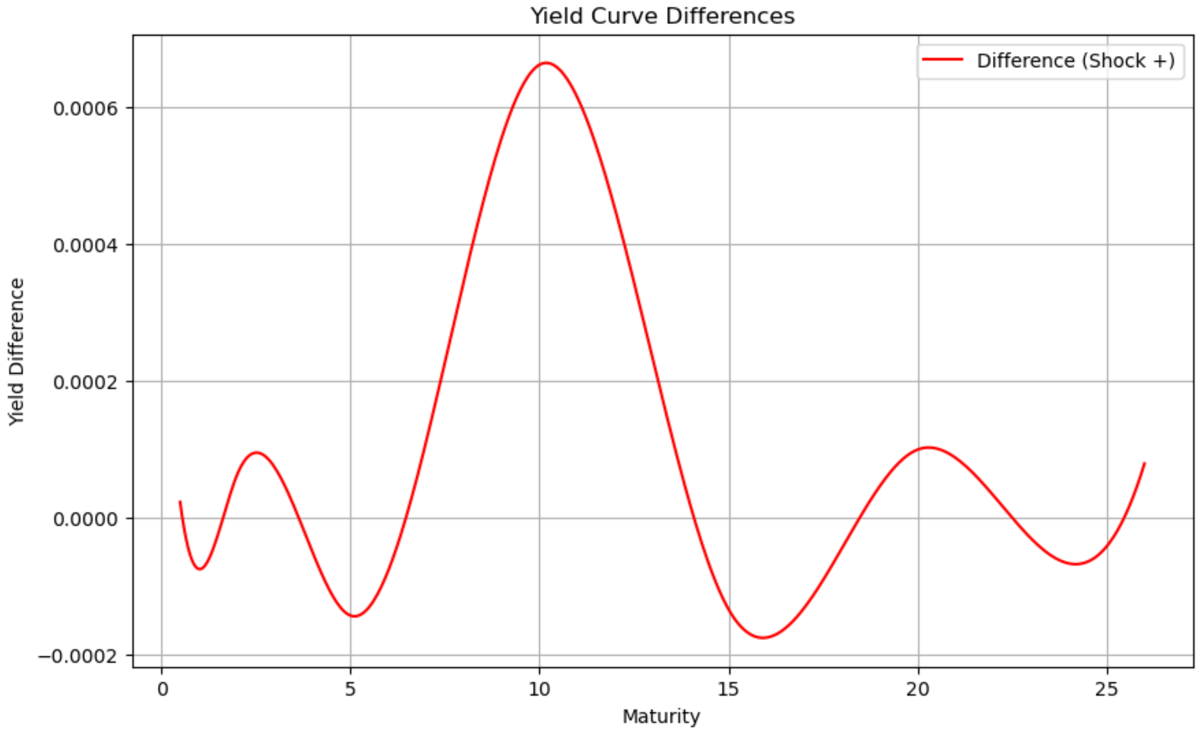
QUESTION 4: Hedging

Q4.1



Shocking the 10-year rate by 50 bps significantly impacts the yield at that maturity, pushing the data point above the cubic spline curve, whereas it was positioned below it in question 2.1. However, an interesting aspect of the cubic spline model becomes evident here: while the 10-year data point shifts due to the rate shock, the overall cubic spline curve remains largely unchanged, failing to adjust fully to fit the new data point.

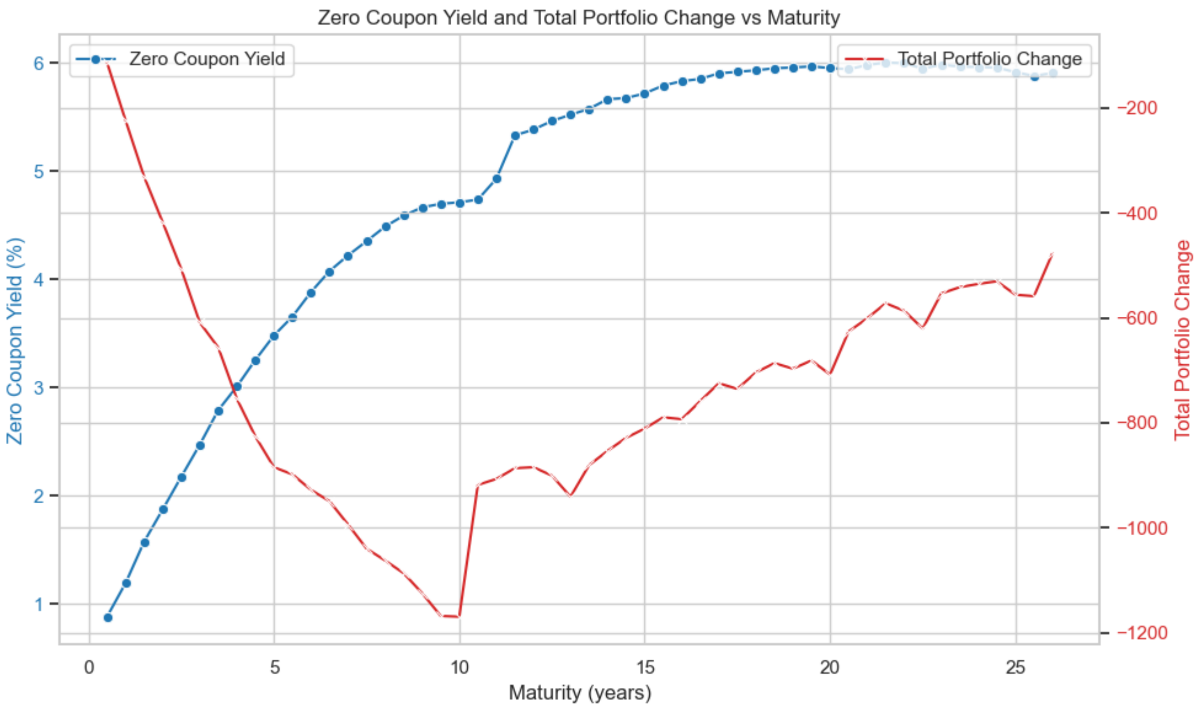
This behavior reflects a fundamental characteristic of cubic splines, which are designed to produce a smooth, continuous curve through specified knots rather than respond dynamically to individual points. As a result, any single-point adjustment—like the 50 bps shock at 10 years—does not shift the curve substantially because the model's shape is constrained by its fixed knots (often at broader intervals, e.g., 5, 10, and 20 years).



Q4.2

The portfolio’s Key Rate Duration (KRD) to a 50 bps upward shock in the 10-year yield results in a value change of -1,169.34. This indicates that the portfolio would lose approximately $1,169 if the 10-year rate increased by 50 basis points, reflecting a negative sensitivity to movements at this key maturity. By calculating each bond’s price adjustment under the rate shock and averaging the changes, we capture the portfolio's specific exposure to the 10-year rate, showing a moderate level of interest rate risk focused around this maturity.

Q4.3

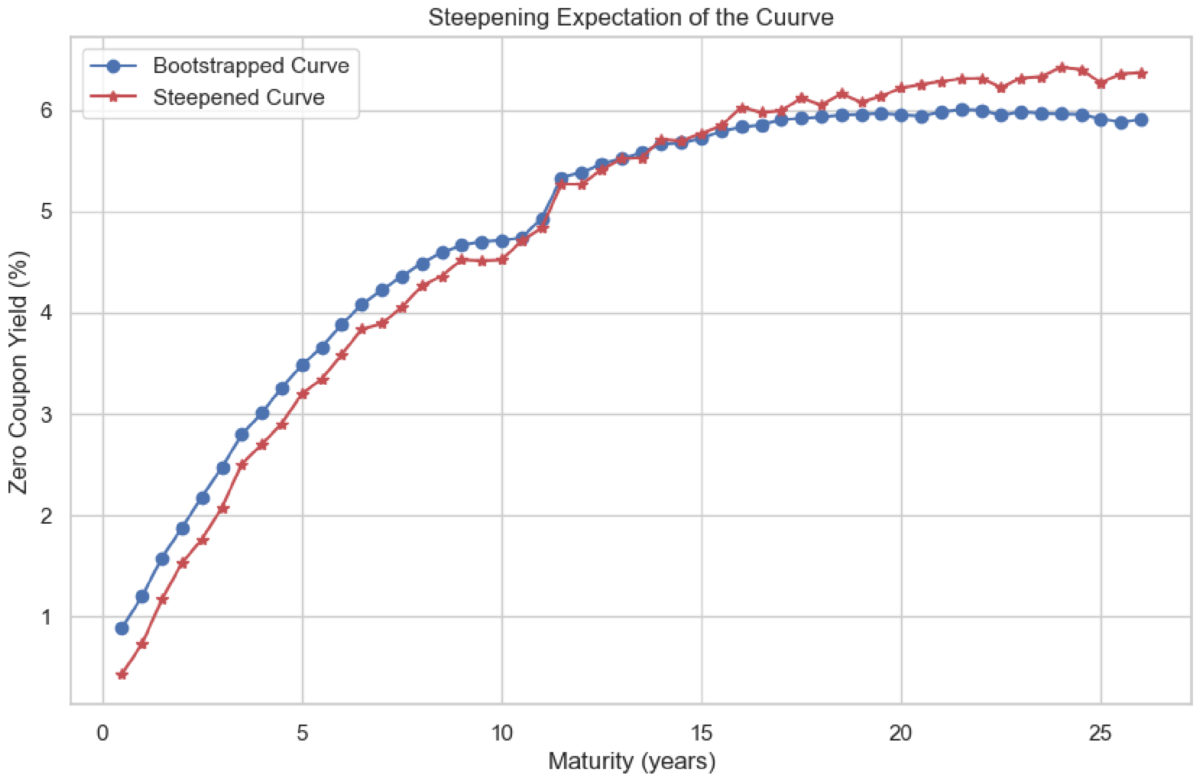


To estimate the Key Rate Durations (KRDs) across all maturities in my portfolio, we applied a 50 bps shock individually to each zero-coupon yield and measured the resulting impact on portfolio value. The results show that the 10-year maturity exerts the highest impact, significantly more than neighboring maturities. This heightened sensitivity is largely due to the lower 4% coupon rate on the 10-year bond compared to, for instance, the 13% coupon on the 10.5-year bond. With a lower coupon, the bond's cash flows are more concentrated in the principal payment, making its duration longer and increasing its exposure to changes in yield. Consequently, the 10-year bond's higher duration amplifies the effect of rate shocks on its price, which in turn results in a more substantial impact on the overall portfolio value compared to higher-coupon, shorter-duration bonds. This analysis highlights how both coupon rates and bond maturities influence the portfolio's response to yield changes across the curve.

Q5 : Portfolio strategy

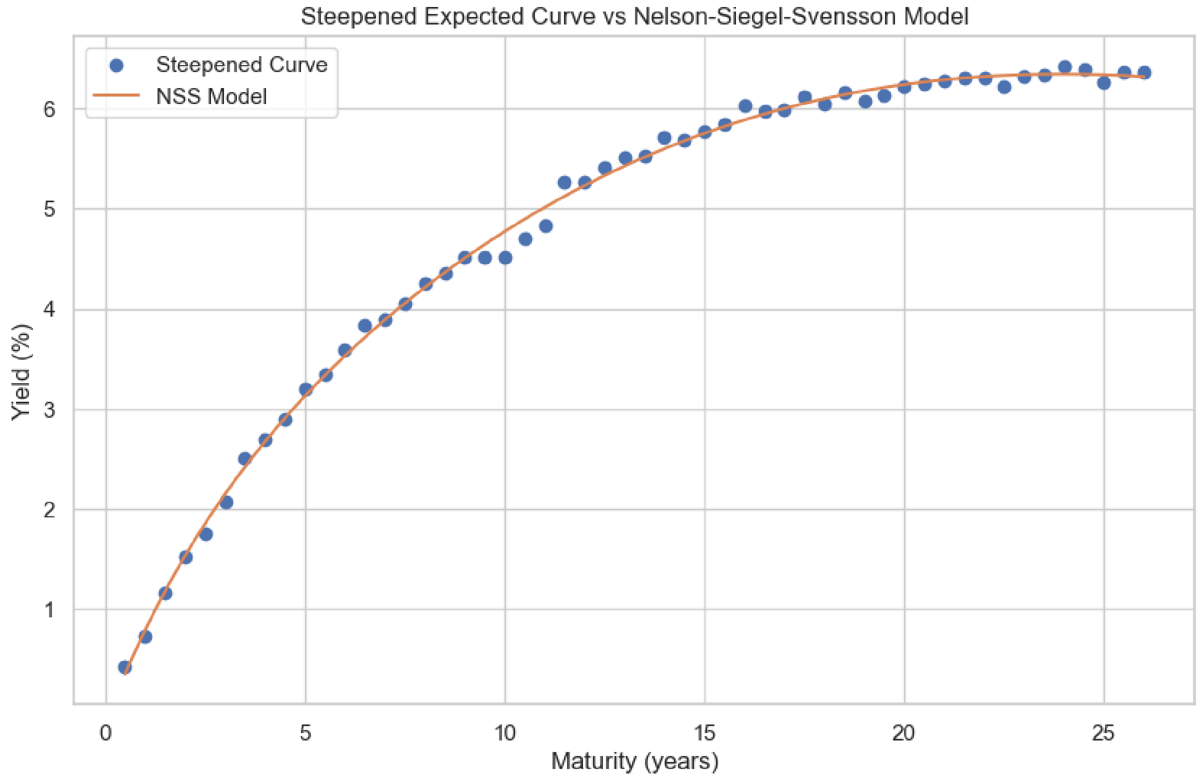
Q5.1

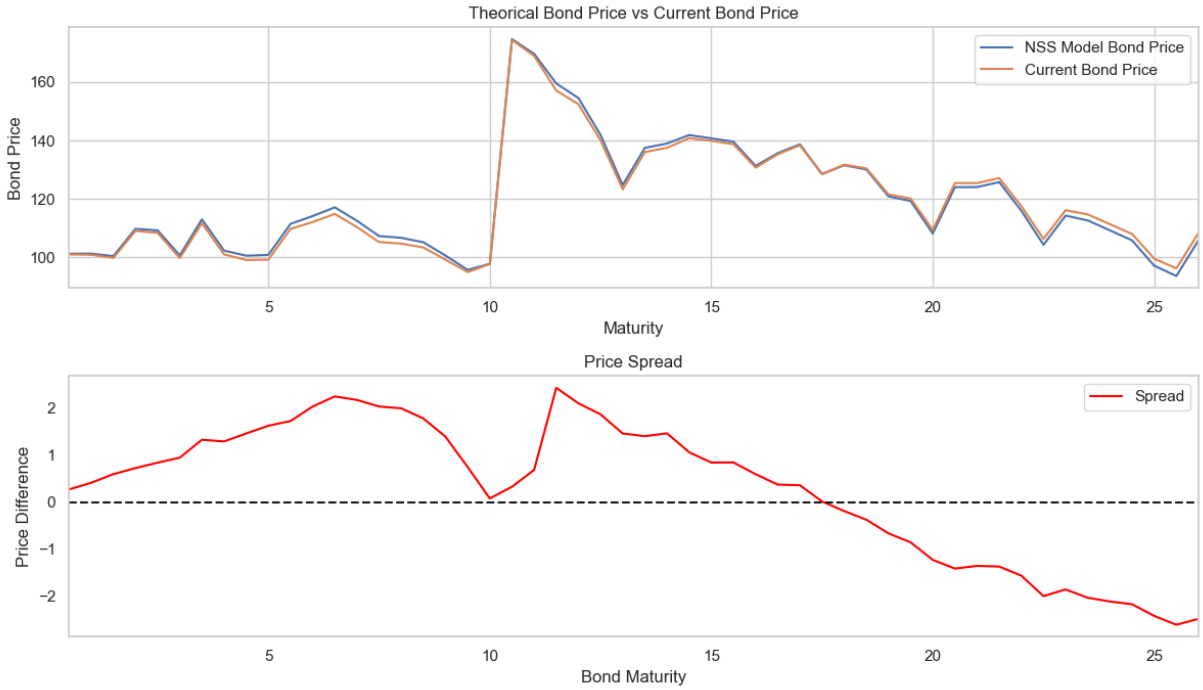
We applied the yield transformation for steepening expectation



The yield transformation produces a steepening of the curve as it is expected

Q5.2





The bonds where the spread is positive are undervalued (the current price is lower than the price given by the NSS model), and the bonds where the spread is negative are overvalued (the current price is higher than the price given by the NSS model). Then, the **cheap bonds** are those where the spread is significantly positive : the bonds that have a maturity between **4 years and 9 years and betwen 11 and 12 years**. On the contrary, **the most expensive bonds** are the ones that have a maturity between **22 and 26 years.**

Q5.3

The long/short strategy we propose is based on the following assumptions:

- **Bond Mispricing Correction**: The current mispricing of bonds is expected to dissipate soon, as arbitrageurs who anticipate yield curve steepening like us are likely to adopt similar strategies. Consequently, bond prices should converge toward those previously calculated using the NSS model applied to the expected steepened yield curve.
- **Fractional Bond Investment**: It is possible to invest in fractional units of bonds by purchasing portions of ETFs that track these bonds or by using fractional bond securities offered by certain brokers.
- **No Financing Costs**: We assume that financing costs are zero.

Here is the proposed allocation for the long and short legs:

Long Leg: Allocate a weight to each undervalued bond based on the magnitude of its spread relative to the total spread of all undervalued bonds. This approach assigns higher weights to bonds with larger spreads, as these bonds are more likely to adjust toward their expected price.

Short Leg: Allocate a weight to each overvalued bond according to the magnitude of its spread relative to the total spread of all overvalued bonds. Consequently, bonds with the largest spreads (in absolute terms) receive the highest weights, reflecting their greater likelihood of price reversion

Long Leg Portfolio Allocation

Maturity	Weight (%)	Allocated Money ($)	Expected Return (%)	Weight Adj Expected Return (%)
11.5	5.859786949	5.8598	0.015465594	0.090625087
6.5	5.421168214	5.4212	0.019555918	0.106015922
7	5.237566295	5.2376	0.019667052	0.103007489
12	5.065384677	5.0654	0.013775308	0.069777233
7.5	4.902732818	4.9027	0.01930075	0.09462642
6	4.90065945	4.9007	0.01811389	0.088770004
8	4.809643803	4.8096	0.019030563	0.091530228
12.5	4.500500577	4.5005	0.013323797	0.059963757
8.5	4.287195797	4.2872	0.0171801	0.073654453
5.5	4.155870663	4.1559	0.015693243	0.065219088
5	3.915622619	3.9156	0.016349916	0.064020103
14	3.527729503	3.5277	0.010634499	0.037515637
13	3.519423899	3.5194	0.011831236	0.041639134
4.5	3.517235357	3.5172	0.014701503	0.051708645
13.5	3.378145495	3.3781	0.010291665	0.034766741
9	3.350081987	3.3501	0.013987115	0.046857982
3.5	3.196483827	3.1965	0.011860006	0.037910318
4	3.10912732	3.1091	0.012749489	0.039639783
14.5	2.554572269	2.5546	0.00752103	0.019213015
3	2.276880708	2.2769	0.009458298	0.021535416
15.5	2.031314802	2.0313	0.006068802	0.012327647
15	2.027002466	2.027	0.0060055	0.012173163
2.5	2.020274942	2.0203	0.007722645	0.015601867
9.5	1.793155849	1.7932	0.007821677	0.014025487
2	1.74229236	1.7423	0.006620025	0.01153402
11	1.650578193	1.6506	0.004049893	0.006684665
1.5	1.432241814	1.4322	0.005943102	0.008511959
16	1.427448227	1.4274	0.004526558	0.006461427
1	0.988326049	0.9883	0.004059167	0.00401178
16.5	0.894033858	0.894	0.002739133	0.002448878
17	0.864162553	0.8642	0.002589191	0.002237482
10.5	0.783401996	0.7834	0.001862781	0.001459307
0.5	0.647259267	0.6473	0.002654973	0.001718456
10	0.182220418	0.1822	0.000772559	0.000140776
17.5	0.030474979	0.0305	9.83058E-05	2.99587E-06

Short Leg portfolio Composition:

Maturity	Weight (%)	Borrowed Money ($)	Expected Return (%)	Weight Adj Expected Return (%)
25.5	-9.763468056	-9.7635	0.02711369	0.264723645
26	-9.294689708	-9.2947	0.022926389	0.213093669
25	-9.059969303	-9.06	0.024315231	0.220295244
24.5	-8.13085841	-8.1309	0.020123815	0.163623891
24	-7.911723619	-7.9117	0.018990078	0.150244252
23.5	-7.615366958	-7.6154	0.0177521	0.135188755
22.5	-7.483557877	-7.4836	0.018813693	0.140793361
23	-6.953978751	-6.954	0.016002482	0.111280918
22	-5.85587819	-5.8559	0.013320513	0.078003302
20.5	-5.29336493	-5.2934	0.011281879	0.059719101
21.5	-5.132039815	-5.132	0.010792856	0.055389367
21	-5.081010311	-5.081	0.010830439	0.055029572
20	-4.600580655	-4.6006	0.011239233	0.051706999
19.5	-3.20735873	-3.2074	0.007136585	0.022889589
19	-2.492444347	-2.4924	0.005481737	0.013662925
18.5	-1.406178903	-1.4062	0.00288186	0.00405241
18	-0.717531437	-0.7175	0.001456457	0.001045054

Final Comments

The expected return of our long/short strategy is **3.08%**.
The strategy could be improved in several ways, notably:

- The short leg is less diversified than the long leg, as its components are concentrated in maturities exceeding 20 years.
- We could apply more selectivity in allocation, particularly for the long leg, since some bonds with insignificant mispricings are currently included. However, this would reduce portfolio diversification.