Exam 5 - Summary

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Contents

Part I

Ratemaking (*Werner & Modlin*)

Chapter 1

INTRODUCTION

For every business having the goal of indefinitely operating, one should try and find the perfect balance of the following fundamental equation :

$$\text{Price} = \text{Cost} + \text{Profit}$$

For most of the non-insurance corporation, this process is relatively straight forward as they control every parameter of the equation. They can always try and reduce their production cost in order to gain profit or improve their competitiveness. The insurance market is different, as the insurer can't control their primary cost of doing business, ie the losses accompanying the insurance contracts they provide. Because of this particularity, insurers tend to write the **fundamental equation** as

$$\text{UW Profit} = \text{Premium} - \text{Losses} - \text{LAE} - \text{UW Expenses.}$$

Where "UW" stands for *underwriting* and "LAE" stands for *loss adjustment expenses*. Though out the chapter, the will be covered in more detail.

1.1 ADD THE CAS STATEMENT'S PRINCIPLES!!!!

1.2 EXPOSURE & PREMIUM

An exposure is the basic unit of risk that underlies the insurance premium. There are four different ways to measure exposures and premiums :

- **Written** Exposure/Premium
  When we are in the written basis, we refer to all exposures and premium arising from policies issued during a specific period of time.

- **Earned** Exposure/Premium
  On a *earned* basis, we refer to the portion of exposures or premiums for which the insurance coverage has been provided, as of a certain point in time.

- **Unearned** Exposure/Premium
  As opposed to an earned basis, on a *unearned* basis, we refer to the portion of exposures or premiums for which the insurance coverage has not been provided, as of a certain point in time.

- **In-force** Exposure/Premium
  When we refer to in-force exposure or premiums, we refer to the exposures or premiums exposed to loss (or for which the insurance coverage is provided) at a specific point in time.

1.3   CLAIMS & LOSSES

The text defines a claim as the demand for indemnification under the coverage of a policy. Note that a claim can be filed by an insured or by a third party alleging injuries or damages. Associated with a claim is a loss, which is the amount of compensation for the claim.

An insurance company usually compiles claims on one of two metrics : **accident date** (date of loss, occurrence date) or **reported date**.

Claims that have occurred but are not yet reported to the insurer constitute the **IBNR (incurred but not reported) claims**. These have to be accounted for (ie as reserve of money) when calculating the **estimated ultimate losses**, as well as the **IBNER (incurred but not enough reported) claims**.

Finally, a reported loss can always be divided into two components : the **paid loss** and the **case reserve**. The paid loss is the amount that is already paid to the claimant, and the case reserve is the best estimation of what is left to be paid in order to close the claim.

We calculate the total estimated ultimate losses as :

$$\textbf{Estimated Ultimate Losses} = \textbf{Reported Losses} + \textbf{IBNR reserve} + \textbf{IBNER reserve},$$

where reported losses $= \sum \left(\text{individual paid losses} + \text{case reserve}\right)$.

In addition to paying money for compensation to the claimant, an insurer also incurs expenses in the process : **Loss adjustment expenses (LAE)**. These are separated into two categories :

- **ALAE** : Allocated loss adjustment expenses;

- **ULAE** : Unallocated loss adjustment expenses.

ALAE are expenses incurred directly attributable to a specific claim, in contrast with ULAE that are expenses incurred from the general process of settling claims (for example, salaries to pay compensation experts). By definition, we have :

$$\textbf{LAE} = \textbf{ALAE} + \textbf{ULAE}$$

1.4 UNDERWRITING EXPENSES

Just like the insurer incurs expenses to settle claims, it also incurs expenses in its underwriting process. These are generally separated into 4 categories :

- Commissions and brokerage;

- General;

- Taxes, licenses and fees;

- Other acquisition.

Commissions and taxes are self-explanatory. We define other acquisition expenses everything other than commission and brokerage expenses that are directly linked to the process of acquiring business (publicity, mailing cost, etc.). The general expenses are every costs associated with the insurance operations, for example maintenance of the company's offices.

1.5 PROSPECTIVE RATEMAKING & EXPERIENCE ADJUSTMENTS

It is common practice in ratemaking to use historical data to predict future expected costs : *this does mean actuaries are setting premium to recoup past losses*, as this would contradict the first principle of the CAS's statement of principles, which is "*A rate is an estimate of the expected value of future costs*".

So we can use historic loss experience in the estimation of future losses. With that said, it is important to remember that adjustments are necessary for the experience to accurately predict future losses. Namely, we have to consider these potential factors :

- Rate changes

- Operational changes

- Inflationary pressures

- Change in the mix of business

- Law changes

1.6 BASIC INSURANCE RATIOS

Here are a few basic ratios that insurers use to monitor and evaluate the appropriateness of its rates.

Frequency

Frequency mesures the rate at which claims occur. It is calculated as :

$$\textbf{Frequency} = \frac{\textbf{Number of Claims}}{\textbf{Number of Exposures}}.$$

Usually, we would use the reported claims and the number of earned exposures. Analysis of frequency models can help measure the effectiveness of underwriting actions, such as deductibles.

Severity

Severity is the measure of average costs per claim, and is calculated as :

$$\textbf{Severity} = \frac{\textbf{Losses}}{\textbf{Number of Claims}}.$$

We would calculate the paid severity using paid losses on closed claims, divided by the number of closed claims, and the reported severity using reported losses and claims. ALAE can be included or excluded from calculations.

Analysis of severity models helps the understanding loss trends.

Pure Premium

$$\textbf{Pure Premium} = \frac{\textbf{Losses}}{\textbf{Number of Exposures}} = \textbf{Frequency} \times \textbf{Severity}.$$

Also known as loss cost or burning cost, it measures the average loss per exposure.

Average Premium

$$\textbf{Average Premium} = \frac{\textbf{Premium}}{\textbf{Number of Exposures}}.$$

The average premium can be calculated using earned premium and exposures as well as written or in force exposure, depending on what is analyzed. Both premium and exposures have to be on the same basis.

If adjusted for rate changes, changes in average premium highlight changes in the mix of business written.

Loss Ratio

$$\text{Loss Ratio} = \frac{\text{Losses}}{\text{Premium}}.$$

A loss ratio measures the portion of each premium dedicated to paying losses. Typically, earned premiums and reported losses used. Sometimes, LAE are included in the losses, forming a loss and LAE ratio, which would be :

$$\text{Loss and LAE Ratio} = \frac{\text{Losses} + \text{LAE}}{\text{Premium}} = \text{LR}(1 + \text{LAE ratio}).$$

Loss Adjustment Ratio

$$\text{LAE Ratio} = \frac{\text{LAE}}{\text{Losses}}.$$

Usually, analysis of LAE Ratio if made to monitor the effectiveness of the claim settlement process.

Underwriting Expense Ratio

If we want to calculate the underwriting expense ratio for all underwriting expenses, than we would do

$$\text{UW Expense Ratio} = \frac{\text{Commission} + \text{Taxes} + \text{Other}}{\text{Written Premium}} + \frac{\text{General Expenses}}{\text{Earned Premium}}.$$

If we want to calculate the ratio for a specific type of underwriting expense, it is important to note that the basis of premiums will depend on the type of underwriting expenses analyzed. In general, the ratio is

$$\text{UW Expense Ratio} = \frac{\text{UW Expenses}}{\text{Premium}}.$$

- For commissions, other acquisitions and taxes, we use *written premiums*, the reasoning being that no matter if the policies remains in force the total duration of its term, we will pay those expenses in full. Hence the use of written premiums.

- For general expenses, we use *earned premiums*, since you want to evaluate theses expenses with the amount of premiums the company is actually earning.

Operating Expenses Ratio (OER)

$$\text{OER} = \text{UW Expense Ratio} + \frac{\text{LAE}}{\text{Earned Premiums}}.$$

This ratio measures the overall portion of premiums used to pay for all the company's expenses except the actual losses coming from claims.

It's important to note that in this definition, earned premiums are used in the calculation of the underwriting expense ratio.

Combined Ratio

$$\textbf{Combined Ratio} = \textbf{Loss Ratio} + \frac{\textbf{Underwriting Expenses}}{\textbf{Written Premiums}} + \frac{\textbf{LAE}}{\textbf{Earned Premiums}}.$$

If we were to use earned premiums instead of written premiums in the underwriting expense ratio, we could also write the combined ratio as

$$\textbf{Combined Ratio} = \textbf{Loss Ratio} + \textbf{OER}.$$

In calculating the combined ratio, the loss ratio should not include LAE, since we would consider theses expenses twice. Obviously, you always want a combined ratio under 100%, otherwise you are losing money in operating the business.

Renewal/Retention Ratio

Retention and renewal ratios are the rate at which insureds renew their policies. There are a small difference in their definition. We define a renewal ratio as

$$\textbf{Renewal Ratio} = \frac{\textbf{Number of Policies Renewed}}{\textbf{All expiring written policies}}.$$

A renewal ratio doesn't take into account cancellations of policies during their term, it only looks at how many policies renewed from the pool of policies that would have come to expire. As opposed to that, a retention ratio is defined as

$$\textbf{Retention Ratio} = \frac{\textbf{Number of Policies Renewed}}{\textbf{Number of Potential Renewal Policies}}.$$

As we can see, a retention ratio only takes into account policies which were available for renewal, hence not considering policies that canceled their coverage during their term. It is a conditional version of a renewal ratio

Close Ratio

$$\textbf{Close Ratio} = \frac{\textbf{Number of Accepted Quotes}}{\textbf{Number of Quotes}}.$$

The close ratio measures the competitiveness of the company's rates on the market for new business.

Chapter 2

RATING MANUALS

Chapter 3

Ratemaking Data

3.1 Data Aggregation

In order to perform ratemaking analysis, policy, claims and accounting data must be aggregated. There are three general abjectives when aggregating :

1. Accurately match losses and premium for the policy

2. Use the most recent data available

3. Minimize the cost of data collection and retrieval

There are four common methods of data aggregation : calendar year, accident year, policy year and reported year.

3.1.1 Calendar year

This method considers all premium, exposure and loss transaction that occur during a twelve month calendar year, *without regard to the effective date of policy insurance, the accident date or the report date of the claim.* Hence, at the end of the calendar year, earned premiums and exposures are fixed (since all future premiums and exposure will be earned in the next calendar year).

For reported loss, when aggregating in calendar year they are equal to all losses paid during the twelve month period, plus the change in case reserve. In math, it looks like :

$$\text{Reported Losses} = \text{Paid Losses} + (\text{Case Reserve}_{fin} - \text{Case Reserve}_{ini})$$

Advantages of the method are :

1. Data is available as soon as the calendar year ends;

2. Very few cost of collection and retrieval.

Disadvantages of the method are :

1. There is a time mismatch between premiums and losses, as losses may include payment and reserve changes from policies issued years ago.

One would use calendar year for lines of business with relatively quick loss developments.

3.1.2   Accident year

Using this aggregation, premiums and exposures follow the same logic as in a calendar year aggregation ; the difference comes in the aggregation of losses. Whereas in calendar year you would only compiled losses that were incurred during the year, in accident year losses are always associated with their respective accident year. So if an accident happens in 2017, losses from this specific event will always be counted in the losses of the 2017 calendar year. This difference makes it so that losses are not set at the end of a calendar year, as they are likely to develop further in time.

Advantages of the method are :

1. Better match between premiums and losses, as premiums earned during a calendar year are compared to losses that occurred in the same year;

2. Few cost of collection and retrieval.

Disadvantages of the method are :

1. Since losses can develop at the end of the year, their development has to be estimated. The other option would be to wait for the development to end, which would take way to much time in certain line of business.

2. As a consequence of the first disadvantage, statistics from a calendar year are bound to evolve as losses from that year develop.

3.1.3   Reported year

This method works exactly the same as accident year, with one small difference. Instead of using the accident year, the reported year of claims is used to aggregate losses. This is primarily used for commercial line products while using claims-made policies. Other that that, it has the same advantages and disadvantages as accident year aggregation..

3.1.4   Policy year

Also known as underwriting year, this method considers all premiums and losses associated with policies written during a twelve-month period. So all the premiums, exposures and losses of a policy written on December 30th 2017 would be considered in the year 2017, even though most (almost all) of its premiums would be earned in the calendar year 2018.

This specificity makes this method the best at matching premiums and losses, as they are always associated with each other. On the other hand, it also makes it the slowest to develop, since premiums from the policy year 2017 wont be fully earned until December 31th 2018, 24 months after the start of the policy year.

Advantages of the method are :

1. Best match possible between premiums and losses

Disadvantages of the method are :

1. Data takes 24 months from the start of the policy year to be fully available.

2. Losses development has to be taken into account, so results from a policy year are bound to evolve until losses from the policy year are done developing.

Chapter 4

EXPOSURES

4.1 CRITERIA FOR EXPOSURE BASES

A good exposure base should meet the three following criteria :

1. Being **directly proportional to the expected loss**;

2. Be **practical**;

3. Consider **preexisting exposure base and historical data**.

4.1.1 Proportional to Expected Loss

All else being equal, the expected loss of a policy with two exposures should be twice as high as the one of a one exposure policy. In general, the factor with the most direct relationship to the losses is used as the exposure base.

For example, for home insurance, the expected loss of a home insured for two years is twice the expected amount of the same home insured one year. We could say that the expected losses also vary with the value of a home, but while a 200 000$ home has higher expected losses, it's nor necessarily twice as high as a 100 000$ home. Hence, the number of house year is selected as exposure base, while the home's value is used as a rating variable.

4.1.2 Practical

The exposure base should be objective and relatively easy to obtain and verify. It should not allow for moral hazard from policyholders or underwriters (manipulating the exposure base for their own benefit).

For example, in auto insurance, the exposure base being the most proportional to expected losses is the yearly mileage of the vehicle. That being said, it's very hard for insurers to verify without reasonable doubt (although the technology is getting there). Hence, the most practical exposure base is still the vehicle year.

4.1.3   Historical Precedence

Any changes to the exposure base can lead to large premium swings for individual insureds and requires major changes in the rating algorithm. Also, the company might not have the historical data in order to use its experience in the ratemaking process.

4.1.4   Common Exposure Bases by Line of Business

Here are the most used exposure bases by line of business.

Line of Business	Typical Exposure Base
Personal Automobile	Earned Car Year
Homeowners	Earned House Year
Workers Compensation	Payroll
Commercial General Liability	Sales Revenue, Payroll, Square Footage, Number of Units
Commercial Business Property	Amount of Insurance Coverage
Physician's Professional Liability	Number of Physician Year
Professional Liability	Number of Professionals (ex: Lawyers, Accountants, etc.)
Personal Articles Floaters	Value of Item

4.2   Aggregation of Exposures (One-year Policies)

When aggregating exposures, only two methods are applicable : calendar and policy year. Here is how one would aggregates the different types of exposures (written, earned, in-force) using both methods.

4.2.1   Written Exposures

Written exposures are the total exposures arising from policies written during the period. For example, if a one-year policy is written on May 21st 2017, this would count as one exposure in calendar year 2017 and policy year 2017. The difference between the two methods of aggregation comes with cancellations of policies. For example, let consider a one-year policy written on June 30th 2017 that canceled on April 1st 2018.

When working in **policy year**, we would book both the original written exposure and the exposure due to cancellation in 2017. At the end of the policy year, this policy would account for $(1 - \frac{3 \text{ remaining months}}{12 \text{ months}}) = \frac{9}{12}$ exposure.

In contrast, when working in **calendar year**, the original written exposure and the exposure due to cancellation would not be booked in the same year, since the events didn't happen in the same calendar year. So this policy would account for 1 exposure in 2017 and $(-\frac{3 \text{ remaining months}}{12 \text{ months}}) = -0.25$ exposures in 2018.

4.2.2   Earned Exposures

Earned exposures represent the portion of the written exposures for which the insurance coverage has already been provided. Usually, we assume that the probability of claims is evenly distributed throughout a year, so that we can use the portion of time that passed since writing of the policy as a basis.

For **policy year**, earned exposures are easy. Assuming that the policy year is completed, earned and written exposures are the same, since they are both counted in the same year. If the policy year isn't over when calculations occur, than we would use the portion of time that passed since the writing of the policy as earned exposures. For example, if calculations of earned exposure for policy year 2017 were made on June 30th 2018, a one-year policy written on October 1st 2017 would have 9/12 exposures.

For **calendar year**, things are a little more complicated. We should only consider the exposure earned in the calendar year. So for the same one-year policy written on October 1st, we would only count the exposure in the months of October, November and December as earned in 2017. The rest of the exposure would be counted in 2018. Obviously, earned exposures are rarely the same as written exposures for a specific policy.

4.2.3   In-Force Exposure

These exposure counts the number of insured risk that are exposed to claiming at a specific point in time. On a typical exposure graph (with time a X-axis and % of policy term expired on the Y-axis) then we'd count all the policies that cross a straight vertical line at a given point

4.3   AGGREGATING WITH OTHER POLICY TERMS

All of the preceding definition and examples were based on one-year policies, but in reality terms can be longer or shorter that a year. Assuming we are working with a policy-year base exposure, here are the main adaptation made when working with other terms :

- When working with **written** exposures, all of the policies represent one-half of the written exposure of a one-year policy. Hence, a 6 months policy would represent 0.5 exposure and a two-year policy would be 2 exposures.

- For **in-force** exposures, each policy can contribute to *one in-force exposure*, no matter the policy term.

4.4   CALCULATION OF BLOCKS OF EXPERIENCE

Companies don't always have detailed information about policies. For example, they might have the informations summarized on a monthly or quarterly basis. When such a situation happens, insurers usually work as if *all policies were written on the mid-point of the period*. This is a good approximation only when policies are written evenly throughout the period. Hence, the shorter the period, the more likely this assumption is accurate.

As a quick example, if informations are summarized by month, we would assume that all of the policies written in that month were effective on the 15th of that month.

Since the assumption is that all policies for a given month are written on the 15th of that month, the written exposures for annual policies will be earned over a 13 month period. That is because there is always a half-month still to be earned after 12 months, coming from the fact that we started on the 15th.

Other than that distinction, the same principles of aggregation covered previously apply.

Chapter 5

PREMIUMS

The process of ratemaking usually takes into account historical premium, but in order to do that several adjustment have to be applied. More specifically :

- Bring historical premiums to the rate currently in effect;

- Develop premiums to ultimate levels, if it is still evolving;

- Project historical premiums to the premium level in effect in the future.

But first, just like exposures, let's review ways of aggregating premiums. As the procedure is really similar, we focus on the differences.

5.1 PREMIUM AGGREGATION

5.1.1 Written Premiums

In principle, it's the same as exposure : all premium written in calendar or policy year counts in that specific year. The only difference is again when policies are canceled. When dealing with cancellation in calendar year, we subtract the proportion of written premium that was left to be paid in that calendar year.

For example, when writing a policy on July 1st 2017 for 900\$, we have 900\$ of written premium in the calendar year 2017. If the policy was to be canceled on April 1st 2018, we would subtract $900(1 - 2/3) = 300$\$ from the written premiums of calendar year 2018. In policy year, we would subtract from policy year 2017.

Of course, as with exposures we have

$$\text{Written Premium} = \text{Earned Premium} + \text{Unearned Premium}.$$

5.1.2   Earned Premiums

It works exactly as exposure. The only difference is that instead of calculating the portion of a policy's written exposure, we take the portion of its written premium earned in a specific calendar or policy year.

5.1.3   In-Force Premiums

In-force premiums of a company represents the total amount of full-time premium for all policies in effect at a given time. This metric should be interpreted with care when comparing two portfolios with written policies on different terms. For an insurer that writes half-year policies would have half of the in-force premiums as the same insurer writing one-year policies (even though after a year, they both could have the same earned exposure).

5.1.4   Calculation of Blocks of Policies

Works the same as with exposures. If we have aggregated data by period, it is customary for an insurer to assume all of the policies were written on mid point of the period. With that, you can calculate written, earned and in-force premiums.

5.2   Adjustment To Premium : Extension of Exposures

This method involves rerating every policy from the historical data to restate historical premium to the amount that would be charged under the current rates.

It is obviously the most accurate current rate level method, but it requires detailed data and good computing power. Also, actuaries must have all of the applicable rating characteristics for every policy in the historical period.

5.3   Parallelogram Method

See Werner & Modlin manual, page 74 to page 80 for full details.

This method assumes that premium is written evenly throughout the time period. It adjusts *the aggregated historical premium* by an average factor to bring the premiums on-level with the current rates.

As premiums are aggregated, we calculate the on-level premiums by aggregate group (calendar year, policy year, etc.). The method differs in the calculations depending on the method of aggregation, but the logic is the same.

Here are the general steps of the parallelogram method :

> 1. For each rate change, calculate the cumulative index (taking in consideration all of the previous rate changes). For example :

Rate Level Group	Effective Date	Rate Change	Cumulative Index
1	Initial	0%	1
2	07/01/2010	5%	1.05
3	01/01/2011	10%	1.155
4	04/01/2012	-1%	1.143

2. Calculate the proportion of the aggregated exposure of the period that correspond to each rate level group. This can usually be done geometrically.

3. With each of the proportion, calculate the weighted average cumulative index for the period, like this (if you don't know how to do a weighted average) :

$$\text{Average Cumulative Index} = \sum_{i=1}^{n} (\text{Proportion}_i)(\text{Cumulative index}_i)$$

4. The on-level factor is

$$\text{On-Level Factor} = \frac{\text{Current Cumulative Rate Level Index}}{\text{Average Rate Level Index}}.$$

5. the On-Level Premium for the aggregated period is

$$\text{On-Level Premium} = \text{Aggregated Premiums} * \text{On-Level Factor}.$$

Exemple 5.3.1
CAS Exam 5, Fall 2013 Question #2 :
All policies have 6-months terms, and are written evenly throughout the year. The rating algorithm is (Base rate $*$ Class Factor + Fees), ans the current rates are :

Base Rate	500$
Class A Factor	1.00
Class B Factor	0.80
Fees	55$

Here is the distribution of exposures for different underwriting period :

Period	Effective Date	Class A written exposures	Class B written exposures
1	01/01/2011 - 06/30/2011	125	50
2	07/01/2011 - 12/31/2011	150	100
3	01/01/2012 - 06/30/2012	175	150
4	07/01/2012 - 12/31/2012	200	200

Using the Extension of Exposures method, calculate the on-level earned premium for calendar year 2012.

Solution:
As we have exposures from 6-months period, we will choose the average written date as evaluation point and assume that the exposures are evenly written. From the evaluation point, we calculate the portion of the period that is earned in calendar year 2012. So we have :

Period	Mid Point	% Earned in 2012	Class A exposures	Class B exposures
1	04/01/2011	0%	125	50
2	10/01/2011	50%	150	100
3	04/01/2012	100%	175	150
4	10/01/2012	50%	200	200

Now that we know % of exposure earned in 2012 for each period, we can calculate the aggregate earned exposure for calendar year 2012 and each class.

$$\text{Class A} : 0(125) + 0.5(150) + 1(175) + 0.5(200) = 350 \text{ Earned Exposures}$$
$$\text{Class B} : 0(50) + 0.5(100) + 1(150) + 0.5(200) = 300 \text{ Earned Exposures}$$

With the earned exposures per class, we can now calculate the on-level earned premium :

$$\text{Class A} : (500 * 1.00 + 55)(350) = 194250\$$$
$$\text{Class B} : (500 * 0.80 + 55)(300) = 136500\$$$

Meaning that, for calendar year 2012, there are $194250 + 136500 = \boxed{330750}$ earned premium.   ∎

Exemple 5.3.2
CAS Exam 5, Fall 2015 Question #1 :
For a book of 6-months policies that uses vehicle-year as its exposure base, the premium per vehicle is 500\$ per 6-months, for all policies effective before August 31st 2014. We have the following data:

Effective date	# of vehicle written on this date
02/01/2013	1100
08/01/2013	800
02/01/2014	600
08/01/2014	300

Knowing a rate change is effective on September 1st 2014, calculate :

1. the written and earned exposures for calendar year 2014

   2. the on-level earned premiums for calendar year 2014, using extension of exposures and parallelogram method.

Solution:
#1 - Written and Earned Exposures for Calendar Year 2014:

Written exposures : $\quad 0.5(600) + 0.5(300) = 450$
Earned exposures : $\quad \frac{1}{6}(0.5)(800) + 0.5(600) + \frac{5}{6}(0.5)(300) = 491.66$

*Note that we are multiplying by 0.5 because the exposure base is vehicle-year, while we are working with 6-months policies .

#2 - Extension of exposures method:
There is a subtle detail in the question that is quite important. We are given that the premium per vehicle on a **6-months term** is 500\$, but the exposure base is the vehicle-**year**. Hence, the premium for a full-year vehicle is $500(2) = 1000\$$. As we calculated the earned exposure on a vehicle-year basis in #1, we need to have a year-base premium to calculate the earned premium.

With that said, to calculate the on-level premium, we re-rate all the policies. That means rating all 491.66 earned premium according to the -18% rate change that occurred on September 1st. Hence, we have :

$$\text{On-Level Premiums} = 1000(1 - 0.18)491.66$$
$$= \boxed{403\ 169\ \$}$$

#2 - Parallelogram method :
When using this method, we have to assume that the policies were written evenly throughout time, even though in this case we know it to be false.

As policies have a 6-months term and the rate change is effective on September 1st, we have 1/9 of the policies written in calendar year 2014 that are on the new rates. Calculations goes as follow :

$$\text{Average cumulative index} = 1/9(1 - 0.18) + 8/9 = 0.98$$
$$\text{On-Level factor} = \frac{0.82}{1/9(1 - 0.18) + 8/9} = \frac{0.82}{0.98} = 0.836739$$
$$\text{On-Level Premium} = 0.836739(491.66)(1000\$) = \boxed{411\ 394\ \$}$$

      ■

5.4   PREMIUM DEVELOPMENT

Premiums might have to be developed in order to have an accurate ratemaking process. For example, is an actuary performs analysis on a policy year before all the policies written in that year have expired, than the actuary does not know which policies may still change or cancel during their remaining life.

Another example is when the the total exposure for a period can change. Let's say the insured pays premium based on an estimate or the total exposure. Once the policy is completed and the actual exposure is known, the final premium owed is calculated. This happens a lot is **workers compensation**.

5.5   EXPOSURE TREND

For business line that use inflation-sensitive exposures bases, it is common practice to project exposures (and thus premiums) to account for future inflation.

5.6   PREMIUM TREND (ONE/TWO-STEP TRENDING)

In addition to inflationary pressures, the average premium can change over time due to changes in the characteristics of the policies written (distributional changes). To account for future distributional changes, we also have to adjust the historical on-level premium to the level expected during the future time period. **Note that one should always start with the on-level premiums before adding any trend, as we don't want to include rate changes in the trend factor**.

THIS IS NOT THE SAME AS ADJUSTING TO CURRENT RATE LEVEL. The premium also have to be adjusted to reflect any premium trend.

Typically, actuaries examine changes in the historical *average* premium per exposure to determine the premium trend.

Trends can be determined using written or earned premiums ; written exposures tend to reflect shifts in the distribution faster than earned premium, but both are valid. The more granular the data used (for example quarterly instead of annual premium), the more responsive will be the statistic.

After determining the premium trend factor, there are two methods for adjusting historical data : **one-step** and **two-step** trending.

5.6.1   One-Step Trending

If we think that the historical trend we stay the same in the future rate effective period, then we would use one-step trending.

We have to determine the length of time to trend the earned premiums of the historical period. The trend period is always defined the same :

From :	Average WRITTEN date of the policies that have earned premiums in the historical period.
To :	Average WRITTEN date of the policies that will be in effect during the rate effective period.

Depending on if the historical period is aggregated by calendar or policy year, the "from"

> date will change. **Always refer to the definition**. The "to" date will stay the same, as the rate effective period is always in policy year.

The length of the period is obviously the difference between the "to" date and the "from" date. To calculate the trended on-level historical premiums, the calculation is :

> **Trended On-Level Premium** = (On-Level Premium)(Trend Factor)$^{\text{(length of trending period)}}$

5.6.2 Two-Step Trending

We would use two-step trending when we expect the premium trend to change over time, meaning that the historical trend won't necessarily apply to the future rate effective period.

In two-step trending, there are two steps (thanks captain obvious):

1. Adjust historical on-level premium to the average premium level of **the most recent point in the trend data**

2. Apply a separate trend factor to project the premium calculate in step #1 to the future rate effective period.

Step #1

In step #1, we have two options. First, the current trend factor can be calculate as

$$\textbf{Current Premium Trend Factor} = \frac{\text{Latest Average WRITTEN on-level premium}}{\text{Historical average EARNED on-level premium}}.$$

When the historical average premium is volatile, the actuary can also choose to evaluate a trend based on several points rather than to use the previous ratio. If this method is used, then the trending factor is determined just like one-step trending, but with the following dates for the period :

From : Average WRITTEN date of the policies that have earned premiums in the historical period.

To : Average WRITTEN date of the **latest period** of the trend data.

Step #2

In step #2, we want to select the amount of average premium that is expected to change annually from the current level to the future rate effective period. We also have to calculate a trend period for which to apply this **future trend factor**, that is defined as :

From : Average WRITTEN date of the **latest period** of the trend data.

To : Average WRITTEN date of the policies that will be in effect during the rate effective period.

Finally, the final trended on-level premium is calculated like this :

$$\text{\textbf{Trended On-Level Premium}} = \text{On-Level Premium}$$
$$* \text{Current Trend Factor}$$
$$* (\text{Future Trend Factor})^{(\text{length of trending period})}$$

Chapter 6

Losses and ALAE

Chapter 7

UW Expenses Ratios

In order to calculate an indicated rate or rate change, we need to take into account underwriting expenses, more specifically how much of the premiums is used up to account for those expenses. Since there is variable and fixed underwriting expenses, we will need a ratio for both types.

7.1 Variable Expense Ratio

This ratio evaluates the ratio of variable expenses to premiums. Just like in the definition of the underwriting expense ratio (in chapter 1), the type of premium used depends on the type of expense. We remember that except for general expenses which uses earned premiums for the ratio, all other type of expenses uses written premiums.

If we assume that all of commissions and taxes are variable, we would calculate the variable expense ratio as :

$$\textbf{Variable Ratio} = \frac{\textbf{Commis} + \textbf{Taxes} + (\textbf{Variable \% })(\textbf{Other})}{\textbf{Written Premium}} + \frac{(\textbf{Variable \%})(\textbf{General})}{\textbf{Earned Premium}}$$

IMPORTANT : when calculating the variable expense ratio, it's important that periods for expenses and premiums match, meaning that **if we use historical expenses (\$), then we use historical premiums (\$) as well.**

7.2 Fixed Expenses Ratio

When having to find a fixed expense ratio, there is two methods possible, which we generally use depending on if the fixed expenses are trending at the same rate as the premiums or not. If they are (fixed expense trend = premium trend), then we would use the **Premium-based method**, if not (fixed expense trend ≠ premium trend) we would use the **Exposure-based method.**

7.2.1 Premium-based method

As stated, this method generally assumes that the fixed expenses trend at the same rate as the premiums. This means that we can use historical premiums to calculate the fixed expense ratio,

without going through the trouble of calculating on-level premiums. Hence, assuming that only general and other expenses are fixed, the definition of the fixed expense ratio is :

$$\textbf{Fixed Expenses Ratio} = \frac{\textbf{(Fixed \%)(Other)}}{\textbf{Histo. Written Premium}} + \frac{\textbf{(Fixed \%)(General)}}{\textbf{Histo. Earned Premium}}$$

7.2.2   Exposure-based method

If expenses are trending at a different rate than premiums, this is the method to use. We calculate the expenses per exposure, that we trend to the futur rate effective period.

The definition of the *trending period* can be a little complex, since we usually make the assumption that general expenses are incurred at the average *earned* premium date, while other expenses are incurred at the average *written* premium date. Usually, the historical period will be in calendar year, making it so that the average written date and the average earned date is the same (the middle of the calendar year). But for the new rate effective period, which is a policy year, the written average date isn't equal to the average earned date : the average earned date is always $1/2$ a policy term after written average date. This makes is so that **the trending period for general expenses is always a 1/2 policy term longer than the trending period for the other expenses.**

When we have the **total trended fixed expense per exposure**, which is the sum of all trended expenses per exposure, we can find the fixed expense ratio by dividing by the **average on-level and trended earned premiums**. This is the main difference from the Premium-based method, where we generally can use the historical premiums instead of on-level.

Part II

Reserving (*Freidland*)

Chapter 8

CASE OUTSTANDING TECHNIQUE

8.1 TECHNIQUE #1

In order to use this technique, we need to have access to cumulative paid and reported claims triangles (from which you can calculate the case outstanding triangle). Here are the steps of the method:

1. **Calculate the INCREMENTAL paid claims triangle.**

2. **Calculate the case outstanding triangle.**

3. **Calculate the ratio of *incremental paid* to *case oustanding*, and select a ratio per maturity.**
   The ratio is defined as :

   $$\text{Ratio}_{i,j} = \frac{\text{Incremental Paid Claims}_{i,j}}{\text{Case Outstanding}_{i,j}}.$$

   This ratio corresponds to the proportion of case outstanding that was paid during the interval $(j - 12, j)$.

   When te triangle is complete, we select a ratio for each maturity. **These will be used to develop the incremental paid claim triangle.**

4. **Calculate the development factors for the case outstanding triangle.**
   This works just like calculating the developing factors for reported or paid claims. We take the ratio of the case outstanding to the previous case outstanding. **We select a developing factor for each maturity, that will be used to develop the case outstanding to ultimate levels.**

5. **Complete the case outstanding triangle**
   We multiply the previous case outstanding with the selected ratio for the current maturity.

This will effectively complete the bottom part of the triangle.

6. **Complete the incremental paid claim triangle**
   We complete the bottom part of the triangle. We use the freshly completed triangle of case outstanding with the ratio defined at step #3. Since we now have case outstanding values for all cells (i,j), we can also calculated all the cells of the incremental paid triangle with the relationship :

$$\text{Incremental Paid Claims}_{i,j} = (\text{Ratio}_{i,j}) \left(\text{Case Outstanding}_{i,j}\right).$$

7. **Sum up the incremental paid claims for each accident year to have the ultimate cumulative paid claims.**

8.2   TECHNIQUE #2

This technique is only usable if *there is no information on the reported and paid claims* for the company; **only the case outstanding are known**.

In this situation, the insurer can use industry benchmark to calculate a development factor for the case outstanding in order to find then ultimate *unpaid* claims. The factor are always applied to the current case outstanding for each accident year.

$$\text{Factor} = 1 + \frac{(\text{Rept CDF} - 1)(\text{Paid CDF})}{\text{Paid CDF} - \text{Rept CDF}}$$

The CDF's are all coming from industry benchmark. The ultimate unpaid claims are defined as:

$$\text{Ultimate Unpaid Claims} = \left(\begin{array}{c}\text{Current Case}\\\text{Outstanding}\end{array}\right)\left(1 + \frac{(\text{Rept CDF} - 1)(\text{Paid CDF})}{\text{Paid CDF} - \text{Rept CDF}}\right)$$

In order to calcultate the IBNR, we would subtract the current case outstanding to the ultimate unpaid claims.

Chapter 9

BERQUIST-SHERMAN TECHNIQUES

Theses techniques are used when an insurer has to deal with operational changes. Berquist and Sherman propose to ways :

- Use data *selection* to isolate or neutralize the impact of the changes;

- Adjust the data to restate historical data as if the changes never occurred;

The first option has more to do with making wise choices considering the situation, but doesn't require a specific methodology. The second option however does, and this is what is known as the Berquist-Sherman techniques. Theses techniques can be used to adjust triangles for changes in **claim settlement rates** and/or **case reserve adequacy**.

9.1 CLAIM SETTLEMENT RATES CHANGES

The assumptions of this method is that changes in the disposal rates are only due to changes in settlement rates, not due to things like changes in prioritization between small and large claims or in the rate of reporting.

Here are the steps of this technique :

1. **Calculate the disposal rate triangle**.
   As we remember, disposal rates for accident year i and age j $(D_{i,j})$ are defined as :

   $$D_{i,j} = \frac{\text{\# of Closed Claim Count}_j}{\text{\# of Ultimate } \textit{Reported} \text{ Claim Count}_i}.$$

2. **From the disposal rate triangle, select disposal rates along one of the diagonals**.
   Usually, we choose the latest disposal rate (the latest diagonal of the triangle). We will denote the selected disposal rate as $\hat{D}_j$, where j can be all of the maturities of the triangle.

3. **Create the adjusted paid claims triangle.**

To do this, we need to determine a relationship between disposal rated of paid claim amount. This is usually done by regression. On the exam, we should expect linear or exponential regression to be used. If the relationship is linear, linear interpolation can be used.

For example, let's say we have the two following triangles :

| Accident | Disposal Rates | | | | Avg Paid Claims | | | |
Years	12	24	36	48	12	24	36	48
2010	0.400	0.730	0.850	**0.920**	200	350	450	520
2011	0.400	0.667	**0.833**		250	400	550	
2012	0.357	**0.679**			240	420		
2013	**0.367**				310			

The amount of paid claims we interpolate between depend on whether the historical disposal rate $D_{i,j}$ is higher or lower than the *selected* disposal rate $\hat{D}_j$. Let's assume that we can use linear interpolation. We will show how to use exponential regression between two points later.

- **If $D_{i,j} < \hat{D}_j$**, just like for AY2012 at maturity 12 where $0.357 < 0.367$, then we will want to increase the cumulative paid claims at AY2012 maturity 12. The logic is that we want to bring $D_{2012,12}$ closer to the level of $\hat{D}_{12}$. In order to increase the level of $D_{i,j}$, we will interpolate the cumulative paid claims $PC_{i,j}$ with the paid claims from the next maturity.

  In our example, that means interpolate $PC_{2012,12}$ with $PC_{2012,24}$. So we'd have

$$\text{Adjusted } C_{(2012,12)} = \left(\frac{\hat{D}_{12} - D_{2012,12}}{D_{2012,24} - D_{2012,12}} \right) PC_{2012,24} + \left(\frac{D_{2012,24} - \hat{D}_{12}}{D_{2012,24} - D_{2012,12}} \right) PC_{2012,12}$$

$$= PC_{2012,12} + (PC_{2012,24} - PC_{2012,12}) \left(\frac{\hat{D}_{12} - D_{2012,12}}{D_{2012,24} - D_{2012,12}} \right)$$

$$= 240 + (420 - 240) \left(\frac{0.367 - 0.357}{0.679 - 0.357} \right)$$

$$= 245$$

- **Else if $D_{i,j} > \hat{D}_j$**, just like for AY2010 maturity 12 where $0.400 > 0.367$, then the interpolation has to decrease the amount of paid claims. Hence, we interpolate between the $PC_{i,j}$ and the paid claims of the *previous* maturity.

  In our example with AY2010 and maturity 12, that means interpolate between $PC_{2010,0} =$

0 and $PC_{2010,12} = 200$. The adjusted paid claims would then be

$$\text{Adjusted } C_{(2010,12)} = \left(\frac{\hat{D}_{12} - 0}{D_{2010,12} - D_{2010,0}} \right) PC_{2010,12} + \left(\frac{D_{2010,12} - \hat{D}_{12}}{D_{2010,10} - 0} \right) PC_{2010,0}$$

$$= PC_{2010,0} + (PC_{2010,12} - PC_{2010,0}) \left(\frac{\hat{D}_{12} - D_{2012,0}}{D_{2012,12} - D_{2012,0}} \right)$$

$$= 0 + (200 - 0) \left(\frac{0.367 - 0}{0.400 - 0} \right)$$

$$= 183$$

Instead of a linear interpolation, we could have had to use exponential regression between two points to find the adjusted paid claims. Here's how to proceed in that situation. We start with the general equation $\boxed{\hat{Y}_{i,j} = a_{i,j} e^{b_{i,j} X_{i,j}},}$ where X is the closed claim count at (i,j), Y is the adjusted paid claims at (i,j) and a and b are the regression parameters.

The same rules apply regarding if we interpolate between the paid claim at previous or next maturity (if $D_{i,j} <$ or $> \hat{D}_j$), but these now apply as to the values used to calculate the b parameter. Assuming that we interpolate to the next maturity, the b parameter would be :

$$b_{i,j} = \frac{\ln(PC_{i,j+12}) - \ln(PC_{i,j})}{nC_{i,j+12} - nC_{i,j}}$$

where $nC_{i,j}$ is the number of closed claims at (i,j).

After finding each $b_{i,j}$ parameter, we would find the $a_{i,j}$ parameters using the culumative paid claims and the number of closed claims at (i,j). If $Y_{i,j}$ is the cumulative paid claims at (i,j), then we have

$$a_{i,j} = \frac{Y_{i,j}}{e^{b_{i,j} X_{i,j}}}$$

With both parameters, we can then calculate the adjusted paid claims, with $\boxed{\hat{Y}_{i,j} = a_{i,j} e^{b_{i,j} X_{i,j}}}$.

4. **When the adjusted paid claims triangle is complete, the last step is to calculate the ultimate claims using the development technique.**

9.2 CASE ADEQUACY ADJUSTMENT

The last section adjusted *paid* claims. In order to makes adjustment to *reported* claims triangle, we need to determine whether there have been changes in case adequacy, and if so, how to *adjust the case outstanding* for these changes. Here are the steps of this technique :

1. **Calculate the adjusted *open* claim count.**
   To calculate the adjusted *open* claim count, we first need to adjust the closed claim count.

We calculate the **adjusted *closed* claim count** by multiplying the **disposal rates** by the **projected unadjusted *reported* claim count** (which we usually get in the question).

$$\begin{pmatrix} \text{Adjusted } Closed \\ \text{Claims Count} \end{pmatrix}_{i,j} = \begin{pmatrix} \text{Disposal} \\ \text{Rate} \end{pmatrix}_{j} \begin{pmatrix} \text{Projected Unadjusted} \\ Reported \text{ Claim Count} \end{pmatrix}_{i}$$

Then, we calculate the **adjusted *open* claim count** by subtracting the adjusted closed claim count to the reported claim count.

$$\begin{pmatrix} \text{Adjusted } Open \\ \text{Claim Count} \end{pmatrix}_{i,j} = \begin{pmatrix} \text{Reported Claim} \\ \text{Count} \end{pmatrix}_{i,j} - \begin{pmatrix} \text{Ajusted } Closed \\ \text{Claim Count} \end{pmatrix}_{i,j}$$

2. **Calculate the severity trend observed in the average paid claim.**
   On the exam, we most likely habe the severity trend given from the question. Otherwise, we calculate this trend with the average unadjusted paid claim triangle, and use regression or % of change to evaluate the change.

3. **Calculate the average case reserve triangle.**
   We calculated the average case reserve (avg $CR_{i,j}$) by dividing *reported claims* minus *paid claims* by the *adjusted open claim counts*.

$$\text{avg } CR_{i,j} = \frac{\text{Reported Claims} - \text{Paid Claims}}{\text{Adjusted } Open \text{ Claim Count}}$$

4. **Calculate the adjusted average case reserves.**
   Using the severity trend calculated in the first step, we calculate the adjusted average case reserve by de-trending the latest case outstanding of each accident year by the severity trend. If we write the latest average case reserve as (avg $CR_{i,j}^{S}$), then we calculate the adjusted average case reserve as :

$$\text{Adjusted avg } CR_{i,j} = \frac{\text{avg } CR_{i,j}^{S}}{(\text{Sev Trend})^{\text{Latest Period}-i}}$$

For example, let's say we have a severity trend of 10% and the following average case reserve triangle, we would have the following adjusted average case reserve:

Accident	Avg Case Reserve			Adj Avg Case Reserve		
Years	12	24	36	12	24	36
2011	1000	3000	**5000**	$1136 = 1375/1.10^2$	$3000 = 3000/1.10^1$	**5000**
2012	1100	**3300**		$1250 = 1375/1.10^1$	**3300**	
2013	**1375**			**1375**		

5. **Calculate adjusted average *reported* claims.**
   We calculate the adjusted average reported claims by multiplying the adjusted case reserve times the open claims count and adding the cumulative paid claims :

$$\begin{array}{c} \text{Adjusted Average} \\ \text{Reported Claims} \end{array} = \left(\begin{array}{c} \text{Adjusted Case} \\ \text{Reserve} \end{array} \right) \left(\begin{array}{c} \text{Open Claim} \\ \text{Count} \end{array} \right) + \begin{array}{c} \text{Cumulative} \\ \text{Paid Claim} \end{array}$$

IMPORTANT: If there is no changes in the settlement rates, there is no adjustment to make on the paid claims. If there are settlement rates changes, then we have to make both adjustment (case outstanding and paid claims).

6. **When the adjusted reported claims triangle is complete, the last step is to calculate the ultimate claims using the development technique.**

Chapter 10

ULAE Estimation

10.1 Dollar-Based Techniques

For this section, we have to know the following notation :

M_i: Paid ULAE for calendar year i ;

W_i: Ratio of ULAE to claims for calendar year i ;

W^*: Selected ratio of ULAE to claims ;

B_i: Claim basis for calendar year i ;

L_i: Ultimate claims for calendar year i ;

R_i: Ultimate cost of claims reported (ie. opened) during calendar year i ;

P_i: Paid claims during calendar year i ;

C_i: Ultimate cost of claims closed during calendar year i ;

U_1: Percent of ultimate ULAE spent on opening (reporting) claims ;

U_2: Percent of ultimate ULAE spent on maintaining (paying) claims ;

U_3: Percent of ultimate ULAE spent on closing claims ;

The relationship of the paid ULAE to claim costs is then defined as :

$$M_i = W_i B_i = W_i \left(U_1 R_i + U_2 P_i + U_3 C_i \right).$$

As seen in the previous definition, we define the ration W_i as $W_i = M_i / B_i$.

10.1.1   Paid-to-Paid (Classical Technique)

The assumptions of this method is that the ratio is generally stable through time and that the cost of future claims management activity will be proportional to the IBNR and case outstanding. Hence, in this technique, selected ratio of ULAE to claims ($\mathbf{W}^*$) is estimated using the ratios ($\mathbf{W_i}$) of *paid ULAE* to *paid claims*:

$$W_i = \frac{B_i}{M_i} = \frac{P_i}{M_i}$$

With all of the ratios W_i, we select $\mathbf{W}^*$.

The technique assumes that half of the ULAE are sustained when a claim is opened, while the other half is sustained when it is closed. Hence, *we apply 50% of the ratio to case outstanding* (since the first half of the expenses is already incurred) and *100% of the ratio to the IBNR* (since it's not yet reported, no expenses has been incurred). This means :

Unpaid ULAE $= W^* \times [50\%(\text{Case Outstanding} + \text{IBNER}) + 100\%(\text{Pure IBNR})]$

Usually, the split between IBNR and IBNER isn't available in the data. In that case, we assume IBNER $= 0$.

The classical approach does not work well for long-tail lines of business, in times when ULAE inflation rates differ from claims inflation rates, when the insurer is significantly growing or shrinking, or when the 50/50 assumption is not appropriate.

10.1.2   Kittel Technique

The only difference between the Kittel approach and the classical approach is in the estimation of the claims basis ($\mathbf{B_i}$). Instead of using only the paid claims, the Kittel recognizes that the steady-state assumption ($R = P$) is *flawed for a growing or shrinking book*.