# Kinematic Analysis and Validation of an Industrial Robot Manipulator

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***Abstract*** **– Robot kinematics deals with traits of redundancy, joint limitation, collision avoidance and singularity manifolds. Hence, kinematic analysis plays a vital role in the positioning task of a robot manipulator. This paper discusses the kinematic analysis to study and synthesize the motion of an ABB make IRB 1200 6DOF Industrial Robot Manipulator. The forward kinematic solution is calculated using DH parameters obtained from the main dimensions of the robot and its inverse kinematics is calculated purely algebraically. The results obtained are validated using ABB's official software, RobotStudio. The experimental studies are also done to verify the kinematic model developed.**

***Keywords— Forward Kinematics, Inverse Kinematics, IRB 1200, RobotStudio.***

## I. Introduction

Industrial robots must have high flexibility to execute different tasks in industries where a human cannot perform. To manipulate objects or perform a task in the work area, the tool attached to the end-effector of the robot is required to follow a predefined trajectory. This requires control of the position of each link and joint angles of the manipulator. With the help of Denavit-Hartenberg (DH) convention proposed in 1955 [1], position analysis (Forward kinematic analysis) can be quickly done which has been widely used for finding the position and orientation of a robot end effector if joint angles are known. On the other hand, the reverse process of finding the joint angles in terms of position and orientation parameters (Inverse Kinematic Analysis) is found complex and time-consuming.

In general, the inverse kinematics problem can be solved analytically using algebraic and geometric approaches and numerical method in contrast. The analytical method is not suited for complex robot configuration due to the formation of lengthy mathematical equations and there is no common structured methodology to find the solution as it varies from robot to robot [2]. Hence numerical analysis is used for complex robot configuration and higher order DOF robots.

Numerical method constitutes various algorithms like Newton-Raphson, Predictor-Corrector etc. to solve the minimization problem. Some of them require Jacobian inverse calculation which is the tricky part if the matrix is ill-conditioned. Another numerical method is to use a non-linear programming algorithm to minimize the error function under various constraints [3]. Since Jacobian inverse is not required, this method is computationally stable but may increase computation time due to highly nonlinear kinematics of robot arm. Most of these algorithms converge to a single solution based on an initial guess.

The complexity of the inverse kinematic problem was also solved using intelligent techniques like Genetic algorithm [4], Fuzzy Logic [5] and Artificial Neural Networks [6]. Later, a hybrid approach of adaptive and Neuro-Fuzzy technique which involves the extract of gains from both the above approach is introduced [7]. Even though the ANFIS shows excellent performance in solving inverse kinematics of 2DOF and 3DOF planar robot [8], it requires high computation time and complex programming structure for higher DOF robots. Generally, the analytical method is preferred to the numerical and intelligent method as it returns the complete solution and it is computationally fast and reliable.

In this paper, the forward and inverse kinematic analysis of IRB 1200 industrial robot is done algebraically. Since the mathematical description of rigid body mechanisms is essential for their motion control, a number of examples for the kinematic analysis of various commonly used industrial robots and other mechanisms can be seen in the literature [9-12]. In section II, a brief description of the robot is given, forward and inverse kinematics modelling is given in section III and IV .The model validation using RobotStudio and experimental validation of the model developed is discussed in section V.

## II. Description of Robot Manipulator

The robotic system considered for forward and inverse kinematic analysis is ABB make IRB 1200 robot, a 6 axis industrial robot manipulator which is highly adaptable to many environments and applications. The robot manipulator weighs 54Kg with 703mm reach and 7Kg payload. The maximum value of the tool centre point (TCP) velocity is 7.3m/s and position repeatability is 0.02mm [13]. The main dimensions are shown in Fig. 1.

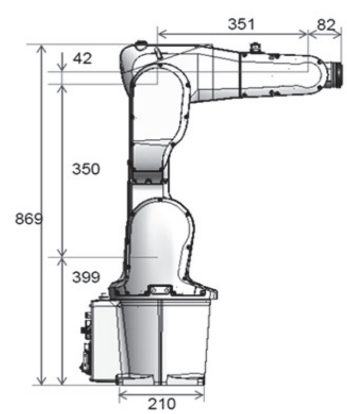


Fig. 1. Main dimensions of IRB 1200 Robot[13]

The robot manipulator uses the first three joints to position the structure and the next three are used to orient the end-effector. The kinematic model shows the relationship between the joint angles and the pose (position and orientation) of the end-effector.

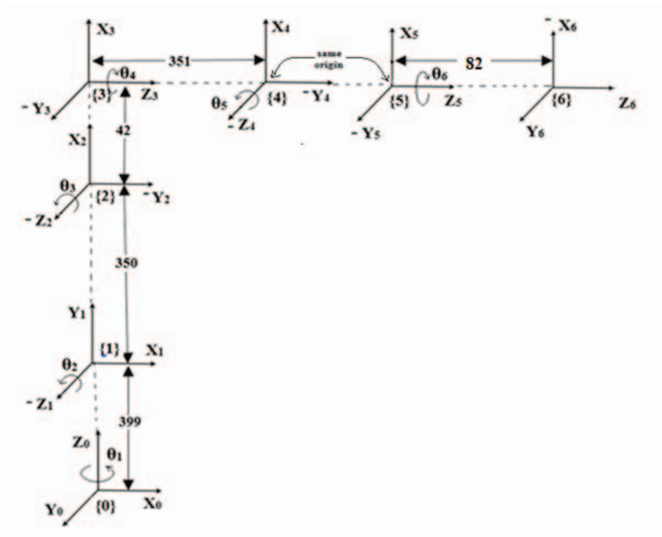


Fig. 2. Frame assignment

TABLE I. DH PARAMETERS OF IRB 1200 ROBOT

| Joint/ Link | θn+1 (degree) | dn+1 (mm) | an+1 (mm) | αn+1 (degree) | Working Range (degree) |
|---|---|---|---|---|---|
| 1 | θ1 | d1 | 0 | -90 | +170 to -170 |
| 2 | θ2-90$^0$ | 0 | a2 | 0 | +135 to -100 |
| 3 | θ3 | 0 | a3 | -90 | +70 to -200 |
| 4 | θ4 | d4 | 0 | +90 | +270 to -270 |
| 5 | θ5 | 0 | 0 | -90 | +130 to -130 |
| 6 | θ6-180$^0$ | d6 | 0 | 0 | +360 to -360 |
| d1=399; a2=350; a3=42; d4=351; d6=82 | | | | | |

## III. MODELLING OF FORWARD KINEMATICS

The forward kinematic model is used to find the pose of the manipulator, given the joint variables. It is a relatively simple problem once the DH parameters are known. DH approach is the standard way for robot representation and its motion modelling that can be used for any robot configuration, regardless of its complexity. First, assign a frame to each joint starting from reference frame {0} fixed to the base and transform this frame to the next. Fig. 2 shows reference frames assigned for the robot manipulator for the analysis of DH parameters. The DH parameters determined and the joint range of IRB 1200 robot are presented in Table I. The represented frames and associated DH parameters are found using Craig's convention [2].

Based on the transformation of reference frames to each joint and DH parameters, individual homogenous transformation matrices relating the frames attached to successive links are found. By using the definition of the link transformation matrix, the matrix ${}^{i-1}T_i$ describes the homogenous transformation from the coordinates system 'i' to 'i-1' which is given by,

$$ {}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1) $$

Finding this matrix for each row of Table I, multiple transformation matrices can be found as shown below.

$$ {}^0T_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} S_2 & C_2 & 0 & a_2 S_2 \\ -C_2 & S_2 & 0 & -a_2 C_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2T_3 = \begin{bmatrix} C_3 & 0 & -S_3 & a_3 C_3 \\ S_3 & 0 & C_3 & a_3 S_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4T_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^5T_6 = \begin{bmatrix} -C_6 & S_6 & 0 & 0 \\ -S_6 & -C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2) $$

These individual matrices are combined to determine the overall transformation matrix which describes the end frame with respect to the base frame. By multiplying the matrices ${}^0T_1$ through ${}^5T_6$ yields ${}^0T_6$ containing the following twelve equations which define the position and orientation of the end effector.

$$ {}^0T_6 = {}^0T_1 . {}^1T_2 . {}^2T_3 . {}^3T_4 . {}^4T_5 . {}^5T_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \begin{aligned} n_x &= -C_1S_{23}C_4C_5C_6 - S_1S_4C_5C_6 - C_1C_{23}S_5C_6 + C_1S_{23}S_4S_6 - S_1C_4S_6 \\ o_x &= C_1S_{23}C_4C_5S_6 + S_1S_4C_5S_6 + C_1C_{23}S_5S_6 + C_1S_{23}S_4C_6 - S_1C_4C_6 \\ a_x &= -C_1S_{23}C_4S_5 - S_1S_4S_5 + C_1C_{23}C_5 \\ p_x &= -C_1S_{23}C_4S_5d_6 - S_1S_4S_5d_6 + C_1C_{23}C_5d_6 + C_1C_{23}d_4 + a_3C_1S_{23} + C_1a_2S_2 \\ n_y &= -S_1S_{23}C_4C_5C_6 + C_1S_4C_5C_6 - S_1C_{23}S_5C_6 - S_1S_{23}S_4S_6 + C_1C_4S_6 \\ o_y &= S_1S_{23}C_4C_5S_6 - C_1S_4C_5S_6 + S_1C_{23}S_5S_6 + S_1S_{23}S_4C_6 + C_1C_4C_6 \\ a_y &= -S_1S_{23}C_4S_5 + C_1S_4S_5 + S_1C_{23}C_5 \\ p_y &= -S_1S_{23}C_4S_5d_6 + C_1S_4S_5d_6 + S_1C_{23}C_5d_6 + S_1C_{23}d_4 + a_3S_1S_{23} + S_1a_2S_2 \\ n_z &= -C_4C_{23}C_5C_6 + S_{23}S_5C_6 + C_{23}S_4S_6 \\ o_z &= C_4C_{23}C_5S_6 - S_{23}S_5S_6 + C_{23}S_4C_6 \\ a_z &= -S_5C_4C_{23} - S_{23}C_5 \\ p_z &= -S_5C_4C_{23}d_6 - S_{23}C_5d_6 - S_{23}d_4 + a_3C_{23} + a_2C_2 + d_1 \end{aligned} \qquad (3) $$

where

$C_1 = Cos(\theta_1); S_1 = Sin(\theta_1); C_{23} = Cos(\theta_2 + \theta_3); S_{23} = Sin(\theta_2 + \theta_3)$

Having derived the direct kinematics of the IRB 1200 as in equation (3), it's now possible to obtain the end-effector position and orientation from the individual joint angles [$\theta_1$, $\theta_2$, $\theta_3$, $\theta_4$, $\theta_5$, $\theta_6$]. These equations specify how to compute the position and orientation of frame {6} relative to frame {0} of the robot. These are the key equations for all kinematic analysis of this robot manipulator. The working area of the robot manipulator plotted in Y-Z plane is shown in Fig. 3.

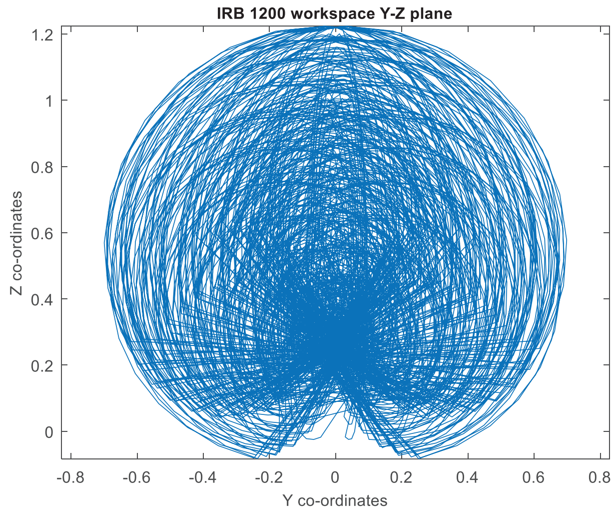


Fig. 3. Workspace of ABB IRB 1200

## IV. MODELLING OF INVERSE KINEMATICS

This section addresses the problem of arm solution for IRB 1200 robot manipulator. To adjust the position and orientation of the end-effector of a robot so as to reach its target object, the inverse kinematic solution is most needed. For a given pose of the end-effector, the joint angles that bring the end-effector in the specified pose is defined by Inverse kinematic model. In other words, given $^0T_6$ as sixteen numeric values, Inverse kinematics solution is the corresponding joint angles $\theta_1$, $\theta_2$, $\theta_3$, $\theta_4$, $\theta_5$ and $\theta_6$[14]. In this section, the inverse kinematic solution of IRB 1200 robot is worked out purely algebraically. From the forward kinematic equations in (3),

$$p_x' = p_x - d_6 a_x = C_1 C_{23} d_4 + a_3 C_1 S_{23} + C_1 a_2 S_2 \quad (4)$$

$$p_y' = p_y - d_6 a_y = S_1 C_{23} d_4 + a_3 S_1 S_{23} + S_1 a_2 S_2 \quad (5)$$

$$\frac{p_y'}{p_x'} = \frac{S_1 (C_{23} d_4 + a_{31} S_{23} + a_2 S_2)}{C_1 (C_{23} d_4 + a_{31} S_{23} + a_2 S_2)} \quad (6)$$

Hence, joint angle $\theta_1$ is given by,

$$\theta_1 = \tan^{-1}\left(\frac{p_y'}{p_x'}\right) \quad (7)$$

Also,

$$p_z' = p_z - d_6 a_z = -S_{23} d_4 + a_3 C_{23} + a_2 C_2 + d_1 \quad (8)$$

Squaring and adding $p_z'$, $p_y'$, $p_z' - d_1$ and rearranging

$$\frac{(p_x')^2 + (p_y')^2 + (p_z' - d_1)^2 - d_4^2 - a_3^2 - a_2^2}{2a_2} = a_3 C_3 - d_4 S_3 \quad (9)$$

$$a_3 C_3 - d_4 S_3 = K$$

where $K = \dfrac{(p_x')^2 + (p_y')^2 + (p_z' - d_1)^2 - d_4^2 - a_3^2 - a_2^2}{2a_2}$

To solve an equation of this type, trigonometric substitution has to be done as shown below.

$$d_4 = \delta Cos\phi; \qquad a_3 = \delta Sin\phi \quad (10)$$

where $\delta = \left(\sqrt{d_4 + a_3}\right)^2$ and $\phi = \tan^{-1}\left(\frac{a_3}{d_4}\right)$

Hence $S_\phi C_3 - S_3 C_\phi = Sin(\phi - \theta_3) = \dfrac{K}{\delta}$ (11)

Therefore, $Cos(\phi - \theta_3) = \pm\sqrt{1 - \dfrac{K^2}{\delta^2}}$ (12)

From (11) and (12),

$$\phi - \theta_3 = \tan^{-1}\left(\frac{K}{\pm\sqrt{a_3^2 + d_4^2 - K^2}}\right) \quad (13)$$

Hence,

$$\theta_3 = \phi - \tan^{-1}\left(\frac{K}{\pm\sqrt{a_3^2 + d_4^2 - K^2}}\right)$$

$$\theta_3 = \tan^{-1}\left(\frac{a_3}{d_4}\right) - \tan^{-1}\left(\frac{K}{\pm\sqrt{a_3^2 + d_4^2 - K^2}}\right) \quad (14)$$

From (14), two solutions for $\theta_3$ will be obtained. Pre-multiplying $^0T_6$ in $\bar{n}, \bar{o}, \bar{a}$ and position format with $^0T_3^{-1}$, a matrix with elements as a function of only known variables will be obtained.

$$^0T_3^{-1} . \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^3T_6 \quad (15)$$

$$^0T_3 = {}^0T_1 . {}^1T_2 . {}^2T_3$$

$$= \begin{bmatrix} C_1 S_{23} & S_1 & C_1 C_{23} & a_3 C_1 S_{23} + a_2 C_1 S_2 \\ S_1 S_{23} & -C_1 & S_1 C_{23} & a_3 S_1 S_{23} + a_2 S_1 S_2 \\ C_{23} & 0 & -S_{23} & a_3 C_{23} + a_2 C_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

If $^0T_3 = \begin{bmatrix} ^0R_3 & ^0P_3 \\ 0 & 1 \end{bmatrix}$, then $^0T_3^{-1} = \begin{bmatrix} ^0R_3^T & -{}^0R_3^T . {}^0P_3 \\ 0 & 1 \end{bmatrix}$.

Hence $^0T_3^{-1}$ is obtained as,

$$^0T_3^{-1} = \begin{bmatrix} C_1 S_{23} & S_1 S_{23} & C_{23} & -a_3 - a_2 C_3 - d_1 C_{23} \\ S_1 & -C_1 & 0 & 0 \\ C_1 C_{23} & S_1 C_{23} & -S_{23} & a_2 S_3 + d_1 S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

To find $^3T_6$, $^0T_3^{-1}$ is multiplied with $^3T_6$ represented in $\bar{n}, \bar{o}, \bar{a}$ and position format to get the following matrix.

$$ {}^3T_6 = {}^0T_3^{-1}\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1S_{23} & S_1S_{23} & C_{23} & -a_3-a_2C_3-d_1C_{23} \\ S_1 & -C_1 & 0 & 0 \\ C_1C_{23} & S_1C_{23} & -S_{23} & a_2S_3+d_1S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18} $$

$$ {}^3T_6 = \begin{bmatrix} C_1S_{23}n_x+S_1S_{23}n_y+C_{23}n_z & C_1S_{23}o_x+S_1S_{23}o_y+C_{23}o_z & C_1S_{23}a_x+S_1S_{23}a_y+C_{23}a_z & C_1S_{23}p_x+S_1S_{23}p_y+C_{23}p_z-a_3-a_2C_3-d_1C_{23} \\ S_1n_x-C_1n_y & S_1o_x-C_1o_y & S_1a_x-C_1a_y & S_1p_x-C_1p_y \\ C_1C_{23}n_x+S_1C_{23}n_y-S_{23}n_z & C_1C_{23}o_x+S_1C_{23}o_y-S_{23}o_z & C_1C_{23}a_x+S_1C_{23}a_y-S_{23}a_z & C_1C_{23}p_x+S_1C_{23}p_y-S_{23}p_z+a_2S_3+d_1S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Also, ${}^3T_6$ is calculated by multiplying ${}^3T_4$, ${}^4T_5$ and ${}^5T_6$ given in (2).

$$ {}^3T_6 = {}^3T_4 . {}^4T_5 . {}^5T_6 = \begin{bmatrix} -C_4C_5C_6+S_4S_6 & C_4C_5S_6+S_{46} & -C_4S_5 & -C_4S_5d_6 \\ -S_4C_5C_6-C_4S_6 & S_4C_5S_6-C_4C_6 & -S_4S_5 & -S_4S_5d_6 \\ -S_5C_6 & S_5S_6 & C_5 & d_6C_5+d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19} $$

By equating the (3,3) elements from (18) and (19),

$$ C_5 = C_1C_{23}a_x + S_1C_{23}a_y - S_{23}a_z \tag{20} $$

Similarly, by equating the (3,4) elements from (18) and (19),

$$ d_6C_5 + d_4 = C_1C_{23}p_x + S_1C_{23}p_y - S_{23}p_z + a_2S_3 + d_1S_{23} \tag{21} $$

Substituting (20) in (21),

$$ d_6(C_1C_{23}a_x + S_1C_{23}a_y - S_{23}a_z) + d_4 = C_1C_{23}p_x + S_1C_{23}p_y - S_{23}p_z + a_2S_3 + d_1S_{23} \tag{22} $$

Rearranging,

$$ C_{23}(C_1p_x + S_1p_y - d_6C_1a_x - d_6S_1a_y) - S_{23}(p_z - d_1 - d_6a_z) = d_4 - a_2S_3; \tag{23} $$

Equation (23) can be written as,

$$ C_{23}M_1 - S_{23}M_2 = M \tag{24} $$

where

$M_1 = C_1p_x + S_1p_y - d_6C_1a_x - d_6S_1a_y$

$M_2 = p_z - d_1 - d_6a_z$

$M = -a_2S_3 + d_4$

Making trigonometric substitution to $M_1$ and $M_2$,

$$ M_1 = \rho Sin\alpha; \quad M_2 = \rho Cos\alpha \tag{25} $$

where $\rho = \sqrt{M_1^2 + M_2^2}$ and $\alpha = \tan^{-1}\left(\frac{M_1}{M_2}\right)$

Substituting (25) in (24),

$$ C_{23}S_\alpha - S_{23}C_\alpha = Sin(\alpha - \theta_{23}) = \frac{M}{\rho} $$

and $Cos(\alpha - \theta_{23}) = \pm\sqrt{1 - \frac{M^2}{\rho^2}}$ (26)

$$ \alpha - \theta_{23} = \tan^{-1}\left(\frac{M}{\pm\sqrt{M_1^2 + M_2^2 - M^2}}\right) \tag{27} $$

$$ \theta_{23} = \alpha - \tan^{-1}\left(\frac{M}{\pm\sqrt{M_1^2 + M_2^2 - M^2}}\right) \tag{28} $$

$$ \theta_{23} = \tan^{-1}\left(\frac{M_1}{M_2}\right) - \tan^{-1}\left(\frac{M}{\pm\sqrt{M_1^2 + M_2^2 - M^2}}\right) $$

$M_1 = C_1p_x + S_1p_y - d_6C_1a_x - d_6S_1a_y$

where, $M_2 = p_z - d_1 - d_6a_z$

$M = -a_2S_3 + d_4$

Equation (28) computes the four values of $\theta_{23}$ according to the four possible combinations for $\theta_1$ and $\theta_3$. Then four possible solutions for $\theta_2$ can be computed as,

$$ \theta_2 = \theta_{23} - \theta_3 \tag{29} $$

where the appropriate solution for $\theta_3$ is used when forming the difference. Equating (1,3) and (2,3) elements from (18) and (19),

$$ -C_4S_5 = S_1a_x - C_1a_y \tag{30} $$

$$ -S_4S_5 = C_1S_{23}a_x + S_1S_{23}a_y + C_{23}a_z \tag{31} $$

Dividing (30) from (31), we get,

$$\frac{S_4}{C_4} = \frac{C_1 S_{23} a_x + S_1 S_{23} a_y + C_{23} a_z}{S_1 a_x - C_1 a_y} \tag{32}$$

As long as $\theta_5 \neq 0$, we can solve for $\theta_4$ as,

$$\theta_4 = \tan^{-1}\left(\frac{C_1 S_{23} a_x + S_1 S_{23} a_y + C_{23} a_z}{S_1 a_x - C_1 a_y}\right) \tag{33}$$

When $\theta_5 = 0$, the manipulator is in singular position. This condition is obtained by finding the Jacobian Matrix of robot and equate its determinant to zero. Considering (3,3) element in (19) and equating to the corresponding element in (18),

$$Cos(\theta_5) = C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z \tag{34}$$

Hence, $\theta_5$ can be solved as

$$\theta_5 = Cos^{-1}(C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z) \tag{35}$$

Similarly equating (3,1) and (3,2) elements of the matrix in (19) with corresponding elements in (18), we get,

$$-S_5 C_6 = C_1 C_{23} n_x + S_1 C_{23} n_y - S_{23} n_z \tag{36}$$

$$S_5 S_6 = C_1 C_{23} o_x + S_1 C_{23} o_y - S_{23} o_z \tag{37}$$

As long as $\theta_5 \neq 0$, $\theta_6$ can be solved as

$$\theta_6 = \tan^{-1}\left(\frac{-(C_1 C_{23} o_x + S_1 C_{23} o_y - S_{23} o_z)}{C_1 C_{23} n_x + S_1 C_{23} n_y - S_{23} n_z}\right) \tag{38}$$

Hence, four inverse kinematic solutions are obtained and four more solutions can be obtained by flipping the manipulator's wrist. The flipped solution is given by,

$$\begin{aligned} \theta_4' &= \theta_4 + 180^0, \\ \theta_5' &= -\theta_5, \\ \theta_6' &= \theta_6 + 180^0 \end{aligned} \tag{39}$$

Out of these eight solutions, the solution which violates the joint angle limitations can be neglected and from the remaining, the one which is adjacent to present manipulator configuration can be chosen as the final solution.

## V. Validation

The following steps are performed to validate and demonstrate the kinematic model developed.

1. Pose selection: Select ten standard poses of the robot for validation.
2. Analytical solution: The mathematical equations to find the kinematic solution of the robot is programmed in MATLAB R2018a and the results for the selected poses are stored.
3. Software validation: The selected poses are simulated in RobotStudio software and compared the pose and joint angle results with the analytical solution.
4. Experimental validation: The robot manipulator is jogged to the selected poses and compared the pose and joint angle results with the analytical solution.

In Fig. 4, 10 standard positions of the IRB 1200 robot manipulator are shown and its corresponding joint angle and wrist centre data are given in Table II.

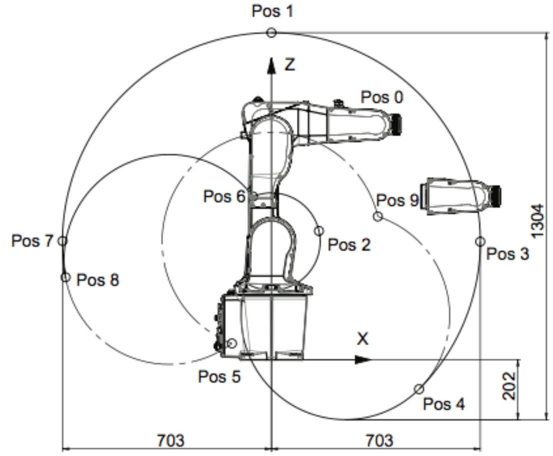


Fig. 4 Positions at wrist center of IRB 1200 Robot.

TABLE II. Wrist Centre Point and angle of axes 2 and 3 for various positions

| Position | Wrist at Centre(mm) | | Joint angles (degrees) | |
|---|---|---|---|---|
| | X | Z | $\theta_2$ | $\theta_3$ |
| Pos0 | 351 | 791 | 0 | 0 |
| Pos1 | 0 | 1102 | 0 | -83 |
| Pos2 | 160 | 434 | 0 | +70 |
| Pos3 | 703 | 398 | +90 | -83 |
| Pos4 | 497 | -99 | +135 | -83 |
| Pos5 | -133 | 55 | -100 | -200 |
| Pos6 | -62 | 550 | -100 | +70 |
| Pos7 | -703 | 400 | -90 | -83 |
| Pos8 | -693 | 278 | -100 | -83 |
| Pos9 | 358 | 488 | +135 | -200 |

### A. Analytical Solution

The homogeneous transformation matrix for each pose is calculated from the forward kinematic equations. The homogenous transformation matrix can be converted into position data as well as the orientation data. The position data indicates the position of the robot tool flange with respect to the world coordinate points and are given in mm. The orientation data extracted from the rotation matrix and expressed in terms of Euler angles. The position in Cartesian co-ordinates and orientation in terms of Euler angles are given in Table III (data for only 4 poses are included).

The inverse kinematic solution corresponding to each pose is also given in Table III. From the eight joint angle solutions obtained, some are not considered due to joint angle limitations. The exact solution corresponding to the transformation matrix is marked in red colour. It is found that one solution will be exactly the same as the one which corresponds to the transformation matrix.

TABLE III. ANALYTICAL SOLUTION

| Sl. No. | Poses | Joint Angles (degrees) | Forward Kinematic Solutions | | | | | | Inverse Kinematic Solutions (degrees) |
|---|---|---|---|---|---|---|---|---|---|
| | | | EZ | EY | EX | X | Y | Z | |
| 1 | Pos0 | [0 0 0 0 0 0] | 0 | 90 | 0 | 0.433 | 0 | 0.791 | Q1= [0 0 0 0 0 0]<br>Q2= [0 0 0 180 0 180]<br>Q3= [0 -96 0 180 96 -180]<br>Q4= [0 -96 0 360 -96 0]<br>Q5= [0 152 -166 180 13.6 -180]<br>Q6= [0 152 -166 360 -13.6 0]<br>Q7= [0 83 -166 180 82 -180]<br>Q8= [0 83 -166 360 -82 0] |
| 2 | Pos1 | [0 0 -83 0 0 0] | 0 | 7 | 0 | 0.0111 | 0 | 1.1839 | Q1= [0 0 -83 180 0 -180]<br>Q2= [0 0 -83 360 0 0]<br>Q3= [0 -13.82 -83 180 13.82 -180]<br>Q4= [0 -13.82 -83 360 -13.82 0]<br>Q5= [0 0.17 -83.35 180 0.17 -180]<br>Q6= [0 0.17 -83.35 360 -0.17 0]<br>Q7= [0 -13.29 -83.35 180 13.64 -180]<br>Q8=[0 -13.29 -83.35 360 -13.64 0] |
| 3 | Pos2 | [0 0 70 0 0 0] | 180 | 20 | 180 | 0.1876 | 0 | 0.3565 | Q1= [0 0 70 0 0 0]<br>Q2= [0 0 70 180 0 180]<br>Q3= [0 -164.43 70 180<br>164343 -180]<br>Q4= [0 -164.43 70 360 164.43 0]<br>Q5= [0 -67.29 123.64 180 13.64 -180]<br>Q6= [0 -67.29 123.64 360 13.64 0]<br>Q7= [0 -204.43 123.64 180 153.78 -180]<br>Q8=[0 -204.43 123.64 360 153.78 0] |
| 4 | Pos3 | [0 0 90 -83 0 0] | -180 | 83 | -180 | 0.7849 | 0 | 0.3879 | Q1= [0 90 -83 0 0 0]<br>Q2= [0 90 -83 180 0 180]<br>Q3= [0 76.17 -83 180 13.82 -180]<br>Q4= [0 76.17 -83 360 -13.82 0]<br>Q5= [0 90.17 -83.35 180 0.17 -180]<br>Q6= [0 90.17 -83.35 360 -0.17 0]<br>Q7= [0 76.70 -83.35 180 13.64 -180]<br>Q8= [0 76.70 -83.35 360 -13.64 0] |

### B. *Validation using RobotStudio*

RobotStudio is a PC application for modelling, offline programming and simulation of robot cells. RobotStudio allows working with an off-line controller, which is a virtual IRC5 controller running locally on the PC. This offline controller is often called as the virtual controller that is identical to the robot's real controller. RobotStudio also allows working with the real physical IRC5 controller, which is the real controller. ABB RobotStudio programs can be directly loaded on the robot's real controller through LAN connection [13].

ABB RobotStudio contains a complete library of virtual models of ABB robots, manipulators and ABB's standard piece of equipment in its database. It helps to design CAD models and import CAD designs. To test the mathematical model developed directly with the RobotStudio, the robot model and the controller has to be loaded and compare the results by jogging the robot in the above-mentioned poses.

The virtual model of the IRB 1200 7Kg robot is loaded into the application and jog the robot in different modes to check the robot movement. Create a joint target with joint angles corresponding to Pos0 and the robot at the joint target can be visualised in the simulation window. Next step is to measure the coordinates of the position and orientation at Pos0. For that, create target and use 'Teach target' command which will store the current location of the robot in terms of position (X, Y, Z) in mm and orientation in degrees [15]. From this, the position and orientation corresponding to each pose can be obtained (See Fig. 5). The same procedure can be repeated for rest of the poses and compare the results stored in ABB RobotStudio with the mathematically calculated co-ordinates given in Table III.

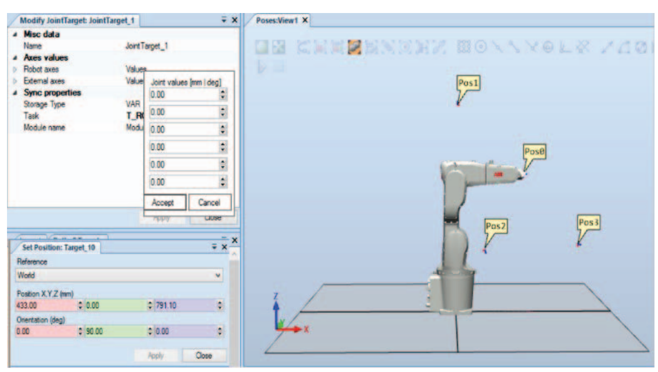


Fig. 5. Pos0 in RobotStudio

### C. Experimental Validation

The experimental setup includes IRB 1200 robot with IRC5 controller which is shown in Fig. 6. The robot is jogged to the selected poses using the Flex Pendant and the pose data corresponding to each set of joint angles is noted. The results obtained are compared with the analytical solutions given in Table III and found the same. Hence the kinematic model is demonstrated and validated experimentally.

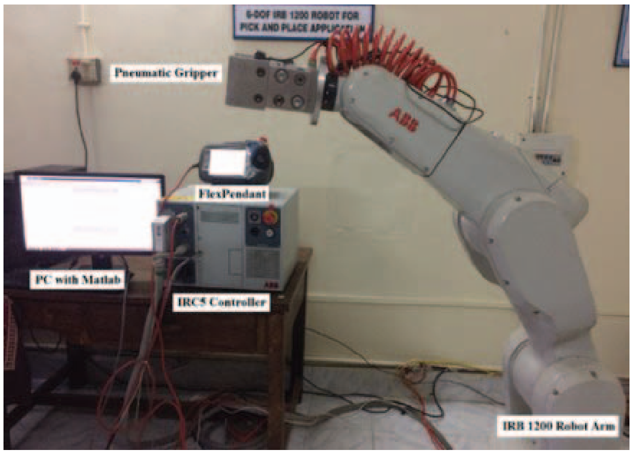


Fig.6 Experimental Setup

The orientation values obtained from the demonstration software and from the hardware is exactly the same as that of the analytical solution. The error in position coordinates while comparing with different validation methods used above are summarised in Table IV. Since all the poses considered here are in x-z coordinates, the y coordinate is always zero and not mentioned in the table.

TABLE IV. POSITION ERRORS

| Poses | | Analytical solution | Position Errors | |
|---|---|---|---|---|
| | | | RobotStudio | Hardware |
| Pos0 | x | 433 | 0 | 0.2 |
| | z | 791 | 0.1 | -0.4 |
| Pos1 | x | 11.1 | -0.02 | 0.1 |
| | z | 1183.9 | 0.09 | 0.1 |
| Pos2 | x | 187.6 | -0.04 | -0.1 |
| | z | 356.5 | 0.08 | 0 |
| Pos3 | x | 784.9 | -0.01 | 0.1 |
| | z | 387.9 | 0.12 | 0 |

## VI. CONCLUSION

The forward and inverse kinematic modelling for ABB IRB 1200 Robot Manipulator was performed analytically. The analytical solution is compared with the results obtained from the RobotStudio software. Since RobotStudio is built on the ABB Virtual Controller, an exact copy of the real software that runs the robots in the real world, a realistic simulation can be performed. Hence RobotStudio results help in the faultless validation of kinematic analysis of all ABB robots. The kinematic analysis method used in this paper can be extended to any robots of similar structure. The industrial robot manipulators are required to have proper knowledge of its joint variables as well as the understanding of kinematic parameters in many fields of applications such as material handling, pick-n-place, planetary and undersea explorations, space manipulation, and hazardous field etc. Moreover, the kinematic analysis is also important in the medical field robotics for rehabilitation and surgery.

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