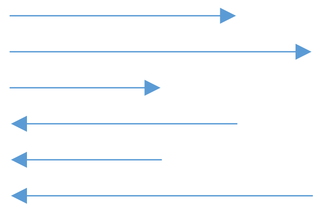
Cheat Sheet – Vector Algebra

Scalar Multiplication: $\lambda\vec{a}$

$\lambda = 1$
$\lambda > 1$
$0 < \lambda < 1$
$\lambda = -1$
$-1 < \lambda < 0$
$\lambda < -1$

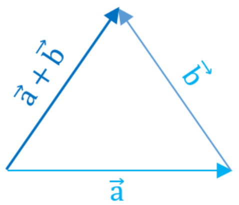


Geometry

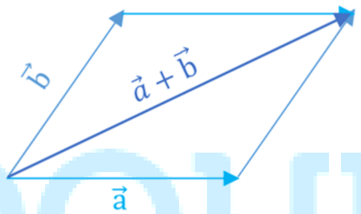
$\vec{a} = \lambda\vec{b} \Leftrightarrow \vec{a} \parallel \vec{b}$ (collinear)
$(\vec{a}, \vec{b} \neq \vec{0})$

Addition: $\vec{a} + \vec{b}$

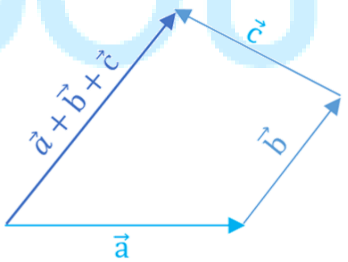
Triangle Law



Parallelogram Law



Polygon Law



Properties

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$
$\vec{a} + \vec{0} = \vec{a}$
$\vec{a} + (-\vec{a}) = \vec{0}$
$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$
$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

Linear Combination: $\lambda_1\vec{a_1} + \lambda_2\vec{a_2} + \cdots + \lambda_n\vec{a_n}$

Geometry

Any vector coplanar with $\vec{a}$ and $\vec{b}$ is
$\lambda_1\vec{a} + \lambda_2\vec{b}$
$(\vec{a}, \vec{b} \neq \vec{0}; \vec{a} \nparallel \vec{b})$

Any vector in space is
$\lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\vec{c}$
$(\vec{a}, \vec{b}, \vec{c} \neq \vec{0}; \vec{a}, \vec{b}, \vec{c}$ are non-coplanar)

Dot Product: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Properties

$\vec{a}.\vec{b} = \vec{b}.\vec{a}$
$\vec{a}.\vec{a} = |\vec{a}|^2$
$(\lambda\vec{a}).\vec{b} = \vec{a}.(\lambda\vec{b}) = \lambda(\vec{a}.\vec{b})$
$\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

Geometry

Projection of $\vec{a}$ on $\vec{b} = \vec{a}.\vec{b}/|\vec{b}|$

$\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ $(\vec{a}, \vec{b} \neq \vec{0})$

Cross Product: $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

Properties

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
$\vec{a} \times \vec{a} = \vec{0}$
$(\lambda\vec{a}) \times \vec{b} = \vec{a} \times (\lambda\vec{b}) = \lambda(\vec{a} \times \vec{b})$
$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Geometry

$\vec{a} \times \vec{b} = 0 \Leftrightarrow \vec{a} \parallel \vec{b}$ $(\vec{a}, \vec{b} \neq \vec{0})$

Any vector $\perp$ to both $\vec{a}$ and $\vec{b}$ is $\lambda(\vec{a} \times \vec{b})$

Area of a parallelogram = $|\vec{a} \times \vec{b}|$
Area of a triangle = $1/2|\vec{a} \times \vec{b}|$

$\vec{a}, \vec{b}$ are the coterminous edges

Scalar Triple Product: $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}.(\vec{b} \times \vec{c})$

Properties

$[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$
$[\lambda\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\ \lambda\vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \lambda\vec{c}] = \lambda[\vec{a}\ \vec{b}\ \vec{c}]$
$[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{c}\ \vec{b}\ \vec{a}] = -[\vec{a}\ \vec{c}\ \vec{b}] = -[\vec{b}\ \vec{a}\ \vec{c}]$
$[\vec{a}\ \vec{b}\ \vec{c}] = 0$ if any two of $\vec{a}, \vec{b}, \vec{c}$ are equal/parallel

Geometry

Volume of a parallelepiped = $[\vec{a}\,\vec{b}\,\vec{c}]$
Volume of a tetrahedron = $\frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}]$

$\vec{a}\,\vec{b}\,\vec{c}$ are the coterminous edges

$[\vec{a}\,\vec{b}\,\vec{c}] = 0 \Leftrightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar $(\vec{a}, \vec{b}, \vec{c} \neq \vec{0})$

Vector Triple Product: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

Geometry

$\vec{a} \times (\vec{b} \times \vec{c})$ is coplanar with $\vec{b}$ and $\vec{c}$
and perpendicular to $\vec{a}$

Vector Quadruple Product

$(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = (\vec{a}.\vec{c})(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})(\vec{b}.\vec{c})$

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\,\vec{b}\,\vec{d}]\vec{c} - [\vec{a}\,\vec{b}\,\vec{c}]\vec{d} = [\vec{c}\,\vec{d}\,\vec{a}]\vec{b} - [\vec{c}\,\vec{d}\,\vec{b}]\vec{a}$